BIRS Workshop, Banff, Alberta, Canada, March 10-15, 2024

MARCONI Lab

Concatenated Coding-Free Massive Unsourced Random via Bilinear Vector Approximate Message Passing

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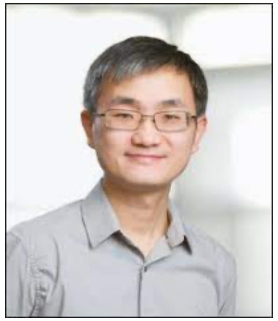
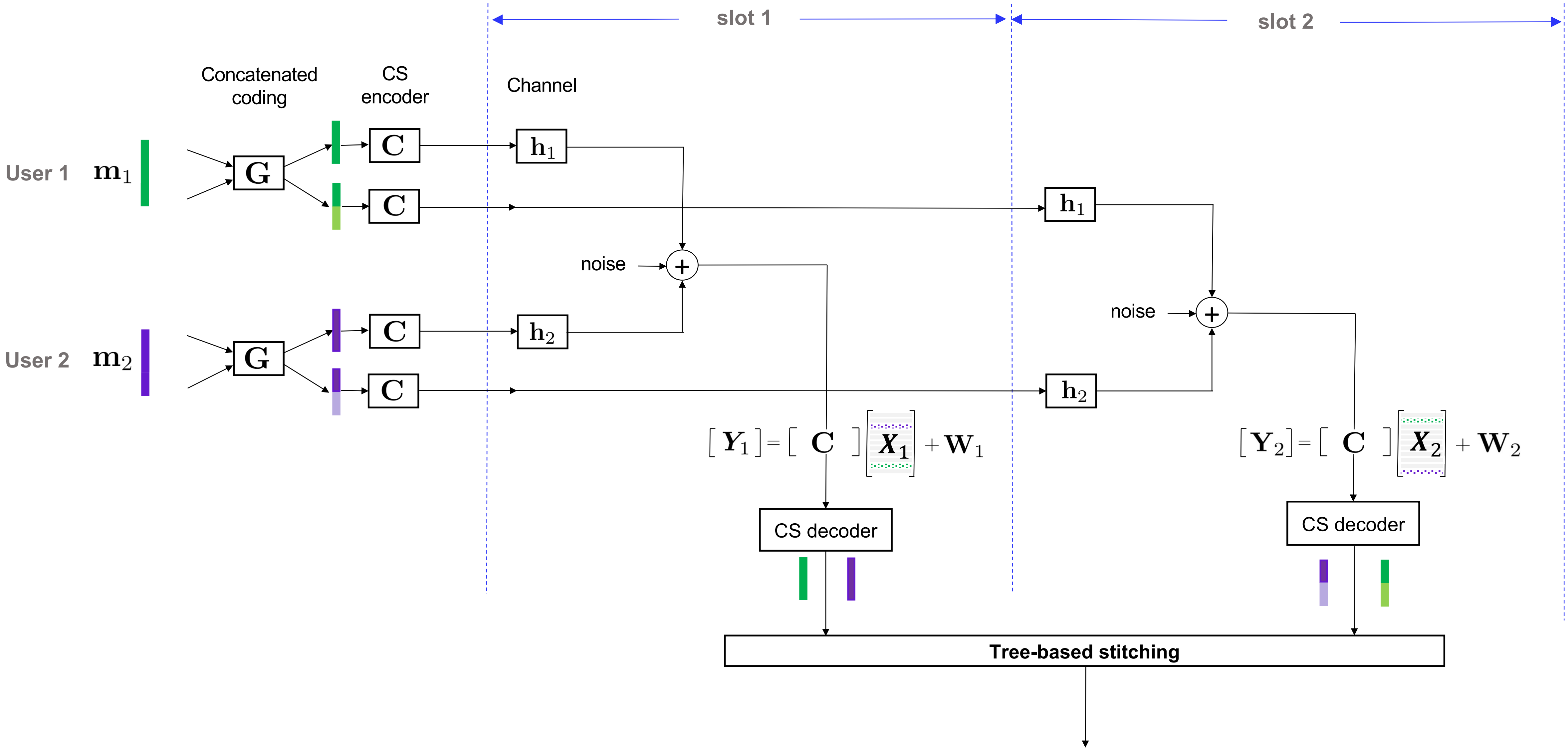
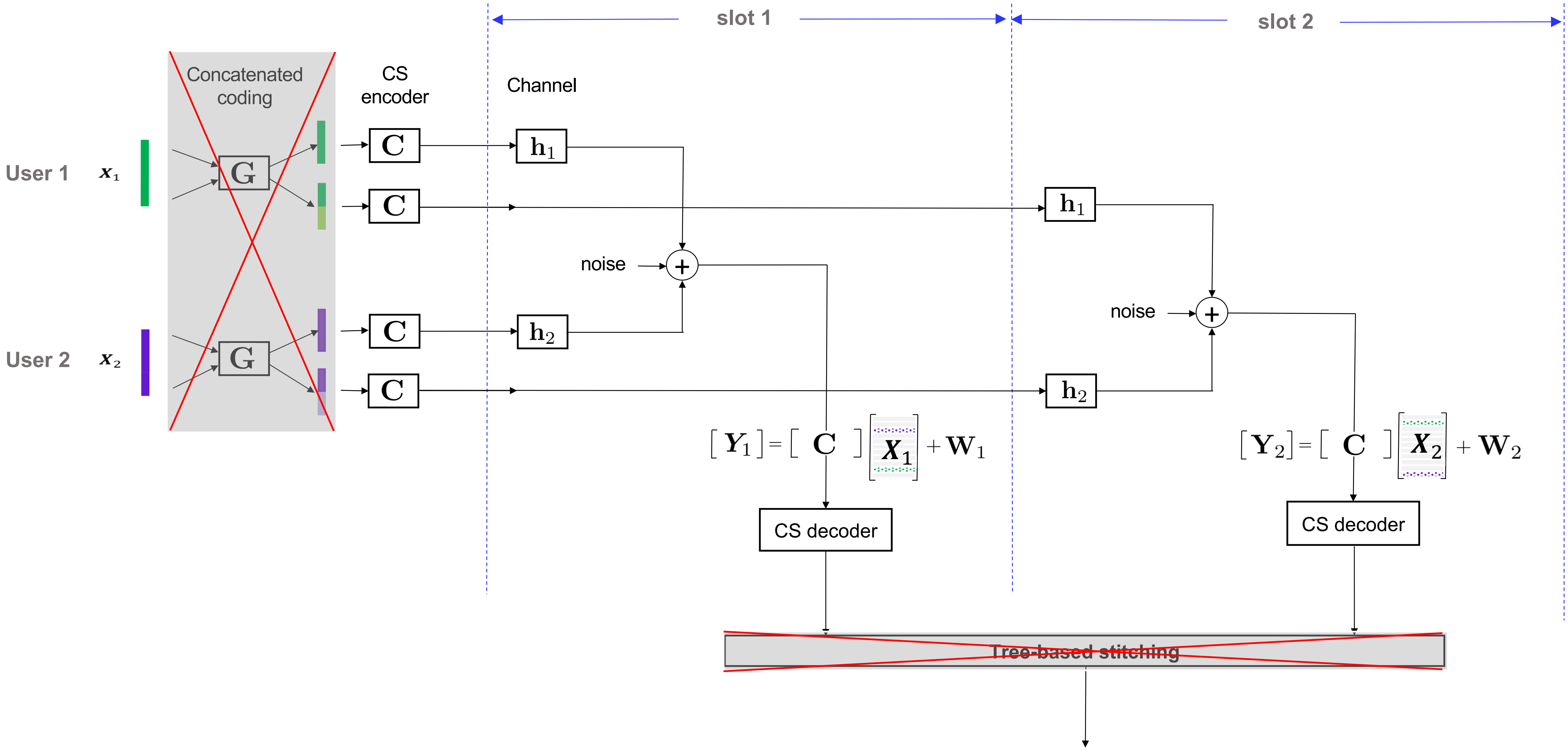


