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Clocks, Algebras and Cosmology

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The problem of TIME in Quantum Gravity.

Invariance under time reparametrizations $\Rightarrow$ The algebra of physical observables is the invariant subalgebra

$$A^G = \{ O \in A \, ; \, [H, O] = 0 \}$$

G - group of time translations

H - hamiltonian

A - algebra of local operators

Similar to SUPERSELECTION charges:

$$A^{ph} = \{ O \, ; \, [Q \, O] = 0 \}$$

Q the S.S charge

In case we declare A^G as the algebra of physical observables we cannot explain the observed changes in time.

Similarly in the case of superselection charges we should conclude that quantum superpositions of states with different charge

$$a\,|q_1\rangle + b\,|q_2\rangle$$

are unobservable.

How to solve this problem?

The solution originally suggested by Aharonov and Susskind in 1967 is:

To add a reference frame quantum system with Hamiltonian H^r and algebra of observables A^r and to define the algebra of physical observables i.e. invariant under reparametrizations as

$$(A_0 \otimes A^r)^{H^r + H}$$

i.e. $\{ \theta \in A_0 \otimes A^r \, ; \, [H + H^r, \theta] = 0 \}$

We will generically refer to the reference frame quantum system with Hamiltonian H^r as a CLOCK and to elements in $(A_0 \otimes A^r)^{H^r + H}$ as clock dressed observables.

We will consider elementary clocks with just one degree of freedom.

The Hilbert space of the system AND the added CLOCK

$$\mathcal{H} \otimes L^2(\mathbb{R}) = L^2(\mathbb{R}, \mathcal{H})$$

with "coordinate" time the real line $\mathbb{R}$.

Coordinate time translations act on $\mathcal{H} \otimes L^2(\mathbb{R})$ as

$H \otimes \mathbb{1}d$ action of system hamiltonian on $\mathcal{H}$

$\mathbb{1}d \otimes H^{\sim}$ action of clock hamiltonian on $L^2(\mathbb{R})$

i.e $H^{\sim} = i\hbar \frac{d}{dt}$

The algebra of the composed system is

$$\mathcal{A} \otimes \mathcal{B}(L^2(\mathbb{R}))$$

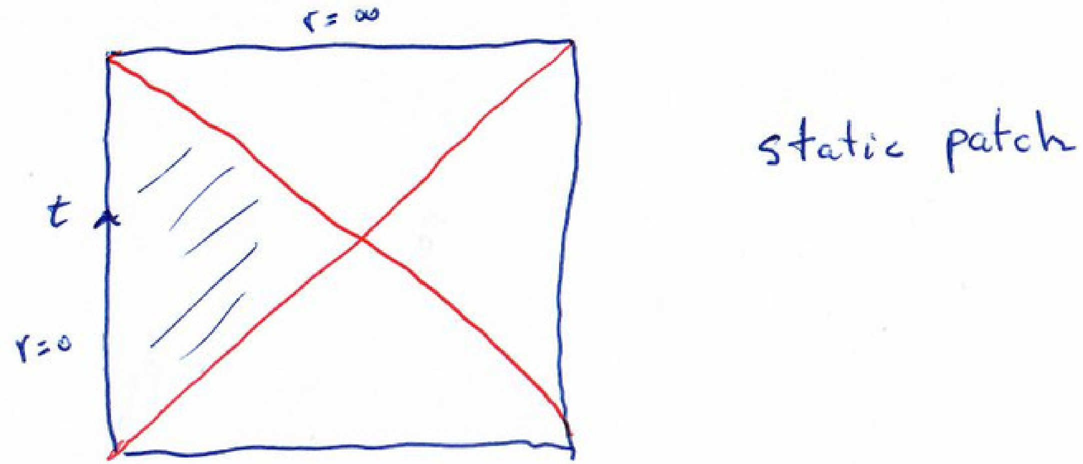
→ "clock observables" i.e bounded operators acting on $L^2(\mathbb{R})$ (clock Hilbert space)

The algebra of gauge invariant observables i.e invariant under time reparametrizations is

$$\left[\mathcal{A} \otimes \mathcal{B}(L^2(\mathbb{R}))\right]^{H+H^c}$$

The problem of TIME in de Sitter.

