

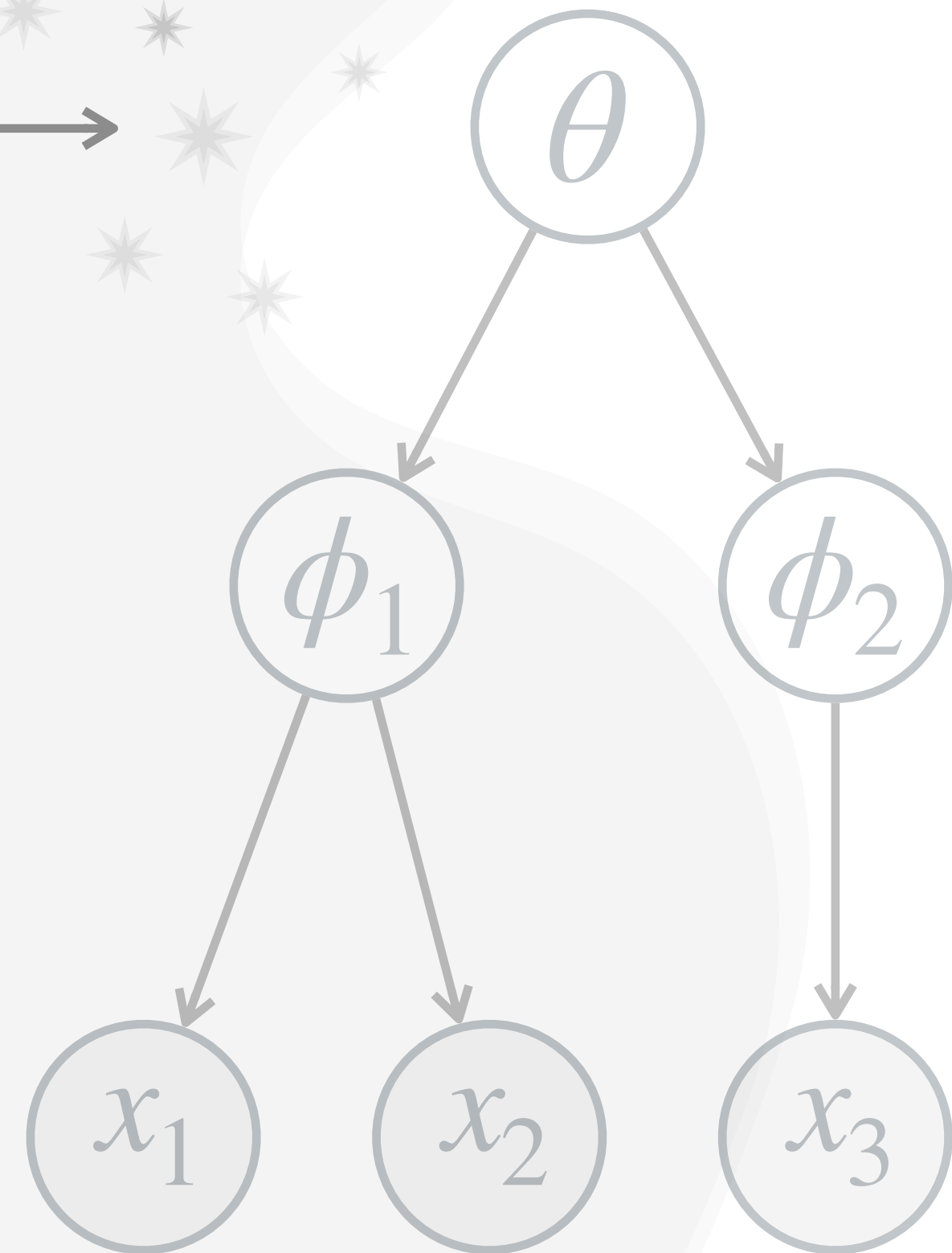
# BAYESIAN APPROACHES IN ASTROPHYSICS

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Systematic Effects and Nuisance Parameters in Particle Physics Data Analyses

Banff International Research Station, 23rd-28th April 2023



SFB 1258

Neutrinos  
Dark Matter  
Messengers



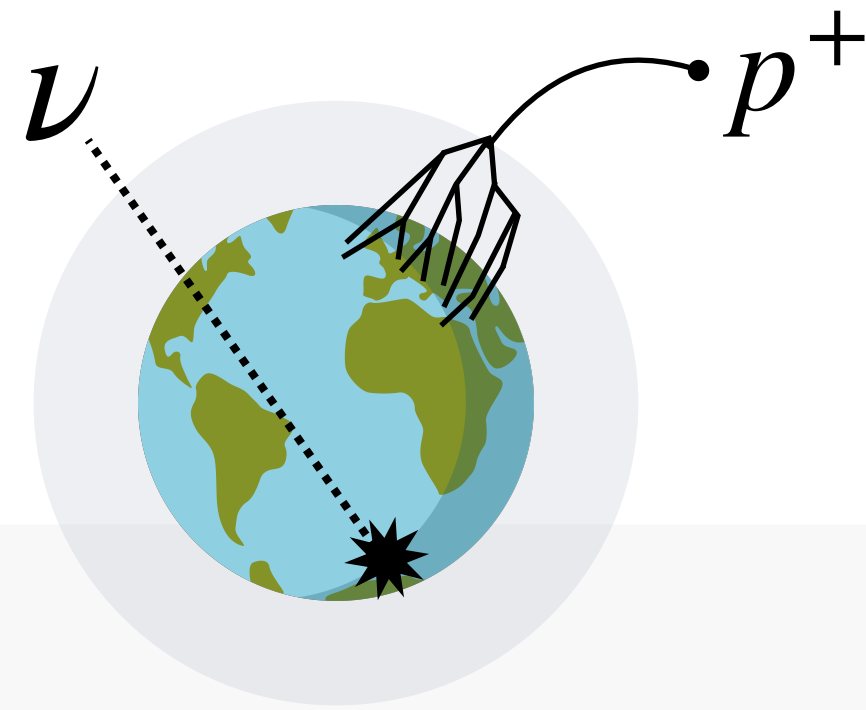
MAX-PLANCK-GESELLSCHAFT



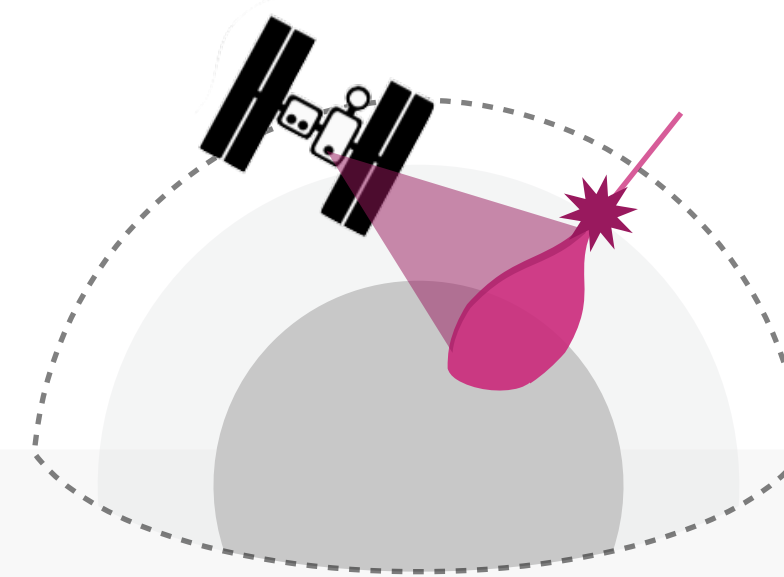
Max-Planck-Institut für Physik  
(Werner-Heisenberg-Institut)

# INTRODUCTION

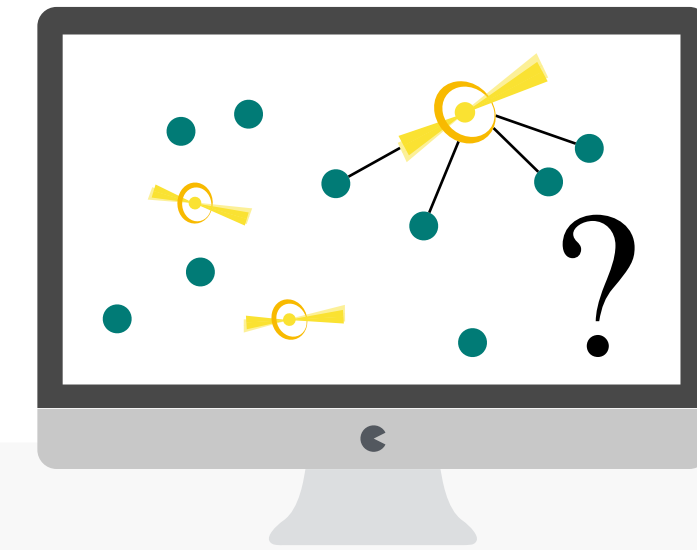
My background:



