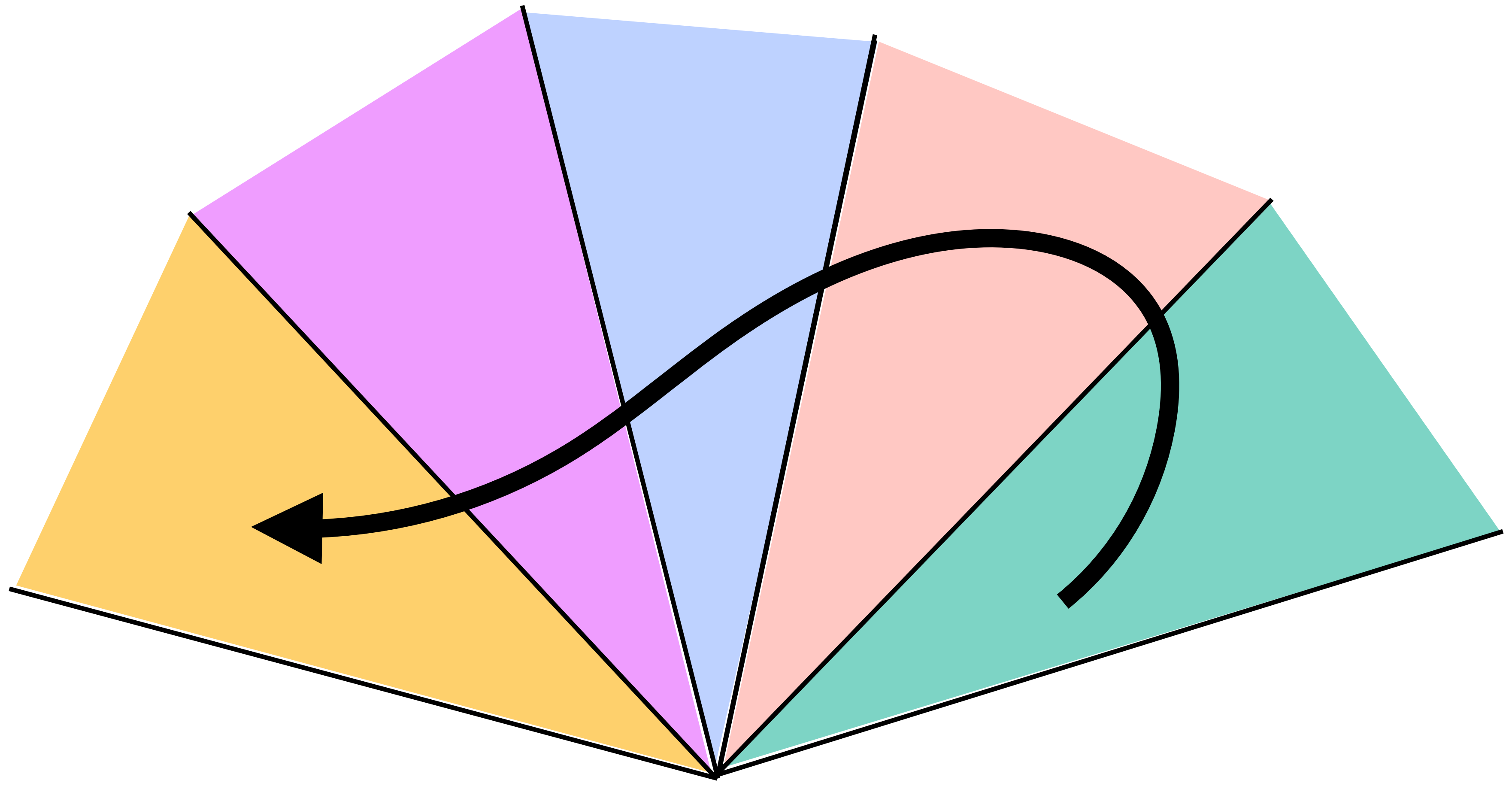
Flops, Topological Invariants, and the Weak Gravity Conjecture



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based on 22XX.XXXX with Ben Heidenreich, Liam McAllister, Jakob Moritz, and Tom Rudelius
Banff Workshop on Geometry and Swampland

Summary

We describe a framework for reconstructing the Kähler moduli spaces of Calabi-Yau threefolds from Gopakumar-Vafa invariants.

We then use this reconstruction to verify the Weak Gravity Conjecture in a large ensemble of geometries.

Outline

1. The Weak Gravity Conjecture (WGC) in geometry

2. Reconstruction of the Kähler moduli space with Gopakumar-Vafa (GV) invariants

3. Testing the Weak Gravity Conjecture

The Weak Gravity Conjecture

[Arkani-Hamed, Motl, Nicolis, Vafa ’06]

Consider a gravitational theory containing a U(1) gauge field.

Then to be a consistent theory of quantum gravity, there should exist a particle with

