Gauged 2-form Symmetries in 6D SCFTs Coupled to Gravity

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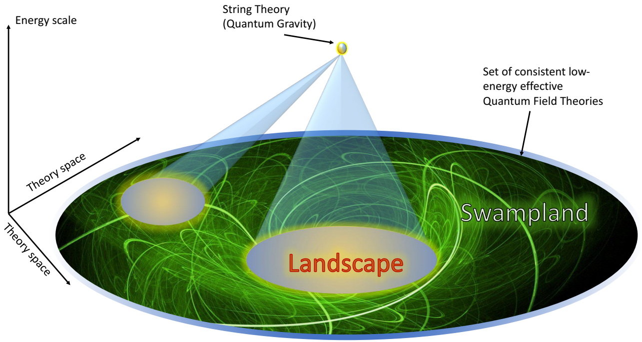
Geometry and the Swampland

based on **2106.13198** with Andreas Braun and Paul-Konstantin Oehlmann
see also **2111.07998** with Finn Bjarne Kohl and Paul-Konstantin Oehlmann

Motivation and summary: The very big picture

Which quantum field theories have a quantum gravity completion?



Palti:19

- Use string theory compactifications, restrict to (super)symmetric QFTs.
- **This talk:** 6D SCFT sectors in N=1,2 supergravity from type IIB/F-theory.

Motivation and summary

- How can SCFTs be coupled to quantum gravity?
- SCFTs are fix points for RG flow $\rightsquigarrow$ classify QFTs (in $D \leq 6$).
- 6D: interacting SCFTs are strongly coupled, and contain tensionless strings.
  Engineer in string/F theory "non-compactifications".
  Witten:95, Seiberg,Witten:96, ... Heckman, Morrison,Vafa:13, Heckman, Morrison,Rudelius,Vafa:15,...
- Stitch together non-compact string geometries
  $\rightsquigarrow$ string/F theory compactifications with SCFT sectors.
  del Zotto,Heckman,Park, Rudelius:15,Garcia-Etxebarria, Heidenreich, Regalado:19, ...
- Symmetries (of all kind) key to SCTF classification.
  E.g. 6D SCFTs can have global, discrete 2-form symmetries.
  In string/F theory, symmetries relate to geometry.

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Fate of global **2-form symmetries** as SCFT sectors are stitched together?

Will see: 2-form symmetry broken or gauged $\sim$ charge lattice embeddings
Agrees with (conjectured) absence of global symmetries in quantum gravity.

Outline

Higher form symmetries

Higher form symmetry: symmetry that acts on extended objects.

Ordinary symmetry G with 1-form current j has

- conserved charge $Q(M^{d-1}) = \int_{M^{d-1}} *j$

  $\rightsquigarrow$ **topological operator** $U_g(M^{d-1})$
- Group law

  $$U_g(M^{d-1})U_{g'}(M^{d-1}) = U_{g''}(M^{d-1})$$

  where $g'' = g'g \in G$
- $U_g(M^{d-1})$ couples to local operators $V(P)$ (particles): for S^{d-1} surrounding P have

  $$U_g(S^{d-1})V(P) = R(g)V(P) \ .$$

Higher form symmetries

Higher form symmetry: symmetry that acts on extended objects.

Ordinary (0-form) symmetry G with 1-form current j has

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Higher form symmetries

Gaiotto,Kapustin,Seiberg,Willet:14

Generalize to p-form symmetry

- $(p+1)$-form current j (if continuous)
- Topological operator $U_g(M^{d-(p+1)})$
- Group law: $U_g(M^{d-(p+1)})U_{g'}(M^{d-(p+1)}) = U_{g''}(M^{d-(p+1)})$, $g'' = g'g \in G$
- $U_g(M^{d-(p+1)})$ couples (via linking) to p-dim operators $V(C_p)$:

1-form symmetry: $V(C_1)$ are line operators

2-form symmetry: $V(C_2)$ are surface operators

6D SCFTs, 2-form symmetries & gravity

6D SCFTs and 2-form symmetries

6D SCFTs: strongly coupled, has tensionless strings. *Witten:95, Seiberg, Witten:96*
Engineered in IIB string/F theory:

Heckman, Morrison,Vafa:13, Heckman, Morrison,Rudelius,Vafa:15, Bhardwaj:15,19...