We will consider QFT on a fixed dS background in the weak gravity limit $G_N \sim 0$

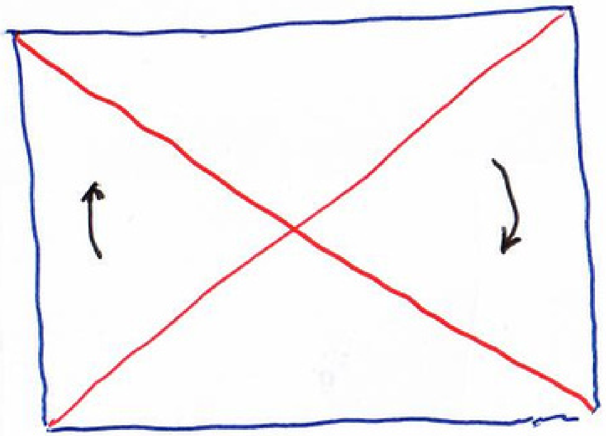


$\mathcal{H}_{dS}$ = QFT Hilbert space (states in $\mathcal{H}_{dS}$ describe full dS)

A_{dS} = algebra of local observables in the static patch

note that the full dS group is represented in $\mathcal{H}_{dS}$

The generator of "time translations" should be associated to a global Killing of dS



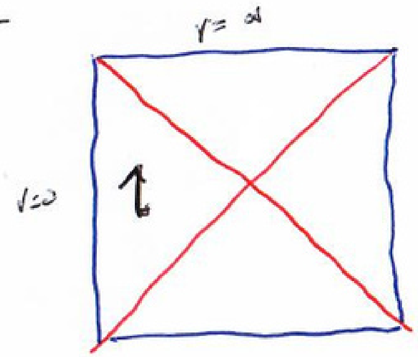
The generator of the corresponding time automorphism on $\mathcal{A}_{dS}$ is the "state dependent" modular hamiltonian h

$$a(t) = e^{iht}\, a\, e^{-iht} \qquad a \in \mathcal{A}_{dS} \quad a(t) \in \mathcal{A}_{dS} \quad \forall t$$

3 - important technical Remarks

1) h is NOT in $\mathcal{A}_{dS}$ $\Leftrightarrow$ time translations define OUTER automorphism

2) H associated to "time translations" in the static patch is ill defined



$$\| H|\psi\rangle \|^2 = \infty \quad |\psi\rangle \in \mathcal{H}_{dS}$$

(due to divergent fluctuations on the cosmological horizon)

3) We can identify a dS invariant state in $\mathcal{H}_{dS}$ as the Bunch-Davis vacuum $|\psi_{dS}\rangle \in \mathcal{H}_{dS}$.

In these conditions the algebra of gauge invariant observables is

$$\mathcal{A}_{dS}^{h} = \{ O \in \mathcal{A}_{dS} : [h\, O] = 0 \}$$

This algebra is TRIVIAL i.e $\mathcal{A}_{dS}^{h} = \{ \mathbb{1}_d \}$

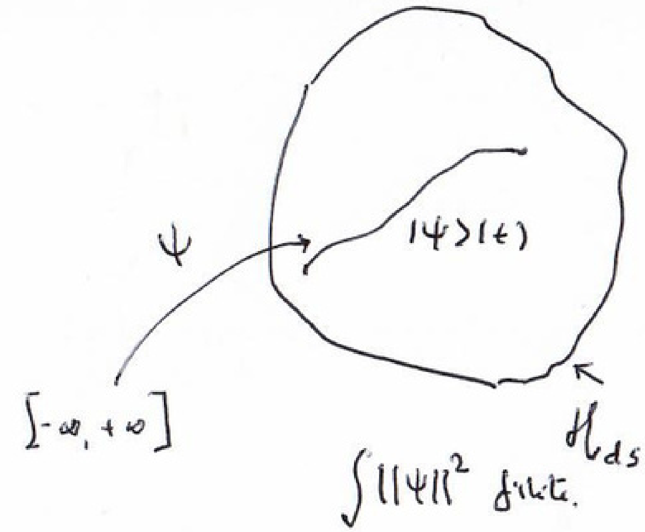
No QFT gauge invariant <u>observables</u> in pure dS

This is the problem of TIME for dS.

