STOCHASTIC MASS TRANSPORTS
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Mathias Beiglböck (Universität Wien),
Martin Huesmann (Universität Münster) ,
Jan Obłój (University of Oxford)

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1 Overview of the Field. Workshop's focus.

Starting with the insights of Brenier/Rüschendorf, McCann, Otto and others in the 90s, the field of optimal transportation has seen a tremendous development in the last 30 years with deep connections forged with many other branches of mathematics and with many fields of applications. A central idea in this development is the interpretation of a coupling as a device to transport mass. While couplings have a long history in probability their reinterpretation as a transport of mass has proven to be a powerful idea within the last 15 years for instance in the field of robust finance, economics, or stochastic optimization. Some of these problems lead to the class of transport problems with probabilistic constraints like martingale, filtration or covariance constraints. It also turns out that certain stochastic processes can be used to construct efficient (non-optimal) transport maps to prove functional inequalities. Motivated by statistical applications optimal transport problems with random measures, especially empirical measures, have received considerable attention. To be able to apply the mathematical results for the motivating applied problems it is often necessary to come up with good computational methods which require a precise understanding of regularity and approximation results.

The family of all these problems is subsumed by the name stochastic mass transport and was the broad focus of this workshop. A particular attention was paid to the possible synergies between different fields and developments. One of our key interests was causal, or adapted, transport which captures the dynamic requirement that the transport respects the evolution of information.

2 Scientific Progress Made

We briefly describe the scientific progress made on some of the central topics of the workshop; by nature this description will not be exhaustive.

Adapted optimal transport. Adapted transport theory and adapted Wasserstein distance aim to extend classical transport theory for probability measures to the case of stochastic processes. A fundamental difference is that it takes the temporal flow of information into account. In contrast to other topologies for stochastic processes, one obtains that probabilistic operations such as the Doob-decomposition, optimal stopping, and stochastic control are continuous with respect to the adapted Wasserstein distance. Moreover, adapted transport enjoys desirable properties similar to classical theory. E.g. it turns the set of stochastic processes into a geodesic space, as discussed in the talk of G. Pammer and allows to construct natural martingale interpolations between given probabilities in convex order as discussed by W. Schachermayer.

Martingale optimal transport and the PCOC problem. The martingale optimal transport (MOT) problem is a transport problem with the additional constraint that the coupling needs to be a martingale. It is naturally related to robust finance via the martingale pricing paradigm. While the one-dimensional discrete time case is well understood there are still many challenging problems in higher dimensions and continuous time versions of the problem. Multi-marginal and infinite marginal versions of the martingale optimal transport problem lead to solutions to the PCOC problem where one searches for martingales with a prescribed family of one-dimensional marginal distributions.

B. Jourdain presented resent results for the approximation of martingale couplings which imply stability of martingale optimal transport as well as weak martingale optimal transport. W. Schachermayer explained in his talk how one can use a particular martingale interpolation between given measures, the stretched Brownian motion, to recover the fundamental decomposition results. Y.-H. Kim showed a remarkable connection between a Brownian martingale transport problem with a free target measure and solutions to the Stefan problem. This connections allows him to establish new well-posedness results as well as uncover new symmetries between the freezing and melting Stefan problem. N. Juillet reported on new canonical solutions to the PCOC problem which do not require any constraints on the marginal distributions. I. Guo reported on other variants of OT, namely semimartingale optimal transport, and how it finds other applications in finance to calibration problems. He explained recent progress in using SOT to calibrate to American options, i.e., to find processes matching prescribed values of optimal stopping problems.

Measure valued processes. Measure valued processes are an infinite dimensional extension of classical stochastic processes which have recently seen significant interest from the mathematical finance community. On the one hand they allow incorporate constraints coming from given market data and no arbitrage conditions directly into the process. On the other hand due to recent advances, classical tools of stochastic analysis like Ito's formula have now been extended to the required generality in the measure valued context. The talks of A. Cox and S. Källblad focused on applications to problems in robust finance while C. Cuchiero explained the connection with energy market modelling.

