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The ultimate fate of apparent horizons in a binary black hole merger

Daniel Pook-Kolb

with Ivan Booth and Robie Hennigar

papers: Pook-Kolb, Hennigar, Booth (PRL 127, 181101 (2021))
Booth, Hennigar, Pook-Kolb (PRD 104, 084083 (2021))
Pook-Kolb, Booth, Hennigar (PRD 104, 084084 (2021))

partly based on previous work with Ofek Birnholtz, Josè Luis Jaramillo, Badri Krishnan, Erik Schnetter

At the Interface of Mathematical Relativity and Astrophysics
April 28, 2022, BIRS / Online

Why look at the interior?

- It's reality*
- It informs mathematics what kind of objects exist
- It carries an imprint of the GW source outside the horizons

The basic picture

- A smooth closed spacelike 2-surface $\mathcal{S}$ is a *marginally outer trapped surface* (MOTS) $\Leftrightarrow$ $\Theta_+ = 0$

  where $\Theta_+ := q^{\alpha\beta}\nabla_\alpha \ell^+_\beta$ = **outward** expansion,

  $q_{\alpha\beta}$ = metric on $\mathcal{S}$, ℓ^α_+ = **outgoing** null normal on $\mathcal{S}$

- $\Theta_+ > 0$ light rays diverge

  $\Theta_+ < 0$ light rays converge

- Apparent Horizon = outermost MOTS

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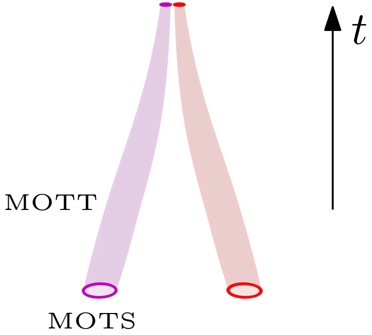

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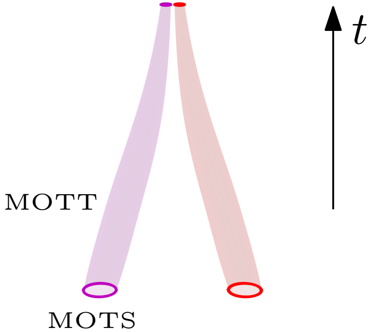

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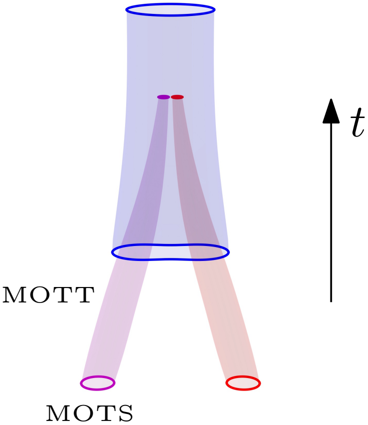

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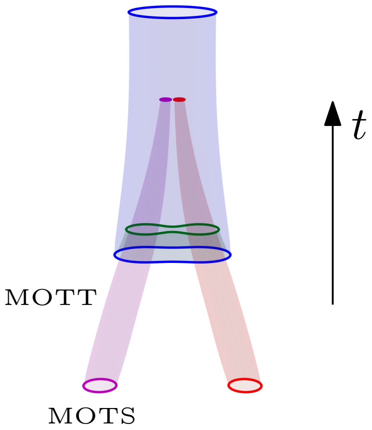

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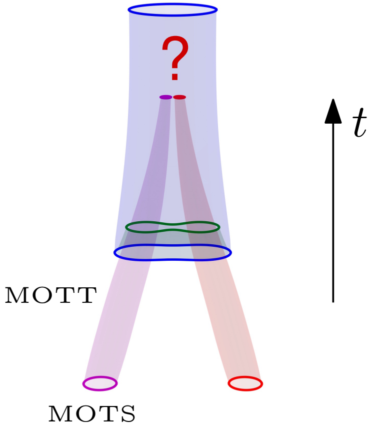
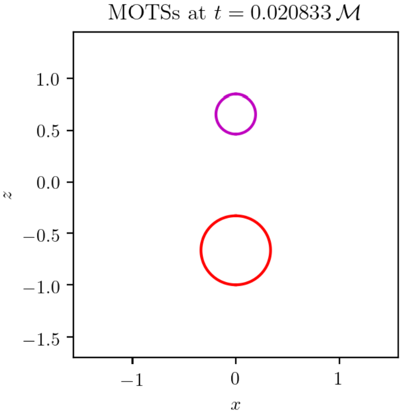