Q:- How to get a non trivial set of gauge invariant physical observables?

(Chandrasekaran, Longo, Penington, Witten answer)

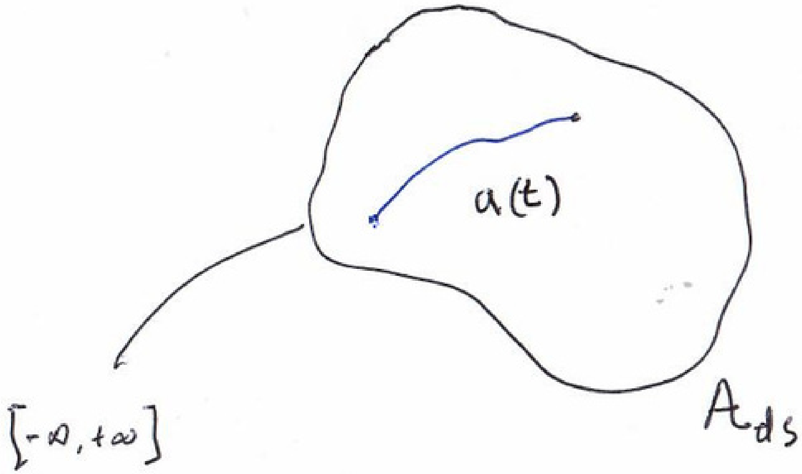
- Let us add a reference frame clock



Hilbert space:

$$L^2(\mathbb{R}, \mathcal{H}_{ds}) = \underbrace{\mathcal{H}_{ds}}_{\text{QFT Hilbert space}} \otimes \underbrace{L^2(\mathbb{R})}_{\text{clock Hilbert space}}$$

Algebra of physical observables:



$$\left[A_{ds} \otimes B(L^2(\mathbb{R})) \right]^{h + H^r}$$

for H^r acting on $L^2(\mathbb{R})$ as $i\hbar \frac{d}{dt}$

Note that the so defined H^r has as spectrum $\mathbb{R}$

The so defined algebra is known as the crossed product of A_{ds} by the action of the modular group and we will denote it $\boxed{A^{cr}_{ds}}$

Some technical remarks:

1) For A_{ds} this algebra is a type II_∞ factor

2) $\exists$ positive faithful linear form $\Phi : A^{cr}_{ds} \to \mathbb{C}$ satisfying the trace property $\Phi(\hat{a}\,\hat{b}) = \Phi(\hat{b}\,\hat{a})$ $\quad \hat{a}\;\hat{b} \in A^{cr}_{ds}$

3) $\hat{a} = e^{i\hat{t}h}\, a\, e^{-i\hat{t}h}$ for $\hat{t}$ acting on $L^2(\mathbb{R})$ as $\hat{t}\, f(t) = t\, f(t)$

$\Rightarrow [\hat{t}, H^{r}] = i\hbar$

4) $\exists\; p \in A^{cr}_{ds}$ $\;(p^2 = p)$ such that $p\, A^{cr}_{ds}\, p$ is type II_1