$$\frac{Q}{M} \geq \left(\frac{Q}{M}\right)_{\text{extremal}}$$

extremality bound of the black hole solutions

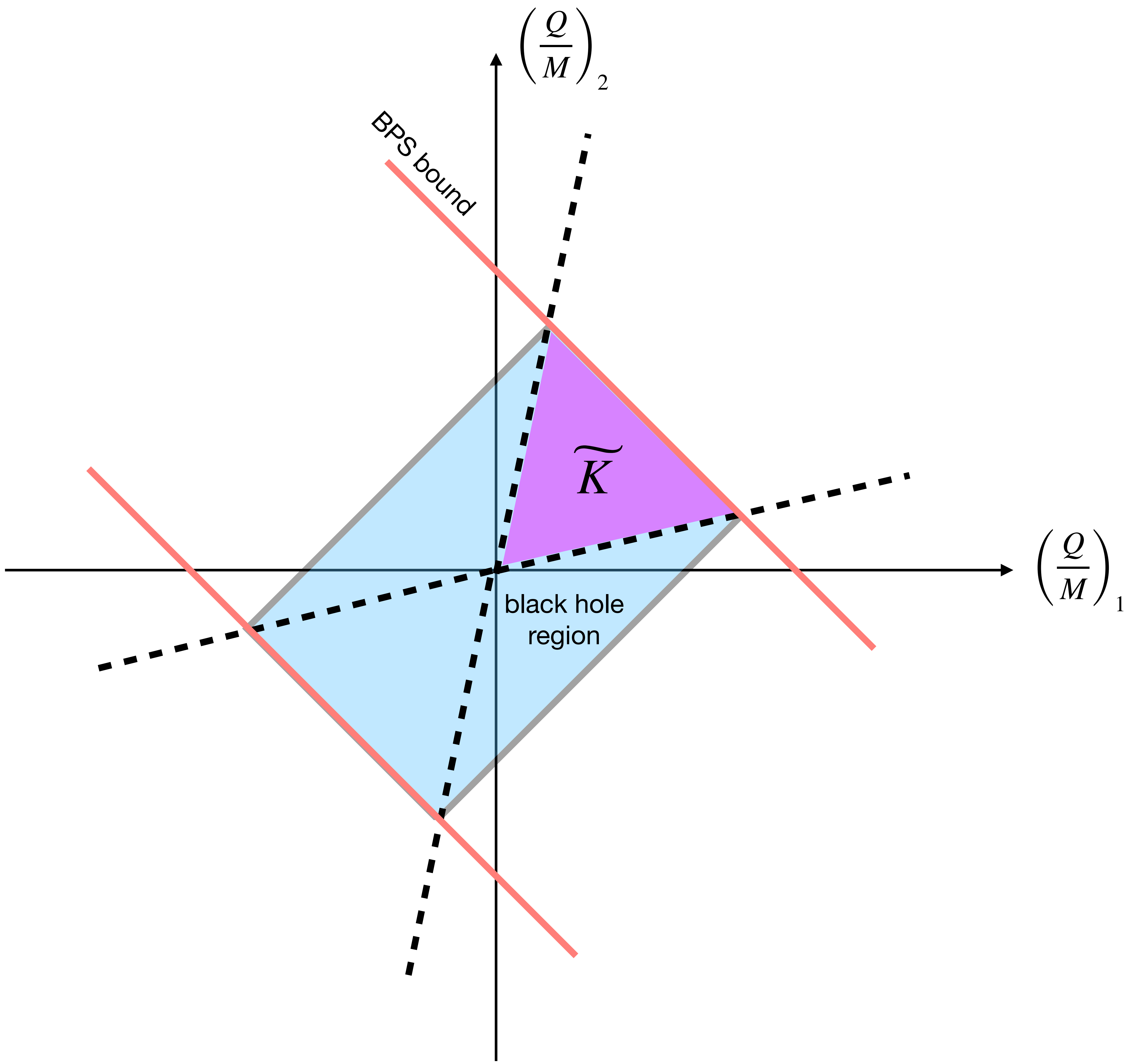
Tower Weak Gravity Conjecture

[Andriolo, Junghans, Noumi, Shiu, ’18; Heidenreich, Reece, Rudelius ’16]

There should exist an infinite number of particles that satisfy the bound.

Hard to check in general, but can check in certain regions of charge space using **BPS states.**

BPS bound vs. extremality bound



The tower WGC implies:

For every direction $\overrightarrow{q}$ in $\widetilde{K}$, there exists an infinite number of holomorphic curves hosting single-particle states with charges $\propto \overrightarrow{q}$.

[Alim, Heidenreich, Rudelius '21]

Required ingredients

1. what are the single particle states?

2. what is $\widetilde{K}$?

Setting for checking the WGC

Consider M-theory compactified on a Calabi-Yau threefold, X.

This gives rise to a five-dimensional effective theory which has the feature that **the polynomial prepotential is exact:**

$$\mathcal{F} = \frac{1}{6}\kappa_{ijk}t^i t^j t^k$$

where the κ_{ijk} are the triple intersection numbers of the Calabi-Yau, and the t^i are the Kähler parameters.

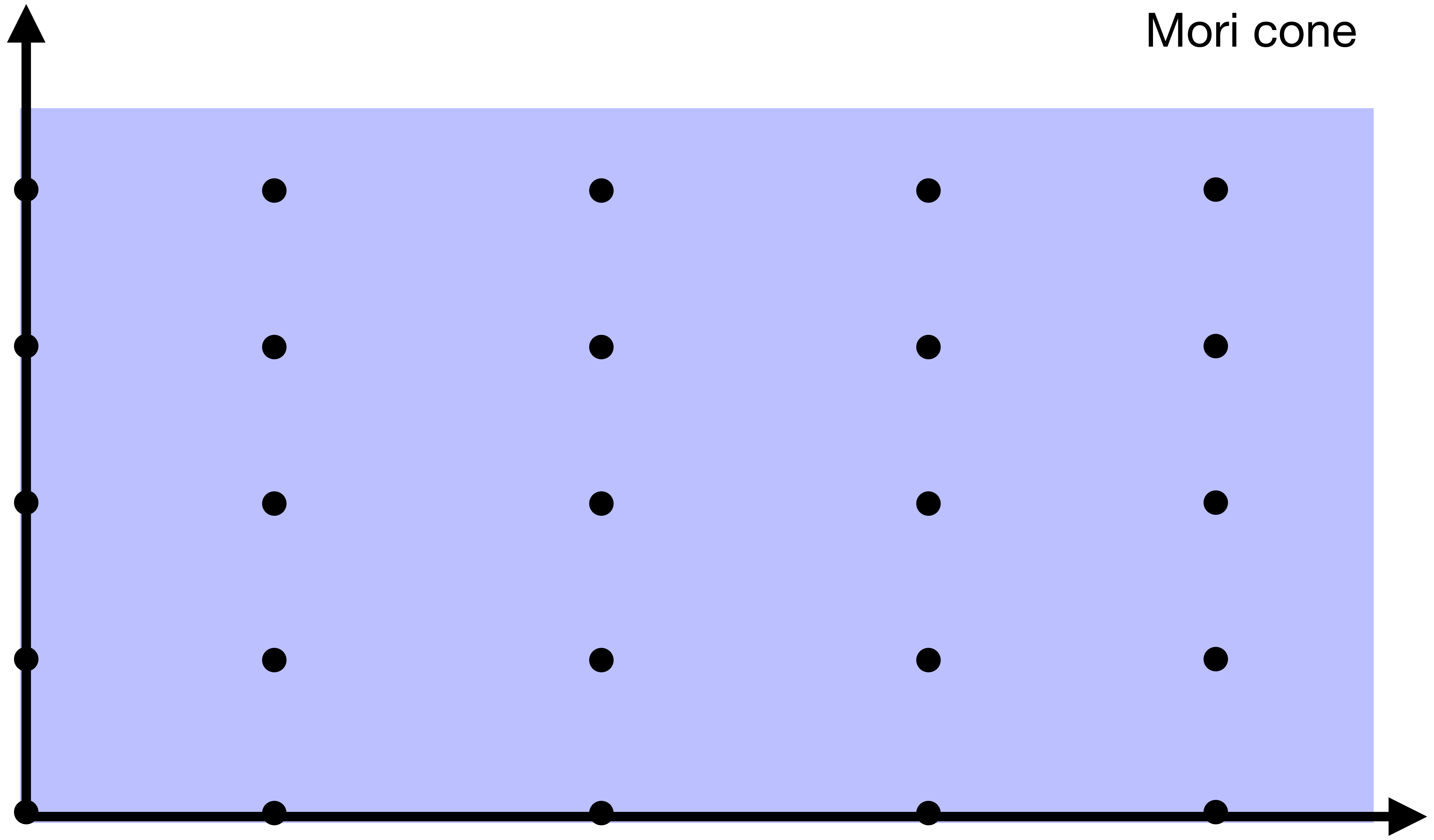
M2 branes wrapped on holomorphic curves are BPS particles (aka they saturate the BPS bound). We will verify that these satisfy the WGC.

