Quantum Spread Complexity in Neutrino Oscillations

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Plan of this talk

Motivation

- Quantum computational complexity estimates the difficulty of constructing quantum states from elementary operations, a problem of prime importance for quantum computation.
- It can also serve to study a completely different physical problem - that of information processing inside black holes.
- Extends the connection between geometry and information.
  Growth of complexity is equal to the growth of black hole interiors.
  [Susskind et al., (2014)]
- It would be intriguing to investigate what characteristics complexity shows in other natural processes of evolution.
- Neutrinos have shown features such as entanglement and nonlocal correlations that proves their efficiency to perform QIP tasks.
  [blasone et al., (2009)], [Formaggio et al., (2016)]
- It gives us motivation to see how complex is an evolution of neutrino system and if complexity can also probe any open issue in the neutrino sector.

Neutrino-properties

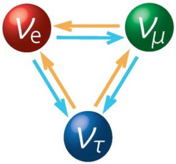
- Postulated first by Wolfgang Pauli to explain how beta decay could conserve energy, momentum and angular momentum(spin)

$$n \to p + e^{-} + \bar{\nu}_e$$

- Spin half, very small mass, no electric charge
- Come in three flavours $\to \nu_e, \ \nu_\mu, \ \nu_\tau$
- Interact only via weak interaction
- Neutrinos $\to$ Left-handed, Anti-neutrinos $\to$ Right-handed

Neutrino Oscillations

- About 65 billion (6.5×10^{10}) neutrinos coming from Sun's interior pass through 1 square centimeter per second. Homestake experiment's observed value was 1/3 of the predicted flux. This lead Pontecorvo to suggest neutrino oscillations.
- Up-down asymmetry of atmospheric muon neutrino flux by IMB and KamioKande experiments gave additional hint of neutrino oscillations. (*T. Kajita* (SK), *A. McDonald* (SNO), 2015)



- Experiments : Solar (e.g. Homestake, Gallex/SAGE, SNO), Atmospheric (e.g. Super Kamiokande), Reactor (e.g. CHOOZ, KamLAND, Daya-Bay, RENO), Accelerator (e.g. T2K, MINOS, NOνA, DUNE (upcoming))

Quantum mechanics in neutrino oscillations

- The three flavor states (eigenstates of weak interaction, which are detectable in lab) of neutrinos, ν_e, ν_μ and ν_τ mix via a 3×3 unitary matrix to form the three mass eigenstates (which are the propagation eigenstates) ν_1, ν_2 and ν_3. Neutrino oscillations occur only if the three corresponding masses, m_1, m_2 and m_3, are non-degenerate.
- In three flavor neutrino oscillation
  Propagation states $\rightarrow \{|\nu_1\rangle\,, \;\; |\nu_2\rangle\,, \;\; |\nu_3\rangle\}$;
  Flavor states $\rightarrow \{|\nu_e\rangle\,, \;\; |\nu_\mu\rangle\,, \;\; |\nu_\tau\rangle\}$
- The general state of a neutrino can be expressed in flavor basis as:

$$|\Psi(t)\rangle = \nu_e(t)\,|\nu_e\rangle + \nu_\mu(t)\,|\nu_\mu\rangle + \nu_\tau(t)\,|\nu_\tau\rangle$$

- Same state in propagation basis looks like:

$$|\Psi(t)\rangle = \nu_1(t)\,|\nu_1\rangle + \nu_2(t)\,|\nu_2\rangle + \nu_3(t)\,|\nu_3\rangle$$

- The coefficients in two representations are connected by a *unitary* matrix

$$\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \\ \nu_\tau(t) \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \\ \nu_3(t) \end{pmatrix}.$$

or,

$$\nu_\alpha(t) = \mathbf{U}\nu_i(t). \tag{1}$$

Quantum mechanics in neutrino oscillations

- A convenient parametrization for **U** or $U(\theta_{12}, \theta_{23}, \theta_{13}, \delta)$ is given by the PMNS matrix

$$U(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{23}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{13}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, θ_{ij} being the mixing angles and δ the CP (Charge-Parity) violating phase.
- The mass eigenstates evolve as

$$\begin{pmatrix} \nu_1(t) \\ \nu_2(t) \\ \nu_3(t) \end{pmatrix} = \begin{pmatrix} e^{-iE_1 t} & 0 & 0 \\ 0 & e^{-iE_2 t} & 0 \\ 0 & 0 & e^{-iE_3 t} \end{pmatrix} \begin{pmatrix} \nu_1(0) \\ \nu_2(0) \\ \nu_3(0) \end{pmatrix},$$

or,

$$\nu_{\mathbf{m}}(t) = \mathbf{E}\nu_{\mathbf{m}}(0) \tag{2}$$

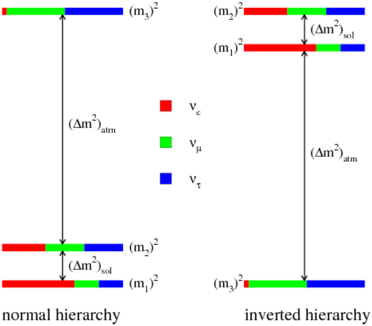
- From 1 and 2, $\nu_{\mathbf{f}}(t) = \mathbf{U}\ \mathbf{E}\mathbf{U}^{-1}\ \nu_{\mathbf{f}}(0) = \mathbf{U}_f\ \nu_{\mathbf{f}}(0)$.

$$\begin{aligned} P_{\alpha\beta} \quad = \quad & \delta_{\alpha\beta} - 4\sum_{i>j} Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}) \sin^2\left(1.27\frac{\Delta_{ij}L}{E}\right) \\ & +2\sum_{i>j} Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}) \sin\left(2.54\frac{\Delta_{ij}L}{E}\right) \end{aligned} \tag{3}$$

where $\Delta_{ij} = m_j^2 - m_i^2 \equiv E_j - E_i$.

Problems not resolved yet . . .