Cosmic ray and neutrino astrophysics



Detector development & data analysis



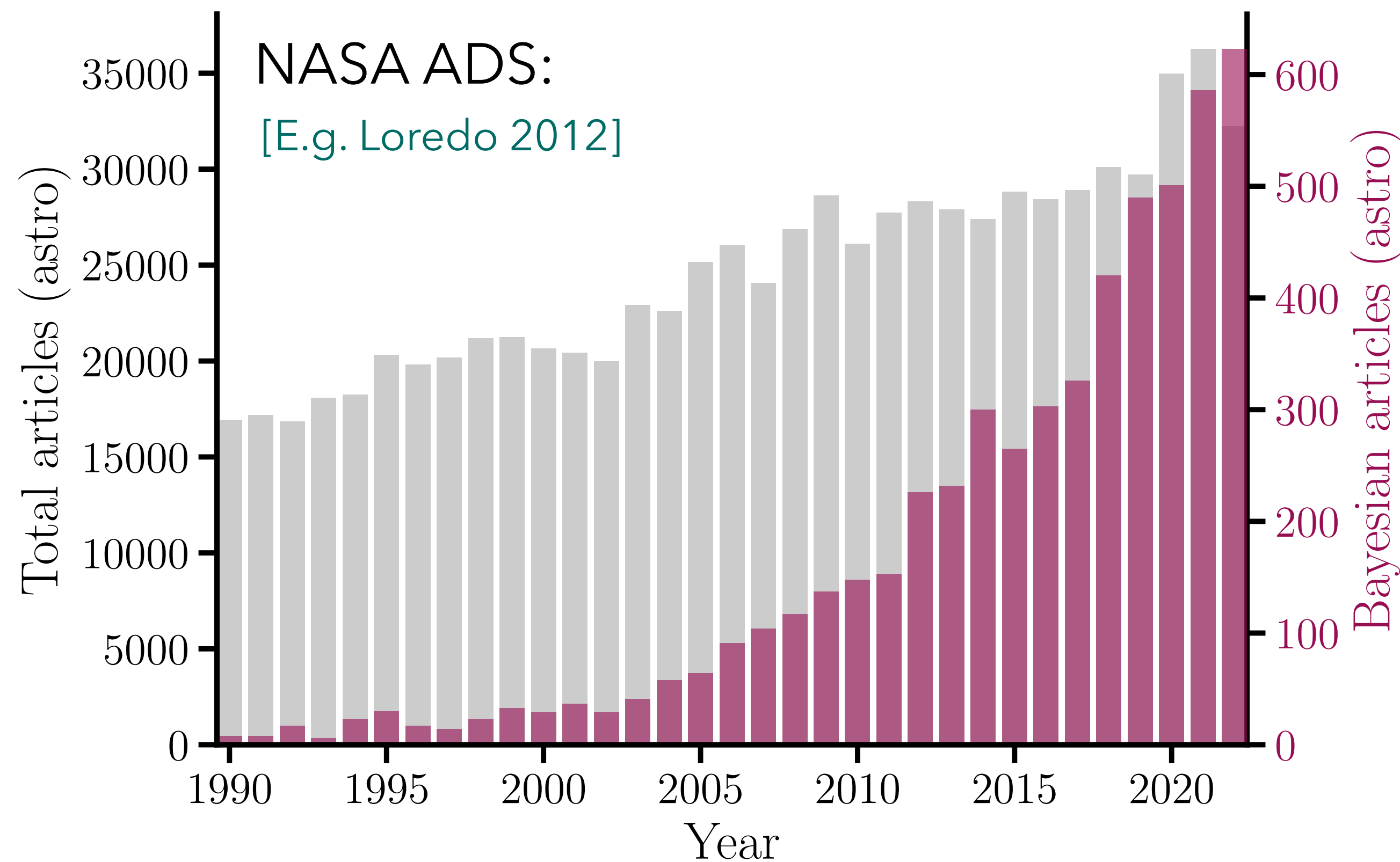
Identifying astrophysical sources

Astroparticle: More frequentist than Bayesian

# INTRODUCTION

Bayesian approaches widely used in astrophysics and cosmology

[Trotta *Contemp. Phys* 2008; Hobson+ 2010; Loredo 2012; Schafer *Annu. Rev. Stat. Appl.* 2015; Feigelson+ *Annu. Rev. Stat. Appl.* 2021]



Driving factors:

- Observational - lack of controlled repeated experiments
- Computational developments
- Increasing complexity
- Lots of data - systematics become increasingly important

# OUTLINE

1

Bayesian view on  
systematic uncertainties

2

General strategies for  
handling systematic  
uncertainties

3

Summary & outlook

Examples  
from  
astrophysics  
and  
cosmology

# BAYESIAN SYSTEMATICS

Nuisance parameters are just parameters

**Marginalisation** summarises the result:

[Dawid 1980; Sinervo, *PHYSTAT* 2003; Sivia+Skilling 2006; Heinrich+Lyons *Annu. Rev. Nucl. Part. Sci.* 2007; Gelman+ BDA 2013]

Nuisance

Interesting

Prior quantifies uncertainty

$$\pi(\theta, \phi | x) = \frac{\pi(x | \theta, \phi) \pi(\theta, \phi)}{\int d\theta \int d\phi \pi(x | \theta, \phi) \pi(\theta, \phi)}$$

$$\pi(\theta | x) = \int d\phi \pi(\theta, \phi | x)$$

Credible regions,  
limits etc.

[Talks by Cousins & Davison]

### Comparison with frequentist

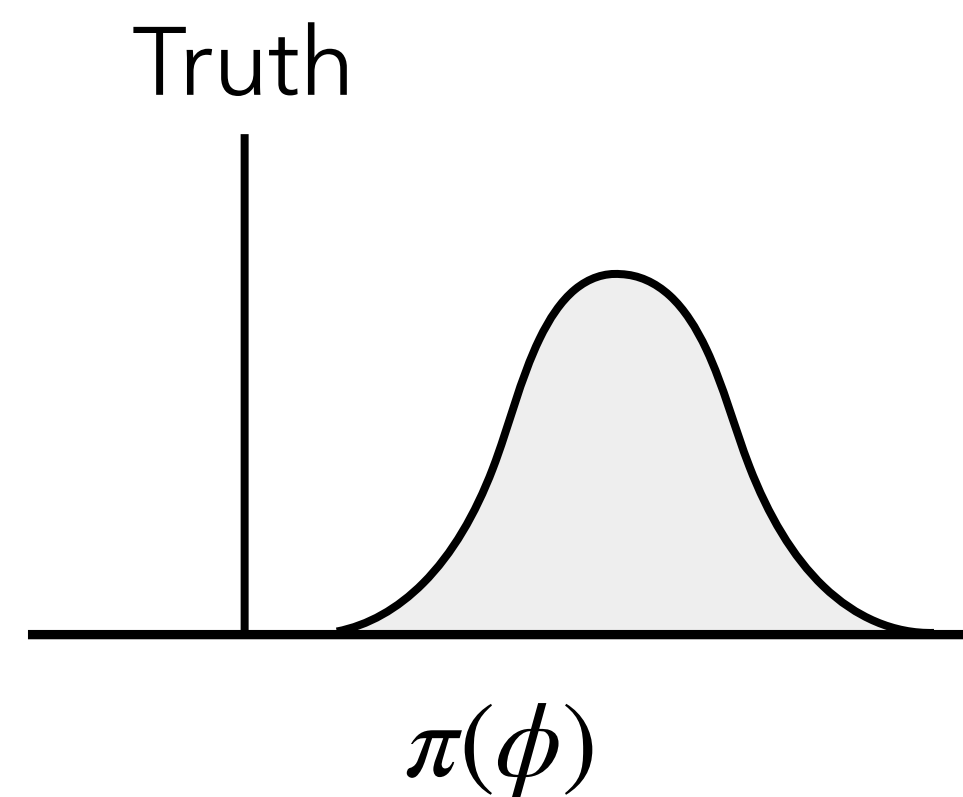
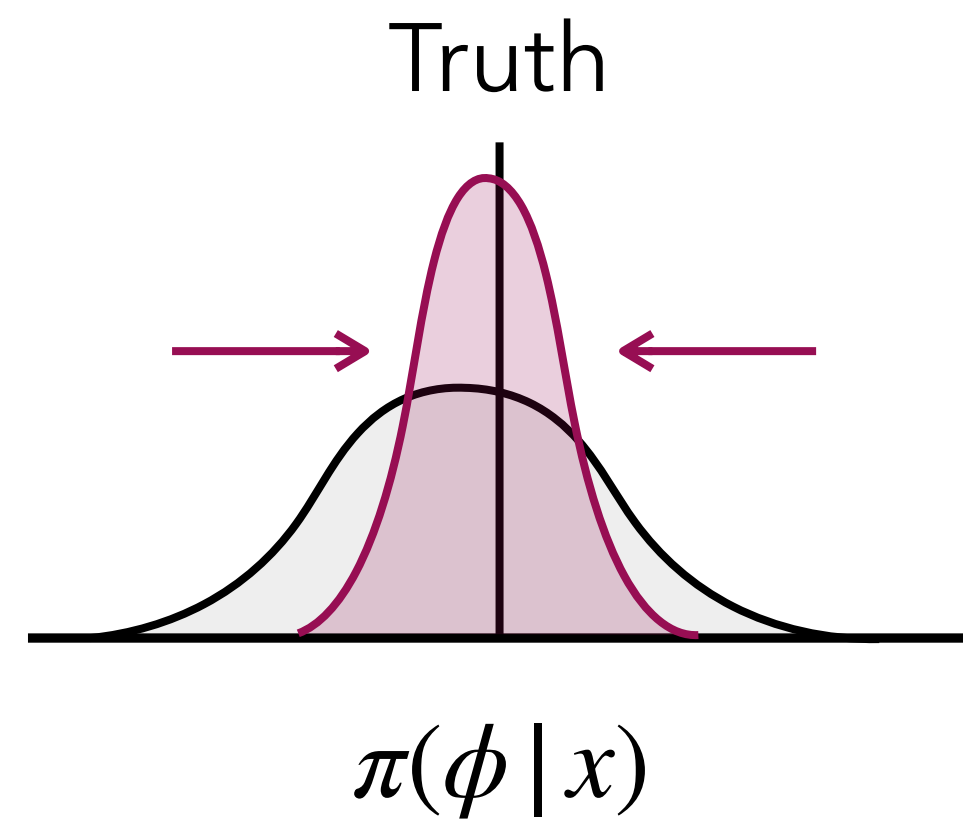
- Clear rationale
- No “special treatment” of nuisance parameters
- Priors necessary
- Coverage not guaranteed

[Loredo *SCMA* 1992; Conrad *Astropart. Phys.* 2015]

# SYSTEMATIC UNCERTAINTIES

[Sinervo *PHYSTAT* 2003; Heinrich+Lyons *Annu. Rev. Nucl. Part. Sci.* 2007]

[Talks by Wardle & Algeri]