BPS states/holomorphic curves

BPS states are M2 branes wrapped on holomorphic curves.
These saturate the **BPS bound:**

$$M \geq |Z(q_i)|$$

BPS states come in cones in the charge space, and are roughly counted by Gopakumar-Vafa invariants



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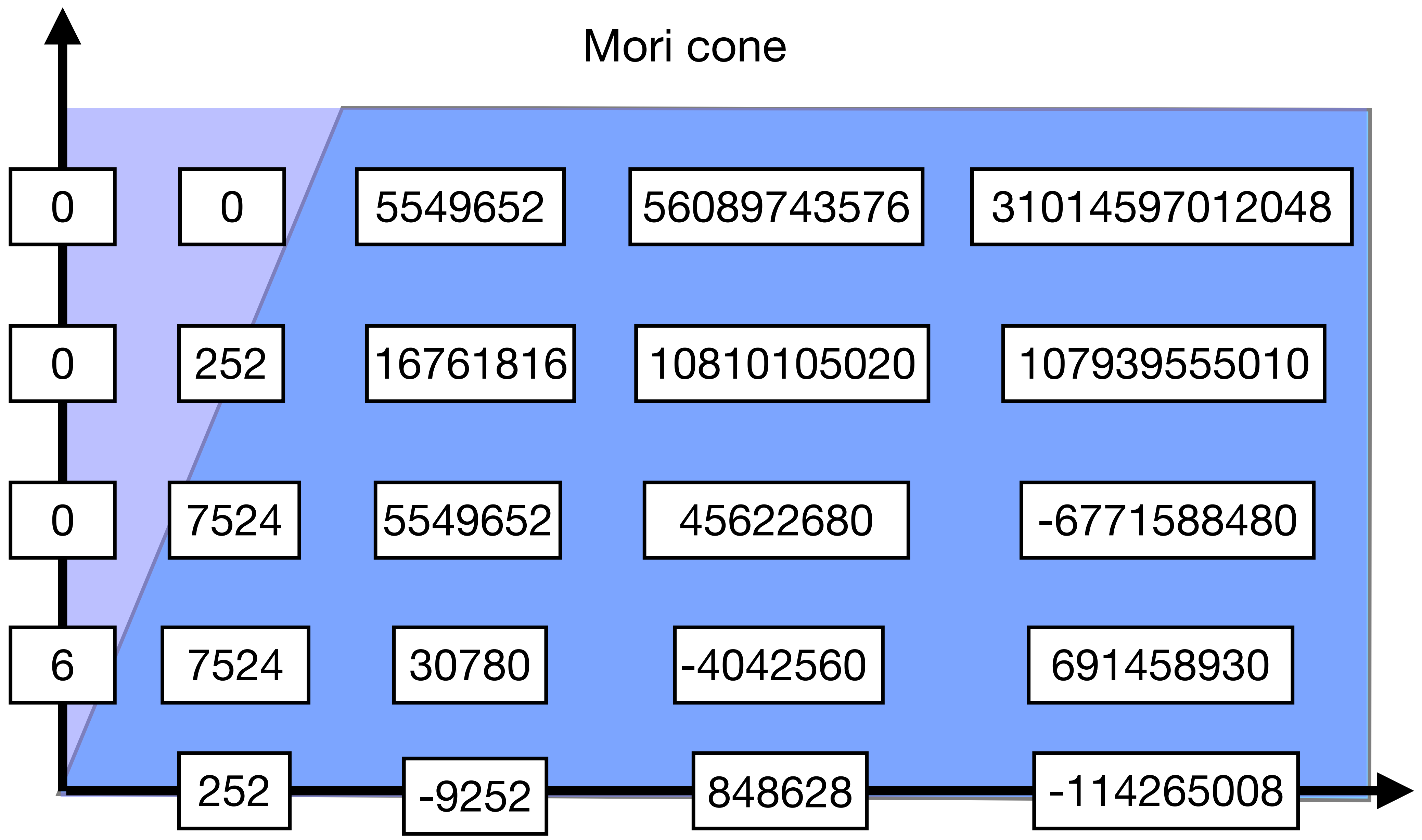

