Many-user multiple access with coding and random user activity

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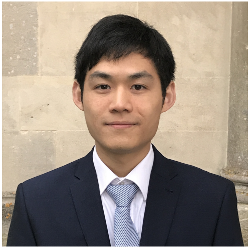
University of Cambridge



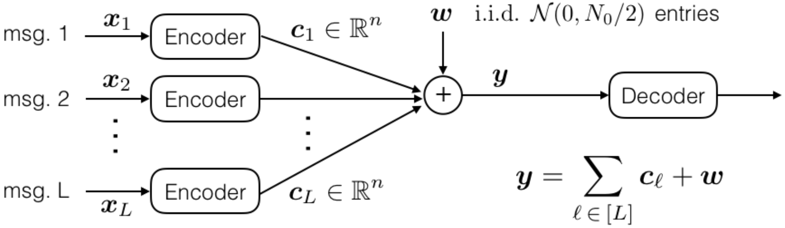
Kuan Hsieh

Xiaoqi (Shirley) Liu

Cynthia Rush

Pablo Pascual Cobo

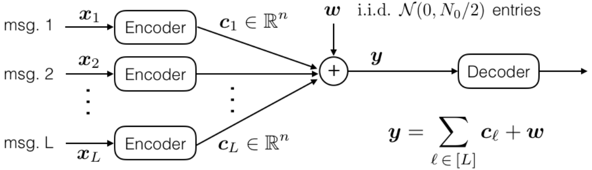
Gaussian multiple-access channel (GMAC)



Modern networks often have

- Very large number of users
- Small data payload for each user

Many-user setting



- User density $\mu = L/n$
- Fixed user payload $\log M$ bits/user
- Energy-per-bit constraint $\|c_i\|^2 \leq E := E_b \log M, \; i \in [L]$
- Per-user probability of error (PUPE) $\frac{1}{L}\sum_i \mathbb{P}(\hat{x}_i \neq x_i)$

[Chen, Chen, Guo, '17], [Ravi, Koch '19] [Polyanskiy '17], [Zadik, Polyanskiy, Thrampoulidis '19] , [Polyanskiy, Kowshik '20]

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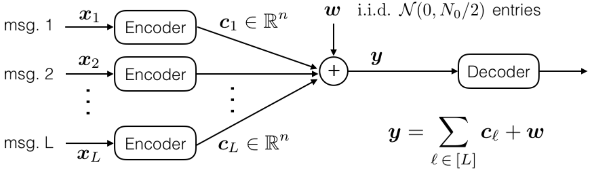

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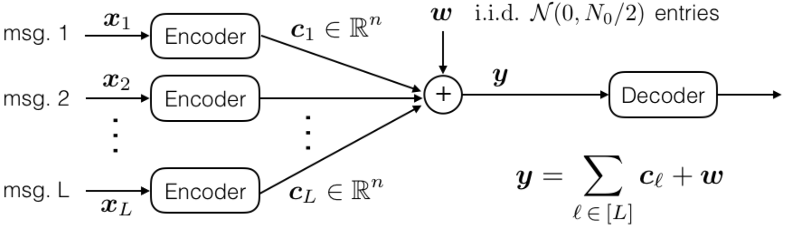
Linear scaling regime

$L, n \to \infty$ with $\mu = L/n$ fixed, E_b and M do not scale with n

What is minimum E_b/N_0 required for a given μ and target PUPE, e.g. 10^{-3} ?

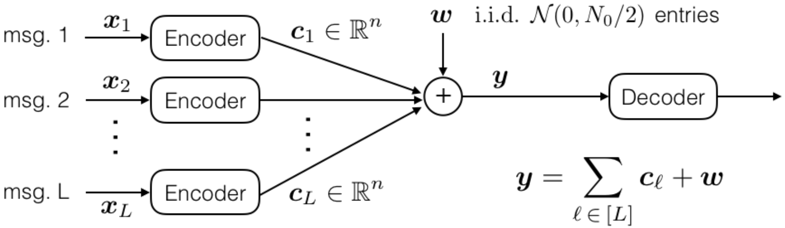
[Chen, Chen, Guo, '17], [Ravi, Koch '19] [Polyanskiy '17], [Zadik, Polyanskiy, Thrampoulidis '19] , [Polyanskiy, Kowshik '20]

GMAC with random user activity



- Only a fraction of users active, decoder may not know the exact number
- Errors: Misdetections, False Alarms, Active-user Errors
- Tradeoff between E_b/N_0 and user density μ for given target error rates

[Ngo et al. '22], [Fengler et al. '20], ...



Different from unsourced random access

[Polyanskiy '17], [Fengler et al. '21], [Amalladine et al. '20], [Polyanskiy, Kowshik '20], [Ngo et al. '22], . . .

Here each user has separate codebook

Previous work

- What can be achieved with random Gaussian codebooks and (infeasible) maximum-likelihood decoding?

