Gravitational singularities and Holographic Complexity[1]

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(Indian Inst. of Technology Hyderabad)

BIRS workshop on "Quantum Information Theory in Quantum Field Theory and Cosmology"

June 9, 2023

[1] w/ J. Ren (SY-S U.) & G. Katoch (IITH) (2303.02752 [hep-th])
w/ E. Rabinovici (Racah) & S. Bolognesi (Pisa), 1802.02045 [hep-th]

Introduction

- Holography: Bulk geometry $\equiv$ Boundary State Entanglement structure (Ryu-Takayanagi' 06, Maldacena-Susskind '13 "*ER=EPR*", Raamsdonk '10)

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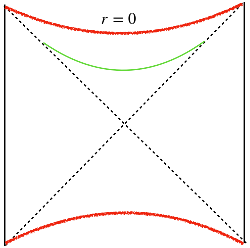

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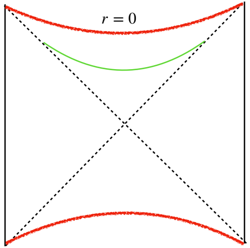

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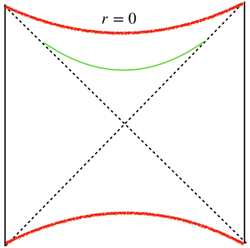

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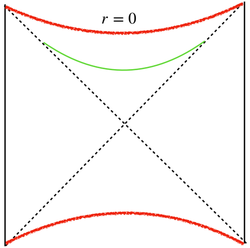

- Recover gravity from boundary state entanglement (Lashkari et. al.' 13, Faulkner et. al. '13 , '17,. . .)
- Comp. Complexity of CFT state $\leftrightarrow$ Spatial volume in bulk
- EAdS-BH at late times:

$$C \sim \text{" ERB volume"}; \ \frac{dC}{dt} \sim T\,S.$$

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- Conclusions and Outlook

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- Quant. mech., $|\psi\rangle = \sum_1^{2^N} \alpha_i |i\rangle$

$$C_{max} \sim 2^N \ !$$

Computational Complexity in Quantum systems

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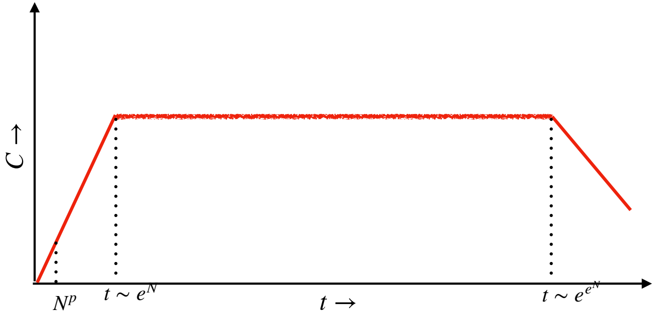
- Time evolution of Complexity



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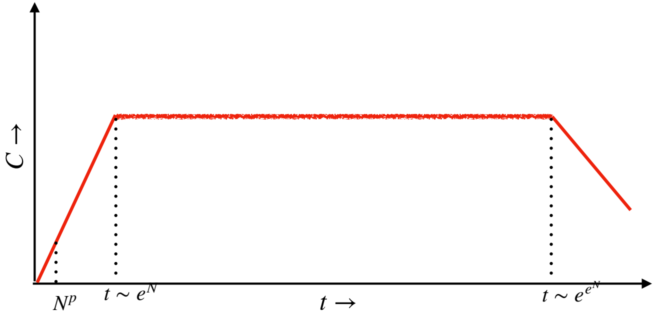

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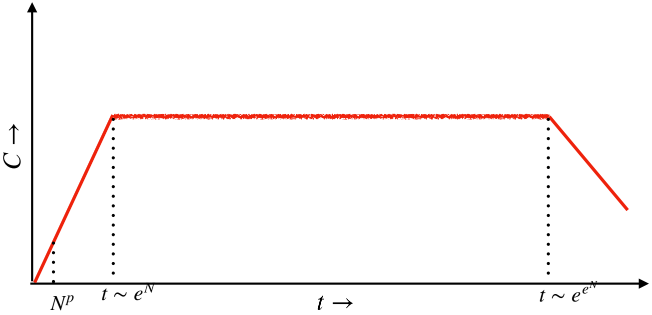