**1**

Systematics that can be constrained

**2**

Uncertain assumptions in measurement/analysis

**3**

Uncertain theoretical framework

Calibration

Data

Model misspecification

Theory/  
phenomenology



# SYSTEMATIC UNCERTAINTIES

1

Systematics that can be constrained

E.g. Calibration of detector, characterisation of the effective area, ancillary measurements

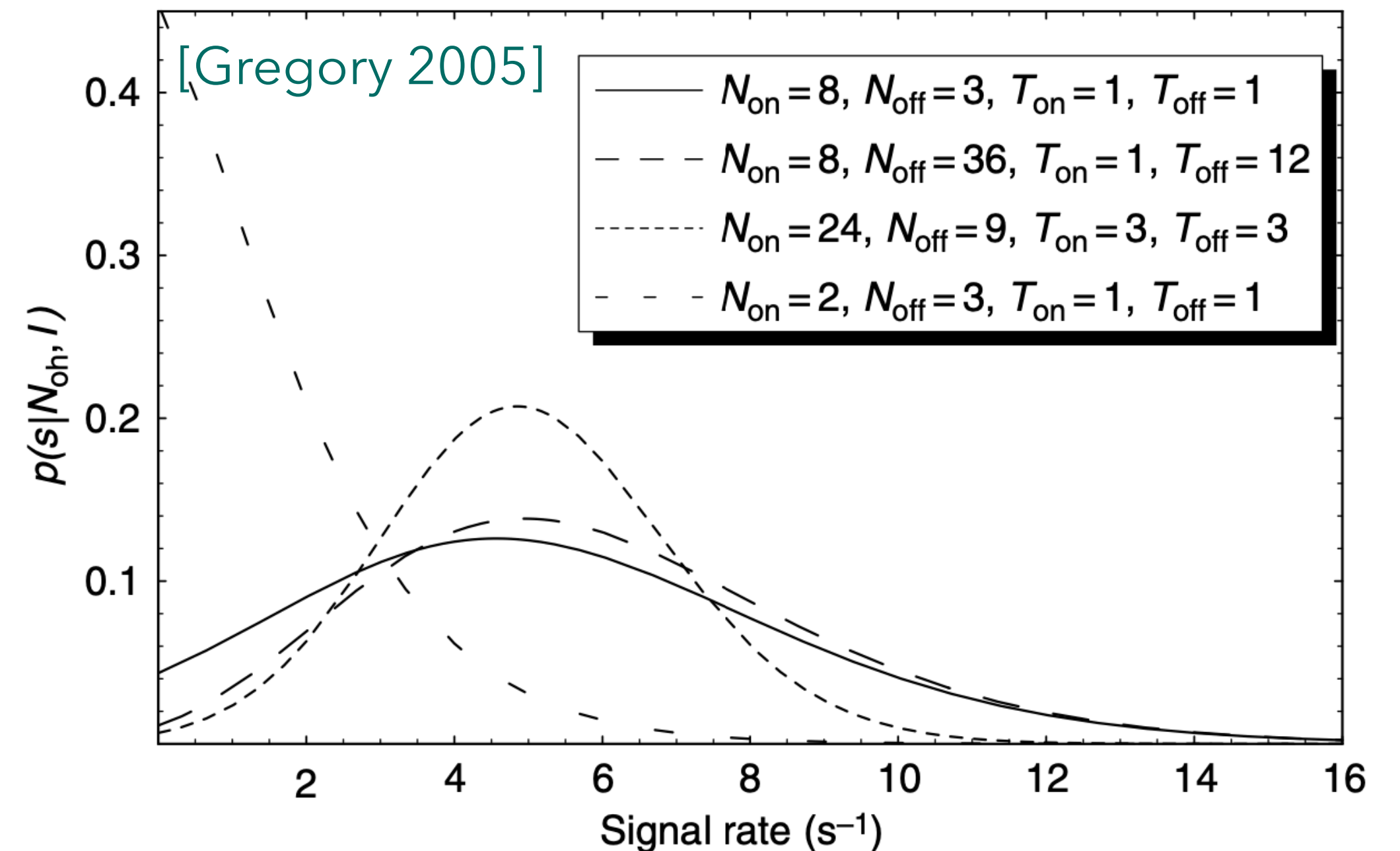
"On/off" signal measurement for low count data

[Helene *NIM* 1983+1984; Prosper *NIMA* 1985; Prosper *PRD* 1988; Loredo *SCMA* 1992; Gregory 2005; Gillessen+Harney *A&A*. 2005; Knoetig *ApJ* 2014; Casadei *ApJ* 2015; Nosek+Nosková *NIMA* 2016]

Astrophysics application: **Gamma-ray data**

- Off: Point away from source and measure  $b$
- On: Point at source and measure  $s + b$
- Marginalise to study source signal
- Quantify significance

$$\pi(s | N_{\text{on}}) = \int_0^{b_{\text{max}}} db \pi(s, b | N_{\text{on}}) \pi(b)$$



# SYSTEMATIC UNCERTAINTIES

2

Uncertain assumptions in measurement/analysis

Not well-constrained by observations, assumptions and simplifications necessary

## Magnetic field modelling in the search for ultra-high-energy cosmic ray (UHECR) sources

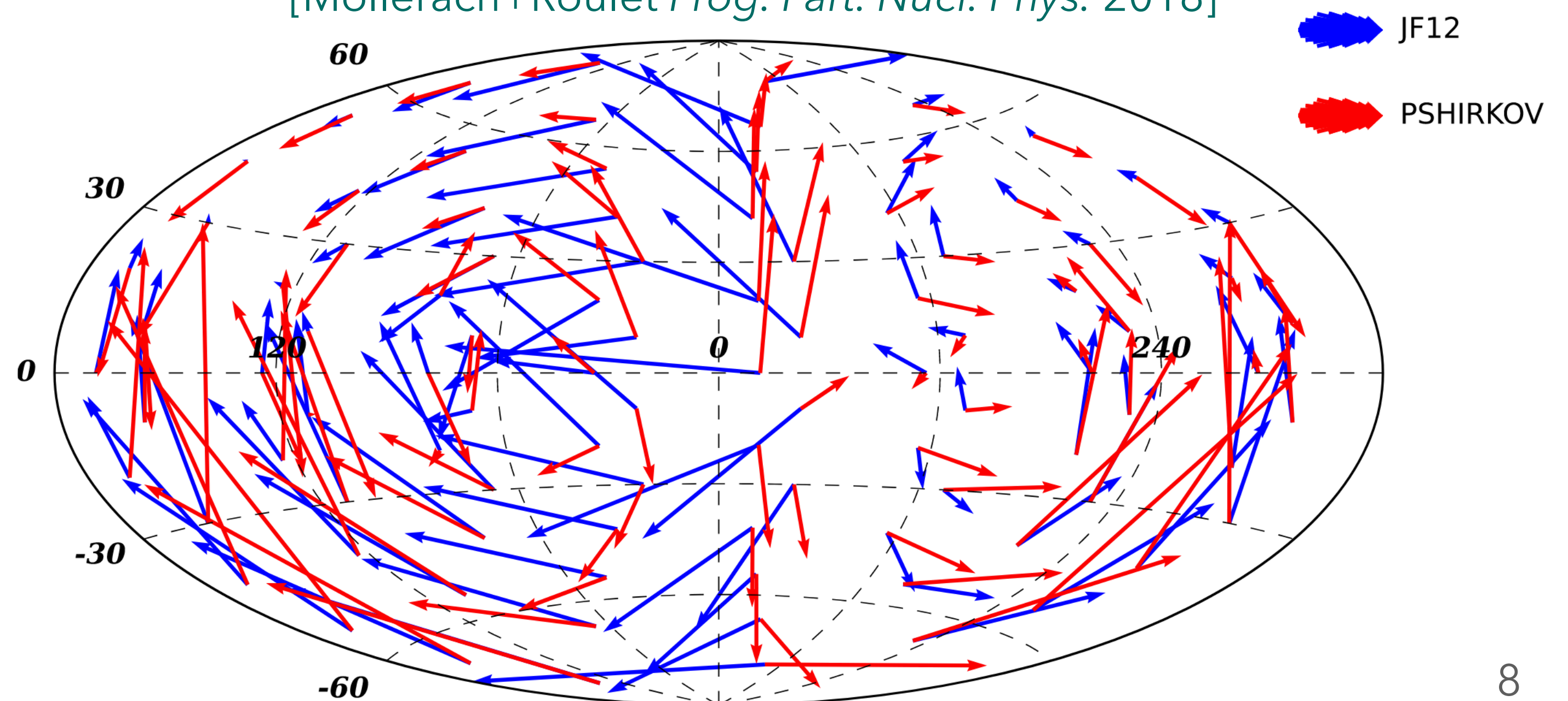
[Pschirkov+ *ApJ* 2011; Janson+Farrar *ApJ* 2012; Unger+Farrar *UHECR* 2018; Batista+ *Front. Astron. Space Sci.* 2019; Jaffe *Galaxies* 2019; Watanabe+ *UHECR* 2022]

- UHECRs are deflected by magnetic fields as they propagate from extragalactic sources
- Galactic magnetic field has a lensing effect that is model dependent

**IMAGINE: Joint fit of all constraints**

[Boulanger+ *JCAP* 2018]

[Mollerach+Roulet *Prog. Part. Nucl. Phys.* 2018]