[Polyanskiy '17], [Zadik, Polyanskiy, Thrampoulidis '19], [Polyanskiy, Kowshik '20]

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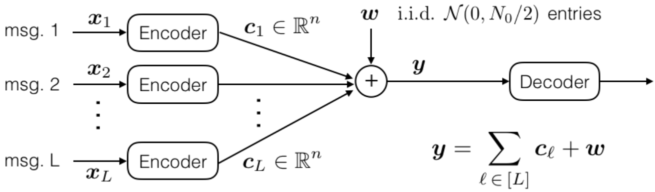
This talk

- What can be achieved with **efficient** coding schemes?
  - SPARC-based and coded CDMA schemes with spatial coupling
  - Approximate Message Passing (AMP) decoding
- GMAC with random user activity
  - Achievability bounds and efficient schemes

Many-user setting



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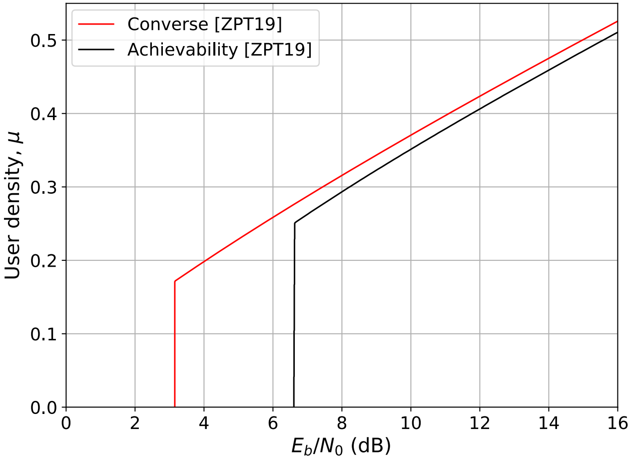
Bounds

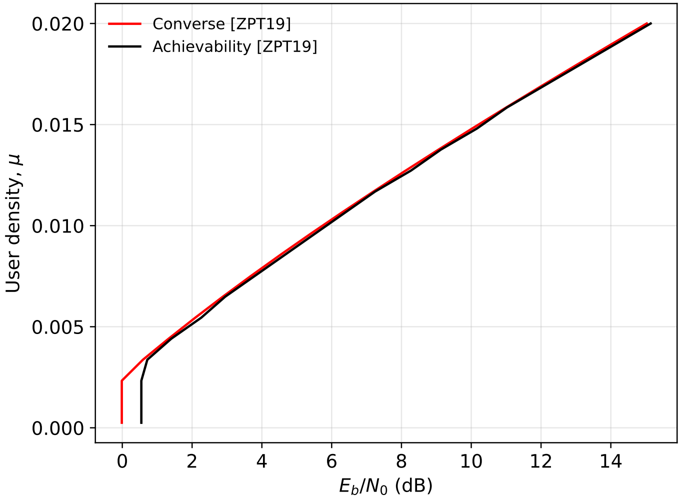


User payload = 8 bits

For each E_b/N_0 value, find max. μ that achieves PUPE $\leq 10^{-3}$

[Zadik, Polyanskiy, Thrampoulidis '19], [Polyanskiy, Kowshik '20]

Bounds

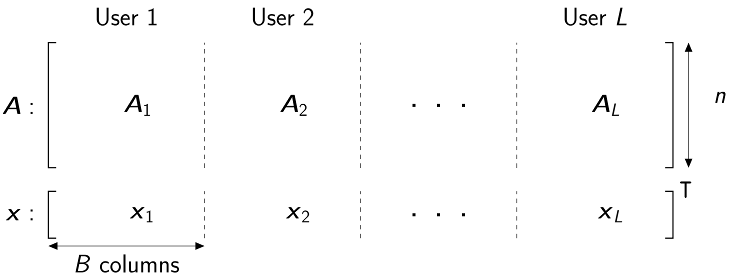


User payload = 200 bits

For each E_b/N_0 value, find max. μ that achieves PUPE $\leq 10^{-3}$

[Zadik, Polyanskiy, Thrampoulidis '19], [Polyanskiy, Kowshik '20]

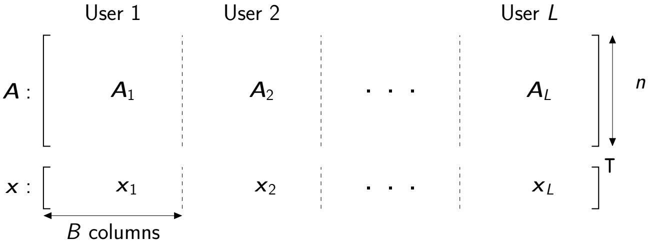
Random linear coding



For each user i, codeword $\boldsymbol{c}_i = \boldsymbol{A}_i \boldsymbol{x}_i$

- Random matrices: $\boldsymbol{A}_i \in \mathbb{R}^{n \times B}$
- User i's message encoded in $\boldsymbol{x}_i \in \mathbb{R}^B \sim P_{\boldsymbol{X}}$

$$\boldsymbol{y} = \sum_i \boldsymbol{A}_i \boldsymbol{x}_i + \boldsymbol{w} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{w}$$



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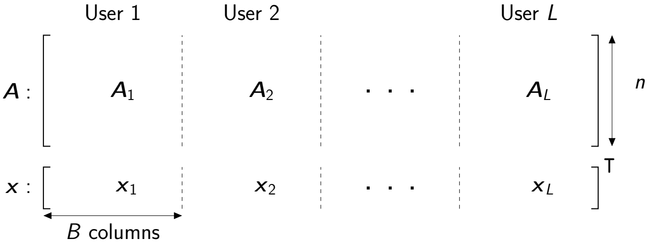
Examples with IID Gaussian $\boldsymbol{A}$