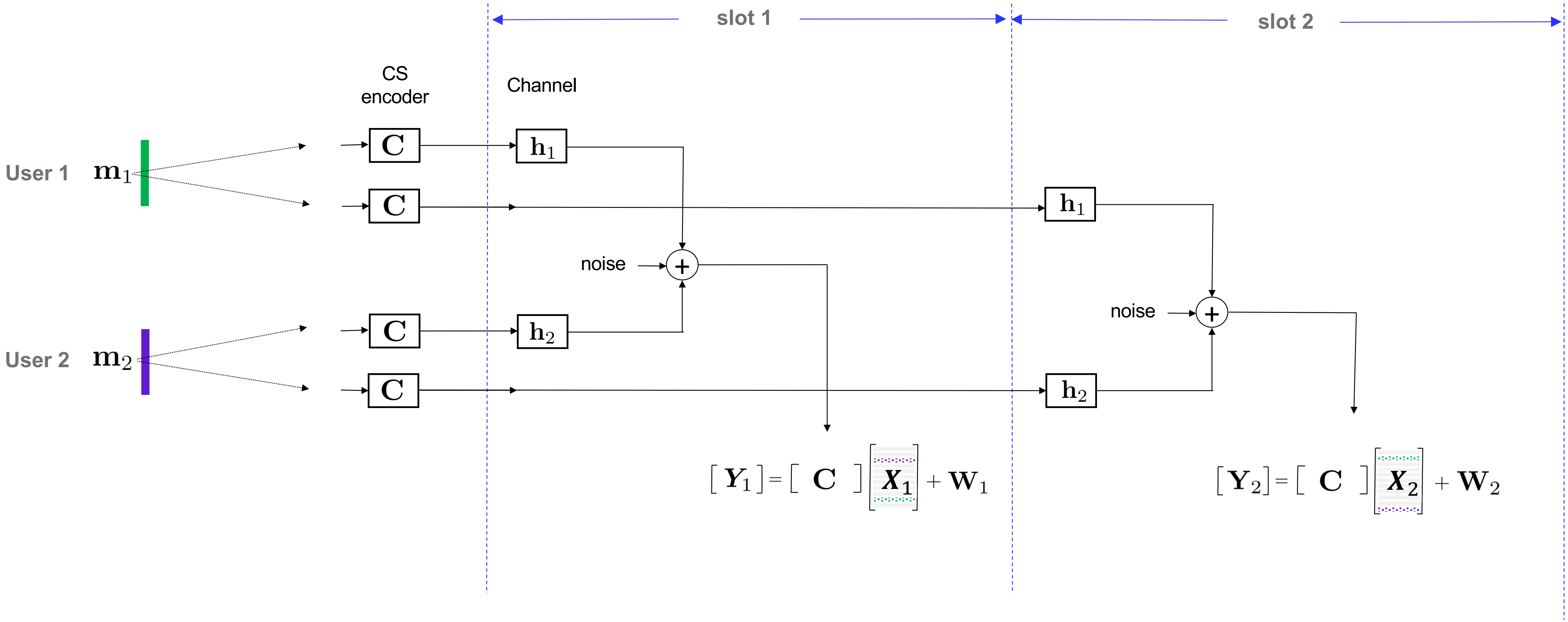
Wei Yu

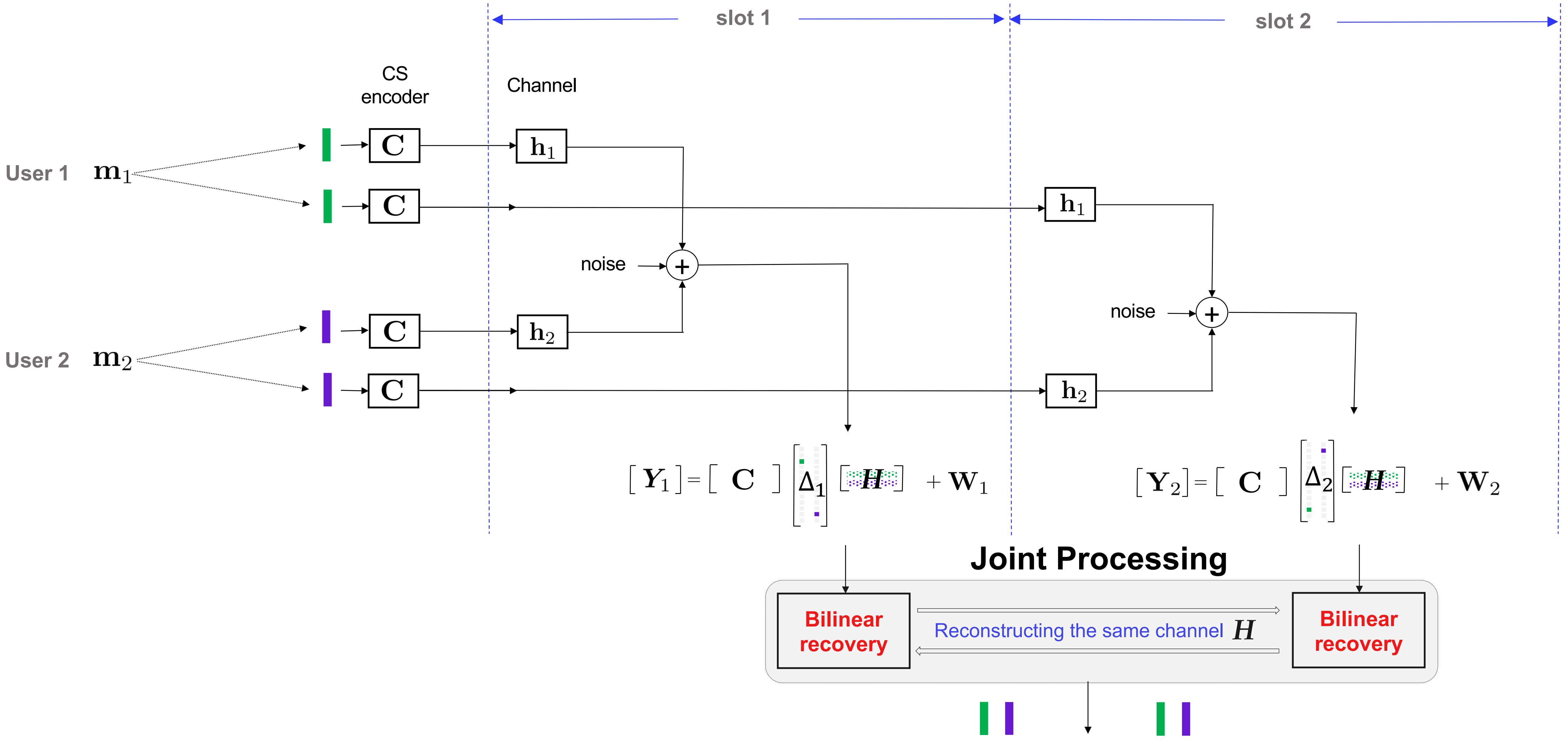
Massive Unsourced Random Access: Slotted transmissions and coupled CS



Massive Unsourced Random Access: Slotted transmissions and bilinear recovery




slot 1
slot 2
CS
encoder
Channel
User 1
$\mathbf{m}_1$
User 2
$\mathbf{m}_2$
C
$\mathbf{h}_1$
$\mathbf{h}_2$
noise
+
$[\boldsymbol{Y}_1] = [\ \mathbf{C}\ ] \boldsymbol{X}_1 + \mathbf{W}_1$
$[\mathbf{Y}_2] = [\ \mathbf{C}\ ] \boldsymbol{X}_2 + \mathbf{W}_2$



slot 1
slot 2
CS
encoder
Channel
User 1
User 2
noise
noise
Joint Processing
Bilinear
recovery
Reconstructing the same channel H
Bilinear
recovery

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Data Encoding

slot l

user k — J bits → $\mathbf{C}$ →

$$\tilde{\mathbf{c}}_{kl} = \left[\begin{array}{cccc} \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_{2^J} \end{array}\right] \boldsymbol{\delta}_{kl} = \mathbf{C}\,\boldsymbol{\delta}_{kl}, \qquad \boldsymbol{\delta}_{kl} = [0, \ldots, 0, 1, 0, \ldots, 0]^\mathsf{T}$$

Received Signal

user 1: $\tilde{\mathbf{c}}_{1l} \rightarrow \mathbf{h}_1 \rightarrow \mathbf{Y}_{1l} = \tilde{\mathbf{c}}_{1l}\,\mathbf{h}_1^\mathsf{T} = \mathbf{C}\,\boldsymbol{\delta}_{1l}\,\mathbf{h}_1^\mathsf{T}$

user k: $\tilde{\mathbf{c}}_{kl} \rightarrow \mathbf{h}_k \rightarrow \mathbf{Y}_{kl} = \tilde{\mathbf{c}}_{kl}\,\mathbf{h}_k^\mathsf{T} = \mathbf{C}\,\boldsymbol{\delta}_{kl}\,\mathbf{h}_k^\mathsf{T}$

user K: $\tilde{\mathbf{c}}_{Kl} \rightarrow \mathbf{h}_K \rightarrow \mathbf{Y}_{Kl} = \tilde{\mathbf{c}}_{Kl}\,\mathbf{h}_K^\mathsf{T} = \mathbf{C}\,\boldsymbol{\delta}_{Kl}\,\mathbf{h}_K^\mathsf{T}$