Transport of random measures. Random versions of optimal transport problems are very natural from a statistical point of view. One of the classical versions is the optimal matching problem which asks for speed of convergence of an empirical measure $\frac{1}{n}\sum_{i=1}^{n}\delta_{X_i}$ to its equilibrium with the model case given by iid uniform samples on the unit cube. But also dependent samples or occupation measures are of interest. While this problem is quite old it receives constant attention in the literature.

M. Goldman explained an approach to the matching problem via linearization of the Monge-Ampere equation proposed in the physics communitiy and how this allows to make significant progress on a refined analysis of the matching problem to show existence of limits and fluctuation results. F. Mattesini showed that the same linearization ansatz allows to show the non-existence of a thermodynamic limit of the solutions to the optimal matching problem in a particular regime in dimension two. J. Jalowy reported on new precise asymptotics for a dependent matching model from random matrix theory.

Computational aspects/EOT/Schrödinger. In view of the applied nature of (probilistic) mass transport, the availability of efficient numeric algorithm plays a crucial role. Famously, entropic regularization of classical optimal transport plays a key role in enabling efficient algorithms with provable convergence, in particular the Sinkhorn algorithm. Morover, this entropic transport problem can be seen as a static instance of the Schrödinger bridge problem and enjoys numerous benefits resulting from regularization such as smoothness and statistical properties.

G. Conforti discussed short time limits and stability properties of the Schrödinger problem. M. Nutz presented results on the convergence of the Sinkhorn algorithm in surprisingly general setups. A. Alfonsi and P. Siorpaes discussed the discretisation of (mult-marginal) transport problems under additional linear constraints.

3 Presentation Highlights

AURELIEN ALFONSI

Approximation of Optimal Transport problems with marginal moments constraints

We investigate the relaxation of Optimal Transport problems when the marginal constraints are replaced by some moment constraints. Using Tchakaloff's theorem, we show that the Moment Constrained Optimal Transport problem (MCOT) is achieved by a finite discrete measure. Interestingly, for multimarginal OT problems, the number of points weighted by this measure scales linearly with the number of marginal laws, which is encouraging to bypass the curse of dimension. We show the convergence of the MCOT problem toward the corresponding OT problem. In some fundamental cases, we obtain rates of convergence in $O(\frac{1}{n})$ or $O(\frac{1}{n^2})$, where n is the number of moments. We will present applications of this approach, in particular for symmetric multimarginal optimal transportation problems arising in physics.

GUILLAUME CARLIER

Convex geometry of finite exchangeable laws and de Finetti style representation with universal correlated corrections

We present a novel analogue for finite exchangeable sequences of the de Finetti, Hewitt and Savage theorem and investigate its implications for multi-marginal optimal transport (MMOT) and Bayesian statistics. If $(Z_1, ..., Z_N)$ is a finitely exchangeable sequence of N random variables taking values in some Polish space X, we show that the law μ_k of the first k components has a representation of the form

$$\mu_k = \int_{\mathcal{P}_{\frac{1}{N}}(X)} F_{N,k}(\lambda)\, \mathrm{d}\alpha(\lambda)$$

for some probability measure α on the set of $\frac{1}{N}$-quantized probability measures on X and certain universal polynomials $F_{N,k}$. The latter consist of a leading term $N^{k-1}/\prod_{j=1}^{k-1}(N-j)\lambda^{\otimes k}$ and a finite, exponentially decaying series of correlated corrections of order N^{-j} ($j = 1, ..., k$). The $F_{N,k}(\lambda)$ are precisely the extremal such laws, expressed via an explicit polynomial formula in terms of their one-point marginals λ. Applications include novel approximations of MMOT via polynomial convexification and the identification of the remainder which is estimated in the celebrated error bound of Diaconis-Freedman between finite and infinite exchangeable laws.