- Random codebooks: $B = M$, each $\boldsymbol{x}_i$ has a single nonzero entry $= \sqrt{E}$



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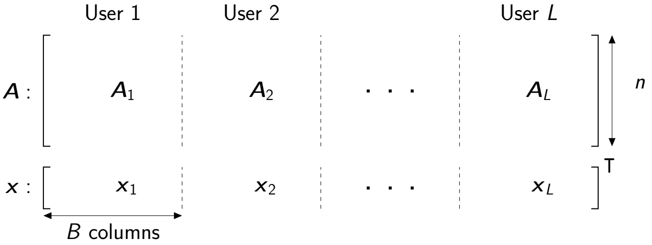
- Random codebooks with binary modulation: $B = M/2$ and each $\boldsymbol{x}_i$ has a single nonzero entry $\in \{\sqrt{E}, -\sqrt{E}\}$
- Random CDMA: $B = 1$, each x_i drawn from M-ary constellation



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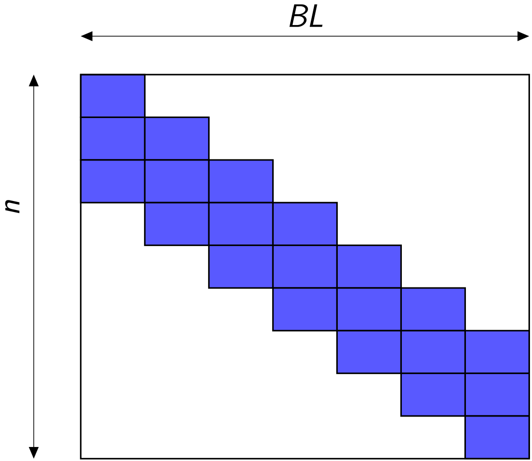
We will also use spatially coupled $\boldsymbol{A}$

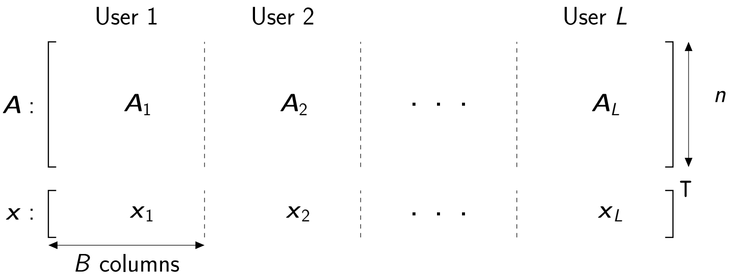
Spatially coupled matrix



Combined codebook matrix $\boldsymbol{A}$

Gaussian entries on band-diagonal, remaining entries zero

IID Gaussian matrix

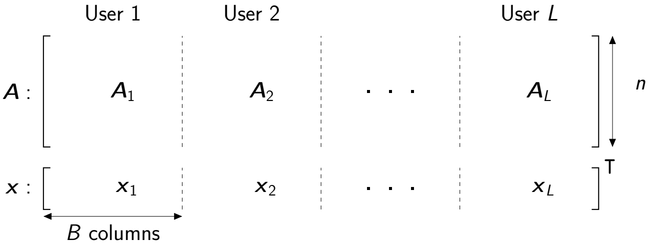


$$A_{jk} \sim_{iid} \mathrm{N}(0, 1/n), \qquad \boldsymbol{x}_i \sim_{iid} P_{\boldsymbol{X}}$$

Decoding task: Recover $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_L$ from

$$\boldsymbol{y} = \sum_i \boldsymbol{A}_i \boldsymbol{x}_i + \boldsymbol{w} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{w}$$

Approximate Message Passing decoder



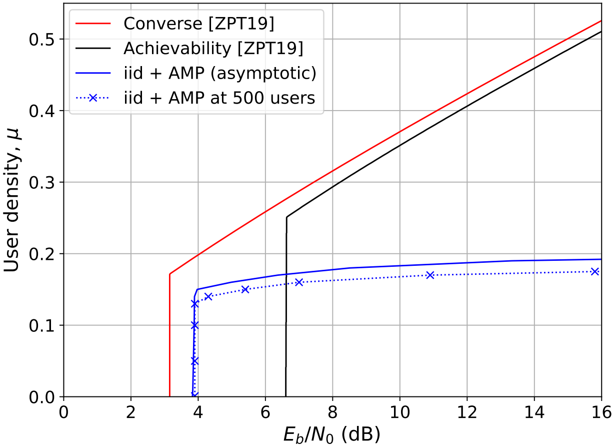
AMP decoder tailored to prior on $\boldsymbol{x} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_L]$

Iteratively produces estimates $\hat{\boldsymbol{x}}^1, \hat{\boldsymbol{x}}^2, \ldots$