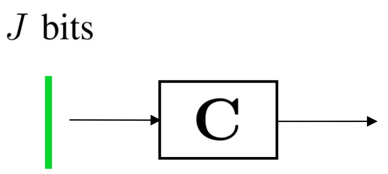
$\oplus \; \boldsymbol{W}_l$

$$\boldsymbol{Y}_l = \sum_{k=1}^{K} \mathbf{Y}_{kl} + \boldsymbol{W}_l = \mathbf{C}\sum_{k=1}^{K} \boldsymbol{\delta}_{kl}\,\mathbf{h}_k^\mathsf{T} + \boldsymbol{W}_l$$

Data Encoding

slot l

user k



$$\tilde{\mathbf{c}}_{kl} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_{2^J} \end{bmatrix} \boldsymbol{\delta}_{kl}, \qquad \mathbf{C} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_{2^J} \end{bmatrix}, \qquad \boldsymbol{\delta}_{kl} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Received Signal

$$\boldsymbol{\Delta}_l = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & 1 & & \vdots \\ \vdots & 0 & & \vdots \\ 0 & \vdots & & \vdots \\ 1 & \vdots & & \vdots \\ 0 & \vdots & & 0 \\ 0 & 0 & \cdots & 1 \end{bmatrix}, \qquad \boldsymbol{H}^\top = \begin{bmatrix} \mathbf{h}_1^\top \\ \mathbf{h}_2^\top \\ \vdots \\ \mathbf{h}_K^\top \end{bmatrix}$$

$$\boldsymbol{Y}_l = \mathbf{C} \sum_{k=1}^{K} \boldsymbol{\delta}_{kl}\, \mathbf{h}_k^\top + \boldsymbol{W}_l = \mathbf{C}\, \boldsymbol{\Delta}_l\, \boldsymbol{H}^\top + \boldsymbol{W}_l$$

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Received Signal

slot l

All users:

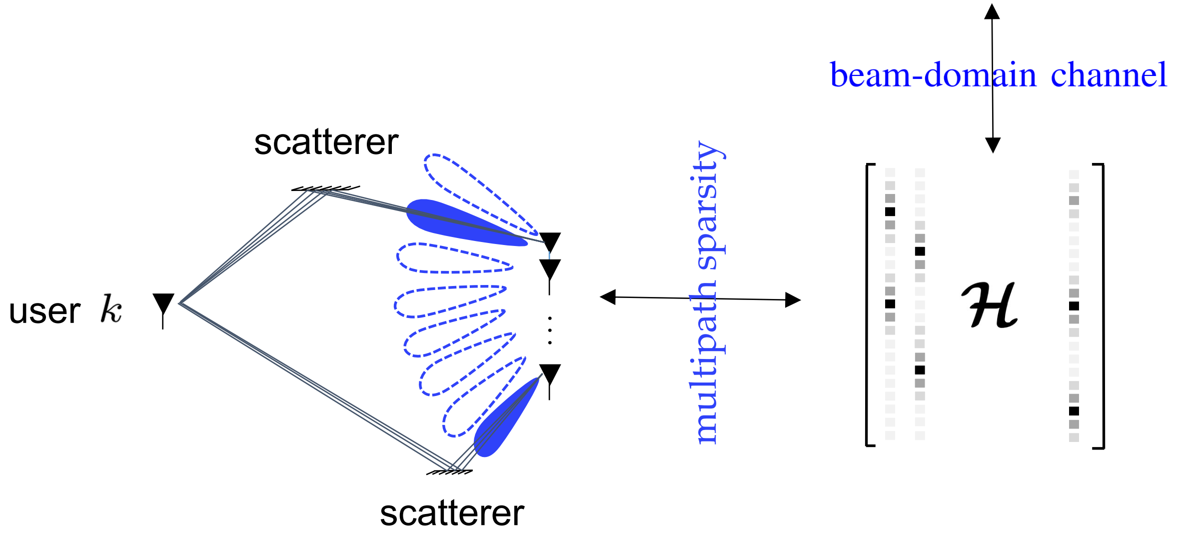
$$\boldsymbol{Y}_l = \mathbf{C}\,\begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & 1 & & \vdots \\ \vdots & 0 & & \vdots \\ 0 & \vdots & \boldsymbol{\Delta}_l & \vdots \\ 1 & \vdots & & \vdots \\ 0 & \vdots & & 0 \\ 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \mathbf{h}_1^{\mathsf{T}} \\ \mathbf{h}_2^{\mathsf{T}} \\ \vdots \quad \boldsymbol{H}^{\mathsf{T}} \\ \mathbf{h}_K^{\mathsf{T}} \end{bmatrix} + \boldsymbol{W}_l$$

Channel deconstruction *

$$\mathbf{F}\mathbf{F}^{\mathsf{H}} = \mathbf{I}$$

spatial DFT matrix

$$\boldsymbol{H}^{\mathsf{T}} = \big(\mathbf{F}\underbrace{\mathbf{F}^{\mathsf{H}}\boldsymbol{H}^{*}}_{\boldsymbol{\mathcal{H}}}\big)^{\mathsf{H}} = \left(\mathbf{F}\,\boldsymbol{\mathcal{H}}\right)^{\mathsf{H}}$$



* A. M. Sayeed, "Deconstructing multiantenna fading channels," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2563–2579, Oct. 2002.

Received Signal

slot l

All users:

$$\boldsymbol{Y}_l = \mathbf{C} \underbrace{\begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & 1 & & \vdots \\ & 0 & & \\ 0 & \vdots & \boldsymbol{\Delta}_l & \\ 1 & & & \\ 0 & & & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 1 \end{bmatrix}}_{\triangleq\, \boldsymbol{U}_l} \underbrace{\begin{bmatrix} \; \boldsymbol{\mathcal{H}}^{\mathsf{H}} \; \end{bmatrix} \mathbf{F}^{\mathsf{H}}}_{\triangleq\, \boldsymbol{V}^{\mathsf{H}}} + \boldsymbol{W}_l$$

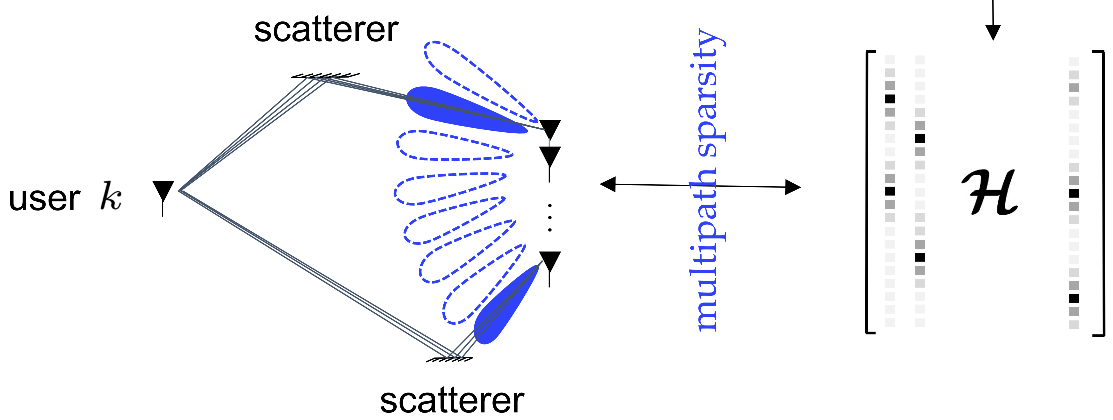
Channel deconstruction *

$$\mathbf{F}\mathbf{F}^{\mathsf{H}} = \mathbf{I}$$

spatial DFT matrix

$$\boldsymbol{H}^{\mathsf{T}} = \big(\mathbf{F}\underbrace{\mathbf{F}^{\mathsf{H}}\boldsymbol{H}^{*}}_{\boldsymbol{\mathcal{H}}}\big)^{\mathsf{H}} = (\mathbf{F}\boldsymbol{\mathcal{H}})^{\mathsf{H}}$$

beam-domain channel

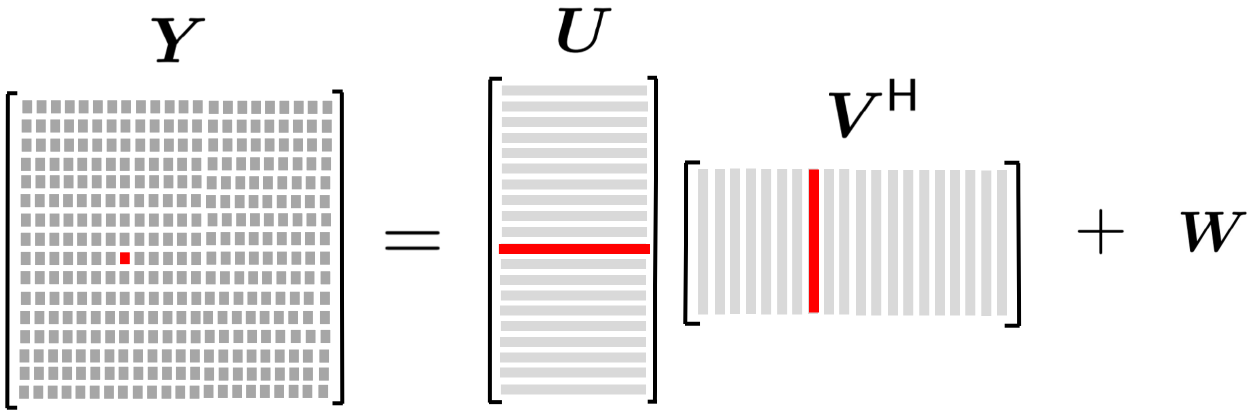


General Purpose BiVAMP

$$\boldsymbol{Y} = \boldsymbol{U}\boldsymbol{V}^{\mathsf{H}} + \boldsymbol{W}$$

* A. M. Sayeed, "Deconstructing multiantenna fading channels," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2563–2579, Oct. 2002.