GIOVANNI CONFORTI

Schrödinger problem: short-time limits and stability.

The aim of this talk is to present two set of results on the Schrödinger bridge problem that can be obtained leveraging logarithmic Sobolev inequalities, both in their integrated and local form. The first result is about the short time (small noise) convergence of the gradient of Schrödinger potentials to the Brenier map. The second set of results is about the stability of Schrödinger bridges and the entropic cost with respect to perturbations in the marginal measures. Based on a joint work with A. Chiarini, G. Greco, and L. Tamanini.

KRZYSZTOF CIOSMAK

Towards multi-dimensional localisation

Localisation is a powerful tool in proving and analysing various geometric inequalities, including isoperimetric inequality in the context of metric measure spaces. Its multi-dimensional generalisation is naturally linked to optimal transport of vector measures and vector-valued Lipschitz maps. I shall present recent developments in this area: a partial affirmative answer to a conjecture of Klartag concerning partitions associated to Lipschitz maps on Euclidean space, and a negative answer to another conjecture of his concerning mass-balance condition for absolutely continuous vector measures. During the course of the talk I shall also discuss an intriguing notion of ghost subspaces related to the above mentioned partitions.

ALEXANDER COX

Controlled measure-valued martingales: a viscosity solution approach

We introduce a stochastic control problem for measure-valued martingales, which are stochastic processes taking values in the space of probability measures and satisfying a martingale-like property. Such control problems arise naturally in a variety of applications, such as the Skorokhod Embedding problem, and informational games with asymmetric information.

Our main results concern characterisation of the value of the problem in terms of the viscosity solution to a corresponding PDE. Joint work with Sigrid Källblad, Martin Larsson and Sara Svaluto-Ferro.

Christa Cuchiero

Measure-valued processes for energy markets

We introduce a framework that allows to employ (non-negative) measure-valued processes for energy market modeling, in particular for electricity and gas futures. Interpreting the process' spatial structure as time to maturity, we show how the Heath-Jarrow-Morton approach (see [6]) can be translated to this framework, thus guaranteeing arbitrage free modeling in infinite dimensions. We derive an analog to the HJM-drift condition and then treat in a Markovian setting existence of (non-negative) measure-valued diffusions that satisfy this condition. To analyze mathematically convenient classes we consider measure-valued polynomial and affine diffusions where we can precisely specify the diffusion part. Indeed, it depends on continuous functions satisfying certain admissibility conditions. For calibration purposes these functions can then be parametrized by neural networks yielding measure-value analogs of neural SPDEs. Exploiting on the one the analytic tractability coming from the affine and polynomial nature and on the other hand stochastic gradient descent methods for neural networks allows for efficient mass transport in infinite dimensions from the initial distribution of the measure-valued process encoding the current forward curve to the distribution of future forward curves implied from option prices. The talk is based on joint work with Luca Di Persio and Francesco Guida

Michael Goldman

On recent progress on the optimal matching problem

The optimal matching problem is a classical random combinatorial problem which may be interpreted as an optimal transport problem between random measures. Recent years have seen a renewed interest for this problem thanks to the PDE ansatz proposed in the physics literature by Caracciolo and al. and partially rigorously justified by Ambrosio-Stra-Trevisan. In this talk I will show how this ansatz combined with subadditivity may be used to give information both on the optimal cost and on the structure of the optimal transport map at various scales. This is based on joint works with L. Ambrosio, M. Huesmann, F. Otto and D. Trevisan.

Ivan Guo

Robust hedging of American options in continuous time

In this work, we look at the problem of robust pricing and hedging American-style options whose underlyings are modelled by continuous semimartingales. Similar to previous related works in discrete time (e.g., [7]), we enlarge the probability space with the stopping decision and use path-dependent optimal transport [8] to establish pricing-hedging dualities for European claims in the enlarged space. To connect this to the original space, we show that martingale measures in the enlarged space can be expressed as convex combinations of martingale measures in the original space coupled with true stopping times. This combination is identified by decomposing the Azéma supermartingale of the associated random time. This in turn allows us to prove the pricing-hedging duality for American options in continuous time.