Can precisely characterize asymptotic error rate as $n, L \to \infty$:

$$\lim_{t\to\infty} \lim_{n\to\infty} \frac{1}{L} \sum_{\ell=1}^{L} \mathbb{1}\{\hat{\boldsymbol{x}}_\ell^t \neq \boldsymbol{x}_\ell\}$$

(Limit taken with user density $L/n = \mu$ fixed)



User payload = 8 bits

For each μ, we find minimum E_b/N_0 that achieves PUPE $\leq 10^{-3}$

Theoretical curve is derived from a **single-user effective channel**

Single-user channel

$$\boldsymbol{S}_\tau = \boldsymbol{X} + \sqrt{\tau}\boldsymbol{G}, \qquad \boldsymbol{X} \sim P_{\boldsymbol{X}}, \ \boldsymbol{G} \sim \mathrm{N}(\boldsymbol{0}, \boldsymbol{I}_B)$$

MAP estimator: $\hat{\boldsymbol{x}}^{\mathrm{MAP}}(\boldsymbol{S}_\tau) = \arg\max_{\boldsymbol{x}' \in \mathcal{X}} \ \mathbb{P}(\boldsymbol{X} = \boldsymbol{x}' \mid \boldsymbol{S}_\tau)$

Prob. of error: $P_e(\tau) = \mathbb{P}(\hat{\boldsymbol{x}}^{\mathrm{MAP}}(\boldsymbol{S}_\tau) \neq \boldsymbol{X})$

Example: Random Gaussian codebooks

$$\hat{\boldsymbol{x}}_j^{\mathrm{MAP}}(\boldsymbol{s}) = \begin{cases} \sqrt{E} & \text{if } s_j > s_k \text{ for all } k \in [B] \backslash j, \\ 0 & \text{otherwise} \end{cases}$$

$$P_e(\tau) = 1 - \mathbb{E}\left[\Phi(\sqrt{E/\tau} + G)^{B-1}\right]$$

Theorem

Consider iid Gaussian $\boldsymbol{A}$ and message vectors $\boldsymbol{x}_i \sim_{iid} P_{\boldsymbol{X}}$. Then, the asymptotic user error rate of the AMP decoder is

$$\lim_{t\to\infty} \lim_{L\to\infty} \frac{1}{L} \sum_{\ell=1}^{L} \mathbb{1}\{\hat{\boldsymbol{x}}_\ell^t \neq \boldsymbol{x}_\ell\} \overset{\text{a.s.}}{=} P_e(\tau^{\mathrm{FP}})$$

where the inner limit is taken with $L/n = \mu$.

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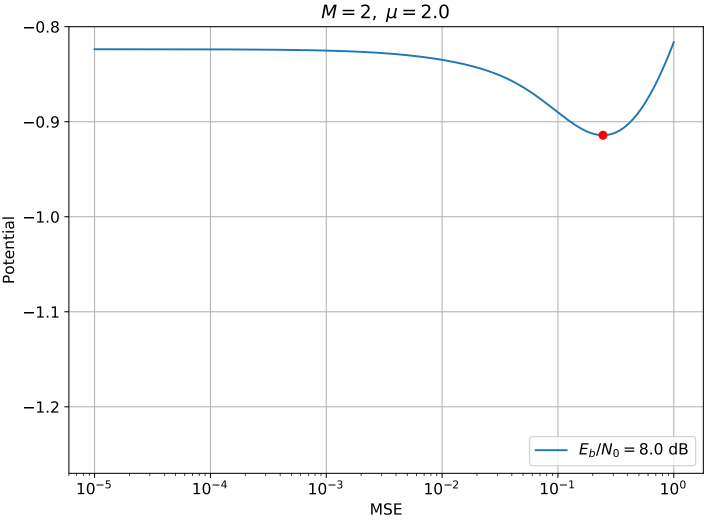
where the inner limit is taken with $L/n = \mu$.

τ^{FP} is the **largest stationary point** of the **potential function**:

$$\mathcal{F}(\tau) = I(\boldsymbol{X}; \boldsymbol{S}_\tau) + \frac{1}{2\mu} \left[\ln\left(\frac{\tau}{N_0/2} \right) - \left(1 - \frac{N_0/2}{\tau} \right) \right]$$

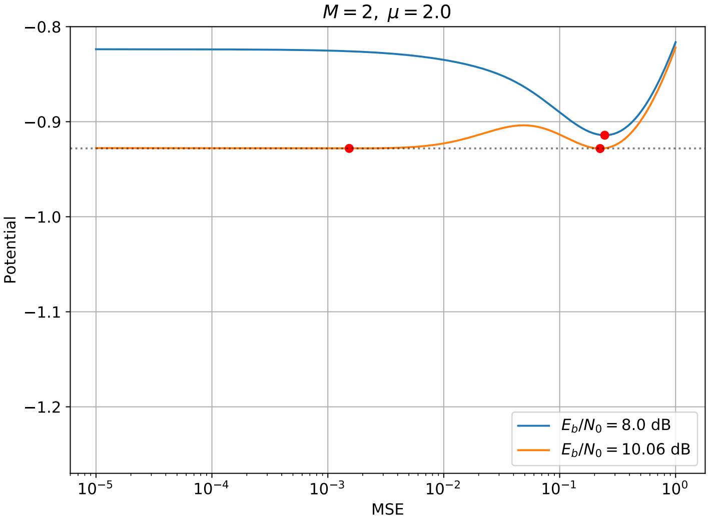
where $\tau \in \left[\frac{N_0}{2}, \frac{N_0}{2} + \mu E\right]$.