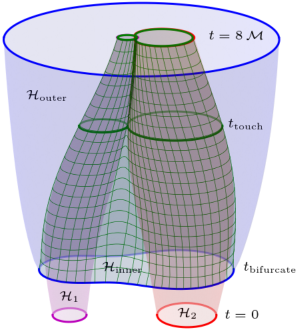
The basic picture

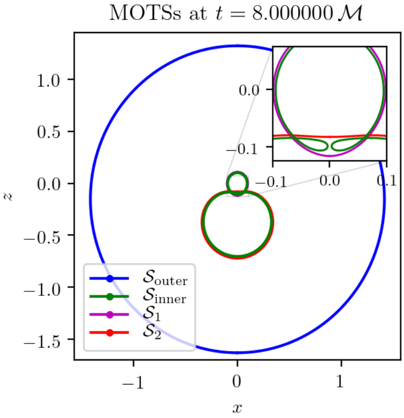
- Brill-Lindquist initial data ($m_1 = 0.5$, $m_2 = 0.8$, $d = 1.3$)



The basic picture

- Brill-Lindquist initial data $(m_1 = 0.5, m_2 = 0.8, d = 1.3)$
- Connected sequence of MOTSs from $\mathcal{S}_{1,2} \to \mathcal{S}_{\text{outer}}$
- Formation of MOTSs that **self-intersect**



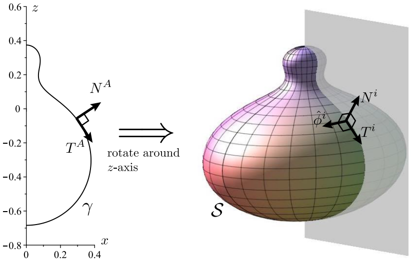


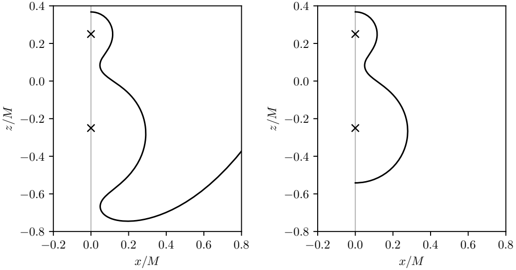
MOTSs without initial guesses – Method I

- A generalized "shooting method"
  (Booth et al., PRD 104, 084083 (2021) and Pook-Kolb et al., PRL 127, 181101 (2021))

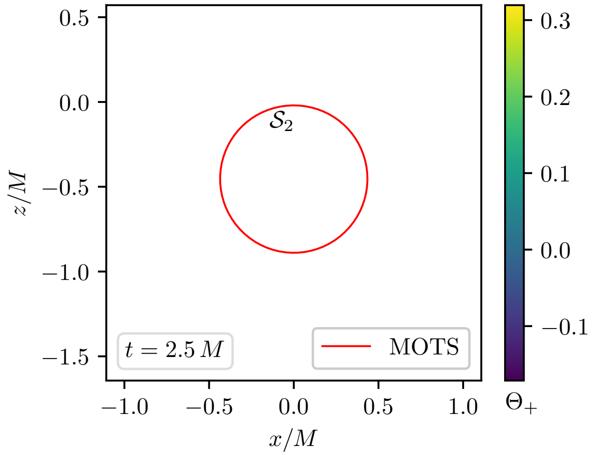
The idea:

- γ determined by two coupled 2nd order ODEs ("MOTSodesic")
- Choose a point on the z-axis and shoot a "$\Theta_+ = 0$ ray"
- γ describes a MOTS
  $\Leftrightarrow$ γ closes upon itself