Massive Unsourced Random Access via Bilinear Recovery: General Purpose BiVAMP

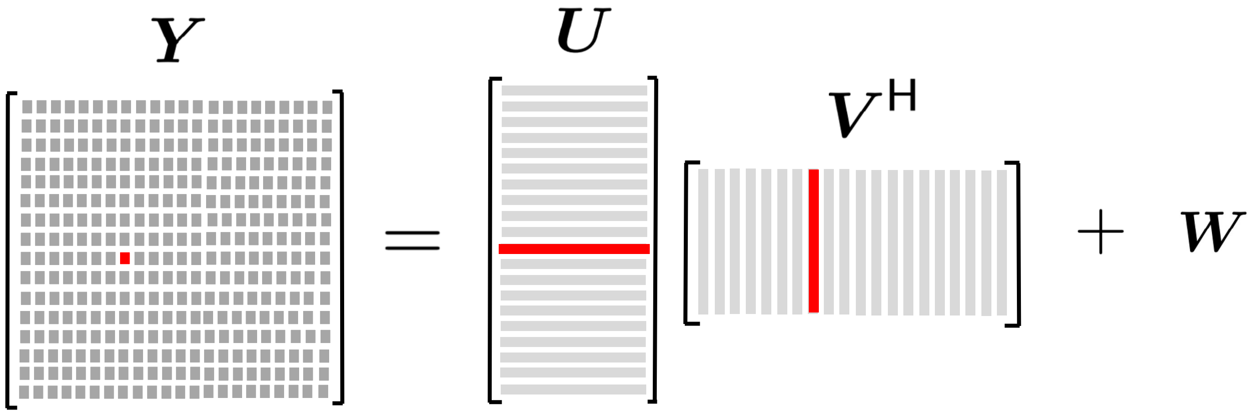


$$p(\boldsymbol{U},\boldsymbol{V}|\boldsymbol{Y}) = p(\boldsymbol{Y}|\boldsymbol{U},\boldsymbol{V})\, p(\boldsymbol{U})\, p(\boldsymbol{V})$$

$$= \prod_{i=1}^{N}\prod_{j=1}^{M} p(y_{ij}|\boldsymbol{u}_i,\boldsymbol{v}_j) \underbrace{\prod_{i=1}^{N} p(\boldsymbol{u}_i) \prod_{j=1}^{M} p(\boldsymbol{v}_j)}_{\text{separable priors}}$$

M. Akrout, et al. “BiG-VAMP: The bilinear generalized vector approximate message algorithm,” Asilomar’22

(Also a more detailed version in Arxiv)

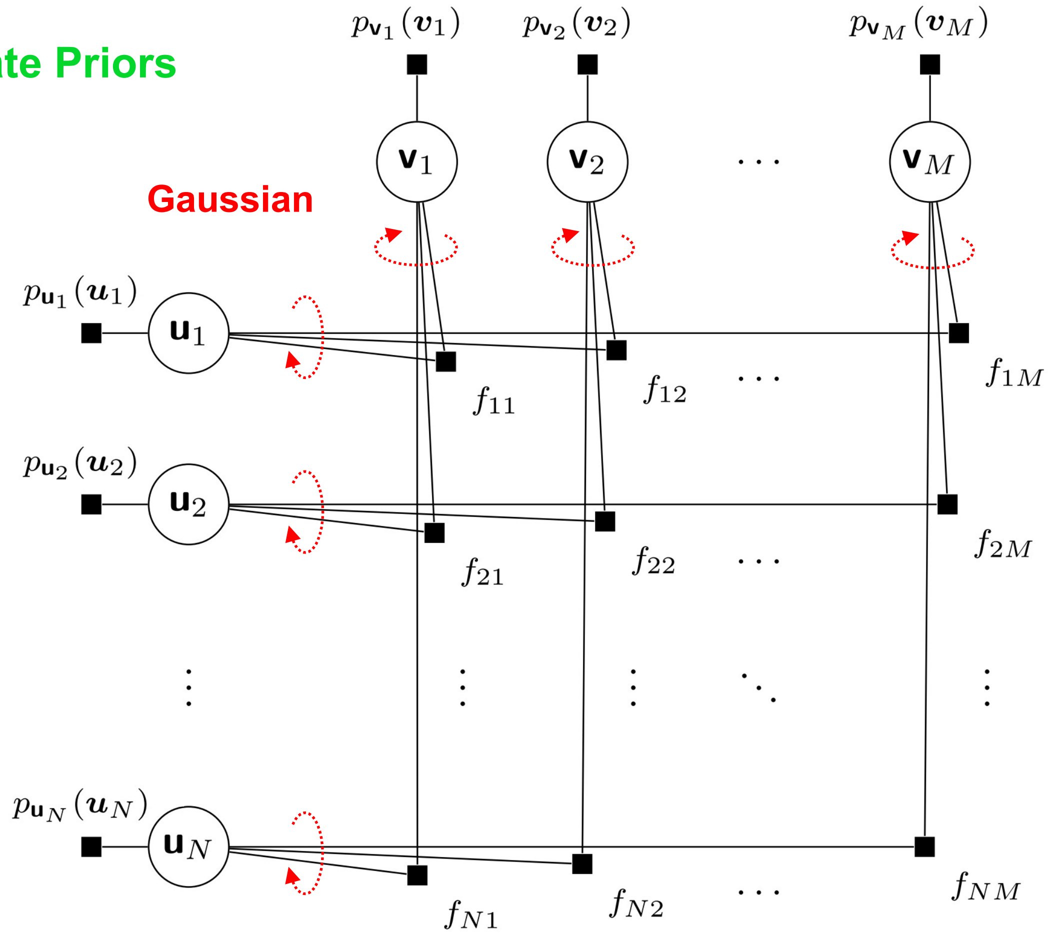


$$p(\boldsymbol{U},\boldsymbol{V}|\boldsymbol{Y}) = \prod_{i=1}^{N}\prod_{j=1}^{M} p(y_{ij}|\boldsymbol{u}_i,\boldsymbol{v}_j) \prod_{i=1}^{N} p(\boldsymbol{u}_i) \prod_{j=1}^{M} p(\boldsymbol{v}_j)$$