Potential function



x-axis is $\left(\tau - \frac{N_0}{2}\right)\frac{1}{\mu}$

Potential function



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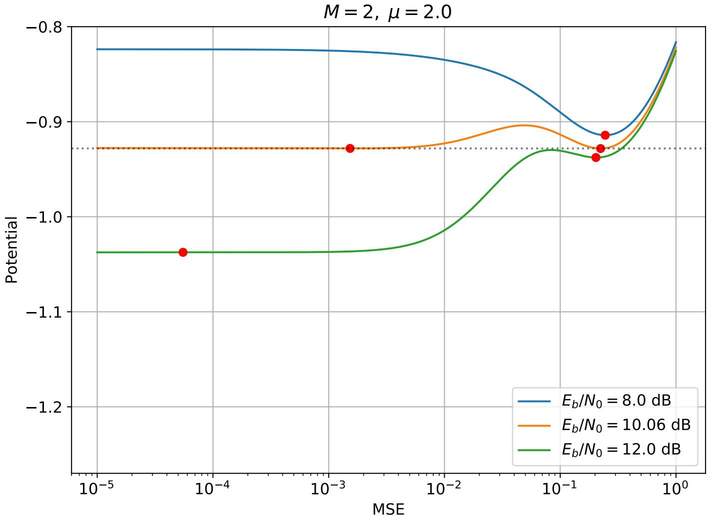

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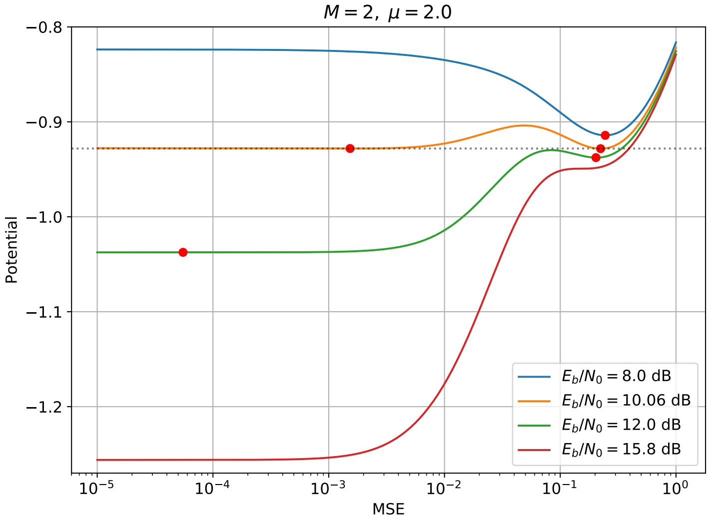
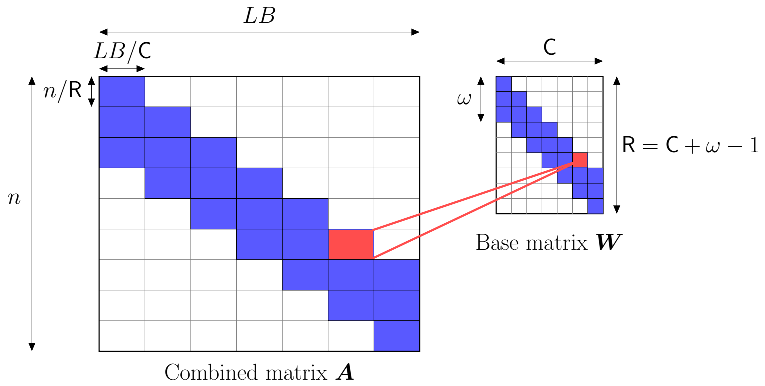


Can we achieve $P_e(\tau^*)$, corresponding to the *global minimum*?

Spatially coupled Gaussian matrix



$\boldsymbol{x} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_L]$ has same form as before: $\boldsymbol{x}_i \in \mathbb{R}^B \sim_{iid} P_{\boldsymbol{X}}$

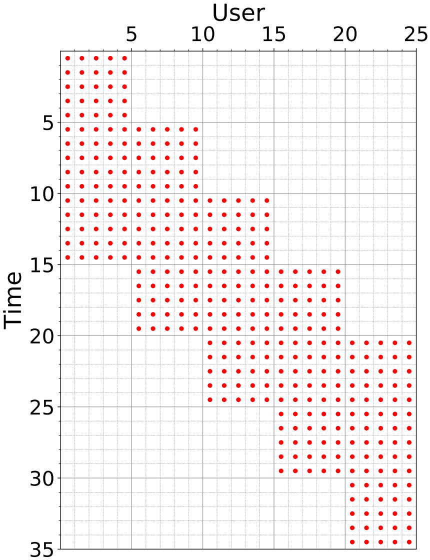
[Kudekar, Pfister '10], [Donoho, Javanmard, Montanari '13] [Barbier and Krzakala '17] [Liang, Ma and Ping '17] [Hsieh, Rush, V.'21] . . .