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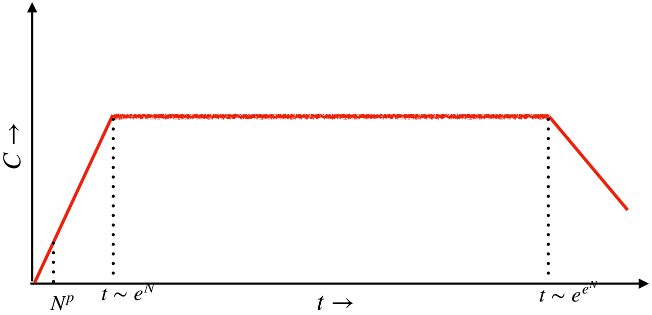

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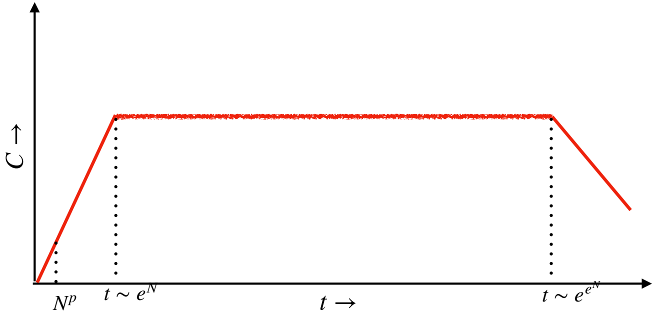

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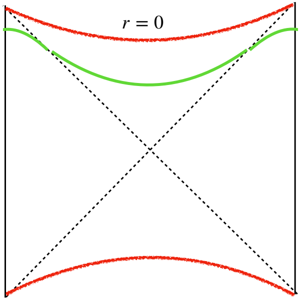
- Initial Growth Slope: $\frac{dC}{dt} \sim TS$

Holography: Complexity and Volumes: CV



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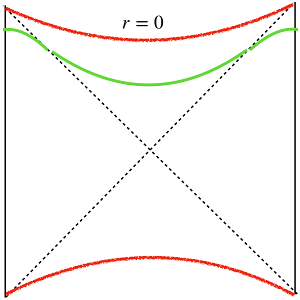

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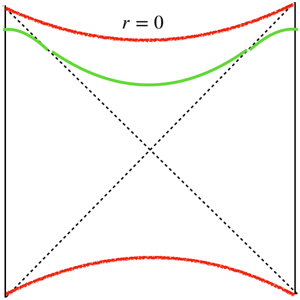
- 

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$$C = \frac{\mathrm{Vol}(\Sigma_{max})}{G_N\, R_c}$$

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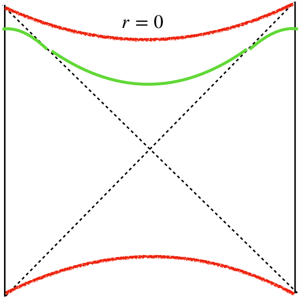
- 

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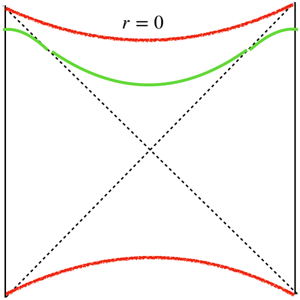

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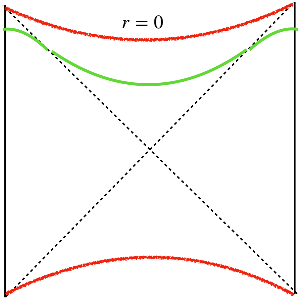
- 

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- However, lesson from BH: lack of entanglement $\Rightarrow$ singular spacetime (firewalls)

Holography: Complexity and WdW Action: CA

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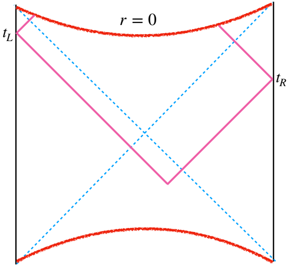

WdW patch for Eternal SAdS

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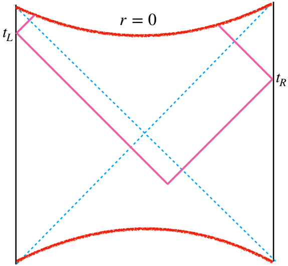
- 

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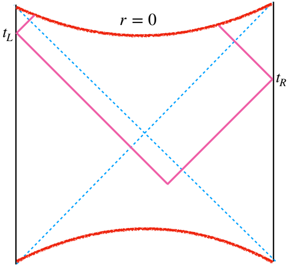
- 

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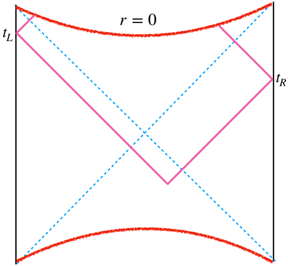

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- Still CV and CA matches perfectly!