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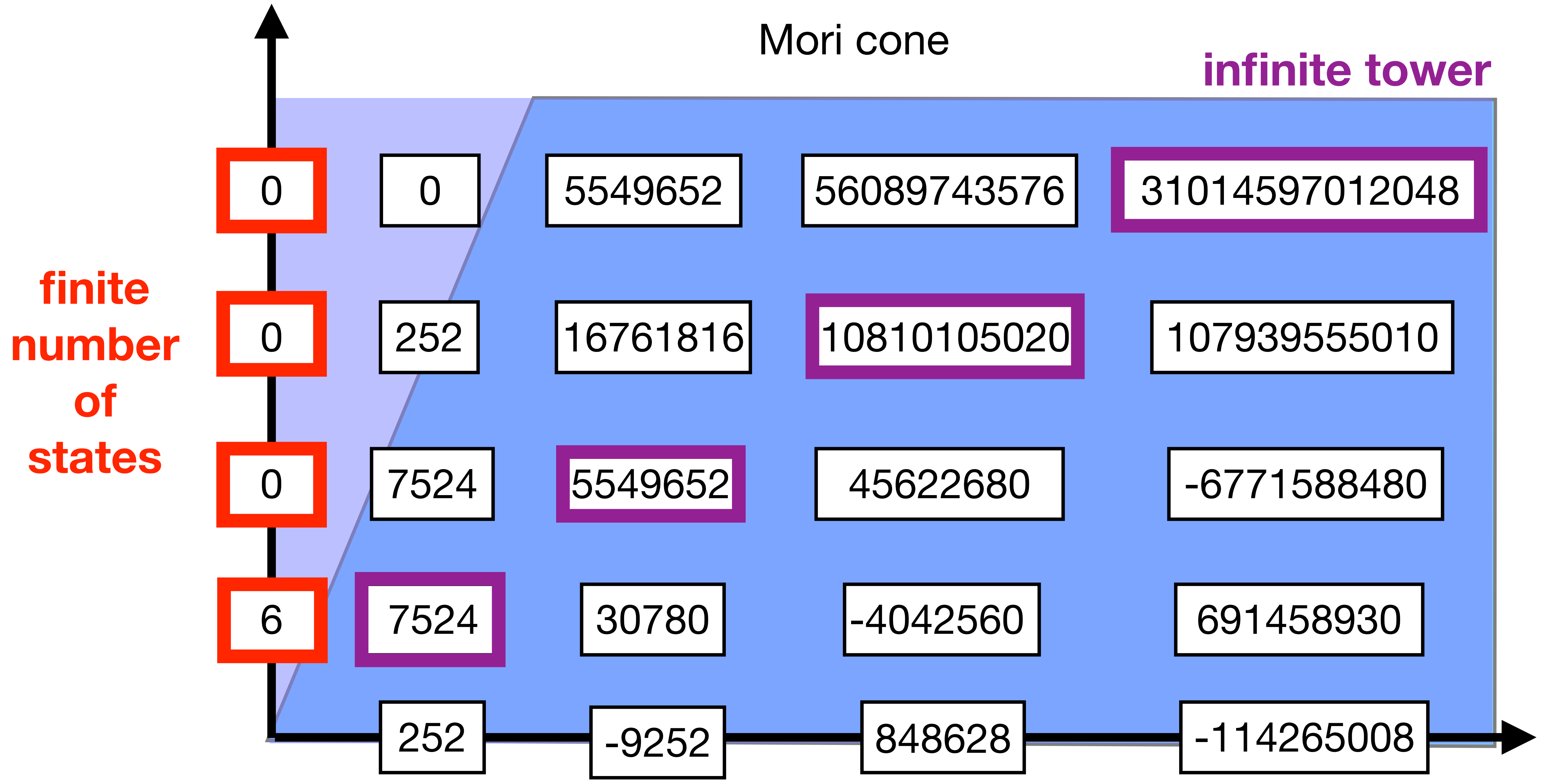
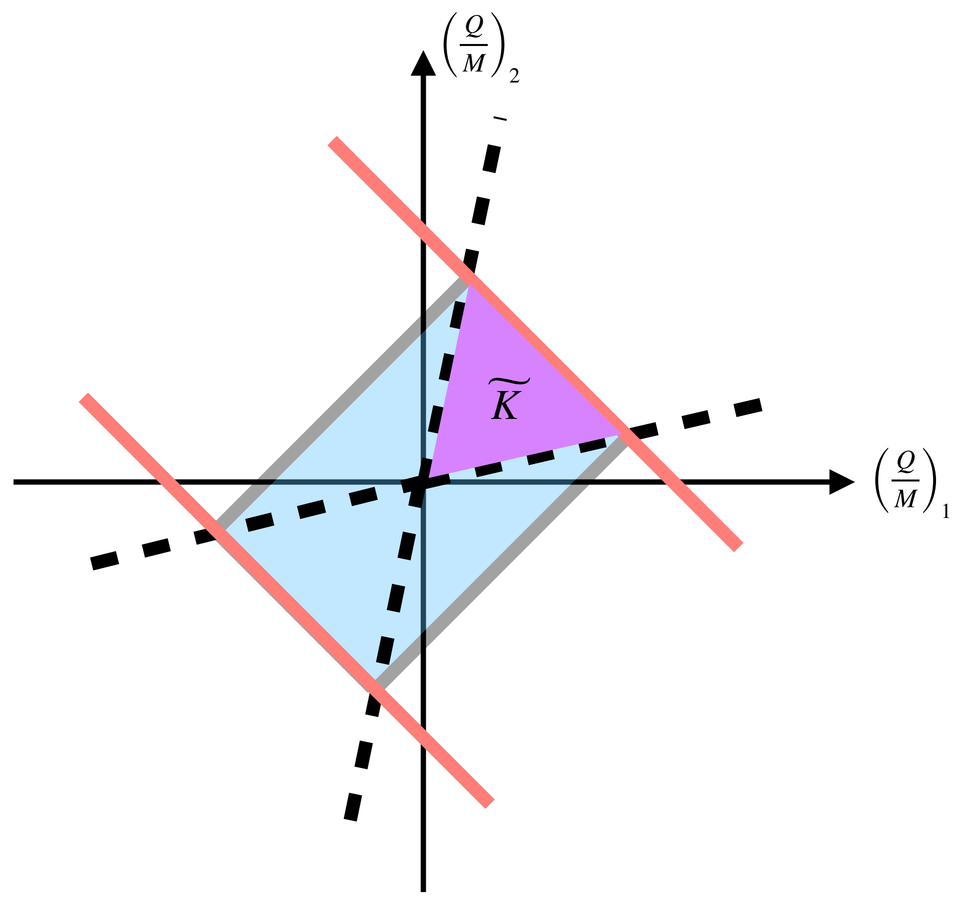


With some effort, one can use CYtools to compute **genus-0** GV invariants for CY threefolds in the Kreuzer-Skarke database. [Hosono, Klemm, Theisen, Yau ’94; Demirtas, Kim, Mcallister, Moritz, Rios-Tascon, to appear]

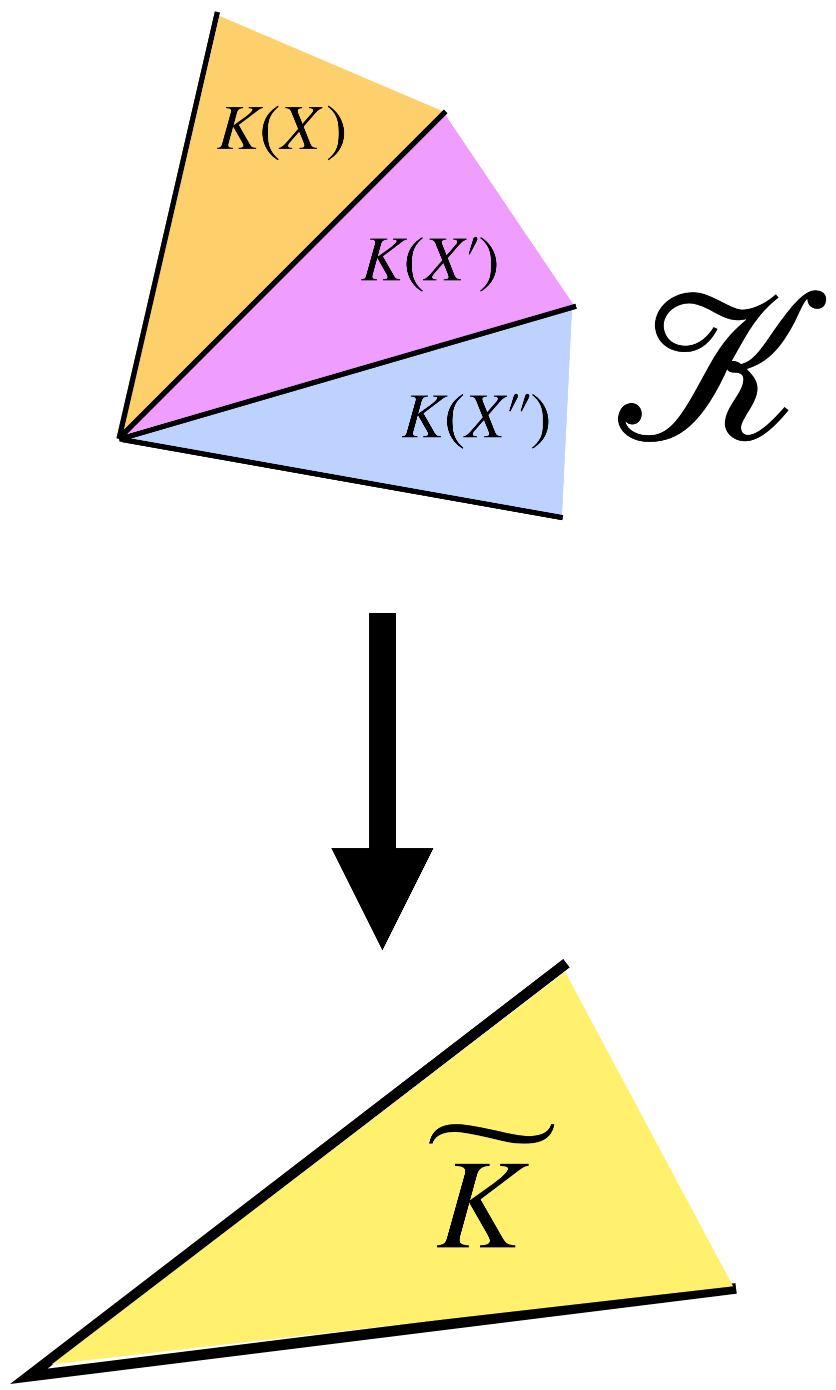
Verifying the tower WGC amounts to verifying that all rays with a finite number of states lies outside $\widetilde{K}$.