Example:

$L = 25$ users

$n = 35$ channel uses

$(\omega = 3, C = 5)$ base matrix



Spatial coupling induces **block-wise time-division with overlap**

Theorem (Threshold Saturation)

Consider spatially coupled Gaussian $\boldsymbol{A}$, message vectors $\boldsymbol{x}_i \sim_{iid} P_{\boldsymbol{X}}$.

For any $\delta > 0$, sufficiently large ω and sufficiently small $\frac{\omega}{\mathsf{C}}$ we have:

$$\lim_{t\to\infty} \lim_{L\to\infty} \frac{1}{L} \sum_{\ell=1}^{L} \mathbb{1}\left\{\hat{\boldsymbol{x}}_\ell^t \neq \boldsymbol{x}_\ell\right\} \leq P_e(\tau^* + \delta) \quad \text{a. s.}$$

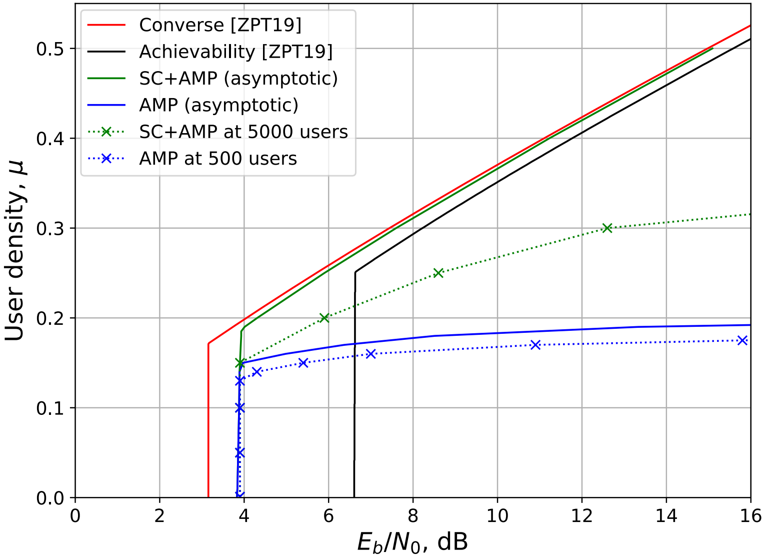
where the inner limit is taken with $L/n = \mu$.

Here τ^* is the **global minimum** of the potential function:

$$\mathcal{F}(\tau) = I(\boldsymbol{X}; \boldsymbol{S}_\tau) + \frac{1}{2\mu}\left[\ln\left(\frac{\tau}{N_0/2}\right) - \left(1 - \frac{N_0/2}{\tau}\right)\right]$$

where $\tau \in \left[\frac{N_0}{2},\ \frac{N_0}{2} + \mu E\right]$.

Hsieh, Rush, V, *Near-Optimal Coding for Many-user Multiple Access Channels*, JSAIT, March 2022.



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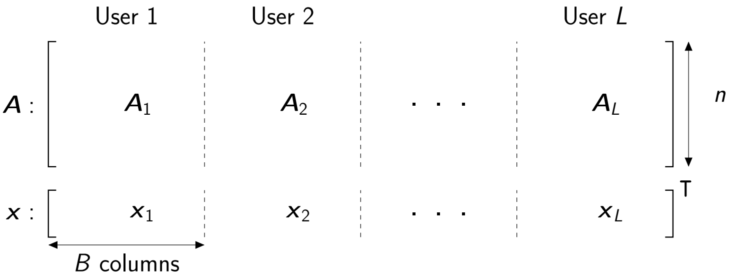

- Complexity of AMP decoder scales exponentially with B
  Multiple transmissions needed for larger payloads ($\sim$ 100 bits)
- User information may be *coded* (e.g., using LDPC outer code)

Coded Binary CDMA

$$\mathbf{A} = \begin{bmatrix} \uparrow & \cdots & \cdots & \uparrow \\ \mathbf{a}_1 & \cdots & \cdots & \mathbf{a}_L \\ \downarrow & \cdots & \cdots & \downarrow \end{bmatrix}, \qquad \mathbf{X} = \begin{bmatrix} \leftarrow & \mathbf{x}_1 & \rightarrow \\ & \vdots & \\ \leftarrow & \mathbf{x}_L & \rightarrow \end{bmatrix}$$

For each user i:

- Signature sequence $\mathbf{a}_i \in \mathbb{R}^{\tilde{n}}$
- Codeword $\mathbf{x}_i$ is row i of the "signal" matrix $\mathbf{X} \in \{\pm\sqrt{E}\}^{L \times d}$
- Each $\mathbf{x}_i$ is a codeword of (k, d) linear code

$$\mathbf{Y} = \sum_{i=1}^{L} \mathbf{a}_i \mathbf{x}_i + \text{ noise } = \mathbf{AX} + \text{ noise}$$