- (non)-compactifications on 2D spaces $B_\Gamma = \mathbb{C}^2/\Gamma$ with $\Gamma \in U(2)$ (decoupled gravity needed for scale invariance)
- strings/defects $\sim$ D3s on compact/non-compact cycles $C_i \subset B_\Gamma$
  tension $\sim \mathrm{vol}_{C_i}$, charge $\sim C_i \cdot C_j = \Omega_{ij}$
- gauge/flavour groups $\sim$ D7s on compact/non-compact cycles $C_i \subset B_\Gamma$
- in F theory, all is encoded as elliptic fibration $T \hookrightarrow X \rightarrow B_\Gamma$

Strings and defects determine 2-form symmetry.

6D SCFTs and 2-form symmetries

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Strings and defects determine 2-form symmetry.

6D SCFTs and 2-form symmetries

Strings and defects: charged under 2-form symmetry $U(1)^r \sim$ gauge fields B_i.

- Λ_S charge lattice of (dynamical) strings (D3 on compact curve)
- Λ_S^* charge lattice of defects / surface operators (D3 on non-compact curve)

Defects screened by dynamical strings $\rightarrow$ 2-form symmetry (partly) broken.

Unless charge lattice Λ_S self-dual, SCFT has global 2-form symmetry:

$$G_S = \Lambda_S^* / \Lambda_S = \frac{\mathbb{Z}^r}{[\Omega_{ij}]\mathbb{Z}^r} \,.$$

Bhardwaj,Jefferson,Kim,Tarazi,Vafa:19, Bhardwaj,Schäfer-Nameki:20, Albertini, del Zotto, Garcia-Etxebarria, Hosseini:20, ...

G_S a.k.a. **defect group**:$\sim$ choice of quantized background 3-form flux.

Tachikawa:13, delZotto,Heckman,Park,Rudelius:15, Garcia-Etxebarria,Heidenreich,Regalado:19

6D SCFTs coupled to gravity

6D SCFT sectors emerge naturally in the moduli space of 6D supergravity:

- Compactify type IIB string/F theory on compact 2D space B
- Intersection lattice Λ_B from compact cycles $C_i \in H^2(B, \mathbb{Z})$
- D3s on cycles $C_i \rightsquigarrow$ tensionless strings as $Vol(C_i) \to 0$
- SCFT sector specified by sublattice $\Lambda_S \subset \Lambda_B$ of shrunken cycles.

So, combine SCFTs so Λ_S "fit" in Λ_B.

Seiberg,Taylor:11, delZotto,Heckman,Morrison,Park:14

6D SCFTs coupled to gravity

Each SCFT sector has a its own "global" 2-form symmetry But global symmetries are absent in quantum gravity.

Misner, Wheeler:57, Banks, Seiberg:10, Harlow, Ooguri:18, ...

So 2-form symmetries must be **broken or gauged**.

6D supergravity and gauged 2-form symmetries

6D supergravity: string charge lattice Λ_B is self-dual

Seiberg, Taylor:11

$$\implies G_S = \Lambda_B^*/\Lambda_B = 1$$

However, it is still possible that $G \subset G_S$ is gauged, at points in moduli space where only some BPS strings are dynamical.

Caveat: Even at such points, there might be other objects that fully break G_S.

6D supergravity and gauged 2-form symmetries

Gauged 2-form group G in (2,0) and (1,0) supergravity

IIB string/F theory on compact 2D space B with string lattice $\Lambda_B = H_2(B, \mathbb{Z})$.