- Neutrino mass hierarchy problem *i.e.*, whether $m_1 \leq m_2 \leq m_3$ or $m_3 \leq m_1 \leq m_2$).
- CP violation ($\delta \neq 0$).
  $P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$
- Absolute mass



Neutrino experimental facilities

We included accelerator ν_μ- neutrino experimental conditions in our study such as

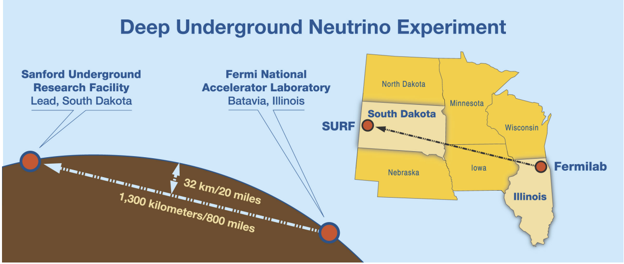
DUNE (L = 1300 Km, E = 1 - 10 GeV, $A = 1.7\times10^{-13}$ eV)

NOνA (L = 810 Km, E = 1 - 4 GeV, $A = 1.7\times10^{-13}$ eV)

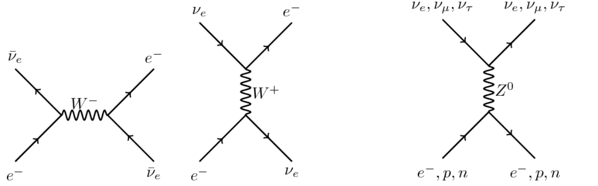
T2K (L = 295 Km, E = 0.1 - 1 GeV, $A = 1.01\times10^{-13}$ eV)

($L \rightarrow$ baseline, $E \rightarrow$ neutrino-energy, $A \rightarrow$ matter density potential)

Source: www.fnal.gov/



Matter effects on neutrino oscillations



(a) The Feynman diagrams for charged current inter- actions

(b) The Feynman diagram for neutral rent interactions

$$H_f = UH_mU^{-1} + V\ diag(1,0,0) + V_{Z_0}\ \mathbb{1}_{3\times 3}.$$

where, $V \rightarrow$ matter density potential due to coherent-forward scattering of ν_e with e^- present in the matter.

Complexity

- How difficult is it to construct a desired target state with the elementary operations (gates) at your end?
- Or, the minimum number of unitaries required to construct a "target state" through a "reference state".
- For a system $|\phi(s)\rangle$, if

$$U_1 U_2 U_3 U_2 |\phi(s)\rangle = U_3 U_1 U_2 U_1 (U_1)^3 U_2 |\phi(s)\rangle ,$$

then the complexity $= 4$.

Complexity of spread of states

Balasubramanian et al., PRD 106, 046007 (2022)

- The complexity of the state can be defined by minimizing the spread of the wavefunction over all possible bases.
- This minimum is uniquely attained by an orthonormal basis produced by applying the Gram-Schmidt procedure.

Schrodinger equation for a system represented by $|\psi(t)\rangle$

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$$

Then, the time evolution of the state $|\psi(t)\rangle$ is obtained as

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle .$$

One can also write

$$|\psi(t)\rangle = \sum_{n=0}^{\infty}\frac{(-it)^n}{n!}H^n|\psi(0)\rangle = \sum_{n=0}^{\infty}\frac{(-it)^n}{n!}|\psi_n\rangle ,$$

where, $|\psi_n\rangle = H^n|\psi(0)\rangle$. Hence, we can see that the time evolved system-state $|\psi(t)\rangle$ is represented as superposition of infinite $|\psi_n\rangle$ states.

Complexity of spread of states

We have $|\psi_n\rangle = H^n\,|\psi(0)\rangle$. These states $\{|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle, \dots\}$ are not orthonomalized. Gram-Schmidt procedure to obtain an ordered orthonomalized basis

$$
\begin{aligned}
|K_0\rangle &= |\psi_0\rangle\,, \\
|K_1\rangle &= |\psi_1\rangle - \frac{\langle K_0|\psi_1\rangle}{\langle K_0|K_0\rangle}\,|K_0\rangle\,, \\
|K_2\rangle &= |\psi_2\rangle - \frac{\langle K_0|\psi_2\rangle}{\langle K_0|K_0\rangle}\,|K_0\rangle - \frac{\langle K_1|\psi_2\rangle}{\langle K_1|K_1\rangle}\,|K_1\rangle\,, \quad \text{and so on.}
\end{aligned}
$$

$\mathcal{K} = \{|K_n\rangle\,, n = 0, 1, 2\dots\} \Rightarrow$ Krylov basis

Cost function to quantify the complexity (Balasubramanian et al., PRD 106, 046007 (2022))
For a time evolved state $|\psi(t)\rangle$ and the Krylov basis defined as $\{|K_n\rangle\}$, the cost function is

$$\chi = \sum_{n=0}^{\infty} n|\langle K_n|\psi(t)\rangle|^2,$$

where $n = 0, 1, 2\dots$ For such Krylov basis the above defined cost function becomes minimum.

Spread complexity in two flavor neutrino oscillations

The evolution of flavor states can be represented by Schrodinger equation as

$$i\frac{\partial}{\partial t}\begin{pmatrix}|\nu_e(t)\rangle\\ |\nu_\mu(t)\rangle\end{pmatrix} = H_f\begin{pmatrix}|\nu_e(t)\rangle\\ |\nu_\mu(t)\rangle\end{pmatrix} \tag{4}$$

where $H_f = UH_mU^{-1}$, U being the mixing matrix and H_m is the Hamiltonian (diagonal) that governs the time evolution of neutrino mass eigenstate

$$H_m = \begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}, \qquad U = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}.$$

$$|\nu_e(0)\rangle = \begin{pmatrix}1\\ 0\end{pmatrix}, \qquad |\nu_\mu(0)\rangle = \begin{pmatrix}0\\ 1\end{pmatrix}$$