- Number of channel uses $n = \tilde{n}d$

AMP Decoder

Starting with initializer $\boldsymbol{X}^0 = 0$, for $t \geq 1$

$$\boldsymbol{Z}^t = \boldsymbol{Y} - \boldsymbol{A}\boldsymbol{X}^t + \frac{1}{n}\boldsymbol{Z}^{t-1}\left[\sum_{\ell=1}^{L} \eta'_{t-1}\left(\boldsymbol{s}_\ell^{t-1}\right)\right]^\top$$

$$\boldsymbol{S}^t = \boldsymbol{A}^\top \boldsymbol{Z}^t + \boldsymbol{X}^t,$$

$$\boldsymbol{X}^{t+1} = \eta_t\left(\boldsymbol{S}^t\right)$$

For each t:

Empirical distribution of rows of $(\boldsymbol{S}^t - \boldsymbol{X}) \to \mathrm{N}(0, \Sigma^t)$

State evolution to iteratively compute $d \times d$ covariance Σ^t

$\Rightarrow \eta^t$ estimates $\boldsymbol{X}$ from observation in Gaussian noise

$$Z^t = Y - AX^t + \frac{1}{n} Z^{t-1} \left[\sum_{\ell=1}^{L} \eta'_{t-1}\left(s_\ell^{t-1}\right) \right]^\top$$

$$S^t = A^\top Z^t + X^t,$$

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Theorem [Liu, Hsieh, V '24]

Let η_t be Lipschitz for $t \geq 1$. Then for each $t \geq 1$,

$$\lim_{L\to\infty} \frac{1}{L} \sum_{\ell=1}^{L} \mathbb{1}\{\hat{\boldsymbol{x}}_\ell^{t+1} \neq \boldsymbol{x}_\ell\} = \mathbb{P}\left(h_t\left(\bar{\boldsymbol{x}} + \boldsymbol{g}^t\right) \neq \bar{\boldsymbol{x}}\right)$$

- $\bar{\boldsymbol{x}} \in \{\pm\sqrt{E}\}^d$ uniformly distributed among 2^k codewords
- $\boldsymbol{g}^t \in \mathbb{R}^d \sim \mathrm{N}(0, \Sigma^t)$ independent of $\bar{\boldsymbol{x}}$

$$\boldsymbol{Z}^t = \boldsymbol{Y} - \boldsymbol{A}\boldsymbol{X}^t + \frac{1}{n}\boldsymbol{Z}^{t-1}\left[\sum_{\ell=1}^{L} \eta'_{t-1}\left(\boldsymbol{s}_\ell^{t-1}\right)\right]^\top$$

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Can extend result to spatially coupled $\boldsymbol{A}$

Choice of denoiser η_t

$$\boldsymbol{Z}^t = \boldsymbol{Y} - \boldsymbol{A}\boldsymbol{X}^t + \frac{1}{n}\boldsymbol{Z}^{t-1}\left[\sum_{\ell=1}^{L}\eta'_{t-1}\left(\boldsymbol{s}_\ell^{t-1}\right)\right]^\top$$

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Bayes-optimal denoiser $\eta_t(\boldsymbol{s}) = \mathbb{E}[\bar{\boldsymbol{x}} \mid \bar{\boldsymbol{x}} + \boldsymbol{g}^t = \boldsymbol{s}]$

Requires averaging over 2^k codewords from (k, d) code

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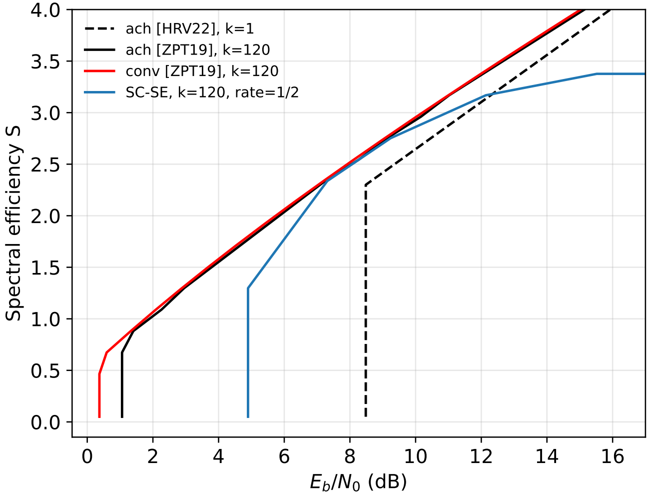
Requires averaging over 2^k codewords from (k, d) code

BP denoiser when $\bar{\boldsymbol{x}}$ drawn from binary LDPC code

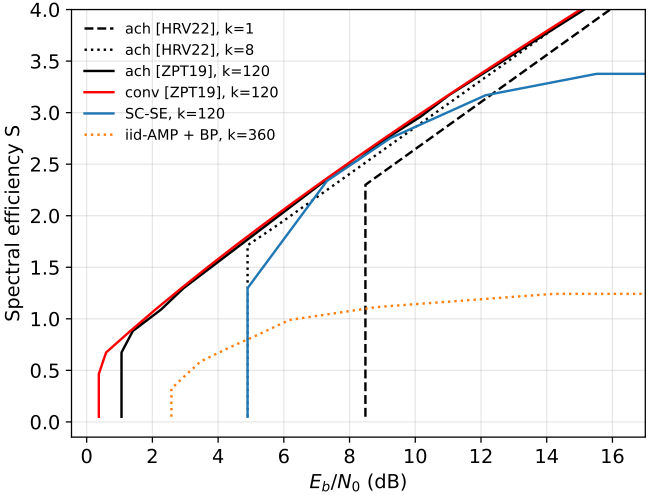
η^{BP}: BP decoding on each row of $\boldsymbol{S}^t$