1. Choose a sublattice $\Lambda_S \subset \Lambda_B$
   $\rightsquigarrow$ SCFT sectors with tensionless strings (at special loci in moduli space)
2. Determine 2-form symmetry of SCFT sectors $\Lambda_S^*/\Lambda_S = \sum_i \Gamma_i^*/\Gamma_i = \sum_i \mathbb{Z}_{n_i}$
3. Strings are invariant under gauged (diagonal) subgroup $G \in G_S$ iff

$$G = (\Lambda_B \cap \Lambda_S^*)/\Lambda_S \neq 1$$

6D supergravity and gauged 2-form symmetries

An unbroken, gauged subgroup $G \in G_S$ requires

$$G = (\Lambda_B \cap \Lambda_S^*)/\Lambda_S = \Lambda_B \cap (\Lambda_S \otimes \mathbb{Q})/\Lambda_S \neq 1$$

$\iff$ $\exists$ element $\eta \in (\Lambda_S \otimes \mathbb{Q})$ s.t. $\eta \notin \Lambda_S$, but $k\eta \in \Lambda_S$ for some $k \in \mathbb{Z}$.

Such elements η live in

$$\boxed{G = \text{tors}(\Lambda_B/\Lambda_S) = \mathbb{Z}_k}.$$

SCFT sectors specified by Λ_S preserve a **gauged 2-form symmetry**
$\iff$ lattice Λ_S is **non-primitively** embedded in Λ_B .

Examples

Examples: (2,0) supergravity

(2,0) examples from IIB string on K3 surface X.

- Unique lattice of BPS strings $\Lambda_B = \Lambda_{(5,21)} = U^{\oplus 5} \oplus (-E_8)^{\oplus 2}$
- SCFT sections: $\Lambda_S := \bigoplus_i \Gamma_i$ with Γ_i ADE root lattice
- Gauged 2-form symmetry

$$G = \text{tors}\left(\Lambda_{5,21} / \bigoplus_i \Gamma_i\right)$$

$\rightsquigarrow$ May classify using lattice theoretic techniques

Font, Fraiman, Grana, Nunez, Parra De Freitas:20,21

- Choosing U-duality frame where X has elliptic fibration, can show

$$G = \text{tors}\left(H^2(X, \mathbb{Z})/\Lambda_S\right) = \text{tors}\left(MW(X)\right)$$

$\rightsquigarrow$ May classify using elliptic fibrations and their Mordell-Weil groups

Nishiyama:96, Aspinwall–Morrison:97, ...,

Braun–Kimura–Watari:13, Kumar:14, Hajouji–Oehlmann:19,...

Examples: (2,0) supergravity

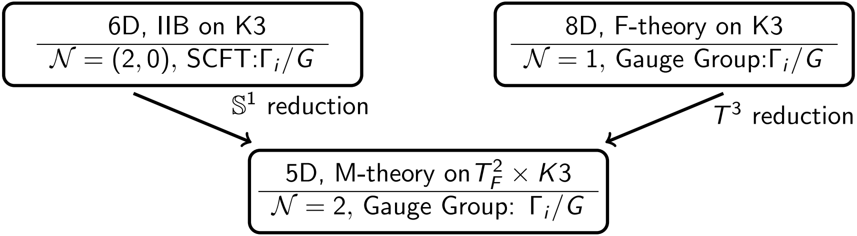
Example

X elliptically fibered K3 surfaces $\sim$ Weierstrass model over $\mathbb{P}^1[z]$:

$$X: \quad y^2 = x(x^2 - z^3(z-1)^3(z-i)^2)$$

- Kodaira $\rightsquigarrow$ 2 type III^* fibres, one type I_0^*
  $\rightsquigarrow$ SCFT sectors have global 2-form symmetry $G_S = \mathbb{Z}_2^{\oplus 4}$.
- Torsional section at $y = x = 0 \rightsquigarrow \mathbb{Z}_2 \subset MW(X)$ $\rightsquigarrow G = \mathbb{Z}_2 \subset G_S$.