5) Φ on $P A^{\omega}_{ds} P \equiv A^{\omega}_{ds,P}$ defines a finite trace $tr(\mathbb{1}) = 1$

and it is a vector state in $P(\mathcal{H}_{ds} \otimes L^2(\mathbb{R}))$ i.e

$$\Phi(\hat{a}) = \langle \hat{\Psi} | \hat{a} | \hat{\Psi} \rangle \qquad |\hat{\Psi}\rangle \in P(\mathcal{H}_{ds} \otimes L^2(\mathbb{R}))$$

$$|\hat{\Psi}\rangle = \int_0^{\infty} |\Psi_{ds}\rangle \, e^{-\beta \varepsilon} |\varepsilon\rangle \, d\varepsilon \qquad |\varepsilon\rangle \text{ spectrum of } P\hat{H}P$$

The state $|\hat{\Psi}\rangle$ is the max entropy state (flat entanglement)

Brief Parenthesis: The problem of Time in Q. M

$\Delta p \, \Delta q \geq \hbar \quad \Leftarrow \quad [p, q] = i\hbar$

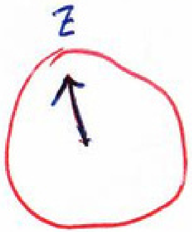
$\Delta E \, \Delta t \geq \hbar \quad \Leftarrow\!\!- - \quad [\hat{t}, H] = i\hbar$

The problem is that if $[\hat{t}, H] = i\hbar \Rightarrow$ Spectrum $H = \mathbb{R}$

i.e no time operator if H is positive.

Thus: How to define a physical clock with H^{clock} positive?

The simplest way to define a clock in QM is to use a quantum system with a "coordinate" observable Z that tells us the time



$$[Z, P_Z] = i\hbar \qquad i\hbar \dot{Z} = [H^{clock}, Z]$$

$$H^{clock}$$

and to use a "clock quantum state" Φ_{clock} such that

$$\frac{\Delta \dot{Z}}{|\langle \dot{Z} \rangle|} \ll 1 \qquad \langle \dot{Z} \rangle = \langle \Phi_{clock} | \dot{Z} | \Phi_{clock} \rangle$$

For this clock time uncertainty is

$$\Delta t = \frac{\Delta Z}{\langle \dot{Z} \rangle}$$

How to define a time operator $\hat{t}$ such that $[\hat{t}, H^{clock}] = i\hbar$?

(Aharonov - Bohm 1961)

Imagine we use as clock a free particle : $H^{clock} = \frac{P_z^2}{2M}$

and define

$$\hat{t} = \frac{1}{2} M \left(Z \frac{1}{P_z} + \frac{1}{P_z} Z \right)$$ formally hermitian

$$[H^{clock}, \hat{t}] = i\hbar$$

But $\hat{t}$ is ill defined for $P_z = 0$

You need to reduce the Hilbert space to states with $\langle \dot{z} \rangle \neq 0$. On this Hilbert space you can use

$$\hat{t} \approx \frac{Z}{\langle \dot{z} \rangle}$$

The problem of Time in Inflationary Cosmology.

The key problem in Inflationary Cosmology is to identify gauge invariant quantum fluctuations during the primordial exponentially expanding dS phase.

In pure dS the set of gauge invariant quantum fluctuations is just the identity i.e non existent

The lesson of the previous discussion is to ADD a clock and to work with the extended algebra

$$\left(A_{dS} \times A_{clock}\right)^{h_{dS} + H^{clock}}$$

for some clock hamiltonian.

Mukhanov-Sasaki gauge invariant observables $\Longleftrightarrow$ clock dressed operators

recall that a typical clock dressed operator has the form

$$\hat{a} = e^{i\hat{t}h}\, a\, e^{-i\hat{t}h}$$

The M-S variable defining scalar curvature fluctuations

$$\zeta = \psi + \frac{H_{hub}\, \rho}{\dot{\rho}}$$

for ψ defined by

$$ds^2 = a^2\left((1+2\phi)d\eta^2 - 2B_i dx^i d\eta - ((1-2\Psi)\delta_{ij} + 2E_{ij})dx^i dx^j\right)$$

η - conformal time.