We have

$$\{|\psi_n\rangle\} = \begin{cases}\{|\nu_e(0)\rangle, H_f|\nu_e(0)\rangle, H_f^2|\nu_e(0)\rangle\dots\} & \textit{for initial } \nu_e \textit{ flavor}\\ \{|\nu_\mu(0)\rangle, H_f|\nu_\mu(0)\rangle, H_f^2|\nu_\mu(0)\rangle\dots\} & \textit{for initial } \nu_\mu \textit{ flavor}\end{cases}$$

After applying Gram-Schmidt procedure we get $\{|K_n\rangle\} = \{|K_0\rangle, |K_1\rangle\}$, *i.e.*,

$$\{|K_n\rangle\} = \begin{cases}\{|K_0\rangle = \begin{pmatrix}1\\ 0\end{pmatrix}, |K_1\rangle = \begin{pmatrix}0\\ 1\end{pmatrix}\} = \{|\nu_e\rangle, |\nu_\mu\rangle\} & \textit{for initial } \nu_e\\ \{|K_0\rangle = \begin{pmatrix}0\\ 1\end{pmatrix}, |K_1\rangle = \begin{pmatrix}1\\ 0\end{pmatrix}\} = \{|\nu_\mu\rangle, |\nu_e\rangle\} & \textit{for initial } \nu_\mu\end{cases}$$

Spread complexity in two flavor neutrino oscillations

For a time evolved state $|\nu_e(t)\rangle = \begin{pmatrix} A_{ee}(t) \\ A_{e\mu}(t) \end{pmatrix} = \begin{pmatrix} \cos^2\theta e^{-iE_1 t} + \sin^2\theta e^{-iE_2 t} \\ \sin\theta\cos\theta(e^{-iE_2 t} - e^{-iE_1 t}) \end{pmatrix}$ (with $\{|K_n\rangle\} = \{|\nu_e(0)\rangle, |\nu_\mu(0)\rangle\}$)

$$\chi_e = \sum_{n=0}^{1} n|\langle K_n|\nu_e(t)\rangle|^2 = P_{e\mu}$$

Similarly, for state $|\nu_\mu(t)\rangle = (A_{\mu e}(t), A_{\mu\mu}(t))^T$ (with $\{|K_n\rangle\} = \{|\nu_\mu(0)\rangle, |\nu_e(0)\rangle\}$)

$$\chi_\mu = P_{\mu e}$$

- The more the oscillation probability of neutrino flavor, the more complex the evolution of the neutrino flavor state.
- Since $P_{e\mu} = P_{\mu e}$ for standard vacuum oscillations, the complexity embedded in this system comes out to be same for both cases of initial flavor, *i.e.*, complexity of the system doesn't depend on the initial flavor of neutrino.

Spread complexity in three flavor neutrino oscillations

We have three types of initial states as $|\nu_e\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $|\nu_\mu\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}$, $|\nu_\tau\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$ with Hamiltonian $H_f = UH_mU^{-1}$, $H_m = diag(0, \Delta m_{21}^2, \Delta m_{31}^2)$ and U $\rightarrow$ 3 $\times$ 3 PMNS mixing matrix

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{13}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Here, Krylov basis $\neq$ flavor basis.

- For initial $|\nu_e\rangle$ state $|K_0\rangle \equiv |\nu_e\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$, other states spanning the Krylov basis take the form

$$|K_1\rangle = N_1 \begin{pmatrix}0\\a_1\\a_2\end{pmatrix} = N_1 \begin{pmatrix} 0 \\ \left(\frac{\Delta m_{21}^2}{2E}\right) U_{e2}^* U_{\mu 2} + \left(\frac{\Delta m_{31}^2}{2E}\right) U_{e3}^* U_{\mu 3} \\ \left(\frac{\Delta m_{21}^2}{2E}\right) U_{e2}^* U_{\tau 2} + \left(\frac{\Delta m_{31}^2}{2E}\right) U_{e3}^* U_{\tau 3} \end{pmatrix},$$

$$|K_2\rangle = N_2 \begin{pmatrix}0\\b_1\\b_2\end{pmatrix} = N_2 \begin{pmatrix} 0 \\ \left(\frac{\Delta m_{21}^2}{2E}\right)\left(\frac{\Delta m_{21}^2}{2E} - A\right) U_{e2}^* U_{\mu 2} + \left(\frac{\Delta m_{31}^2}{2E}\right)\left(\frac{\Delta m_{31}^2}{2E} - A\right) U_{e3}^* U_{\mu 3} \\ \left(\frac{\Delta m_{21}^2}{2E}\right)\left(\frac{\Delta m_{21}^2}{2E} - A\right) U_{e2}^* U_{\tau 2} + \left(\frac{\Delta m_{31}^2}{2E}\right)\left(\frac{\Delta m_{31}^2}{2E} - A\right) U_{e3}^* U_{\tau 3} \end{pmatrix}$$

Spread complexity in three flavor neutrino oscillations

$$\boxed{\begin{gathered}\chi_e = P_{e\mu}(t)(N_1^2|a_1|^2 + 2N_2^2|b_1|^2) + P_{e\tau}(t)(N_1^2|a_2|^2 + 2N_2^2|b_2|^2) + 2\Re(N_1^2 a_1^* a_2 A_{e\mu}(t)A_{e\tau}(t)^*) \\ + 4\Re(N_2^2 b_1^* b_2 A_{e\mu}(t)A_{e\tau}(t)^*)\end{gathered}}$$

with

$$A = \frac{\begin{pmatrix} \left(\Delta m_{21}^2\right)^3 |U_{\alpha 2}|^2(1-|U_{\alpha 2}|^2) + \left(\Delta m_{31}^2\right)^3 |U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2) \\ \qquad - \left(\Delta m_{21}^2\right)\left(\Delta m_{31}^2\right)|U_{\alpha 2}|^2|U_{\alpha 3}|^2\left(\Delta m_{21}^2 + \Delta m_{31}^2\right)\end{pmatrix}}{\left(\Delta m_{21}^2\right)^2 |U_{\alpha 2}|^2(1-|U_{\alpha 2}|^2) + \left(\Delta m_{31}^2\right)^2 |U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2) - 2\left(\Delta m_{21}^2\right)\left(\Delta m_{31}^2\right)|U_{\alpha 2}|^2|U_{\alpha 3}|^2},$$