$(2,0)$ theories: crosscheck via 5D duality



IIB on elliptic $K3$: $\Lambda_S = \sum_i \Gamma_i \in ADE$; 2-form gauging $G =$tors$(MW(K3))$

- $\mathbb{S}^1$ **reduction:** 6D strings become W-bosons of 5D **gauge group** Γ_i/G
  $2 \to 1$-form symmetry: 5D gauge group is **non-simply connected**

Bhardwaj–Schäfer-Nameki:20

- **Dual to**: M-theory on $T^2_F \times K3$
- There exists a **second F-theory lift** to 8D *Cvetic–Dierigl–Lin–Zhang:20*

Examples: (1,0) supergravity

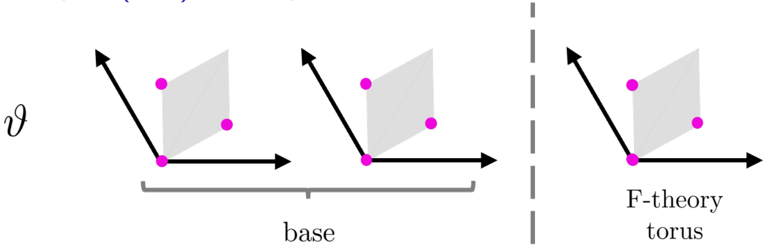
(1,0) examples from F-theory on elliptic CY 3-fold X with base B.

- Different (partly classified) choices for B, so Λ_B not unique.
- Also admit gauge and flavour groups.
- SCFT sectors: $\Lambda_S := \bigoplus_i \Gamma_i$ but Γ_i need not be ADE root lattice

Examples constructed using

- Toroidal orbifolds *Fischer, Ratz, Torrado, Vaudrevange:13, Bailin,Love:99, Donagi,Wendland:08,Forste,et.al:06, Dillies:06*
- Toric bases *Morrison,Taylor:12,12, Martini,Taylor:14, Taylor, Wang:15*

Simple (1,0) example from toroidal orbifold



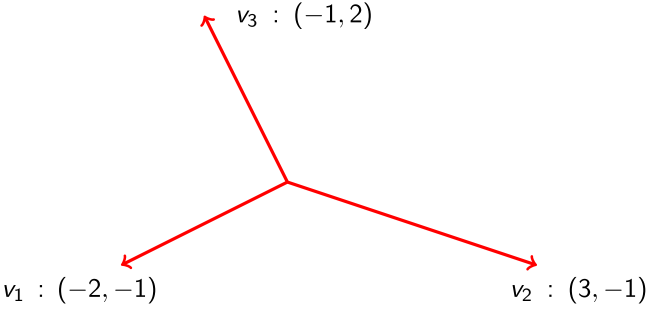
Toroidal orbifold $X_3 = (\mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2_F)/\mathbb{Z}_3$

F-theory on orbifolds

Hayashi, Jefferson, Kim, Ohmori, Vafa'19 , Kohl, Larfors, Oehlmann'21

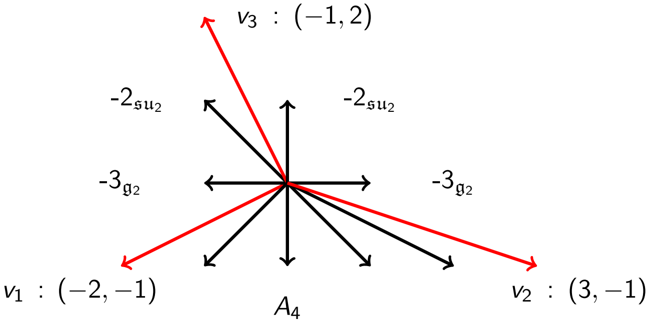
- For $\mathbb{Z}_\mathbf{3}$: $3 \times 3 \times 3$ twisted sector fields
- In $B = T^4/\mathbb{Z}_3 : 9 \times (-3)_{SU(3)}$ non-Higgsable clusters w. 2-form symmetry $\mathbb{Z}_3$ $\rightsquigarrow$ putative global 2-form sym. $G_S = \mathbb{Z}_3^9$...
- ... of which diagonal subgroup $G = \mathbb{Z}_3$ is gauged

