Problem session 3: Banff, 17.3.2023

Shiyue Li

① set $V_\cdot^n = \mathbb{C}\{\text{perms in } S_n\}$ ← S_n acts on this by conjugation

$= \bigoplus_{i=0}^{\infty} \{\text{perms with } i \text{ cycles}\}$

Is this equivariantly log-concave in the sense of Gedeon-Proudfoot-Young?

Remark: $\dim V_i^n = c(n,i)$, the Stirling numbers of the first kind. The corresponding polynomial is real-rooted, so this sequence is log-concave.

② Is $\mathrm{Poin}(V_\cdot^n)$ (equivariantly) real rooted?

③ Is there a combinatorial proof for ① or ②? Ask Alex Fink for details

Lucia Lopez de Medrano

④ If we have a tropical fan F, does there always exist a tropical manifold V with $\mathrm{rec}(V) = F$? [NO]

⑤ Let V_1, V_2 be tropical manifolds with $\mathrm{rec}(V_1) = \mathrm{rec}(V_2)$. Must $\mathrm{rec}(csm_k(V_1)) = \mathrm{rec}(csm_k(V_2))$? [NO]

⑥ [Modification of ④] Given F a tropical fan, is there a tropical manifold V s.t. $\mathrm{rec}(V)$ is a multiple of F? or F with different weights? Ask Lucia Lopez de Medrano for details.

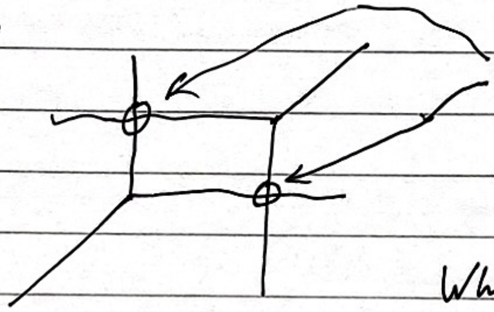
Hunter Spink

(7) We know $\deg(\Delta_M \wedge \beta^{r-1}) = \overset{T(1,0)}{\cancel{\text{Beta invariant of } M}}$ ← this counts bases of external activity 0.

So if $L \subseteq \mathbb{C}^n$, and we have an $(r-1)$ dimensional "reciprocal linear space" Λ,

then $|L \cap \Lambda| = T_M(1,0) = |\text{Bases of external activity } 0|$

this bijection has a combinatorial interpretation (see B[redacted]ST or Huh-Katz)



is natural bijection with bases of external activity 0.

What happens when we cross a wall?
Do we have a monodromy?
Conjecture: the monodromy is transitive.

(8) Study the monodromy of ~~two~~ general fans intersecting under the fan displacement rule, assuming they have complementary dimension.
~~Conjecture~~:

Matt Larson

(9) $R_M(u, 0)$ ⇝ f-vector of independence complex
$R_M(u, -1)$ ⇝ ~~broken cir~~ " " broken circuit complex
$U_0(u, 0)$ ⇝ " independence complex
What about $U_0(u, -1)$? Is this the f-vector of something?
$f_n u^n + \dots + f_1 u + f_0$
Conjecture: $f_i \leq f_{n-i}$ for any delta matroid.

Nick Proudfoot

(10) Let M be a rank-3 matroid, and consider the lattice of flats. Expand the poset s.t. the meet of any 2 points at rank 2 is of rank 1, and the join of 2 things at rank 1 is rank 2...

~~[illegible]~~

This is a formal operation on such posets.

The final closure is an infinite projective geometry, into which the matroid is embedded.

Do all rank-3 matroids give the same answer.

This gives an invariant of matroids

What does it look like?

What information about the matroid does it encode?