Cosmological Singularities in the bulk[2]

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SR, Rabinovici and Bolognesi (1802.02045[hep-th])

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- Generic idea: Time-dependent deformations of CFTs
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- Marginal: Coupling or boundary metric gains time-dependence (Kasner, Topological Crunch)

$$ds^2 = \frac{l^2}{z^2}\left(dz^2 - dt^2 + h_{ij}(t,x)\, dx^i\, dx^j\right), i,j = 1,..,d$$

$$h^K_{ij}(t,x) = \mathrm{diag}\left(\left(\frac{t}{l}\right)^{2p_1}, \dots, \left(\frac{t}{l}\right)^{2p_d}\right),$$
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- Relevant: Time dependent Mass scale, $M(t) = M \sec t$ (dS/Crunch)

$$ds^2_{bulk} = d\rho^2 + f^2(\rho, M)\, ds^2_{dS_d}$$

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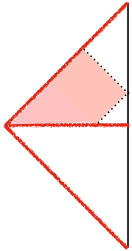
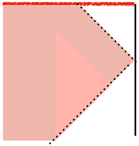
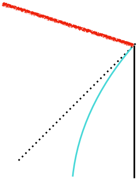

- *AdS Kasner* *Topological Crunch* *dS Crunch*

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$$C(t) \sim N^2\, \Lambda^d\, V_x \frac{|t|}{l} + \Lambda^{d-2} N^2 \frac{V_x}{l\, t},\; N^2 \sim \frac{l^d}{G_N}.$$

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$$C_\infty \sim N^2 \, V_{S^d} \, \Lambda^d \cos\left(\frac{t}{l}\right) + N^2 \frac{V_{S^d}}{l^2} \Lambda^{d-2} \frac{\sin^2 t/l}{\cos t/l}.$$

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$$C \sim N^2 V\left(\Lambda^{d-1} - M(t)^{d-1}\right) + N^2 l_-\,\Omega_{d-1} r(t)^{d-1}$$

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- Every case: Complexity decreases as we approach the singularity!

Complexity Estimates: CA

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- Kasner

$$C_{\mathcal{V}} \sim N^2\,\Lambda^{d-1}\,V_x\frac{|t|}{l} + N^2\Lambda^{d-3}\frac{V_x}{tl} + O\left(\Lambda^{d-5}\right)$$

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- dS/Crunch

$$\frac{dC_{\mathcal{V}}}{dt} \sim \left(\frac{\pi}{2} - t_*\right)^{-d},$$

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- Topological Crunch

$$C_{\mathcal{V}} \sim N^2\Lambda^{d-1} l^d \cos\left(\frac{t}{l}\right) + N^2\Lambda^{d-3} l^{d-3} \sin^2\left(\frac{t}{l}\right) \sec\left(\frac{t}{l}\right)$$

$$C_{\mathcal{A}} \sim N^2\Lambda^{d-1} l^d \cos\left(\frac{t}{l}\right) + N^2\Lambda^{d-3} l^{d-3}\left[\sin^2\left(\frac{t}{l}\right)\sec\left(\frac{t}{l}\right) + ..\cos\left(\frac{t}{l}\right)\right]$$

- dS/Crunch

$$\frac{dC_{\mathcal{V}}}{dt} \sim \left(\frac{\pi}{2} - t_*\right)^{-d},$$

$$\frac{dC_{\mathcal{A}}}{dt} \sim \left(\frac{\pi}{2} - t_*\right)^{-d}.$$

- dS/Crunch: Subleading terms are also different!

Complexity of Cosmological Crunches: Universal features

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- Time rate of change of complexity contains a UV divergent time-dependent piece for CFT metric being time-dependent
- Coefficient of the rate of change determined by the subleading term (YGH term for $C \propto \mathcal{A}$).