$\widetilde{K}$: where BHE=BPS



We want to find the cone where spherically symmetric BPS black holes can exist.

Alim, Heidenreich, and Rudelius ('21) proved that this is contained in the cone $\widetilde{K}$, defined in the following way.



Definition: The **extended Kähler cone,** $\mathscr{K}$**,** is the union of Kähler cones of all Calabi-Yaus birationally equivalent to X.

Definition: $\widetilde{K}$ is the cone obtained by applying the "dual coordinate map" to $\mathscr{K}$:

$$\tau_i = \frac{1}{2}\kappa_{ijk}t^j t^k$$

Where the κ_{ijk} are the triple intersection numbers of each CY X_i .

Upshot: need to find the extended Kähler cone.

Reconstructing Kähler moduli space with GV invariants

Main point: given only the GV invariants of a Calabi-Yau X, we can reconstruct the extended Kähler cone.

How? The GV invariants tell us which curves can be shrunk without inducing an infinite number of massless states.

First, let’s describe abstractly how this works.

Then we’ll apply it in our example from earlier.

Flops and the extended Kähler cone

Let's compactify our 5D theory on a circle to obtain a **4D theory in type IIA.**

The prepotential now takes the form [Candelas, de la Ossa, Font, Katz, Morrison '93; Gopakumar, Vafa '98]

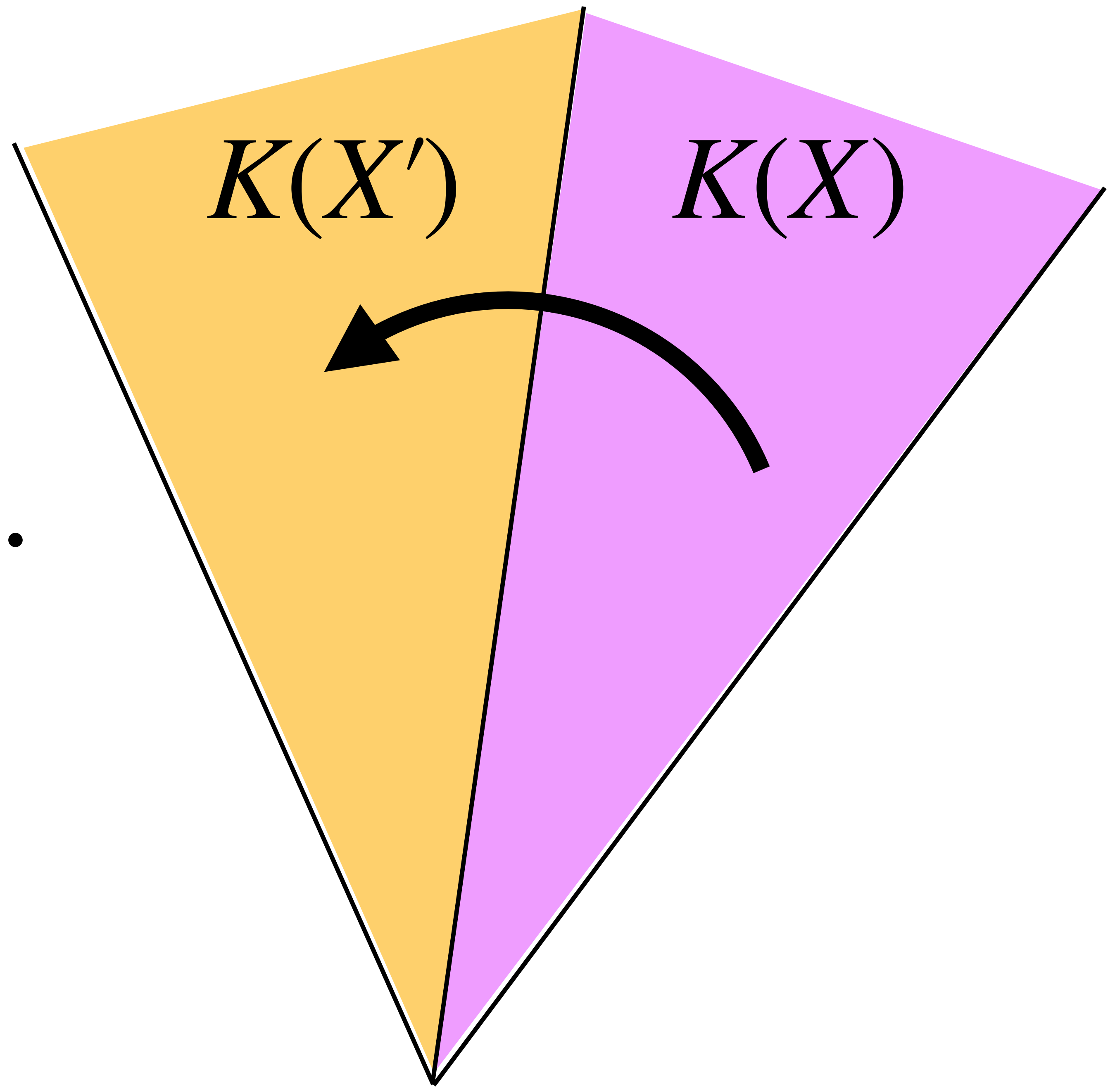
$$\mathcal{F}_X = p(\kappa_{abc}, t^a, t^b, t^c) - \frac{1}{(2\pi i)^3} \sum_{[\mathcal{C}]\in \mathrm{Mori}(X)} n^0_{[\mathcal{C}]} \mathrm{Li}_3(e^{2\pi i [\mathcal{C}]\cdot \vec{t}})$$

Where the sum runs over the curve classes in the Mori cone and $n^0_{[\mathcal{C}]}$ is the genus-0 GV invariant of the curve class $[\mathcal{C}]$.

Flops and the extended Kähler cone

Now suppose that there is a ray in the Mori cone with only a single curve class with non-vanishing GV invariant, $\mathcal{C}_0$.

Let's change the Kähler parameter $\text{vol}(\mathcal{C}_0)$ so that it formally becomes negative: $\text{vol}(\mathcal{C}_0) \to -\text{vol}(\mathcal{C}_0)$



By doing this, we enter the Kähler cone of a **new** Calabi-Yau, X', which is birationally equivalent to X .

Let's now see why that is.