f_{ij}

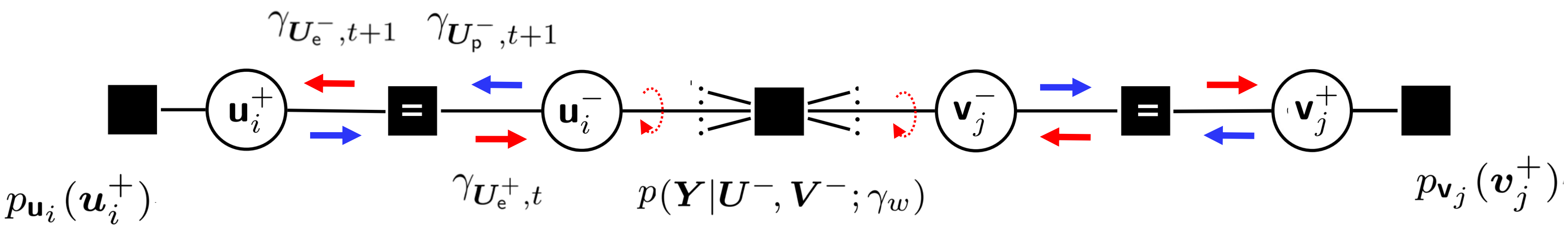
Conjugate Priors

Gaussian

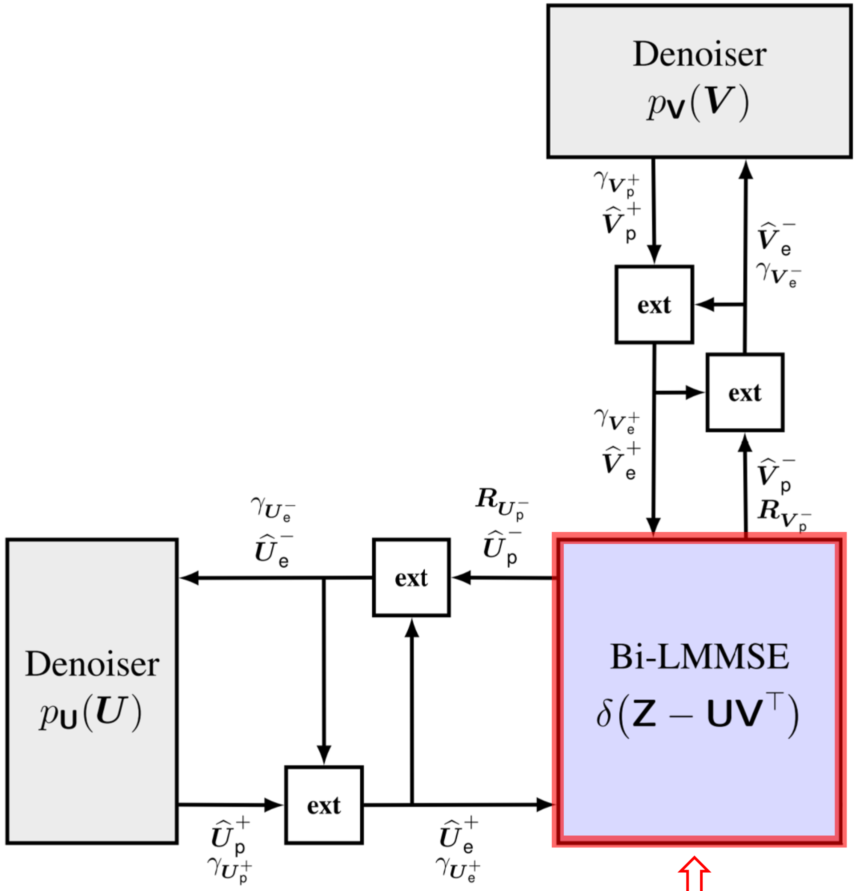


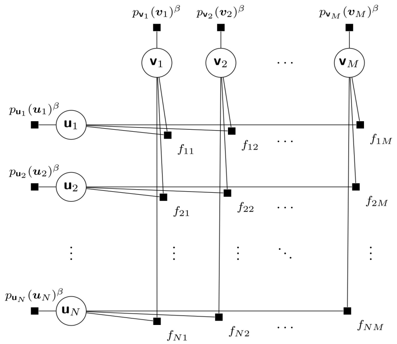
Non-conjugate Prior

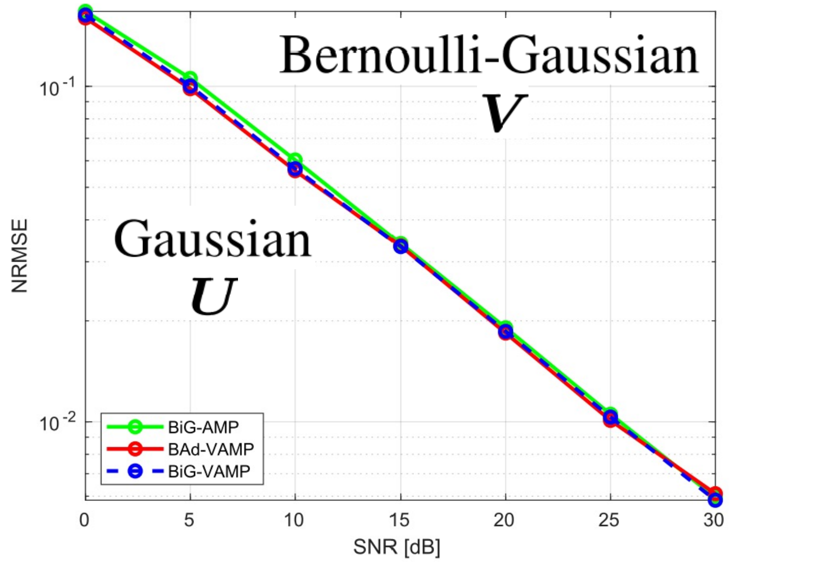
$$\widehat{\boldsymbol{u}}_{i,\mathsf{e},t+1}^{-} = \gamma_{\boldsymbol{U}_{\mathsf{e}}^{-},t+1}^{-1}\left(\gamma_{\boldsymbol{U}_{\mathsf{p}}^{-},t+1}\widehat{\boldsymbol{u}}_{i,\mathsf{p},t+1}^{-} + \gamma_{\boldsymbol{U}_{\mathsf{e}}^{+},t}\widehat{\boldsymbol{u}}_{i,\mathsf{e},t}^{+}\right)$$

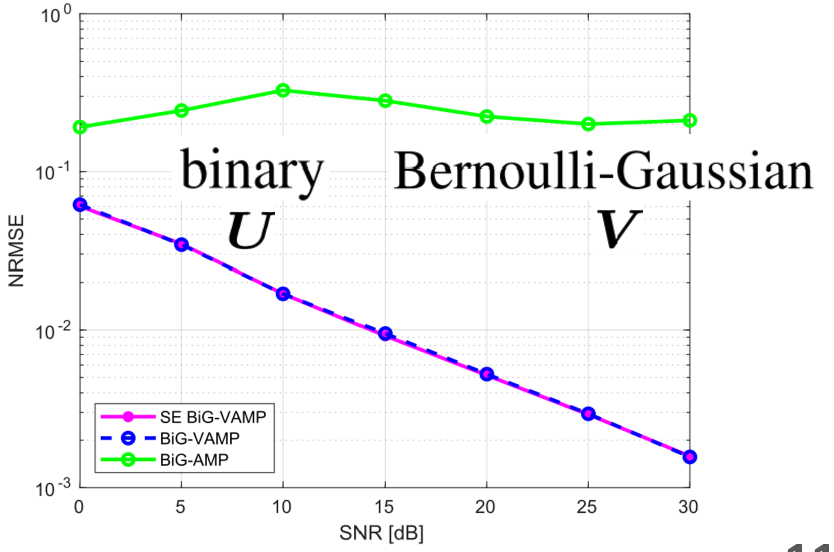


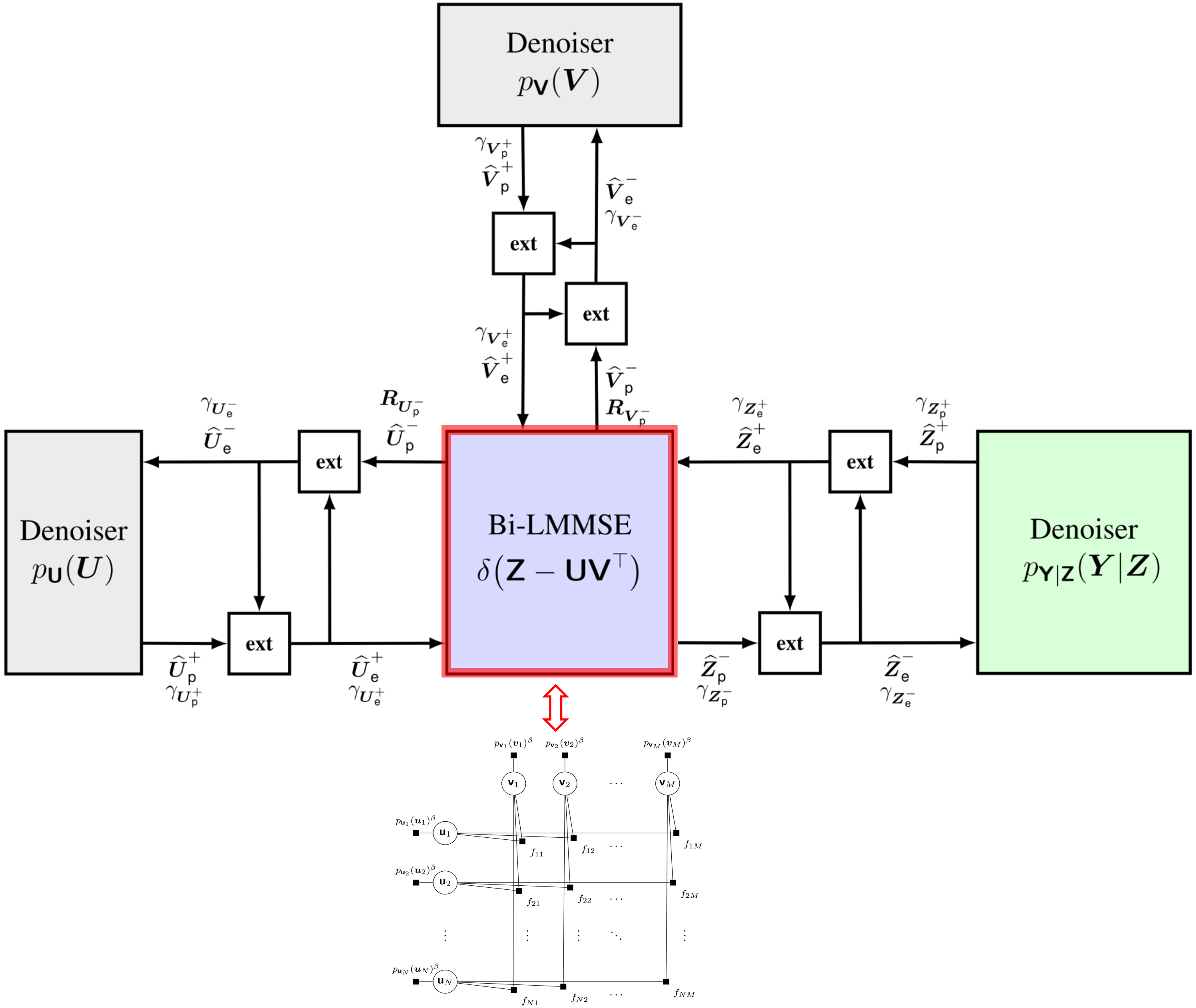
Massive Unsourced Random Access via Bilinear Recovery: General Purpose BiVAMP