# SYSTEMATIC UNCERTAINTIES

3

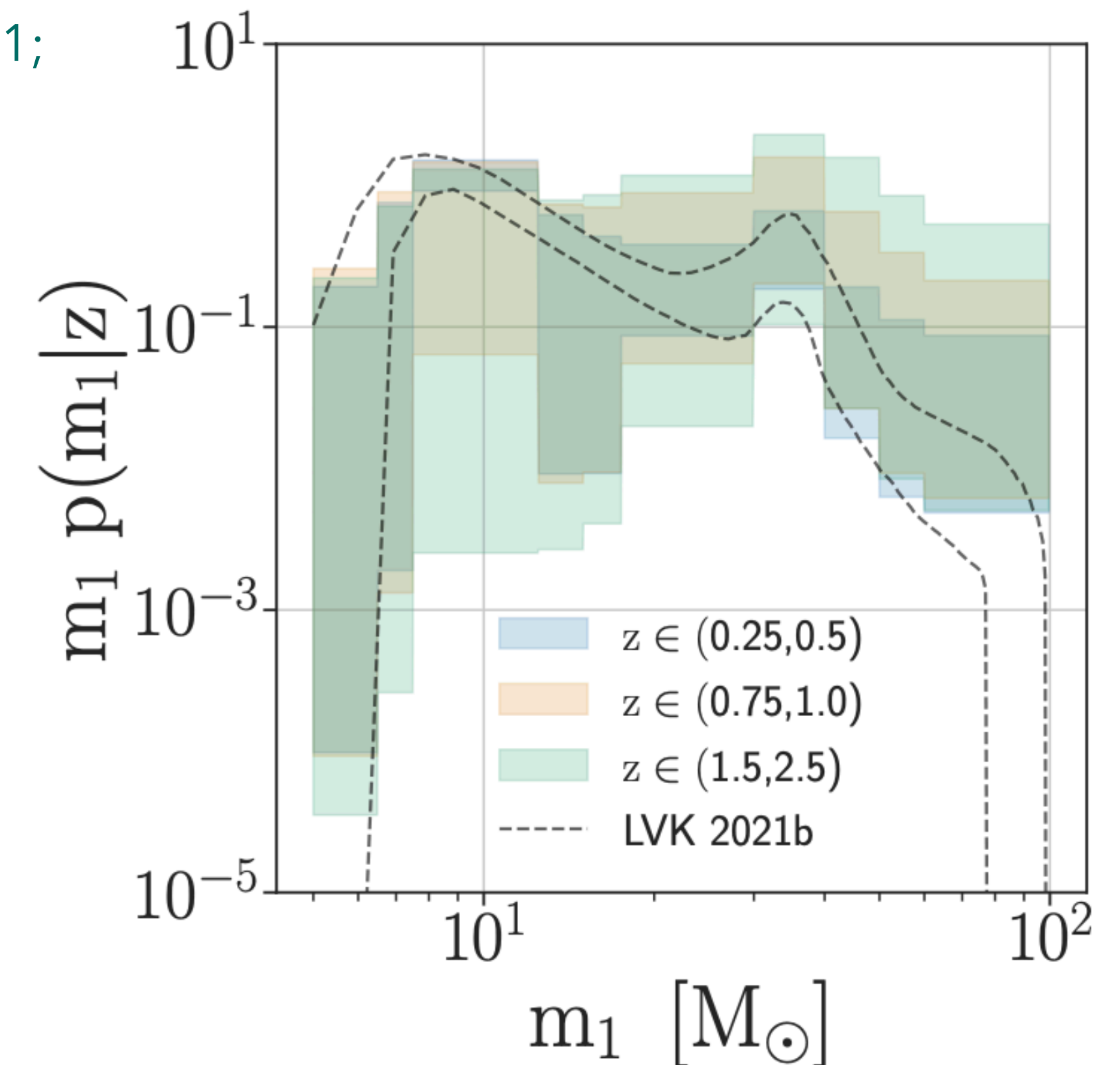
Uncertain theoretical framework

Empirical functional forms, choice of priors

Inference of binary black hole population properties from gravitational wave data

[Mandel+ *MNRAS* 2016; Abbott+ *ApJL* 2021; Tiwari+ *Class. Quantum Grav.* 2021; Callister+ *ApJL* 2022; Callister+Farr 2023; Ray+ 2023]

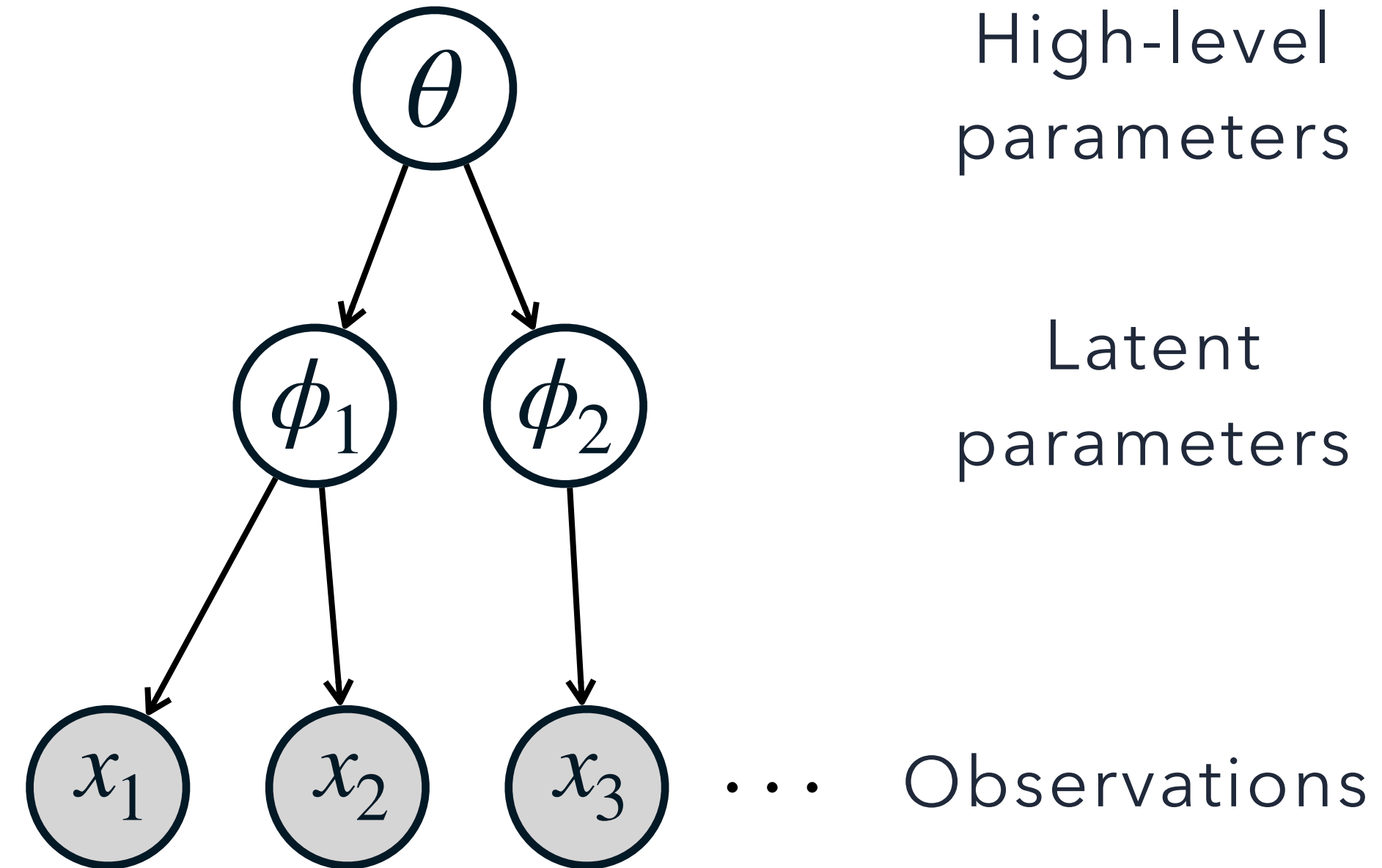
- Mass distribution and evolution informs physics of black hole formation
- Usual: power law + Gaussian
- **Alternative: Non-parametric**
- E.g. Binned **Gaussian process** to capture correlation between mass and redshift distributions [Ray+ 2023]



# BAYESIAN STRATEGIES

Forward or generative modelling

- **Identify** known and expected **systematics**
- Map out **complexity**
- Contains all information needed to derive the **likelihood function**
- Prior predictive checks to **evaluate prior choices**
- Testbench to **evaluate possible simplifications** or assumptions