Flops and the extended Kähler cone

Start with the prepotential:

$$\mathcal{F}_X = p(\kappa_{abc}, t^a, t^b, t^c) - \frac{1}{(2\pi i)^3} n^0_{[\mathcal{C}_0]} \mathrm{Li}_3(e^{2\pi i [\mathcal{C}_0]\cdot \vec{t}}) - \frac{1}{(2\pi i)^3} \sum_{[\mathcal{C}]\neq \mathcal{C}_0} n^0_{[\mathcal{C}]} \mathrm{Li}_3(e^{2\pi i [\mathcal{C}]\cdot \vec{t}})$$

Now take $\mathrm{vol}(\mathcal{C}_0) \to -\mathrm{vol}(\mathcal{C}_0)$, i.e. $[\mathcal{C}_0] \cdot \vec{t} \to -[\mathcal{C}_0] \cdot \vec{t}$.

$$\mathcal{F}_X \to p(\kappa_{abc}, t^a, t^b, t^c) - \frac{1}{(2\pi i)^3} n^0_{[\mathcal{C}_0]} \mathrm{Li}_3(e^{-2\pi i [\mathcal{C}_0]\cdot \vec{t}}) - \frac{1}{(2\pi i)^3} \sum_{[\mathcal{C}]\neq \mathcal{C}_0} n^0_{[\mathcal{C}]} \mathrm{Li}_3(e^{2\pi i [\mathcal{C}]\cdot \vec{t}})$$

Use polylogarithm identity: $\frac{\mathrm{Li}_3(e^{-2\pi i [\mathcal{C}_0]\cdot \vec{t}})}{(2\pi i)^3} = \frac{\mathrm{Li}_3(e^{2\pi i [\mathcal{C}_0]\cdot \vec{t}})}{(2\pi i)^3} + \frac{1}{6}([\mathcal{C}_0]\cdot \vec{t})^3 + \mathcal{O}(t^2)$

$$\mathcal{F}_X \to p(\kappa'_{abc}, t^a, t^b, t^c) - \frac{1}{(2\pi i)^3} n^0_{[\mathcal{C}_0]} \mathrm{Li}_3(e^{2\pi i [\mathcal{C}_0]\cdot \vec{t}}) - \frac{1}{(2\pi i)^3} \sum_{[\mathcal{C}]\neq \mathcal{C}_0} n^0_{[\mathcal{C}]} \mathrm{Li}_3(e^{2\pi i [\mathcal{C}]\cdot \vec{t}}) \equiv \mathcal{F}_{X'}$$

So we obtain the prepotential for a **new Calabi-Yau,** X' .

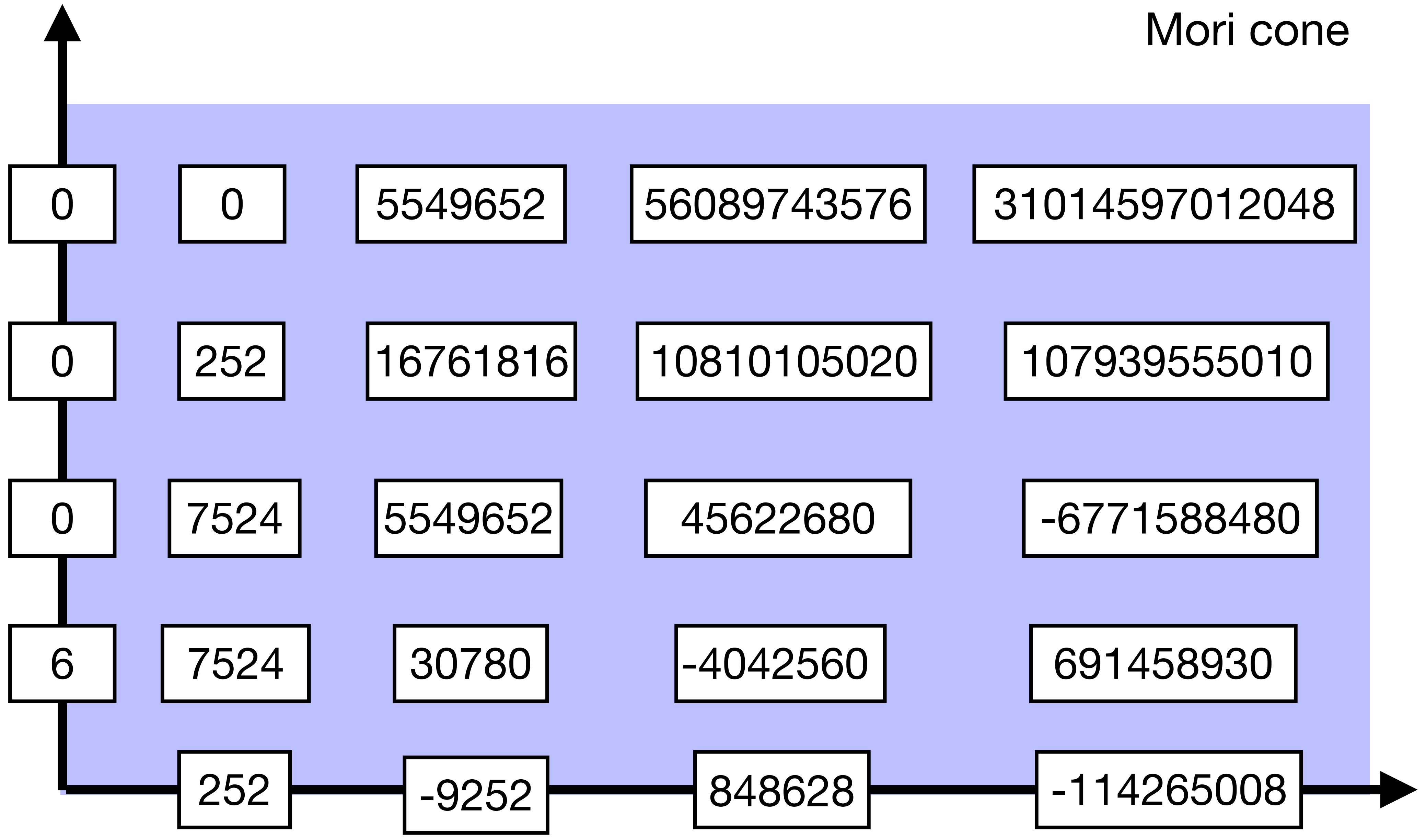
What have we learned?

- An isolated generator of the Mori cone with a finite number of non-vanishing GV invariants can be **flopped** to obtain a new Calabi-Yau.
- Subtlety: if a **divisor** also shrinks when we shrink a curve, then we have reached the boundary of moduli space in M-theory. [Witten '96]
- By performing all available flops to uncover all Calabi-Yaus which are birationally equivalent to the original Calabi-Yau, we can determine the extended Kähler cone.

Let's now apply this machinery in a concrete example.