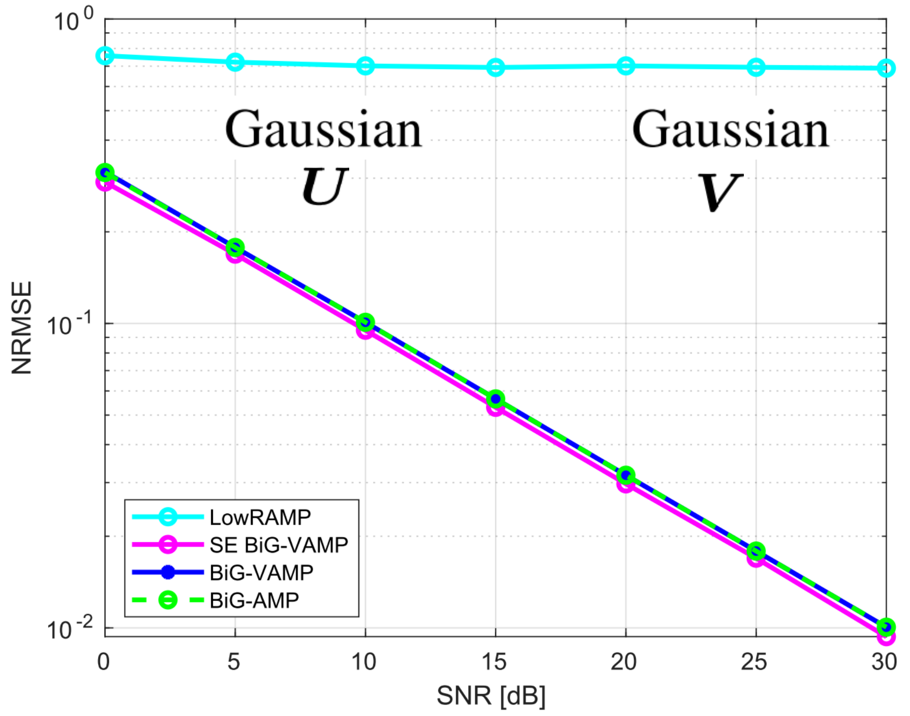












Matrix completion

Selection rate = **10%**

Rank = **3**

Received Signal

slot l

All users:

$$\boldsymbol{Y}_l = \mathbf{C}\begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & 1 & & \vdots \\ 0 & 0 & & \\ 1 & \vdots & \boldsymbol{\Delta}_l & \\ 0 & & & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 1 \end{bmatrix}\begin{bmatrix} \\ \boldsymbol{\mathcal{H}}^{\mathsf{H}} \\ \\ \end{bmatrix}\mathbf{F}^{\mathsf{H}} + \boldsymbol{W}_l$$

All slots

All users:

$$\begin{bmatrix} \boldsymbol{Y}_1 \\ \boldsymbol{Y} \\ \vdots \\ \boldsymbol{Y}_L \end{bmatrix} = \begin{bmatrix} \mathbf{C} & & & \\ & \mathbf{C} & & \\ & & \boldsymbol{\mathcal{C}} & \\ & & & \mathbf{C} \end{bmatrix}\begin{bmatrix} \boldsymbol{\Delta}_1 \\ \boldsymbol{\Delta} \\ \vdots \\ \boldsymbol{\Delta}_L \end{bmatrix}\begin{bmatrix} \\ \boldsymbol{\mathcal{H}}^{\mathsf{H}} \\ \\ \end{bmatrix}\begin{bmatrix} \mathbf{F}^{\mathsf{H}} \end{bmatrix} + \boldsymbol{W} = \underbrace{\boldsymbol{\mathcal{C}}\boldsymbol{\Delta}}_{\boldsymbol{U}}\underbrace{(\mathbf{F}\boldsymbol{\mathcal{H}})^{\mathsf{H}}}_{\boldsymbol{V}^{\mathsf{H}}} + \boldsymbol{W}$$

Massive Unsourced Random Access via Bilinear Recovery: Customizing BiVAMP

$$\begin{bmatrix} \boldsymbol{Y}_1 \\ \boldsymbol{Y} \\ \vdots \\ \boldsymbol{Y}_L \end{bmatrix} = \begin{bmatrix} \mathbf{C} & & & \\ & \mathbf{C} & & \\ & & \boldsymbol{\mathcal{C}} & \\ & & & \mathbf{C} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Delta}_1 \\ \boldsymbol{\Delta} \\ \vdots \\ \boldsymbol{\Delta}_L \end{bmatrix} \begin{bmatrix} \boldsymbol{\mathcal{H}}^{\mathsf{H}} \end{bmatrix} \begin{bmatrix} \mathbf{F}^{\mathsf{H}} \end{bmatrix} + \boldsymbol{W} = \underbrace{\boldsymbol{\mathcal{C}}\boldsymbol{\Delta}}_{\boldsymbol{U}} \underbrace{(\mathbf{F}\boldsymbol{\mathcal{H}})^{\mathsf{H}}}_{\boldsymbol{V}^{\mathsf{H}}} + \boldsymbol{W}$$

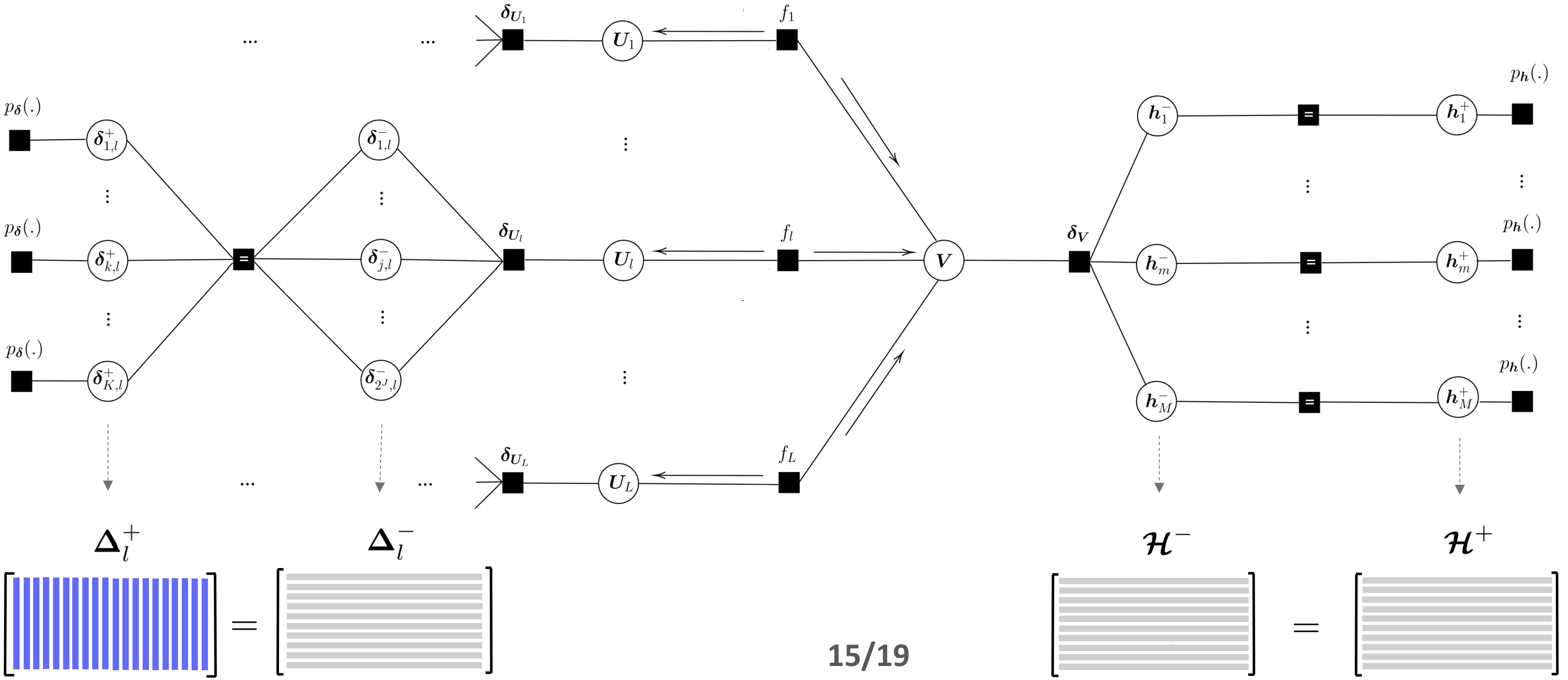
$$p(\boldsymbol{\Delta}, \boldsymbol{\mathcal{H}}, \boldsymbol{U}, \boldsymbol{V} | \boldsymbol{Y})$$