(1,0) example from toric base



F-theory compactification on orbifold base $B = \mathbb{P}^2/\mathbb{Z}_5$

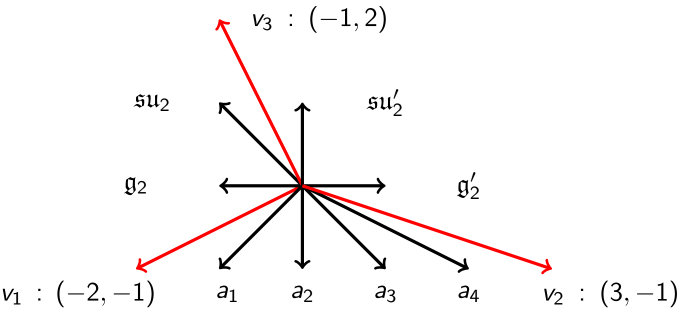
Examples: (1,0) supergravity from toric base



F-theory compactification on orbifold base $B = \mathbb{P}^2/\mathbb{Z}_5$. SCFT data:

- exceptional divisors D_{res} on minimal resolution B_{res}
- gauge groups A_4 (2,0) theory and two (-3,-2) non-Higgsable clusters $\rightsquigarrow G_S = \mathbb{Z}_5^3$.

Examples: (1,0) supergravity from toric base



Go to tensor branch to read off (diagonal) $\mathbb{Z}_5$ gauging

- Toric geometry $\rightarrow$ linear relation between curves, e.g.

$$5[x_1] = 5[x_2] + (3[a_1] + [a_2] - [a_3] - 3[a_4]) - (2[g_2] + [su_2]) + (2[g_2'] + [su_2'])$$

- ...so $D_5 \sim [x_1] - [x_2]$ is torsional at blow-down locus.
- To preserve this, strings may only wrap curves $C \in B$ with $C \cdot D_5 = 0 \bmod 5$

Conclusions and outlook

Conclusions

- 6D SCFTs have global, discrete 2-form symmetries $G_S = \Lambda_S^*/\Lambda_S$.
- When coupled to gravity, the 2-form symmetry must be broken or gauged.
- Gauged 2-form symmetry $G \subset G_S$ requires Λ_S embeds non-primitively in Λ_B

$$\boxed{G = \text{tors}(\Lambda_B/\Lambda_S) \neq 1}.$$

- Applies to (1,0) and (2,0) supergravity, also Little string theory
- Duality check with 5D gauged 1-form symmetry support (2,0) theory result.
- Is G broken by other effects?
  6D gauged 2-form symmetry is supported by duality and reduction to 5D gauged 1-form symmetry in many examples

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Outlook

- Field theory analysis and anomaly cancellation
  Cvetic,Dierigl,Lin,Zhang:20,21, Apruzzi,Dierigl,Lin:20, Tarazi,Vafa:21
- Classify the allowed 2-form gauge groups.
  (2,0) theories and classifications: MW groups, Narain lattice embeddings
  Nishiyama:96; Font,Fraiman,Grana,Nunez,Freitas:20,21; Kim,Tarazi,Vafa:20, ...
  (1,0) theories: classification possible in toric setting. cf. *Morrison,Taylor:12*

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Outlook

- Field theory analysis and anomaly cancellation
- Classify allowed 2-form gauge groups.
- Consequences for Swampland/Landscape?

Thank you for listening!