Jonas Jalowy

The Wasserstein distance between complex eigenvalues and the Circular Law

It is well known that the expected Wasserstein distance between the empirical measure of n i.i.d. points and the uniform measure is of order $\sqrt{\log n/n}$. However, the repulsive behavior of complex eigenvalues of random matrices forces the point process to be more evenly spread. This phenomenon will be illustrated by simulations and shall be quantified in terms of the Wasserstein distance.

BENJAMIN JOURDAIN

Approximation of martingale couplings on the real line in the adapted weak topology (joint work with M. Beiglböck, W. Margheriti and G. Pammer)

When approximating in Wasserstein distance the two marginals of a martingale coupling by probability measures in the convex order, it is possible to construct a sequence of martingale couplings between these probability measures converging in adapted Wasserstein distance to the original coupling. We deduce the stability with respect to the marginal distributions of the Weak Martingale Optimal Transport problem in dimension one.

NICHOLAS JUILLET

A martingale exactly fitting an infinite family of given marginals

I will present a systematic construction for families of real random variables $(X_t)_{t\geq 0}$ that 1) are a martingale 2) go through any given family of marginals $(\mu_t)_{t\geq 0}$ —provided the necessary condition that μ_t is increasing (for t) in convex order. Our construction method is based on the shadow martingale couplings introduced in a previous work with Mathias Beiglböck. We show optimality properties and a Choquet representation for the constructed martingale. The talk is based on joint work with Martin Brückerhoff and Martin Huesmann.

SIGRID KÄLLBLAD

Measure-valued martingales: analysis and applications

In this talk we consider a class of probability measure-valued process satisfying an additional martingale condition on its dynamics, called measure-valued martingales (MVMs). We will discuss various structural properties of such processes; in particular, we present an appropriate version of Itô's formula for controlled MVMs and an existence result for an SDE featuring MVMs. We also illustrate how problems of this type arise in a number of applications paying particular attention to their connection to some functional inequalities. The talk is based on joint work with Alex Cox, Martin Larsson and Sara Svaluto-Ferro.

YOUNG-HEON KIM

The Stefan problem and free targets of optimal Brownian martingale transport

We discuss an optimal Brownian stopping problem from a given initial distribution where the target distribution is free and is conditioned to satisfy a given density height constraint. The solutions to this optimization problem then generate solutions to the Stefan problem, a free boundary problem of the heat equation that describes supercooled fluid freezing (St1) or ice melting (St2), depending on the type of cost for optimality. The freezing (St1) case has not been well understood in the literature beyond one dimension, while our result gives a well-posedness of weak solution in general dimensions, with naturally chosen initial data. We also give a new connection between the freezing and melting Stefan problems. This is joint work with Inwon Kim (UCLA).

MICHAEL KUPPER

Wasserstein perturbation of Markovian semigroups

We deal with a class of time-homogeneous continuous-time Markov processes with transition probabilities bearing a nonparametric uncertainty. The uncertainty is modelled by considering perturbations of the transition probabilities within a proximity in Wasserstein distance. As a limit over progressively finer time periods, on which the level of uncertainty scales proportionally, we obtain a convex semigroup satisfying a nonlinear PDE. In standard situations, the nonlinear transition operators arising from nonparametric uncertainty coincide with the ones related to parametric drift uncertainty. The results are illustrated with Wasserstein perturbations of Lévy processes, Ornstein-Uhlenbeck processes, and Koopman semigroups. The talk is based on joint works with Daniel Bartl, Stephan Eckstein, Sven Fuhrmann, and Max Nendel.