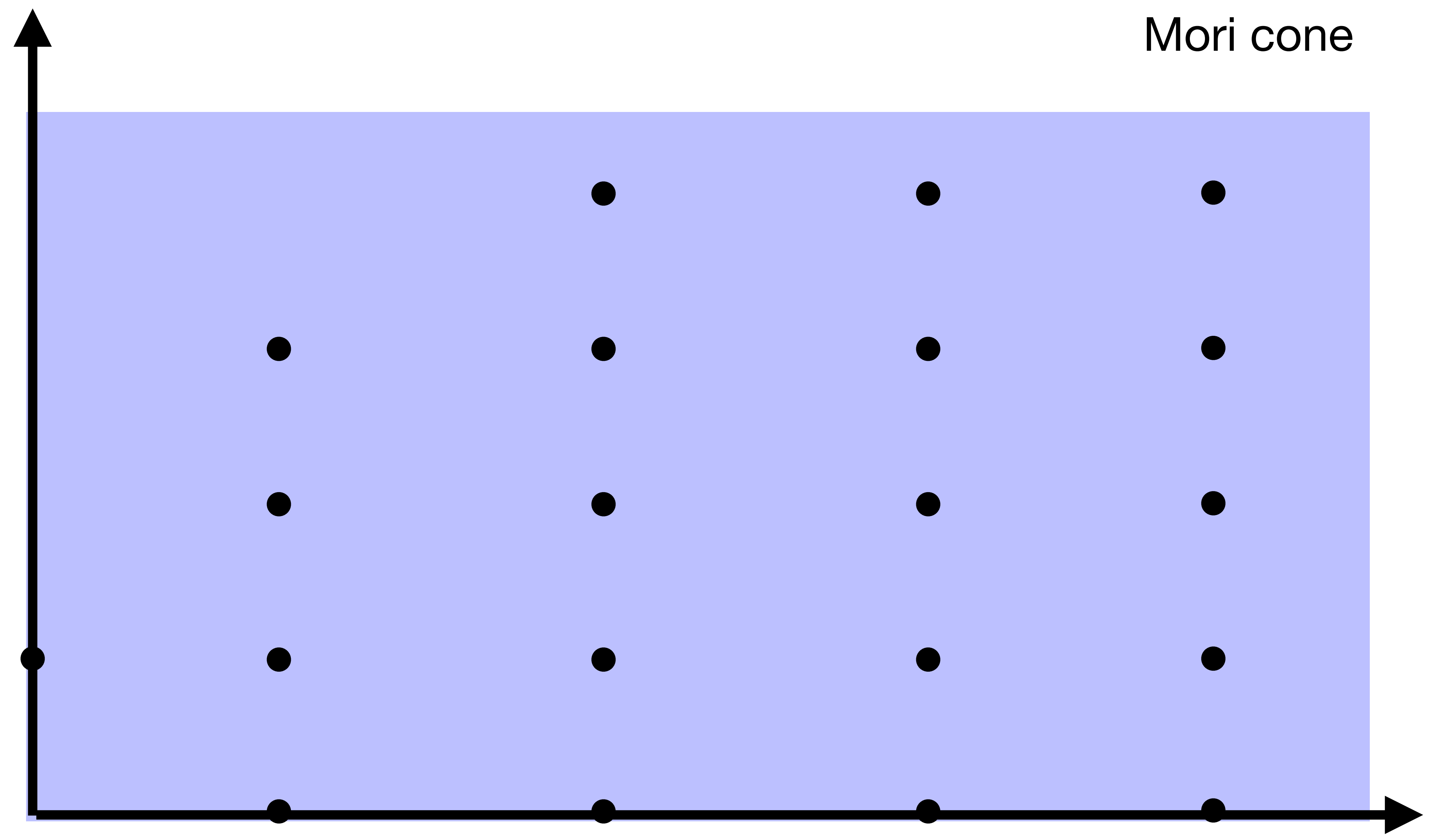
Flops

A Calabi-Yau threefold, X, with $h^{1,1} = 2$



Flops

For visual clarity, let's put a dot at every site with a non-vanishing GV invariant:



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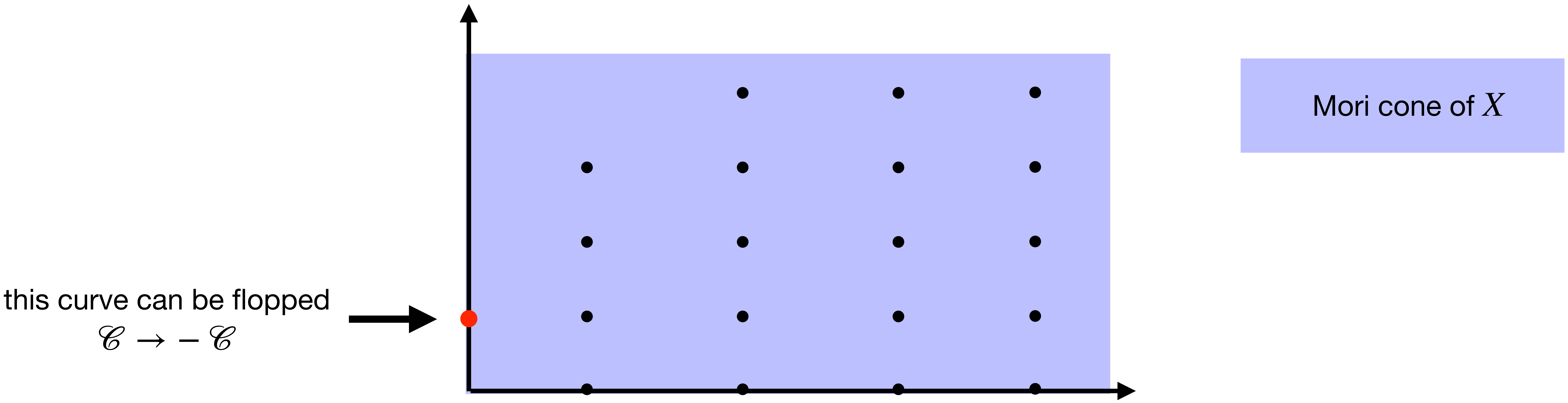
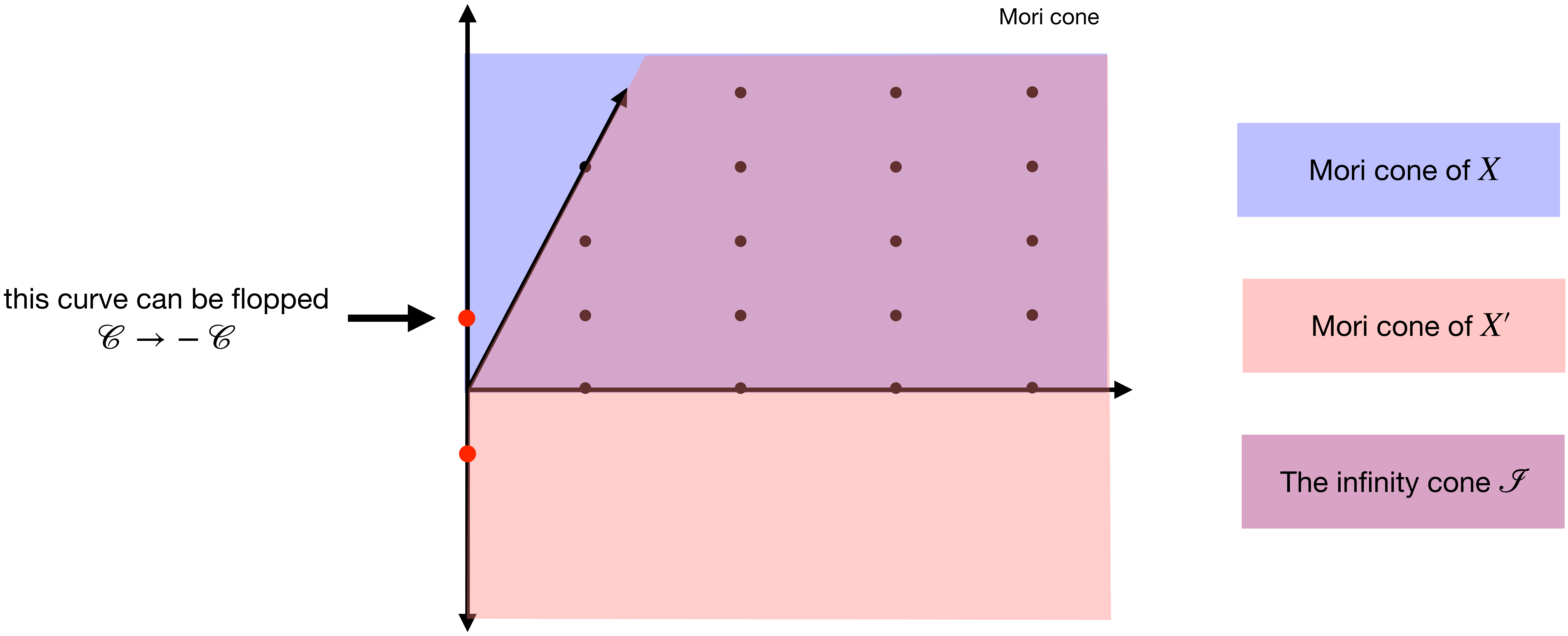