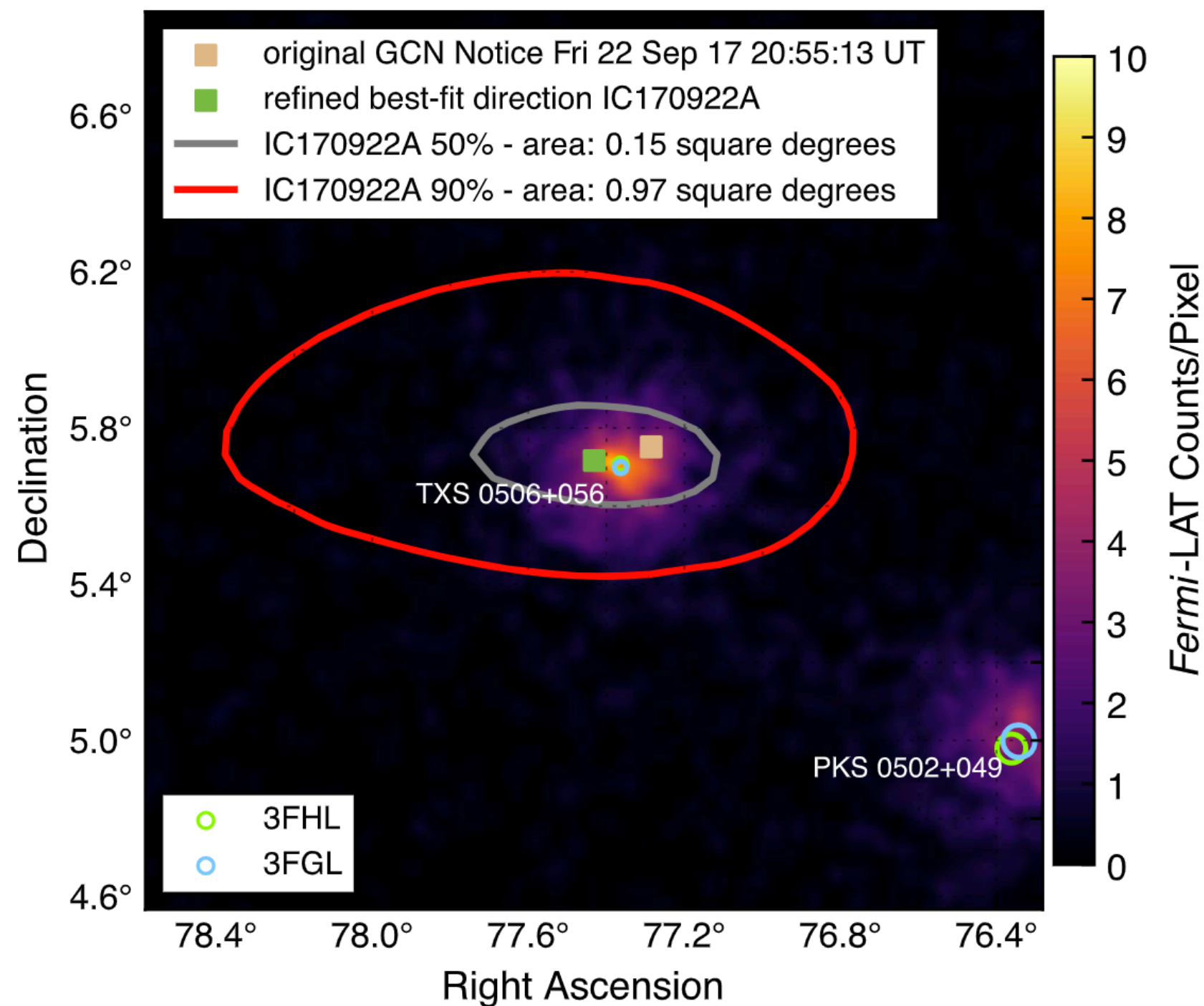


# BAYESIAN STRATEGIES

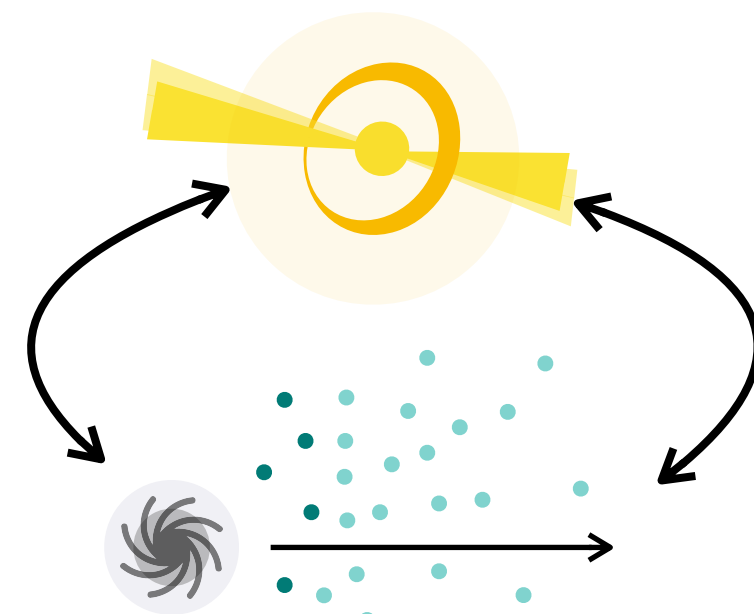
Forward or generative modelling

Example: Evaluating a possible blazar-neutrino connection from a population perspective

Individual association



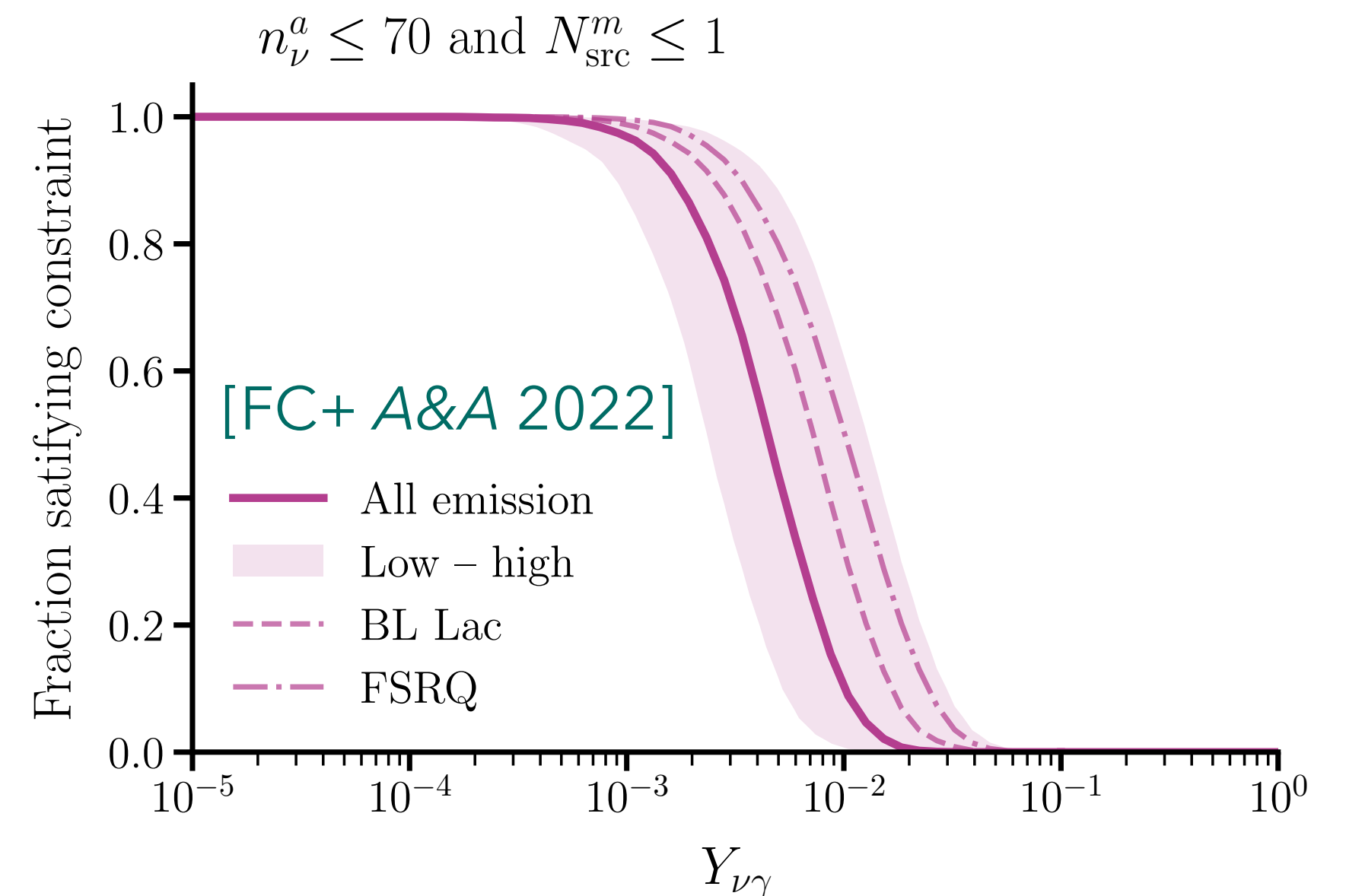
[Aartsen+ Science 2018]



Key assumption for  $\sim 3\sigma$  result: Proportionality between gamma-ray and neutrino fluxes

$$F^\nu = Y_{\nu\gamma} F^\gamma$$

What does this imply for the population?