DANIEL LACKER

New results on quantitative propagation of chaos for mean field diffusions

This talk discusses new quantitative results on mean field limits for large n-particle diffusion systems. The 11distance" between the marginal law of k particles and its limiting product measure is shown to be $O((\frac{k}{n})^2)$ at each time, as long as the same is true at time zero. A simple Gaussian example shows that this rate is optimal. The best previously known bound was $O(\frac{k}{n})$ and was widely believed to be optimal. Various "distances" are possible here, with relative entropy playing a key role: The new approach relies on establishing transport inequalities for the limiting measure, as well as a differential inequality which bounds the k-particle entropy in terms of the $(k+1)$-particle entropy, derived from a form of the BBGKY hierarchy.

TONGSEOK LIM

Generalized Shapley axioms and value allocation in cooperative games via Hodge theory on graphs

Lloyd S. Shapley introduced a set of axioms in 1953, now called the Shapley axioms, and showed that the axioms characterize a natural allocation among the players who are in grand coalition of a cooperative game. Recently, A. Stern and A. Tettenhorst showed that a cooperative game can be decomposed into a sum of component games, one for each player, whose value at the grand coalition coincides with the Shapley value. The component games are defined by the solutions to the naturally defined system of least squares – or Poisson – equations via the framework of the Hodge decomposition on the hypercube graph.

In this talk we propose a new set of axioms which characterizes the component games. Furthermore, we realize them through an intriguing stochastic path integral driven by a canonical Markov chain. The integrals are natural representation for the expected total contribution made by the players for each coalition, and hence can be viewed as their fair share. This allows us to interpret the component game values for each coalition also as a valid measure of fair allocation among the players in the coalition. Finally, we extend the path integrals on general graphs and discover an interesting connection between stochastic integrations and Hodge theory on graphs.

FRANCESCO MATTESINI

There is no invariant cyclically monotone Poisson matching in 2d

The optimal matching problem is a classical random variational problem that received interest in the last 30 years. We show that there exists no cyclically monotone invariant matching of two independent Poisson processes in the critical dimension $d = 2$. Our argument relies on a recent harmonic approximation theorem together with the two-dimensional local asymptotics for the bipartite matching problem, for which we provide a new self-contained proof based on martingale arguments. Joint work with M. Huesmann (WWU Münster) and F. Otto (MPI Leipzig).

ROBERT MCCANN

On the Monopolist's Problem Facing Consumers with Nonlinear Price Preferences

The principal-agent problem is an important paradigm in economic theory for studying the value of private information; the nonlinear pricing problem faced by a monopolist is a particular example. In this lecture, we identify structural conditions on the consumers' preferences and the monopolist's profit functions which guarantee either concavity or convexity of the monopolist's profit maximization. Uniqueness and stability of the solution are particular consequences of this concavity. Our conditions are similar to (but simpler than) criteria given by Trudinger and others for prescribed Jacobian equations to have smooth solutions. By allowing for different dimensions of agents and contracts,nonlinear dependence of agent preferences on prices, and of the monopolist's profits on agent identities, we improve on the literature in a number of ways. The same mathematics can also be adapted to the maximization of societal welfare by a regulated monopoly. In the classical case of bilinear preferences, we introduce a new duality for certifying solutions, which leads to a free boundary formulation for the missing region in the square example of Rochet and Chone. This represents joint work with Kelvin Shuangjian Zhang.

DAN MIKULINCER

The Brownian Transport Map

The existence of a transport map from the standard Gaussian leads to succinct representations for, potentially complicated, measures. Inspired by results from optimal transport, we introduce the Brownian transport map that pushes forward the Wiener measure to a target measure in a finite-dimensional Euclidean space. Using tools from Itô's and Malliavin's calculus, we show that the map is Lipschitz when the target measure satisfies an appropriate convexity assumption. This facilitates the proof of several new functional inequalities. In other settings, where a globally Lipschitz transport map cannot exist, we derive Sobolev estimates which are intimately connected to several famous open problems in convex geometry. Joint work with Yair Shenfeld.