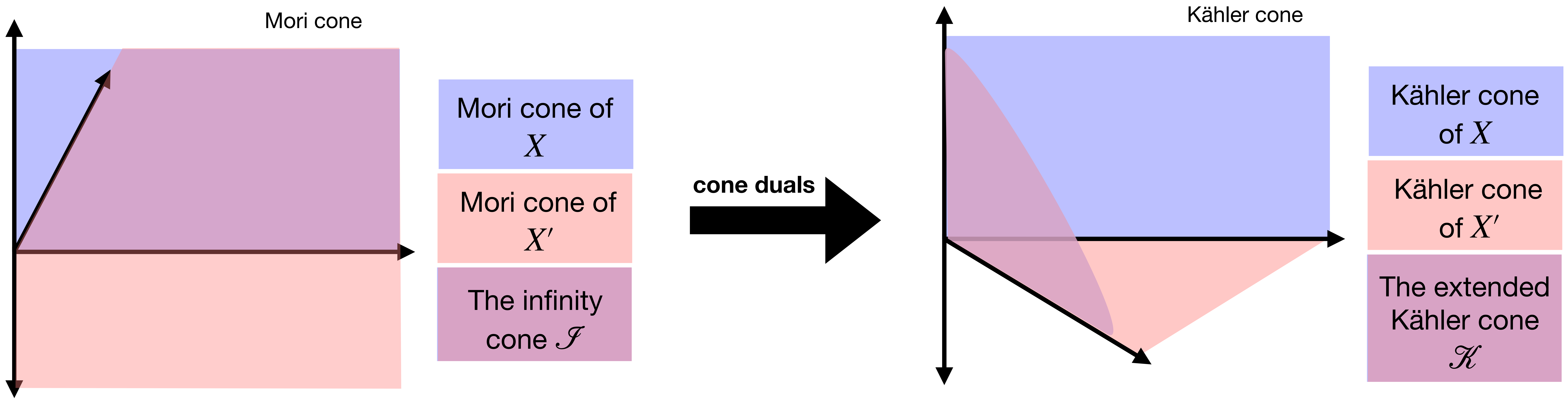
- Taking $\mathscr{C} \to -\mathscr{C}$ gives rise to a new CY, X', which is birationally equivalent to X.
- When you take vol$(\mathscr{C}) \to 0$, only a finite number of states become massless.
- The number of states that become massless is equal to the GV invariant of $\mathscr{C}$

Flops

For visual clarity, let's put a dot at every site with a non-vanishing GV invariant:

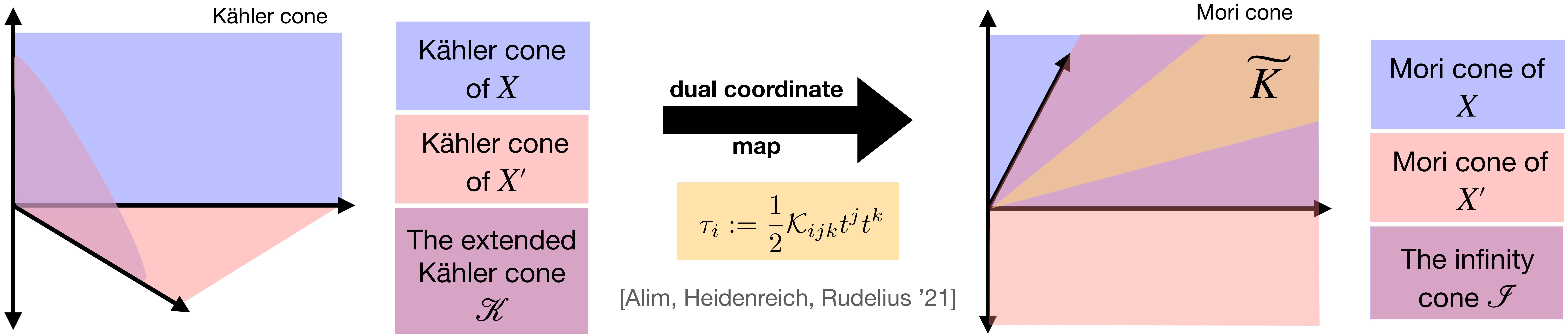


Obtaining the extended Kähler cone



The infinity cone is the dual of the extended Kähler cone!

Checking the Weak Gravity Conjecture



- $\widetilde{K}$ is contained in the infinity cone $\mathscr{I}$
- Every ray in $\widetilde{K}$ has an infinite number of BPS states
- The tower WGC is satisfied!

Checking the WGC in a large ensemble

For a given Calabi-Yau, we now have an **algorithm to verify the tower WGC:**

1. Compute the GV invariants.
2. Determine the extended Kähler cone by performing all possible flops.
3. Apply the dual coordinate map to the extended Kähler cone to obtain $\widetilde{K}$.
4. Ask: is every site in $\widetilde{K}$ populated by a non-vanishing GV invariant?

We performed these steps in over 1000 Calabi-Yaus constructed as hypersurfaces in toric varieties and found that the tower WGC was always satisfied.

Conclusions

- We used genus-0 Gopakumar-Vafa invariants to reconstruct the Kähler moduli space of Calabi-Yau threefolds

- With the full Kähler moduli space and collection of BPS states, we verified the Weak Gravity Conjecture in a plethora of examples

Food for thought

- All of the examples checked appear to satisfy the WGC with **genus-0** GV invariants—why?
- All of the examples checked actually satisfy the stronger **lattice** WGC
- What about the infinite towers *outside* of $\widetilde{K}$?
- Didn't have time to tell you about cool features of moduli space that you can uncover using this method (see work to appear this month with Kim, McAllister, Moritz, and Stillman)

thank you!