Effects of different oscillation parameters

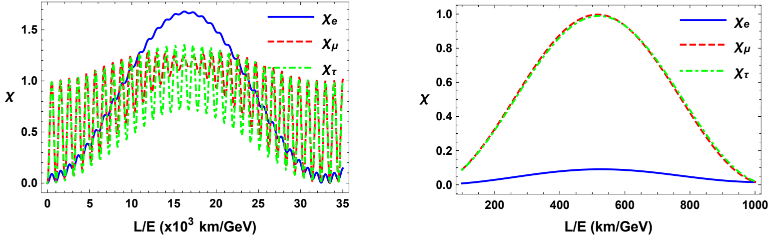


Figure: Complexity plotted with respect to the distance L over energy E traveled by neutrinos in vacuum and in case if the initial flavor is ν_e (blue solid line), ν_μ (red dashed line) and ν_τ (green dot-dashed line) for CP-violating phase $\delta = 0^o$. Here, mixing parameters $\theta_{12} = 33.64^o$, $\theta_{13} = 8.53^o$, $\theta_{23} = 47.63^o$, $\Delta m^2_{21} = 7.53 \times 10^{-5}\ eV^2$ and $\Delta m^2_{31} = 2.45 \times 10^{-3}\ eV^2$ are considered.

- The rapid oscillation pattern seen in the left panel (zoomed-in in the right panel) is due to Δm^2_{31} mass-squared difference in the oscillation phase, while the longer oscillation pattern is due to Δm^2_{21} in the oscillation phase. The oscillation length is $\sim 10^3$ km at $E = 1$ GeV for Δm^2_{31} and $\sim 3 \times 10^4$ km at $E = 1$ GeV for Δm^2_{21}.
- In the general case the complexity is maximum if the neutrino is produced initially as ν_e, however, this happens only at a very large L/E value of $\sim 1.6 \times 10^4$ km/GeV.
- In current experimental setups (right panel), which covers roughly one oscillation length for Δm^2_{31}, the initial ν_e flavor provides the least complexity among all neutrino flavors.

Effects of CP-violating parameter δ

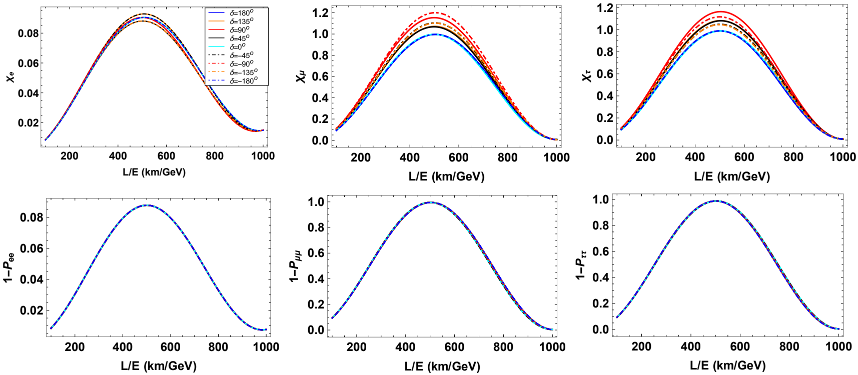


Figure: Complexities and 1-$P_{\alpha\alpha}$ with respect to L/E.

- Complexity mimics the features of the total oscillation probability $1 - P_{\alpha\alpha}$.
- However, it is visible that χ_α for all three flavors provide more information regarding the CP-violating phase δ.

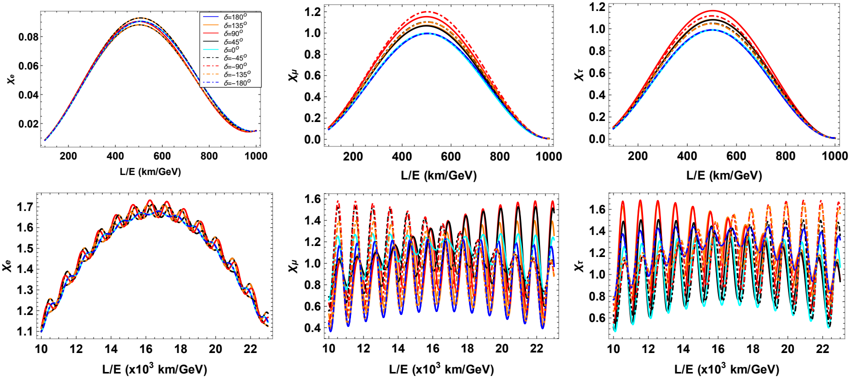


Figure: Complexity for small L/E range (upper panels), large L/E range (lower panels) with respect to L/E for initial flavor is ν_e (left), ν_μ (middle) and ν_τ (right) for different values of the CP-phase δ depicted by different colors.

- For large L/E range the complexities are maximized and the corresponding $\delta = +90^o$ or -90^o for χ_μ and χ_τ, and at $\delta = \pm 90^o$ for χ_e where CP is maximally violated.
- In the limited L/E range χ_μ and χ_τ are maximized at $\delta = -90^o$ (red-dashed line) and at $\delta = +90^o$ (red-solid line), respectively. However, χ_e is maximized at $\delta = +135^o$ and at -45^o.