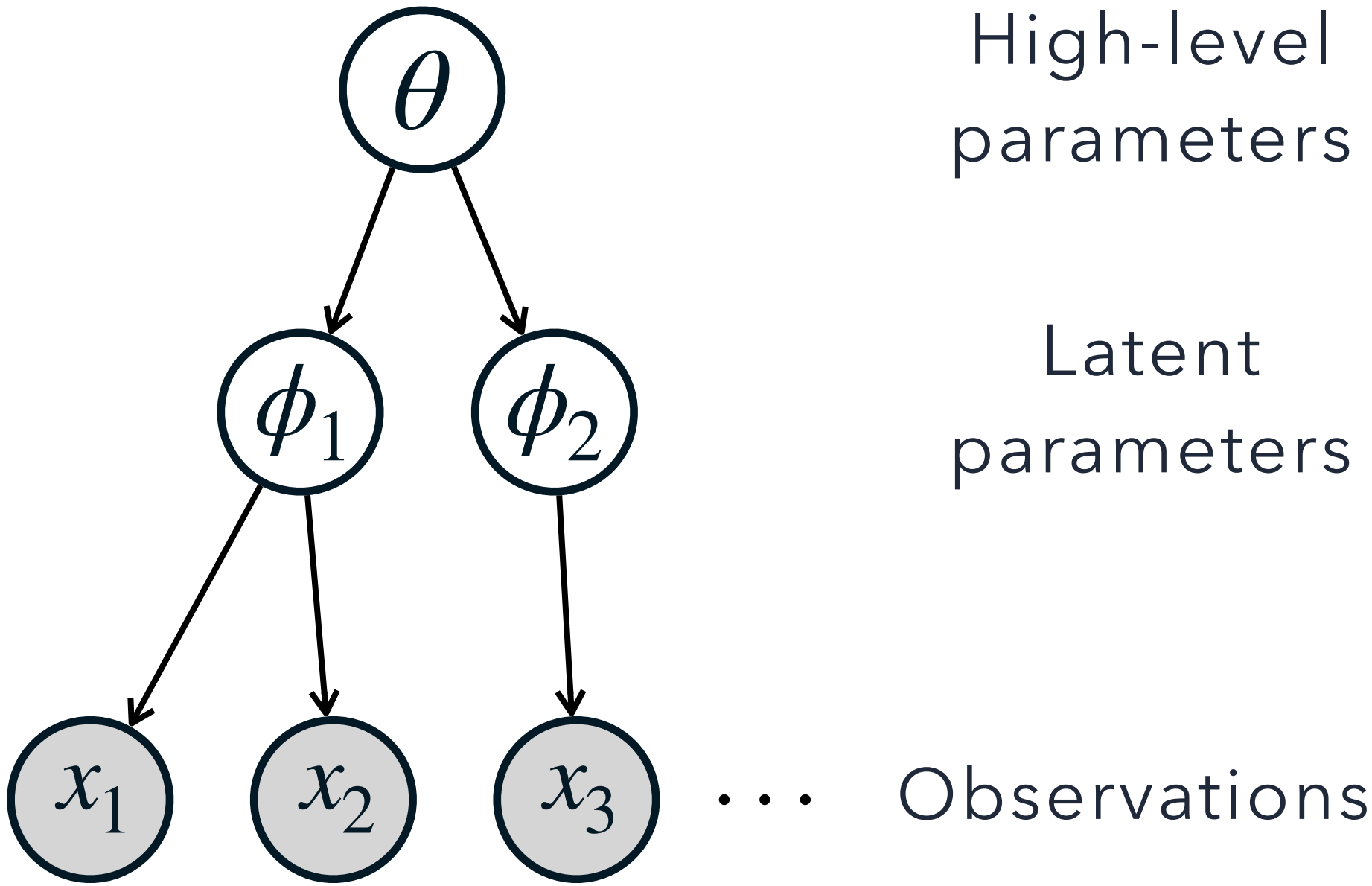
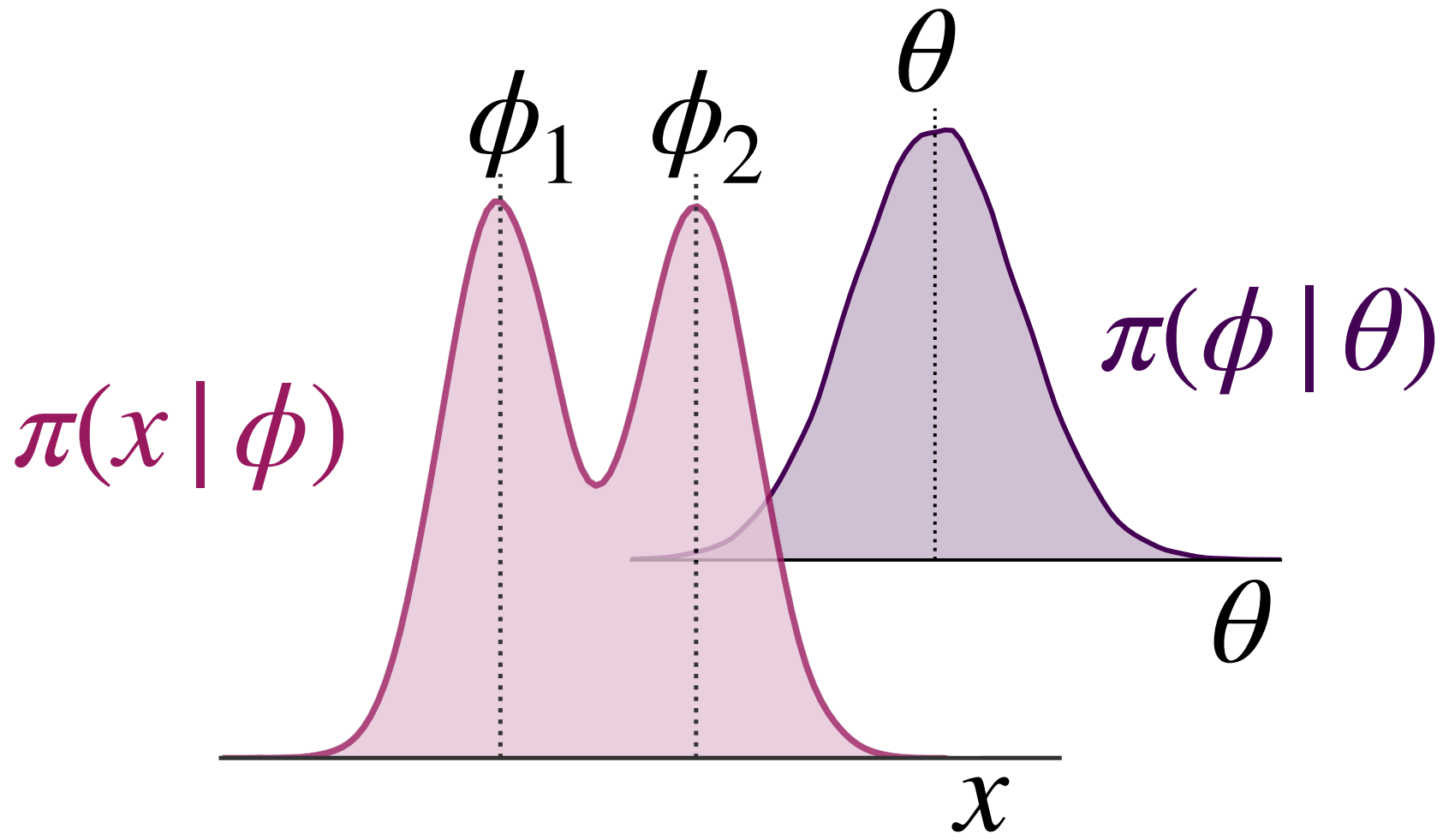


# BAYESIAN STRATEGIES

## Hierarchical modelling

Organise parameters into a hierarchy that describes the forward model

$$\mathcal{L}(x, \theta) = \pi(x | \phi) \pi(\phi | \theta) \dots$$



Goal: "Good enough" model considering available constraints



# BAYESIAN STRATEGIES

## Hierarchical modelling

Widely applicable to a broad range of problems

[See Loredo ACNA 2013]

## Fitting lines and correlations

[Kelly *ApJ* 2007; Andreon+Hurn *Stat. Data Min.* 2012]

## Supernova cosmology and $H_0$ estimation

[Shariff+ *ApJ* 2016; Feeney+ *MNRAS* 2018; Hinton+ *ApJ* 2019]

## Gravitational wave astronomy

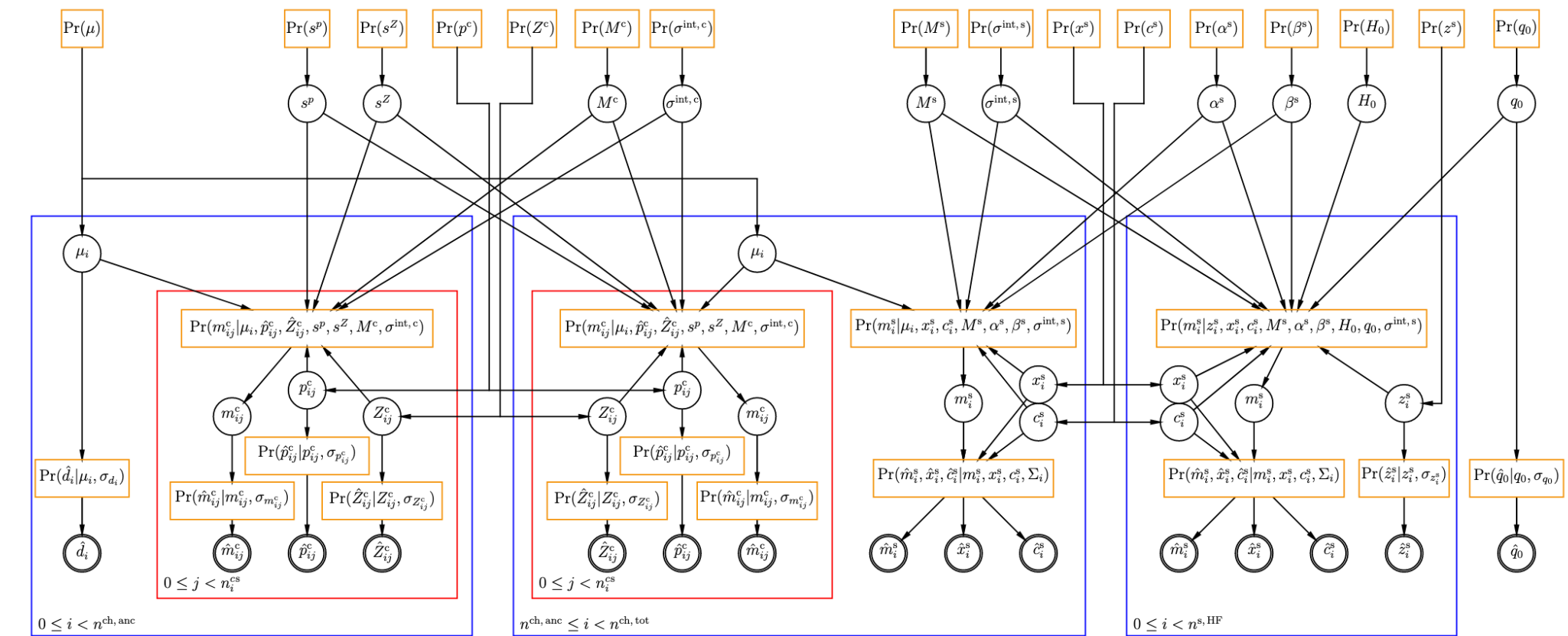
[Thrane+Talbot *PASA* 2019 + refs therein]