MARCEL NUTZ

Stability of Entropic Optimal Transport and Convergence of Sinkhorn's Algorithm

We discuss entropically regularized optimal transport and its stability with respect to the marginals. A qualitative result (for weak convergence) is obtained using the geometric notion of c-cyclical monotonicity and a quantitative result (for Wasserstein distance) is obtained by control theoretic methods. These results can be applied to deduce convergence of Sinkhorn's algorithm for unbounded cost functions such as the quadratic cost and find a convergence rate in Wasserstein sense. Based on joint works with Espen Bernton, Stephan Eckstein, Promit Ghosal, Johannes Wiesel.

SOUMIK PAL

Gradient flows on graphons

Wasserstein gradient flows often arise from mean-field interactions among exchangeable particles. In many interesting applications however, the "particles" are edge weights in a graph whose vertex labels are exchangeable but not the edges themselves. We investigate the question of optimization of functions over this different class of symmetries. This leads us to gradient flows or curves of maximal slopes on the metric space of continuum graphs called graphons. This is a generalization of Wasserstein calculus to higher order exchangeable structures.

GUDMUND PAMMER

The Wasserstein space of stochastic processes & computational aspects

Wasserstein distance induces a natural Riemannian structure for the probabilities on the Euclidean space. This insight of classical transport theory is fundamental for tremendous applications in various fields of pure and applied mathematics. We believe that an appropriate probabilistic variant, the adapted Wasserstein distance AW, can play a similar role for the class FP of filtered processes, i.e. stochastic processes together with a filtration. In contrast to other topologies for stochastic processes, probabilistic operations such as the Doob-decomposition, optimal stopping and stochastic control are continuous w.r.t. AW. We also show that (FP, AW) is a geodesic space, isometric to a classical Wasserstein space, and that martingales form a closed geodesically convex subspace. Finally we consider computational aspects and provide a novel method based on the Sinkhorn algorithm. The talk is based on articles with Daniel Bartl, Mathias Beiglböck and Stephan Eckstein.

WALTER SCHACHERMAYER

Martingale Transport, De March-Touzi Pavings, and Stretched Brownian Motion

In classical optimal transport, the contributions of Benamou-Brenier and McCann regarding the time-dependent version of the problem are cornerstones of the field and form the basis for a variety of applications in other mathematical areas. For μ, ν probability measures on R^d, increasing in convex order, stretched Brownian motion [1] provides an analogue for the martingale version of this problem. In dimension $d = 1$ it was shown in [2] that any martingale transport decomposes into at most countably many invariant intervals and that this decomposition is universal. Extensions of this result to $d \geq 2$ were studied in [4], [5], [3]. We show that the dual optimization problem attached to a stretched Brownian motion induces the universal DeMarch–Touzi paving [3] of R^d. Joint work with M. Beiglböck, J. Backhoff, and B. Tschiderer

ANDRÉ SCHITLING & MATTHIAS ERBER

Covariance-modulated optimal transport and gradient flows

We study a variant of the dynamical optimal transport problem in which the energy to be minimised is modulated by the covariance matrix of the current distribution. Such transport metrics arise naturally in mean field limits of recent particle filtering methods for inverse problems. We show that the transport problem splits into two separate minimisation problems: one for the evolution of mean and covariance of the interpolating curve and one its shape. The latter consists in minimising the usual Wasserstein length under the constraint of maintaining fixed mean and covariance along the interpolation. We analyse the geometry induced by this modulated transport distance on the space of probabilities as well as the dynamics of the associated gradient flows. Those show better convergence properties in comparison to the classical Wasserstein metric in terms of exponential convergence rates independent of the Gaussian target. Also on the level of the gradient flows a similar splitting into the evolution of moments and shapes of the distribution can be observed. This is joint work of Martin Burger, Matthias Erbar, Franca Hoffmann, Daniel Matthes and André Schlichting.