Matter effects on complexity

- For any initial flavor ν_α

$$|K_0\rangle_\alpha^{matter} = |K_0\rangle_\alpha^{vacuum}$$
$$|K_1\rangle_\alpha^{matter} = |K_1\rangle_\alpha^{vacuum},$$

- $|K_2\rangle$ contains the effects of constant matter density

$$|K_2\rangle_e = N_{2e}^m (0, b_1^m, b_2^m)^T$$

where,

$$b_1^m = \left(\frac{\Delta m_{21}^2}{2E}\right)\left(\frac{\Delta m_{21}^2}{2E} + V - B_e\right) U_{e2}^* U_{\mu 2} + \left(\frac{\Delta m_{31}^2}{2E}\right)\left(\frac{\Delta m_{31}^2}{2E} + V - B_e\right) U_{e3}^* U_{\mu 3},$$
$$b_2^m = \left(\frac{\Delta m_{21}^2}{2E}\right)\left(\frac{\Delta m_{21}^2}{2E} + V - B_e\right) U_{e2}^* U_{\tau 2} + \left(\frac{\Delta m_{31}^2}{2E}\right)\left(\frac{\Delta m_{31}^2}{2E} + V - B_e\right) U_{e3}^* U_{\tau 3}.$$

Matter effects on complexity

- Similarly, for the initial ν_μ flavor

$$|K_2\rangle_\mu = N^m_{2\mu}(d^m_1, 0, d^m_2)^T,$$

where,

$$d^m_1 = \left(\frac{\Delta m^2_{21}}{2E}\right)\left(\frac{\Delta m^2_{21}}{2E} + V - B_\mu\right) U_{e2}U^*_{\mu 2} + \left(\frac{\Delta m^2_{31}}{2E}\right)\left(\frac{\Delta m^2_{31}}{2E} + V - B_\mu\right) U_{e3}U^*_{\mu 3}$$

$$d^m_2 = \left(\frac{\Delta m^2_{21}}{2E}\right)\left(\frac{\Delta m^2_{21}}{2E} - B_\mu\right) U^*_{\mu 2}U_{\tau 2} + \left(\frac{\Delta m^2_{31}}{2E}\right)\left(\frac{\Delta m^2_{31}}{2E} - B_\mu\right) U^*_{\mu 3}U_{\tau 3},$$

Matter effects on complexity

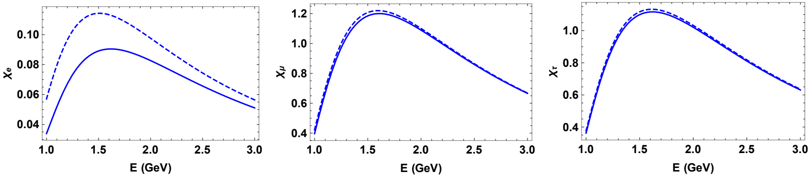


Figure: Cost function χ_e (left), χ_μ (middle) and χ_τ (right) w. r. t. neutrino-energy E is shown. Here, $L = 810$ km, $\delta = -90^o$ and higher octant of θ_{23} is considered. Solid and dashed curves represent the case of vacuum and matter oscillations, respectively. $V = 1.01 \times 10^{-13}$ eV.

- Matter effect increases complexity of the system in all cases of initial flavors of the neutrino, most significantly for ν_e as expected.

Spread complexity in neutrino oscillation experiments

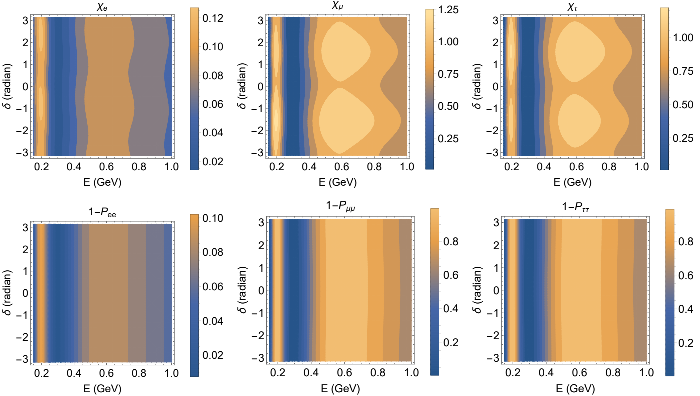


Figure: T2K: Cost function (upper panel) and 1-$P_{\alpha\alpha}$ (lower panel) in the plane of $E - \delta$ in case of initial flavor ν_e (left), ν_μ (middle) and ν_τ (right). Here, $L = 295$ km and mixing parameters $\theta_{12} = 33.64^o$, $\theta_{13} = 8.53^o$, $\theta_{23} = 47.63^o$, $\Delta m^2_{21} = 7.53 \times 10^{-5}\ eV^2$ and $\Delta m^2_{31} = 2.45 \times 10^{-3}\ eV^2$ are considered.

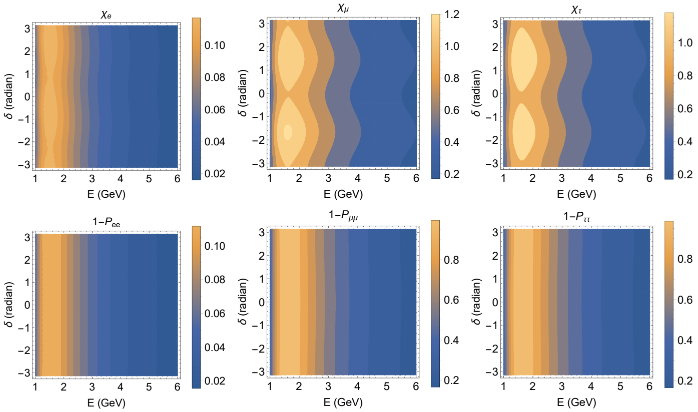


Figure: NOνA: Cost function (upper panel) and 1-$P_{\alpha\alpha}$ (lower panel) in the plane of $E-\delta$ in case of initial flavor ν_e (left), ν_μ (middle) and ν_τ (right). Here, $L = 810$ km, and higher octant of θ_{23} (47.63^o) is considered.

- For both the experiments, the maxima of χ_μ and χ_τ are found at $\delta \approx -\pi/2$ and $\delta = \pi/2$, respectively.
- This means that the matter effect just enhances the magnitude of complexities, however, the characteristics of χ_α with respect to δ are almost similar for both T2K and NOvA experiments.