## Photometric redshifts

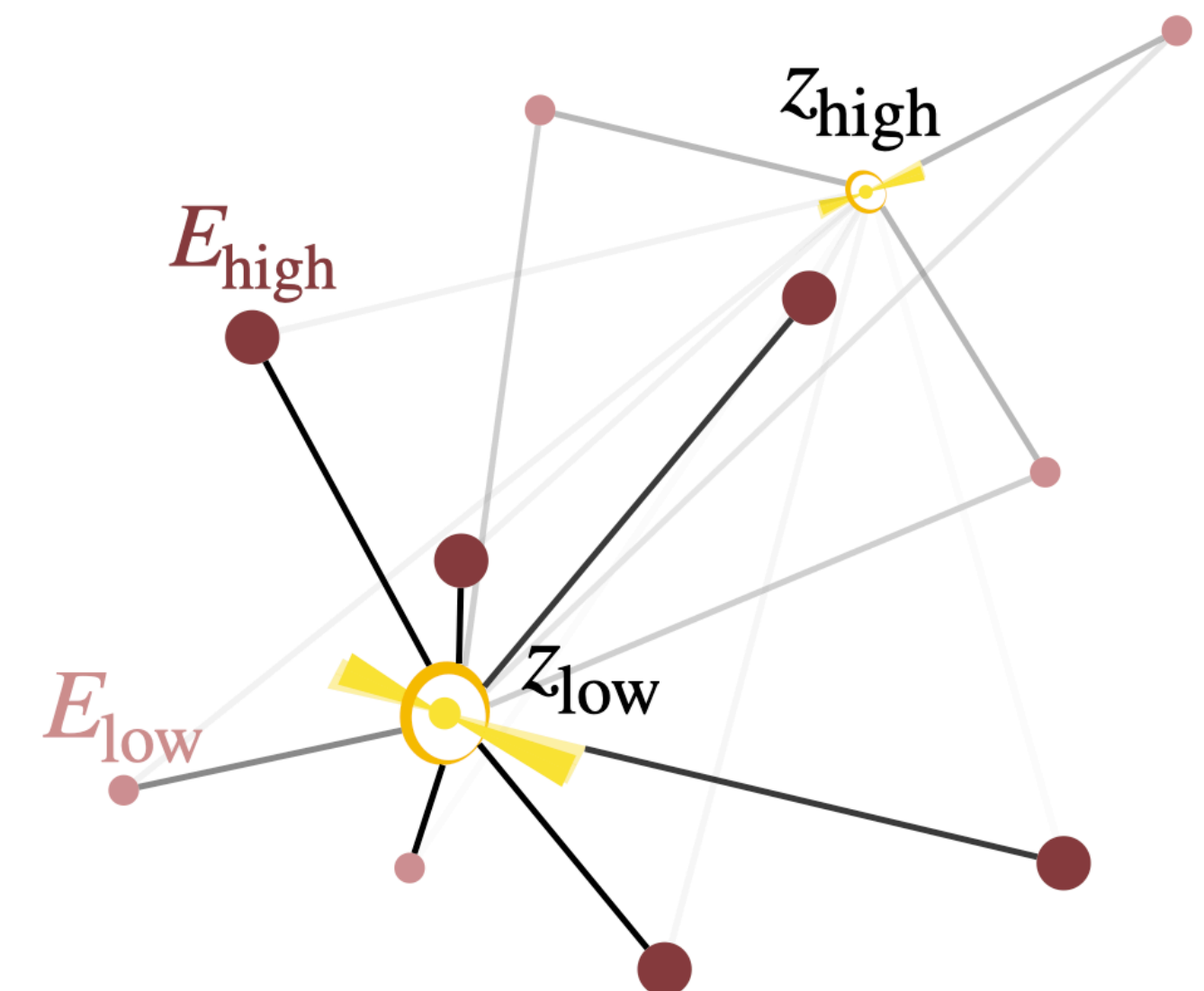
[Malz *PRD* 2021; Newman+Gruen *Annu. Rev. Astron. Astrophys.* 2022; Malz+Hogg *ApJ* 2022; Leistedt+ *ApJS* 2023]

## Astrophysical association and cross-matching

[Soiaporon+ *Ann. Appl. Stat.* 2012; Budavári+Loredo *Annu. Rev. Stat. Appl.* 2015; FC+Mortlock *MNRAS* 2019]



[Feeney+ *MNRAS* 2018]



# BAYESIAN STRATEGIES

Inference in high-dimensional parameter spaces

## Markov chain Monte Carlo

Algorithm to numerically approximate high-dimensional integrals (e.g. expectation values, variances of parameters)

Exact convergence in the limit of infinite samples

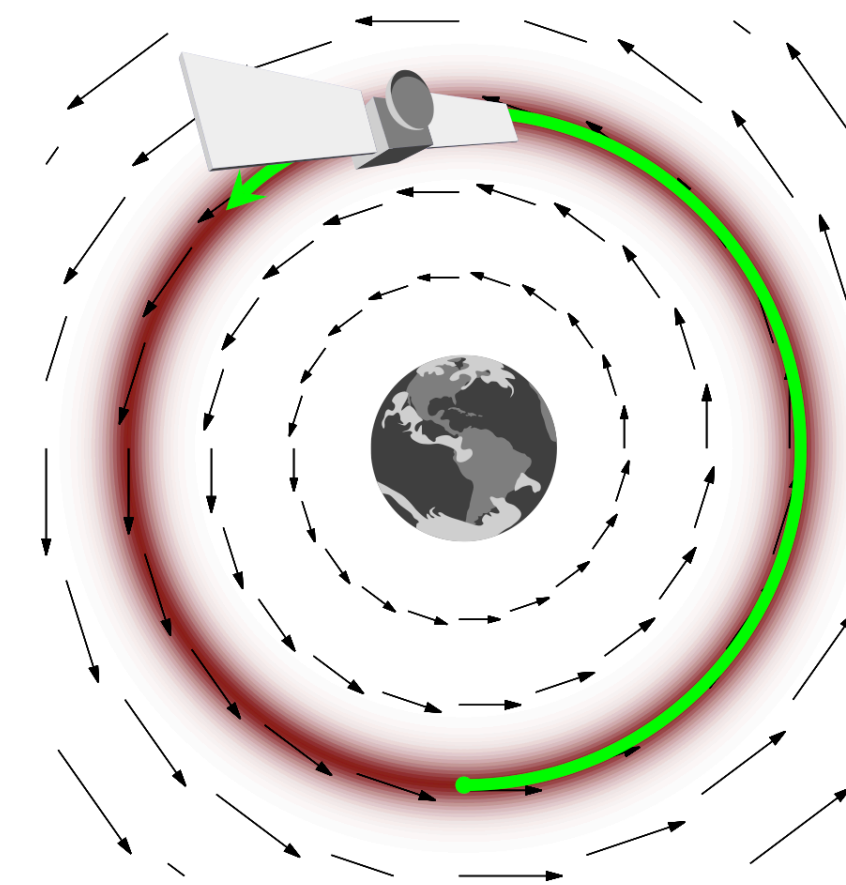
$$\int_{\Theta} d\theta f(\theta)p(\theta) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^N f(\theta_n)$$

## Hamiltonian Monte Carlo

A type of Markov chain Monte Carlo that uses Hamiltonian dynamics to move efficiently through high-dimensional parameter spaces

$$\theta \longrightarrow (\theta, p) \quad \frac{d\theta}{dt} = \frac{\partial H}{\partial p}$$

$$H(\theta, p) \equiv \log P(\theta, p) \quad \frac{dp}{dt} = -\frac{\partial H}{\partial \theta}$$



[Betancourt 2017]

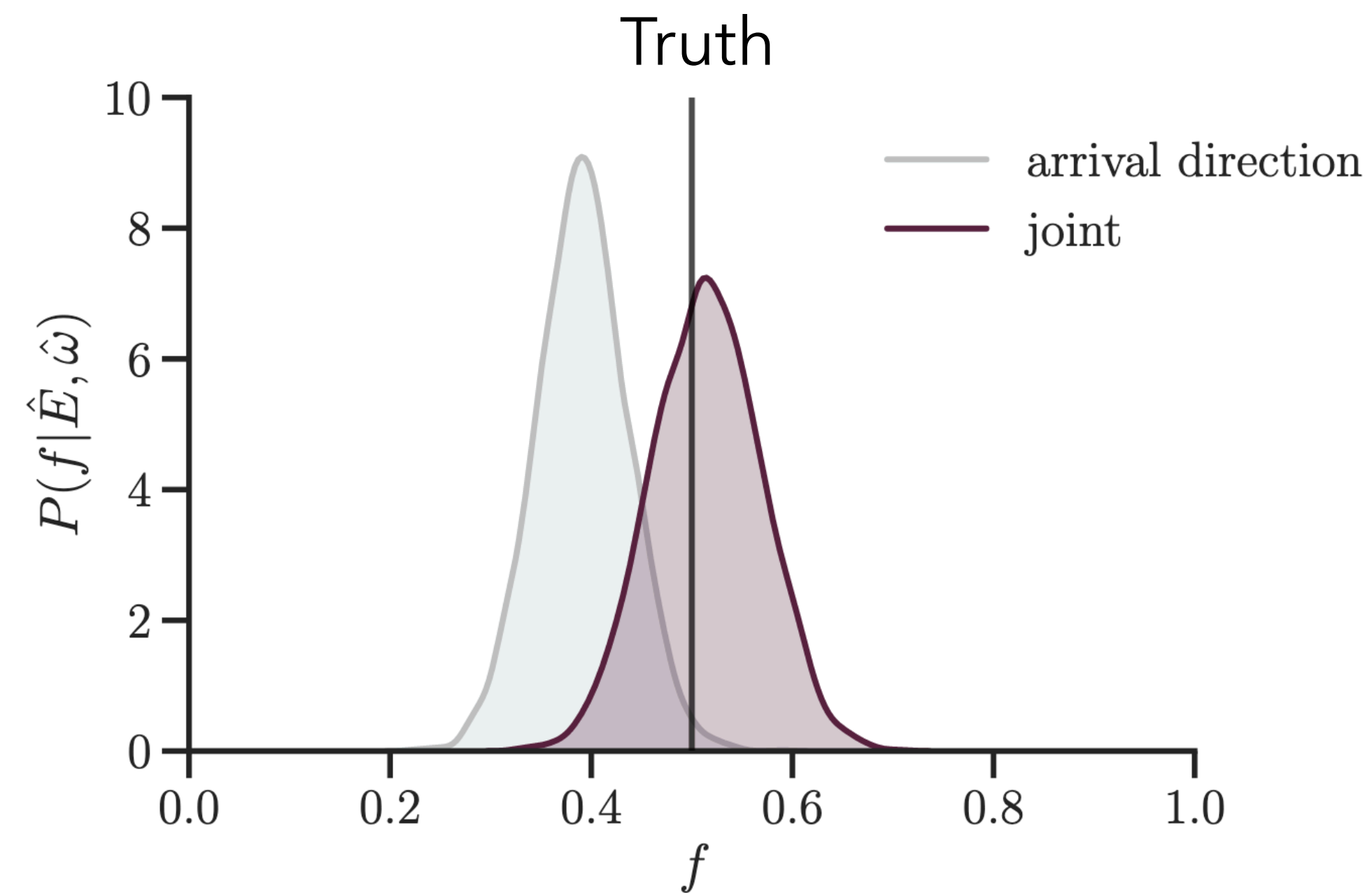
Large numbers of free parameters possible