ζ is the clock dressed operator

$$\zeta = e^{i h_{ds} \hat{t}} \Psi e^{-i h_{ds} \hat{t}}$$

for a "clock operator" $\hat{t} = \dfrac{\varphi}{\langle \dot{\varphi} \rangle}$ and $[h_{ds}\ \Psi] = H_{ds}$.

We recognize in this clock operator $\hat{t}$ the "Inflaton clock"

The power spectrum

$$\mathcal{P} = \langle \zeta^2 \rangle = \langle \psi | \hat{t}^2 | \psi \rangle$$

for $|\psi\rangle \in H_{ds} \otimes H^{clock}$

Can we define a model independent Natural clock in dS?

Bogoliubov Clock

- Use conformal time on planar patch as "coordinate time"

$$H^{clock} \rightarrow i\hbar \frac{d}{d\eta}$$

- Use dS – Bogoliubov transform to define the representation of H^{clock} on clock Hilbert space: $L^2(R) \otimes L^2(R)$ $\quad a_k a_k^+ \quad a_{-k} a_{-k}^+$

  k comoving momentum.

- Define as "clock states" squeezed states in $L^2(R) \otimes L^2(R)$ for each comoving momentum k

$$|k\eta\rangle = \sum_n e^{in\phi(k\eta)} c(k\eta)^n \, |n_k, n_{-k}\rangle$$

$$[\phi\, N] = -i\hbar$$

ϕ is conjugated to N (# operator)

ϕ play the role of $\hat{t}$ for this squeezed dS clock!

The power spectrum $\mathcal{P} = \langle \Psi_{clock} | \zeta^2 | \Psi_{clock} \rangle$

for ζ dressed by this $\hat{t}$ is given by

$$\mathcal{P}_{k\eta} = \left[\langle k\eta | (\partial\phi)^2 | k\eta \rangle \right]^{1/2}$$

The evaluation of this quantity leads to concrete predictions of inflationary parameters $(1-n_s) = 0.0318$.

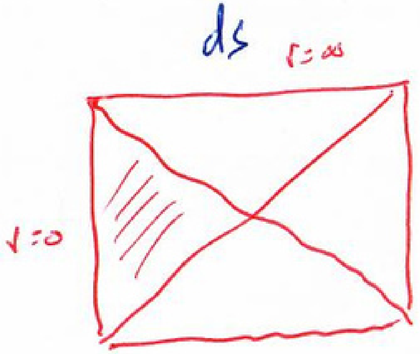
In summary:

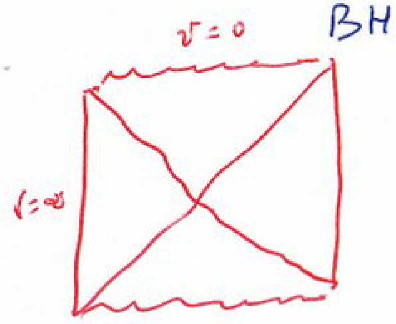
Cosmological scalar fluctuations are elements of

$$\left(A_{dS} \otimes A_{\substack{Bogoliubov \\ clock}} \right)^{h_{dS} + H_{clock}}$$

i.e operators dressed by the dS-Bogoliubov clock

Final Comment — dS / BH





(Chandrasekaran, Penington Witten)

A_{dS} $\xrightarrow{+\text{clock}}$ $\tilde{A}_{dS}$ $\xrightarrow{P_{dS}}$ $\tilde{A}_{dS}$, $\hat{t}_p^{(dS)}$

$P^2 = P$ projection in $\tilde{A}_{dS}$

A_{BH} $\quad III_1$

$\downarrow$

$\tilde{A}_{BH}$ $\quad \hat{t}^{BH}$ II_∞ → conjugated to ADM mass

$\downarrow P_{BH}$

$\tilde{A}_{BH}$ $\quad \hat{t}_p^{(BH)}$ II_1

What is the meaning of this II_1 factor for BH's

Suggestion:

Hawking's time defined by the BH Bogoliubov transformation. ?

Thank you