- In the T2K and NOvA experimental setups, where only ν_μ beams are produced, the only relevant complexity is χ_μ.
- For both the T2K and NOvA χ_μ is maximized at $\delta \approx -1.5$ radian at the relevant experimental energies. The T2K best-fit value of $\delta = -2.14^{+0.90}_{-0.69}$ radian is consistent with this expectation.
- The NOvA best-fit, however, is at $\delta \approx 2.58$ radian which is far away from the maximum χ_μ in the lower-half plane of δ but is still within a region of high χ_μ value in the upper-half plane of δ.
- $P_{\mu e}$, which is the only oscillation probability accessible to the T2K and NOvA setups, it becomes maximum at $\delta \approx -1.5$ radian. This is compatible with T2K best-fit but is in odd with the NOvA best-fit.
- Complexity provides correct prediction for the δ in experimental setups.

Effects of neutrino mass ordering

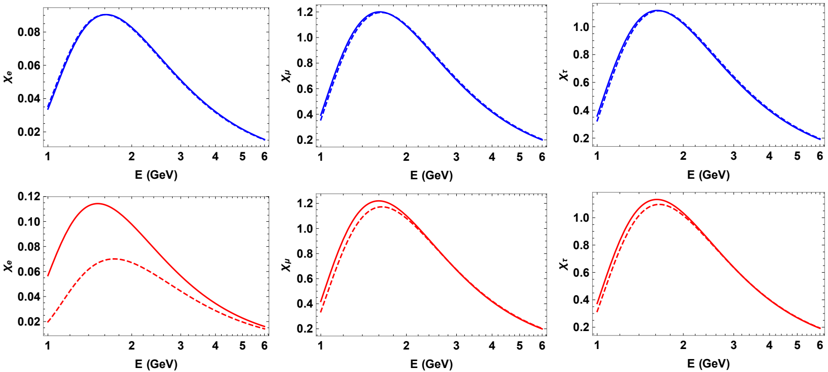


Figure: NOvA: Complexity with respect to neutrino-energy E in case of initial flavor ν_e (left), ν_μ (middle) and ν_τ (right) with $L = 810$ km and $\delta = -90^o$. The upper and lower panel represent the case of vacuum and matter oscillations, respectively. Solid curves are associated with NH and dashed curves depict the IH.

- Complexity can distinguish between the effects due to normal (NH) ($+\Delta_{31}$) and inverted hierarchy (IH) ($-\Delta_{31}$) in the presence of non-zero matter potential.

Summary & Conclusions

- We examined the spread complexity of neutrino states in two- and three-flavor oscillation scenarios.
- In the two-flavor scenario, complexity and transition probabilities yield equivalent information.
- In case of three-flavor oscillation, initial flavor state evolves into two mixed final states. Hence, the complexity contains additional information regarding open issues related to neutrinos, compared to the total oscillation probability.
- Remarkably, we found that the complexity is maximized for a value of the phase angle for which CP is also maximally violated. T2K experimental data also favors this phase angle, which is obtained from flavor transition.
- Quantum spread complexity emerges as a potent and novel quantity for investigating neutrino oscillations. It successfully reproduces existing results, also demonstrates the potential to serve as a theoretical tool for predicting new outcomes in future experiments.

Thank you for your attention!

BACKUP SLIDES

Comparing the effects of neutrino mass ordering for neutrinos & antineutrinos

For antineutrino $\rightarrow \{V \rightarrow -V,\ \delta \rightarrow -\delta\}$

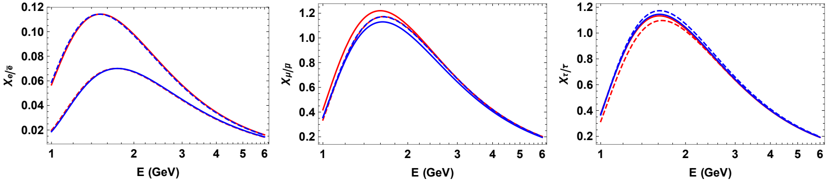


Figure: NOvA: Complexities and $P_{\mu e}$ with respect to neutrino-energy E where red and blue curves represent neutrino and antineutrino case, respectively, with solid (normal ordering) and dashed (inverted ordering) lines. Here $L = 810$ km and $\delta = -90^o$ are considered.

- For both neutrino and antineutrino, the effects of NH and IH are significantly distinguishable for all three flavors.
- In case of χ_e, red-solid line (neutrinos for NH) and blue-dashed line (antineutrinos for IH) exhibit more complexity, $i.e.$, complete swap between the NH (IH) hierarchy and ν ($\bar{\nu}$).
- For χ_μ and χ_τ the maximum is achieved in case of neutrinos with NH and Antineutrinos with IH, respectively.

Spread complexity in three flavor (vacuum) neutrino oscillations

- Similarly, for initial $|\nu_\mu\rangle$, $|K_0\rangle \equiv |\nu_\mu\rangle = (0,1,0)^T$, then we get

$$|K_1\rangle = N_{1\mu}\begin{pmatrix} c_1 \\ 0 \\ c_2 \end{pmatrix} = N_{1\mu}\begin{pmatrix} \left(\frac{\Delta m_{21}^2}{2E}\right) U_{\mu 2}^* U_{e2} + \left(\frac{\Delta m_{31}^2}{2E}\right) U_{\mu 3}^* U_{e3} \\ 0 \\ \left(\frac{\Delta m_{21}^2}{2E}\right) U_{\mu 2}^* U_{\tau 2} + \left(\frac{\Delta m_{31}^2}{2E}\right) U_{\mu 3}^* U_{\tau 3} \end{pmatrix},$$