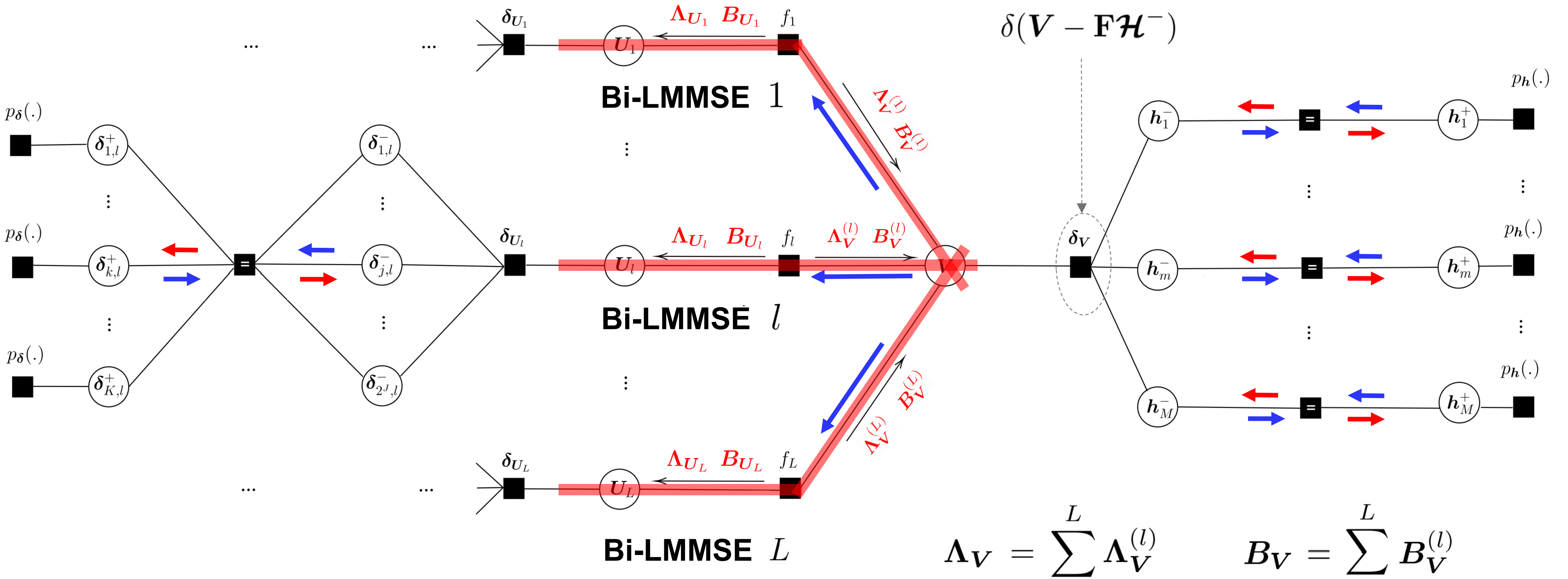
$$\propto \; p(\boldsymbol{Y}|\boldsymbol{U}, \boldsymbol{V})\delta(\boldsymbol{U} - \boldsymbol{\mathcal{C}}\boldsymbol{\Delta})p(\boldsymbol{\Delta})\delta(\boldsymbol{V} - \mathbf{F}\boldsymbol{\mathcal{H}})p(\boldsymbol{\mathcal{H}})$$

$$= \left(\prod_{l=1}^{L} p(\mathbf{Y}_l|\boldsymbol{U}_l, \boldsymbol{V})\delta(\boldsymbol{U}_l - \mathbf{C}\boldsymbol{\Delta}_l)p(\boldsymbol{\Delta}_l)\right)\delta(\boldsymbol{V} - \mathbf{F}\boldsymbol{\mathcal{H}})p(\boldsymbol{\mathcal{H}})$$

$$= \left(\prod_{l=1}^{L} p(\mathbf{Y}_l|\boldsymbol{U}_l, \boldsymbol{V})\delta(\boldsymbol{U}_l - \mathbf{C}\boldsymbol{\Delta}_l)\prod_{k=1}^{K} p_{\boldsymbol{\delta}}(\boldsymbol{\delta}_{k,l})\right)\delta(\boldsymbol{V} - \mathbf{F}\boldsymbol{\mathcal{H}})\prod_{m=1}^{M} p_{\boldsymbol{h}}(\boldsymbol{\hbar}_m)$$

$$\left(\prod_{l=1}^{L} p(\boldsymbol{Y}_l|\boldsymbol{U}_l,\boldsymbol{V})\delta(\boldsymbol{U}_l - \mathbf{C}\boldsymbol{\Delta}_l)\prod_{k=1}^{K} p_{\boldsymbol{\delta}}(\boldsymbol{\delta}_{k,l})\right)\delta(\boldsymbol{V}-\mathbf{F}\boldsymbol{\mathcal{H}})\prod_{m=1}^{M} p_{\boldsymbol{h}}(\boldsymbol{h}_m)$$





$$\boldsymbol{\Lambda}_{\boldsymbol{V}} = \sum_{l=1}^{L} \boldsymbol{\Lambda}^{(l)}_{\boldsymbol{V}} \qquad \boldsymbol{B}_{\boldsymbol{V}} = \sum_{l=1}^{L} \boldsymbol{B}^{(l)}_{\boldsymbol{V}}$$

$$\boldsymbol{B}_{\boldsymbol{V}}\,\boldsymbol{\Lambda}_{\boldsymbol{V}}^{-1} = \mathbf{F}\boldsymbol{\mathcal{H}}^{-}\boldsymbol{N}_{\boldsymbol{V}}$$

LMMSE Estimation

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# of antennas = **50** # of bits = **100** # channel uses = **3600**

Infinite resolution

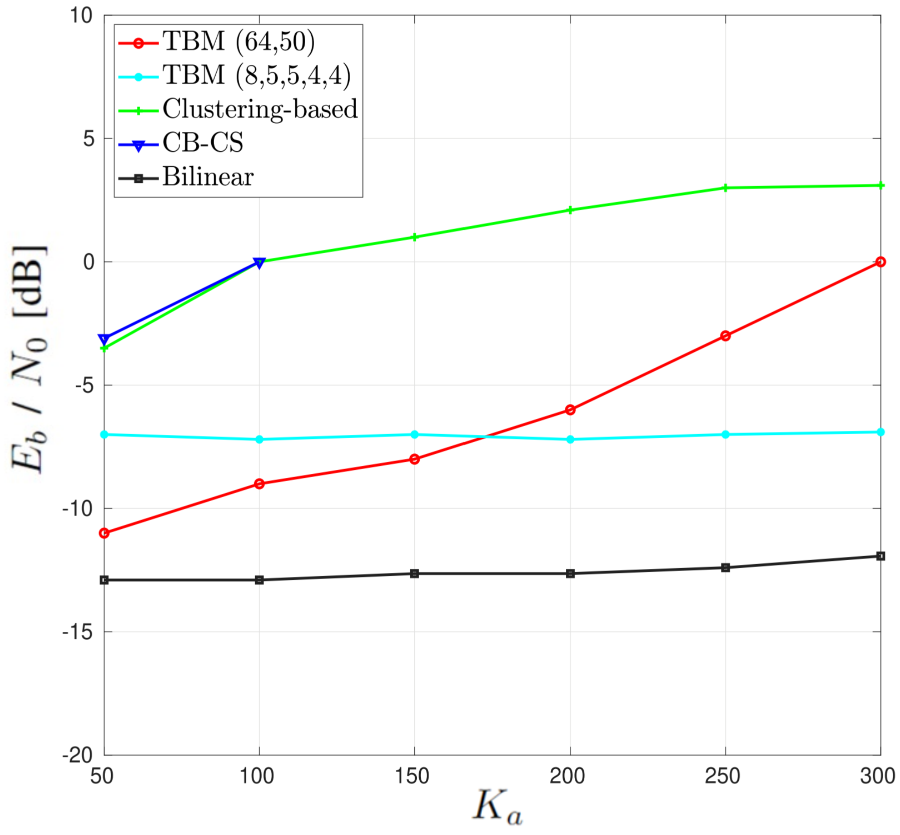


Fig. 2: Minimum E_b/N_0 as a function of the number of active users at a target error probability of 10^{-1}.