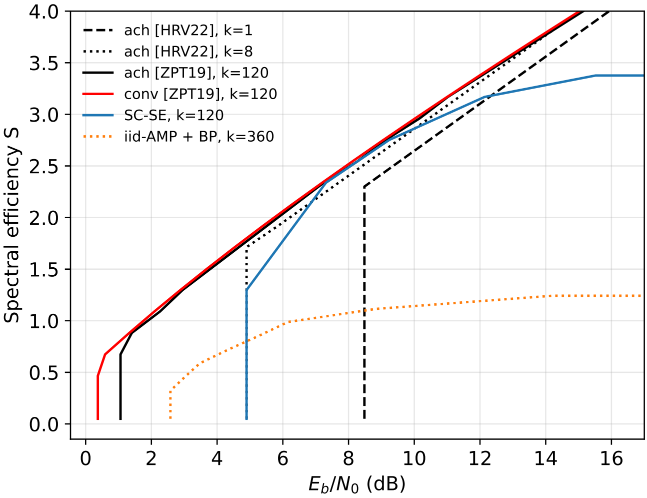
AMP with BP denoisers: [Amalladine et al. '22], [Ebert et al., '23]

User payload = 120 bits, Target BER 10^{-4}



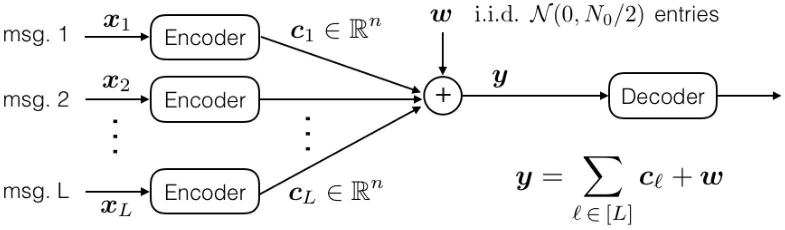
User payload = 360 bits, Target BER 10^{-4}





Coded binary CDMA is (almost) all you need!

Many-user GMAC with random user activity



- Each user active with probability α
- Errors: Misdetections, False Alarms, Active-user Errors
- Tradeoff between E_b/N_0 and user density μ for given target error rates

CDMA-based coding

$$\boldsymbol{A} = \begin{bmatrix} \uparrow & \cdots & \cdots & \uparrow \\ \boldsymbol{a}_1 & \cdots & \cdots & \boldsymbol{a}_L \\ \downarrow & \cdots & \cdots & \downarrow \end{bmatrix}, \qquad \boldsymbol{X} = \begin{bmatrix} \leftarrow & \boldsymbol{x}_1 & \rightarrow \\ & \vdots & \\ \leftarrow & \boldsymbol{x}_L & \rightarrow \end{bmatrix}$$

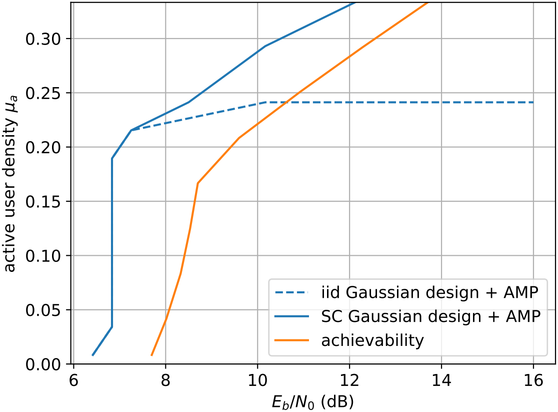
- If user i is inactive, $\boldsymbol{x}_i = \boldsymbol{0}$. Otherwise $\boldsymbol{x}_i \in \{\pm\sqrt{E}\}^d$

$$\boldsymbol{Y} = \sum_{i=1}^{L} \boldsymbol{a}_i \boldsymbol{x}_i + \text{ noise } = \boldsymbol{A}\boldsymbol{X} + \text{ noise}$$

- AMP decoder with denoiser tailored to row-wise prior on $\boldsymbol{X}$

Target $\max\{p_{\mathrm{MD}}, p_{\mathrm{FA}}\} + p_{\mathrm{AUE}} < 0.01$

User payload $k = 6$, and $\alpha = 0.3$



- Achievability bounds: random codebooks and ML decoding
- Proof techniques build on [Ngo et al. '22] unsourced setting

Ongoing and Future Work

- Coded CDMA with longer LDPC codes
- Improved bounds for random user activity
- Extension to **unsourced** random access

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K. Hsieh, C. Rush, and R. Venkataramanan, *Near-optimal coding for many-user multiple access channels*, IEEE Journal on Selected Areas in Information Theory, March 2022

X. Liu, K. Hsieh, and R. Venkataramanan, *Coded many-user multiple access via AMP*, https://arxiv.org/abs/2402.05625, 2024

X. Liu, P. Pascual Cobo, and R. Venkataramanan, *Many-user multiple access with random user activity* (coming soon)