Complexity of Cosmological Singularities: Takeaway

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- Perhaps two distinct bulk geometric constructions are two different CFT complexities as well
- Universal features for decrease of complexity, contrasts w/ local probes (point probes/strings - blue shifting)
- Perhaps one can attempt a parallel with the classic BKL work regarding universality

Timelike singularities (2303.02752 [hep-th] w/ J. Ren & G. Katoch)

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- Solutions to effective holographic theories at zero temperature have typically naked *timelike* singularities
- Such singularities are generically resolved by lifting them to higher dimensions or eventually by the inclusion of the stringy states.
- Gubser criterion: *Naked singularities allowed in geometries are those which can be obtained as deformations/limits of regular black holes* [Gubser '01, Kiritsis et. al. '10,...]

Warm up example: Negative mass SAdS

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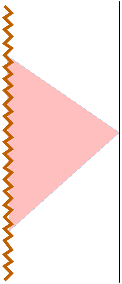
- 

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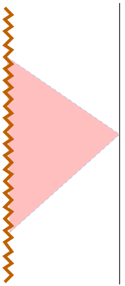

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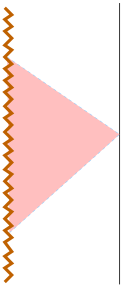

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- **Overall action complexity (also $\mathcal{C}_V$) is less than empty global AdS! (criterion)**

Timelike Kasner AdS

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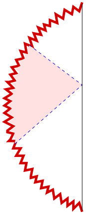
- Deformation of planar BH, an exact solution to AdS SUGRA equations (J. Ren: **1603.08004[hep-th]**)

$$ds^2 = \frac{R^2}{z^2}\left(\frac{dz^2}{f(z)} - f^{\alpha}(z)dt^2 + f^{\beta}(z)dx^2 + f^{\gamma}(z)dy^2\right), \quad f(z) = 1 - \frac{z^3}{z_0^3}$$



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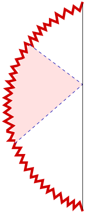

-
$$\mathcal{C}_A = \frac{l^2}{16\pi^2 G_N}\frac{V_{xy}}{\delta^2} - \frac{l^2}{32\pi\, G_N}\frac{V_{xy}}{z_0^2}\frac{(3-\alpha)\Gamma\left(\frac{1}{3}\right)\sec\left(\frac{\pi\alpha}{2}\right)}{\Gamma\left(\frac{5-3\alpha}{6}\right)\Gamma\left(\frac{\alpha+1}{2}\right)}\ .$$

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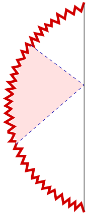

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- **Action complexity lower than the empty (Poincaré) AdS: in sync with Gubser criterion**

Naked Singularities in ES systems

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- Timelike Naked singular solutions in the Einstein-Scalar system (**J. Ren: 1910.06344 [hep-th]**)

$$ds^2 = f(r)(-dt^2 + d\mathbf{x}^2) + \frac{dr^2}{f(r)}, \quad f(r) = r^2 \left(1 + \frac{b}{r}\right)^{\frac{2\delta^2}{1+\delta^2}} = e^{\delta\phi},$$

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- $$\mathcal{C}_{\mathcal{A}} = \frac{V_{xy}}{8\pi G_N}\left(\frac{\Lambda^2}{2L^2} + \frac{\Lambda Q}{(\delta^2+1)\,L^2} + \frac{6\delta^2 Q^{\frac{3-\delta^2}{\delta^2+1}}}{(3\delta^2-1)\,L^2}\epsilon^{\frac{3\delta^2-1}{\delta^2+1}} + O(\Lambda^0)\right)$$

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- **Overall $\mathcal{C}_A$ is positive and larger than pure AdS for $\delta > 1/\sqrt{3}$. For $\delta < 1/\sqrt{3}$, $\mathcal{C}_A$ is negative and (IR) divergent!In sync Gubser criterion!**

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- Action Complexity test for singularities: Having less complexity compared to the empty AdS backgrounds is not allowed in a UV complete QG theory (in sync with Gubser criterion)
- Volume complexity not a reliable tool to probe timelike singularities.
- Need to conduct a more comprehensive survey of other nakedly timelike singular geometries in future to confirm $\mathcal{C}_A$ criterion.