Low resolution (i.e., Quantized)

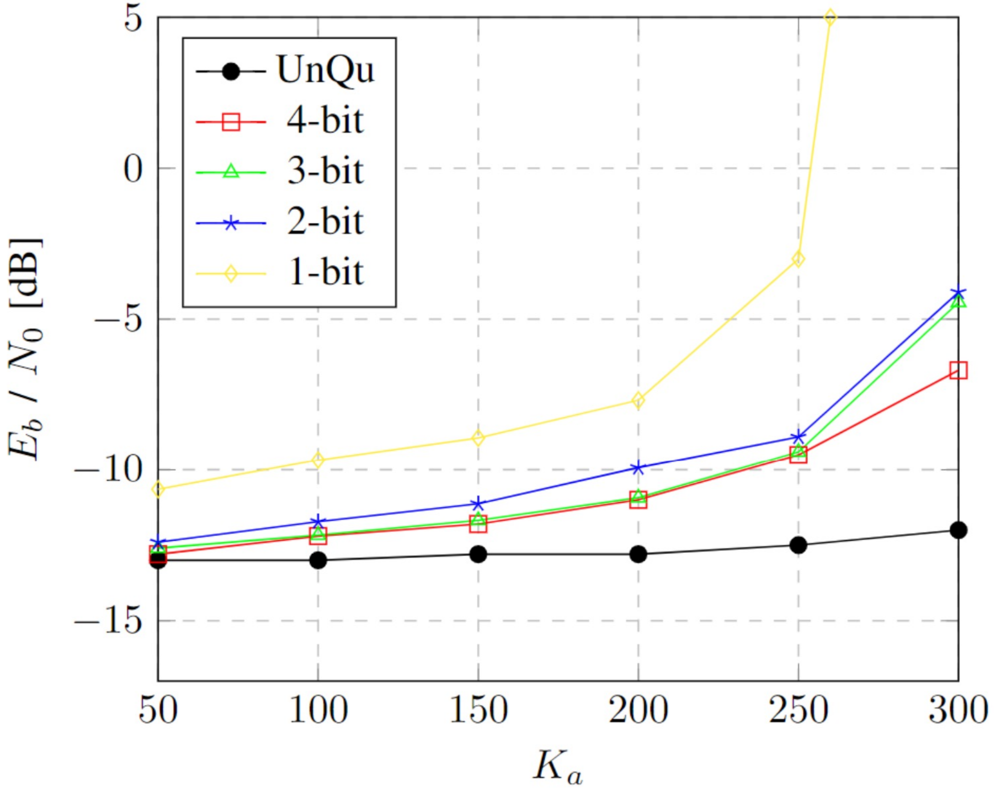
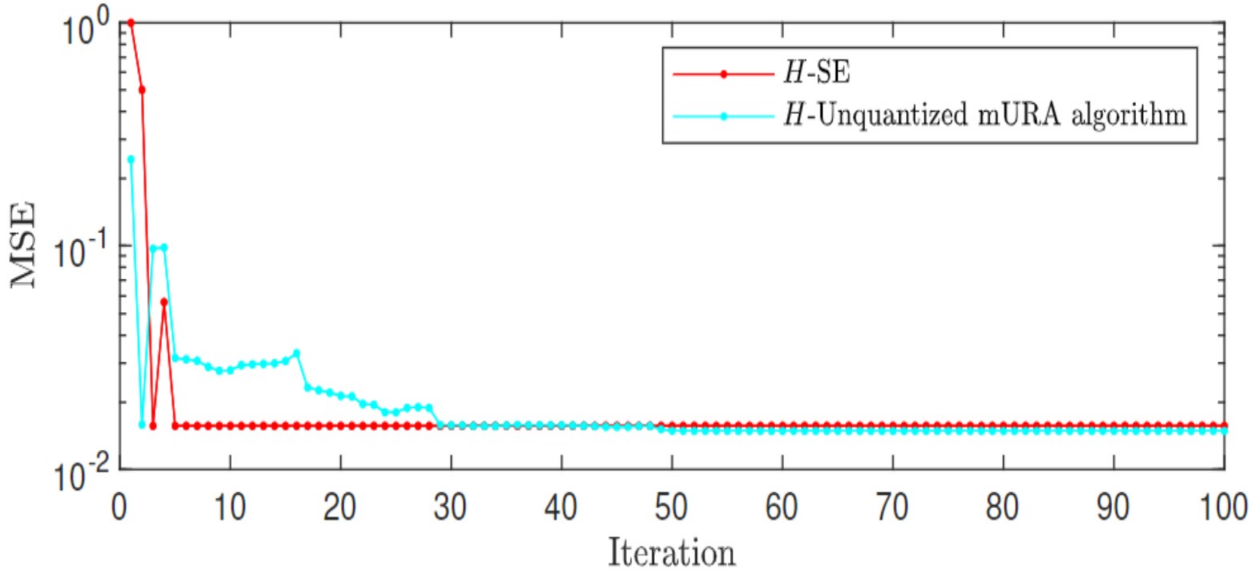


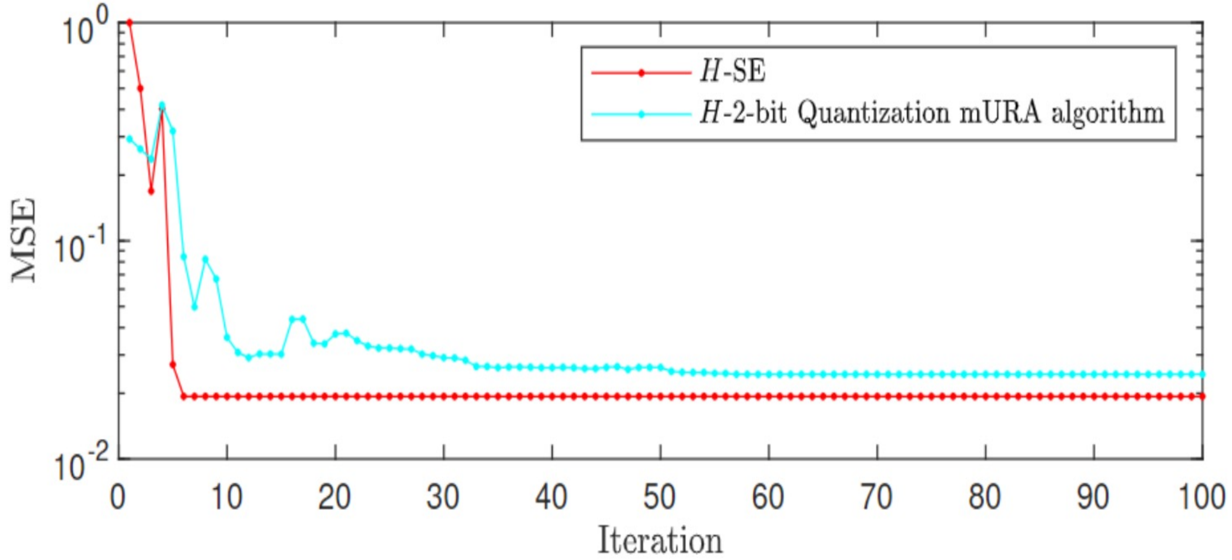
Fig. 10: E_b/N_0 needed as a function of the number of active users at a target error probability of 10^{-1}

# of antennas = **50**   # of bits = **100**   # channel uses = **3600**

Infinite resolution



Low resolution (i.e., Quantized)



Conclusion

Bilinear recovery is essential for concatenated coding-free mURA

Joint processing is **inevitable** to operate at extremely low SNRs