$$|K_2\rangle = N_{2\mu}\begin{pmatrix} d_1 \\ 0 \\ d_2 \end{pmatrix} = N_{2\mu}\begin{pmatrix} \left(\frac{\Delta m_{21}^2}{2E}\right)\left(\frac{\Delta m_{21}^2}{2E} - A\right) U_{\mu 2}^* U_{e2} + \left(\frac{\Delta m_{31}^2}{2E}\right)\left(\frac{\Delta m_{31}^2}{2E} - A\right) U_{\mu 3}^* U_{e3} \\ 0 \\ \left(\frac{\Delta m_{21}^2}{2E}\right)\left(\frac{\Delta m_{21}^2}{2E} - A\right) U_{\mu 2}^* U_{\tau 2} + \left(\frac{\Delta m_{31}^2}{2E}\right)\left(\frac{\Delta m_{31}^2}{2E} - A\right) U_{\mu 3}^* U_{\tau 3} \end{pmatrix}$$

$$\chi_\mu = P_{\mu e}(t)(N_{1\mu}^2|c_1|^2 + 2N_{2\mu}^2|d_1|^2) + P_{\mu\tau}(t)(N_{1\mu}^2|c_2|^2 + 2N_{2\mu}^2|d_2|^2) + 2\Re(N_{1\mu}^2 c_1^* c_2 A_{\mu e}(t) A_{\mu\tau}(t)^*) + 4\Re(N_{2\mu}^2 d_1^* d_2 A_{\mu e}(t) A_{\mu\tau}(t)^*).$$

Spread complexity in three flavor neutrino oscillations

- In case of $|K_0\rangle \equiv |\nu_\tau\rangle = (0,0,1)^T$,

$$|K_1\rangle = N_{1\tau}(e_1, e_2, 0)^T = N_{1\tau}\begin{pmatrix} \left(\frac{\Delta m^2_{21}}{2E}\right) U^*_{\tau 2} U_{e2} + \left(\frac{\Delta m^2_{31}}{2E}\right) U^*_{\tau 3} U_{e3} \\ \left(\frac{\Delta m^2_{21}}{2E}\right) U^*_{\tau 2} U_{\mu 2} + \left(\frac{\Delta m^2_{31}}{2E}\right) U^*_{\tau 3} U_{\mu 3} \\ 0 \end{pmatrix},$$

$$|K_2\rangle = N_{2\tau}(f_1, f_2, 0)^T = N_{2\tau}\begin{pmatrix} \left(\frac{\Delta m^2_{21}}{2E}\right)\left(\frac{\Delta m^2_{21}}{2E} - A\right) U^*_{\tau 2} U_{e2} + \left(\frac{\Delta m^2_{31}}{2E}\right)\left(\frac{\Delta m^2_{31}}{2E} - A\right) U^*_{\tau 3} U_{e3} \\ \left(\frac{\Delta m^2_{21}}{2E}\right)\left(\frac{\Delta m^2_{21}}{2E} - A\right) U^*_{\tau 2} U_{\mu 2} + \left(\frac{\Delta m^2_{31}}{2E}\right)\left(\frac{\Delta m^2_{31}}{2E} - A\right) U^*_{\tau 3} U_{\mu 3} \\ 0 \end{pmatrix}.$$

$$\chi_\tau = P_{\tau e}(t)(N_1^2|e_1|^2 + 2N_2^2|f_1|^2) + P_{\tau\mu}(t)(N_1^2|e_2|^2 + 2N_2^2|f_2|^2) + 2\Re(N_1^2 e_1^* e_2 A_{\tau e}(t) A_{\tau\mu}(t)^*) + 4\Re(N_2^2 f_1^* f_2 A_{\tau e}(t) A_{\tau\mu}(t)^*).$$

Here,

$$A = \frac{\left[\begin{array}{c} \left(\Delta m^2_{21}\right)^3 |U_{\alpha 2}|^2(1 - |U_{\alpha 2}|^2) + \left(\Delta m^2_{31}\right)^3 |U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) \\ - \left(\Delta m^2_{21}\right)\left(\Delta m^2_{31}\right) |U_{\alpha 2}|^2 |U_{\alpha 3}|^2 \left(\Delta m^2_{21} + \Delta m^2_{31}\right) \end{array}\right]}{\left(\Delta m^2_{21}\right)^2 |U_{\alpha 2}|^2(1 - |U_{\alpha 2}|^2) + \left(\Delta m^2_{31}\right)^2 |U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) - 2\left(\Delta m^2_{21}\right)\left(\Delta m^2_{31}\right) |U_{\alpha 2}|^2 |U_{\alpha 3}|^2},$$

Spread complexity in three flavor neutrino oscillations

$$N_{1\alpha} = \Bigg(\left(\frac{\Delta m_{21}^2}{2E}\right)^2 |U_{\alpha 2}|^2 (1 - |U_{\alpha 2}|^2) + \left(\frac{\Delta m_{31}^2}{2E}\right)^2 |U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2)$$

$$-2 \left(\frac{\Delta m_{21}^2}{2E}\right) \left(\frac{\Delta m_{31}^2}{2E}\right) |U_{\alpha 2}|^2 |U_{\alpha 3}|^2 \Bigg)^{-1/2},$$

$$N_{2\alpha} = \Bigg(\left(\frac{\Delta m_{21}^2}{2E}\right)^2 \left(\frac{\Delta m_{21}^2}{2E} - A\right)^2 |U_{\alpha 2}|^2 (1 - |U_{\alpha 2}|^2)$$

$$+ \left(\frac{\Delta m_{31}^2}{2E}\right)^2 \left(\frac{\Delta m_{31}^2}{2E} - A\right)^2 |U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2)$$

$$-2 \left(\frac{\Delta m_{21}^2}{2E}\right) \left(\frac{\Delta m_{31}^2}{2E}\right) \left(\frac{\Delta m_{21}^2}{2E} - A\right) \left(\frac{\Delta m_{31}^2}{2E} - A\right) |U_{\alpha 2}|^2 |U_{\alpha 3}|^2 \Bigg)^{-1/2}$$