Uncertainty quantification for free

# BAYESIAN STRATEGIES

## Model checking: Fitting simulated data

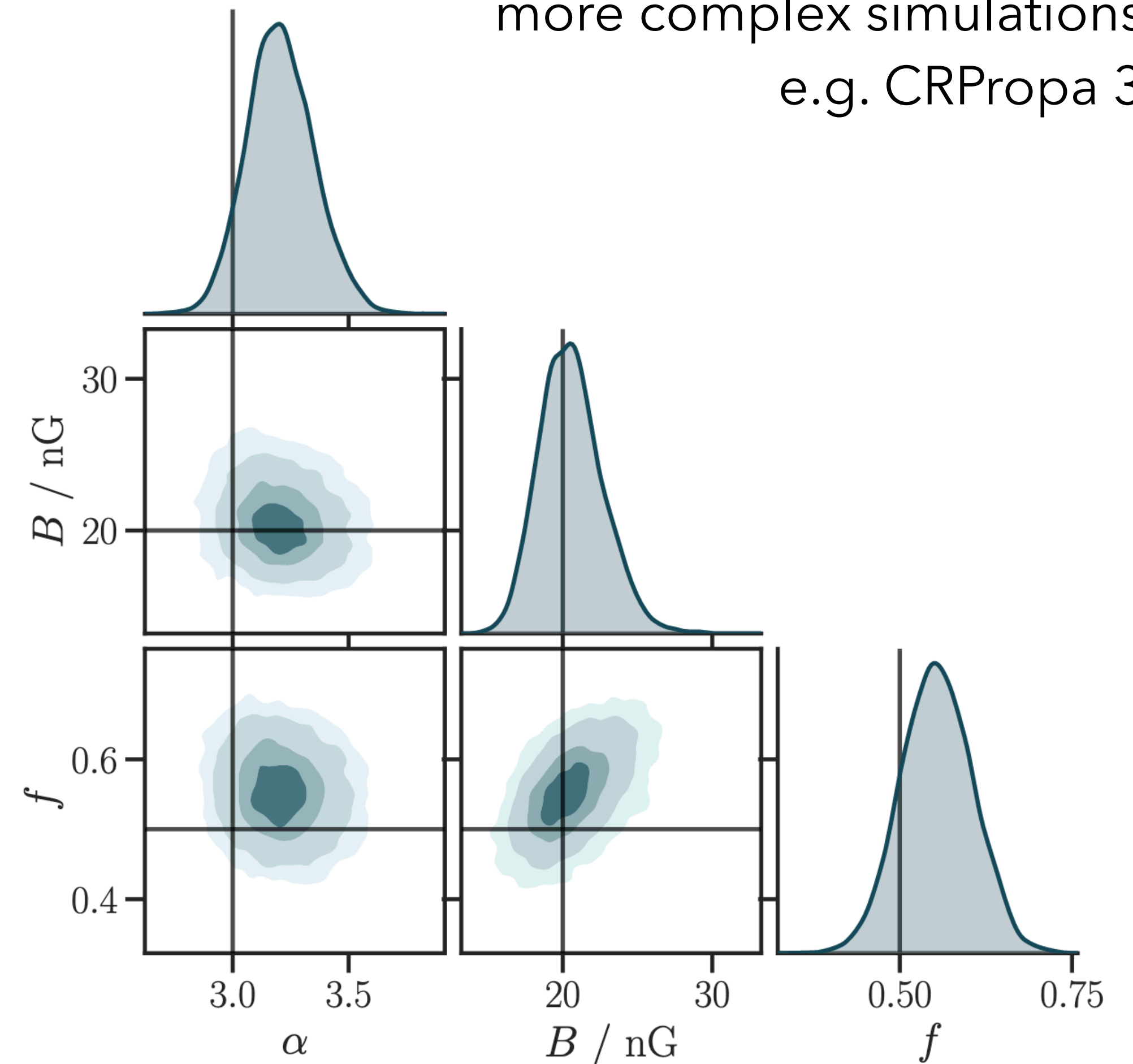
- Consistency check, test for model expansion
- Analysis of fraction ( $f$ ) of UHECRs from set of sources
- Data: detected energies ( $E$ ) and arrival directions ( $\omega$ )



Bias if energies left out of the fit

Examples from [FC+Mortlock MNRAS 2019]

Key parameters recovered when fitting more complex simulations e.g. CRPropa 3





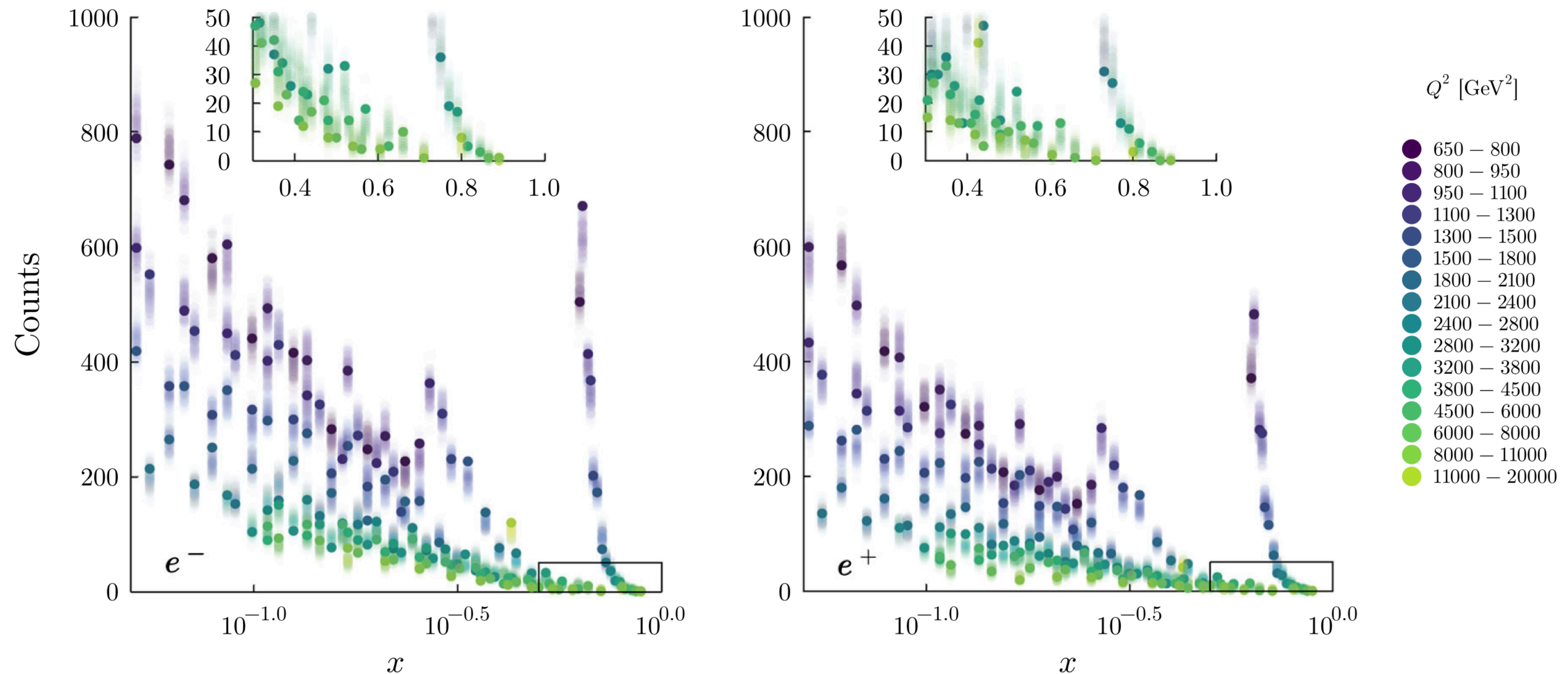
# BAYESIAN STRATEGIES

## Model checking: Posterior predictive checks

- Goodness of fit, test for model expansion
- Parton density extraction from high- $x$  ZEUS data
- Data: Counts in  $x$ - $Q^2$  bins

Example from [Aggarwal+ PRL 2023]

$$\pi(n_{\text{rep}} | n) = \int d\Theta \pi(n_{\text{rep}} | \Theta) \pi(\Theta | n)$$





# BAYESIAN STRATEGIES

## Model averaging

[Hoeting *Statist. Sci.* 1999; Parkinson+Liddle *Stat. Data Min.* 2012]

- Discrete models
- All possible models known (M-closed)
- Model uncertainty quantifiable
- Non-trivial to evaluate evidence via MCMC
- Nested sampling is widely used

[Buchner *Stat. Surv.* 2023]

$$\pi(\theta | x) = \frac{\sum_k \pi(\theta | x, \mathcal{M}_k) \pi(\mathcal{M}_k | x)}{\sum_k \pi(\mathcal{M}_k | x)}$$

Evidence  $\rightarrow$

Model prior  $\rightarrow$

# PRACTICAL ASPECTS

More parameters  $\Rightarrow$  increasingly computationally expensive

Implementation of strategies can be challenging in different scenarios

E.g. Visualisation of predictive checks, likelihood difficult/impossible to write down  
[Talks by Lee & Shen]

Data may not be available in a way that allows for complete modelling

[Loredo ACNA 2013]

Overfitting possible if using the same data for model building and inference

[Betancourt 2020]

Limited protection against “Unknown unknowns”

[Hatfield 2022]

# SUMMARY & OUTLOOK



Bayesian methods are conceptually more straightforward, but can still be challenging to implement (not just priors)



It is worth it to make the most of our data!

Some thoughts:

- Availability of reusable data and reproducible methods is key
- Complementarity with frequentist approaches - why not do both?