YAIR SHENFELD

Transportation along Langevin dynamics

The existence of Lipschitz transport maps between probability measures leads to transportation of functional inequalities from the source to the target measure. Finding such maps, however, is a non-trivial problem. One successful example is Caffarelli's contraction theorem: The optimal transport map is 1-Lipschitz if the source measure is Gaussian and the target measure is strongly log-concave. On the other hand, if this stringent assumption on the target measure is omitted, much less is known about the Lipschitz properties of the optimal transport map. We show, together with Dan Mikulincer, that these questions can be resolved if we work with transportation of measure along Langevin dynamics rather than with optimal transport. In particular, we show that the Langevin dynamics yields Lipschitz transport maps from the Gaussian to target measures which are either semi-log-concave with bounded support, or mixtures of Gaussians. In turn, this transport map leads to improved functional inequalities. Time permitting, I will talk about further work in progress with Max Fathi, Dan Mikulincer, and Joe Neeman.

PIETRO SIORPAES

How to discretize some optimal transport problems with linear constraints

Given a vector $\mu = (\mu_i)_{i=1}^n$ of probabilities, consider an optimization problem of the form

$$\mathcal{P} := \inf E[c(X)], \qquad \text{(P)}$$

where the infimum runs over all the (laws π of) random vectors $X = (X_i)_{i=1}^n$ with marginals $X_i \sim \mu_i$, and which satisfy some additional linear constraints. When μ is finitely supported, (P) is a linear program, and thus is well understood and can be solved numerically with high efficiency. It is then of interest to approximate general μ with some finitely supported μ^k which satisfy the same linear constraints, ideally in a way that the optimal values $\mathcal{P}(\mu^k) \to \mathcal{P}(\mu)$ converge as $k \to \infty$ and so do the corresponding minimisers $\pi^k \to \pi$, and possibly μ^k satisfy some optimality property and/or some additional constraint. We consider the case where X is required to be a vector-valued martingale, thus constructing a quantization method which preserves the convex order on measures on a separable Banach space. This is joint work with Marco Massa.

DARIO TREVISAN

Quantitative Gaussian Approximation of Randomly Initialized Deep Neural Networks

Given any deep fully connected neural network, initialized with random Gaussian parameters, we bound from above the quadratic Wasserstein distance between its output distribution and a suitable Gaussian process. Our explicit inequalities indicate how the hidden and output layers sizes affect the Gaussian behaviour of the network and quantitatively recover the distributional convergence results in the wide limit, i.e. if all the hidden layers sizes become large. Joint with A. Basteri (U. Pisa)

JOHANNES WIESEL

Measuring association with Wasserstein distances

Let $\pi \in \Pi(\mu, \nu)$ be a coupling between two probability measures μ and ν on a Polish space. In this talk we propose and study a class of nonparametric measures of association between μ and ν, which we call Wasserstein correlation coefficients. These coefficients are based on the Wasserstein distance between ν and the disintegration of π with respect to the first coordinate. We also establish basic statistical properties of this new class of measures: we develop a statistical theory for strongly consistent estimators and determine their convergence rate in the case of compactly supported measures μ and ν. Throughout our analysis we make use of the so-called adapted/bicausal Wasserstein distance, in particular we rely on results established in [9]. Our approach applies to probability laws on general Polish spaces.

4 Summary and outlook

The meeting was held in a hybrid format which naturally limited the usual capacity for spontaneous discussions and exchanges. Nevertheless, despite most of the participants being scattered around the globe in different time zones, all the talks were very well attended and the format worked well, allowing for some questions and discussion. Those who travelled to Banff were able to take part in many discussions and reported great satisfaction with being able to participate in person.

Scientifically, the workshop brought many people together for the first time. There was a great wealth of topics and problems discussed but with a strong common thread and clear potential for cross-fertilisation. The field felt rich and often nascent - a great number of talks finished with open questions and conjectures. There was a strong feeling that the workshop opened up a discussion. We are hopeful to continue the discussion via a subsequent workshop and are excited at the prospect of developments which might be presented next time round.

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