Matter effects on complexity

$$B_e = \Bigg[\left(\Delta m^2_{21}\right)^2 \left(\Delta m^2_{21} + 2EV\right) |U_{e2}|^2(1 - |U_{e2}|^2) + \left(\Delta m^2_{31}\right)^2 \left(\Delta m^2_{31} + 2EV\right) |U_{e3}|^2$$
$$(1 - |U_{e3}|^2) - \left(\Delta m^2_{21}\right) \left(\Delta m^2_{31}\right) |U_{e2}|^2|U_{e3}|^2 \left((\Delta m^2_{21} + 2EV) + (\Delta m^2_{31} + 2EV)\right)\Bigg]$$
$$\Bigg[2E \Bigg[\left(\Delta m^2_{21}\right)^2 |U_{e2}|^2(1 - |U_{e2}|^2) + \left(\Delta m^2_{31}\right)^2 |U_{e3}|^2(1 - |U_{e3}|^2)$$
$$- 2 \left(\Delta m^2_{21}\right) \left(\Delta m^2_{31}\right) |U_{e2}|^2|U_{e3}|^2\Bigg]\Bigg]^{-1}.$$

For initial ν_μ and ν_τ state the constant B_α is

$$B_\alpha = \Bigg[\left(\Delta m^2_{21}\right)^3 |U_{\alpha 2}|^2(1 - |U_{\alpha 2}|^2) + \left(\Delta m^2_{31}\right)^3 |U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) - \left(\Delta m^2_{21}\right) \left(\Delta m^2_{31}\right)$$
$$|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \left(\Delta m^2_{21} + \Delta m^2_{31}\right) + 2EV \Bigg(\left(\Delta m^2_{21}\right)^2 |U_{e2}|^2|U_{\alpha 2}|^2 + \left(\Delta m^2_{31}\right)^2 |U_{e3}|^2|U_{\alpha 3}|^2$$
$$+2 \left(\Delta m^2_{21}\right) \left(\Delta m^2_{31}\right) \Re(U^*_{e2} U_{\alpha 2} U_{e3} U^*_{\alpha 3})\Bigg)\Bigg] \Bigg[2E \Bigg[\left(\Delta m^2_{21}\right)^2 |U_{\alpha 2}|^2(1 - |U_{\alpha 2}|^2) + \left(\Delta m^2_{31}\right)^2$$
$$|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) - 2 \left(\Delta m^2_{21}\right) \left(\Delta m^2_{31}\right) |U_{\alpha 2}|^2|U_{\alpha 3}|^2\Bigg]\Bigg]^{-1},$$

Matter effects on complexity

$$N^m_{2e} = \Bigg(\left(\frac{\Delta m^2_{21}}{2E}\right)^2 |U_{e2}|^2 (1 - |U_{e2}|^2) \left[\left(\frac{\Delta m^2_{21}}{2E} + V - B_e\right)^2 \right] + \left(\frac{\Delta m^2_{31}}{2E}\right)^2 |U_{e3}|^2 (1 - |U_{e3}|^2) \left[\left(\frac{\Delta m^2_{31}}{2E} + V - B_e\right)^2 \right] - 2 \left(\frac{\Delta m^2_{21}}{2E}\right) \left(\frac{\Delta m^2_{31}}{2E}\right) \left(\frac{\Delta m^2_{21}}{2E} + V - B_e\right) \left(\frac{\Delta m^2_{31}}{2E} + V - B_e\right) |U_{e2}|^2 |U_{e3}|^2 \Bigg)^{-1/2},$$

$$N^m_{2\mu} = \Bigg(\left(\frac{\Delta m^2_{21}}{2E}\right)^2 |U_{\mu 2}|^2 \left[\left(\frac{\Delta m^2_{21}}{2E} + V - B_\mu\right)^2 |U_{e2}|^2 + \left(\frac{\Delta m^2_{21}}{2E} - B_\mu\right)^2 |U_{\tau 2}|^2 \right] + \left(\frac{\Delta m^2_{31}}{2E}\right)^2 |U_{\mu 3}|^2 \left[\left(\frac{\Delta m^2_{31}}{2E} + V - B_\mu\right)^2 |U_{e3}|^2 + \left(\frac{\Delta m^2_{31}}{2E} - B_\mu\right)^2 |U_{\tau 3}|^2 \right] + 2 \left(\frac{\Delta m^2_{21}}{2E}\right) \left(\frac{\Delta m^2_{31}}{2E}\right) \left[\left(\frac{\Delta m^2_{21}}{2E} + V - B_\mu\right) \left(\frac{\Delta m^2_{31}}{2E} + V - B_\mu\right) \Re(U^*_{\mu 2} U_{e2} U_{\mu 3} U^*_{e3}) + \left(\frac{\Delta m^2_{21}}{2E} - B_\mu\right) \left(\frac{\Delta m^2_{31}}{2E} - B_\mu\right) \Re(U^*_{\mu 2} U_{\tau 2} U_{\mu 3} U^*_{\tau 3}) \right] \Bigg)^{-1/2},$$

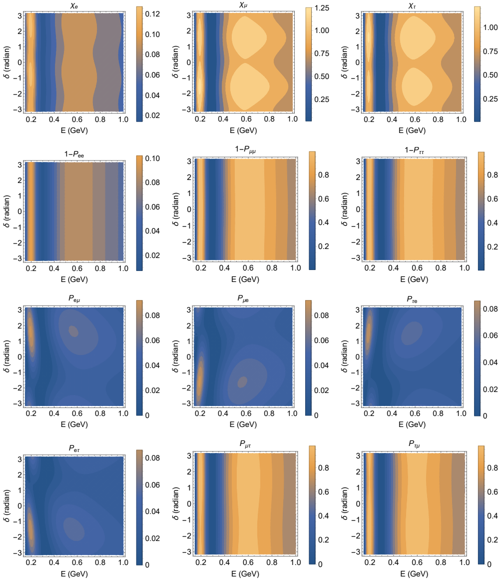


Figure: T2K: Cost function (upper panel) and 1-$P_{\alpha\alpha}$ (lower panel) in the plane of $E - \delta$ in case of initial flavor ν_e (left), ν_μ (middle) and ν_τ (right). Here, $L = 295$ km is considered.