MOTSs without initial guesses – Method II



- Construct a *family* $\mathcal{F}$ of surfaces of *constant expansion*: $\Theta_+ = \text{const}$
- Any $\Theta_+ = 0$ surface $\mathcal{S} \in \mathcal{F}$ is a MOTS

(source: Pook-Kolb et al., PRD 104, 084084 (2021))

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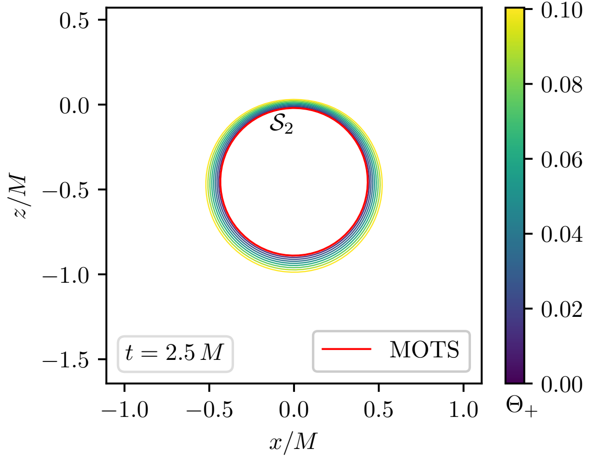

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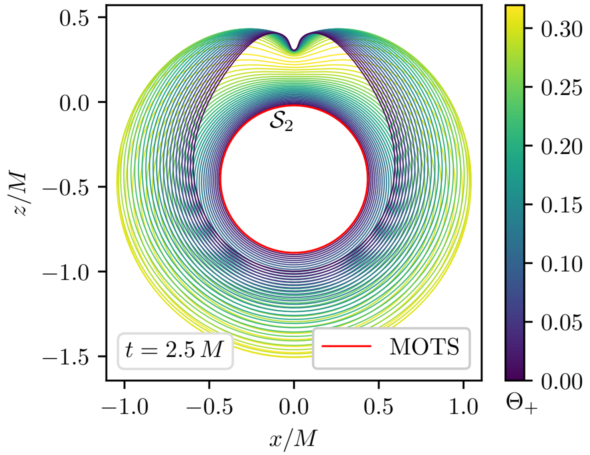

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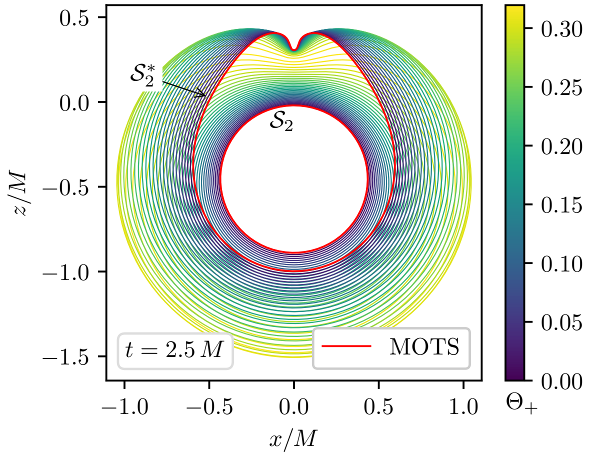

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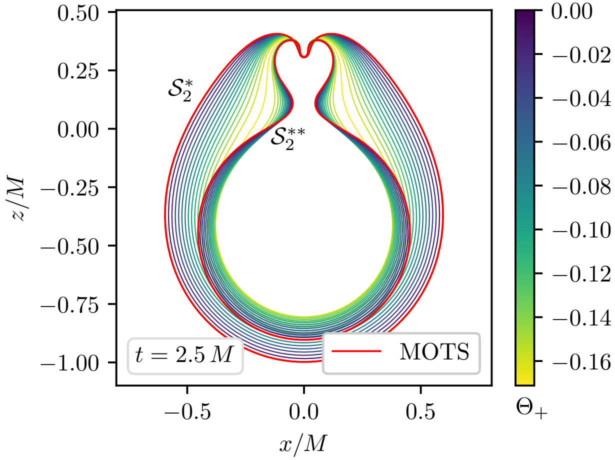

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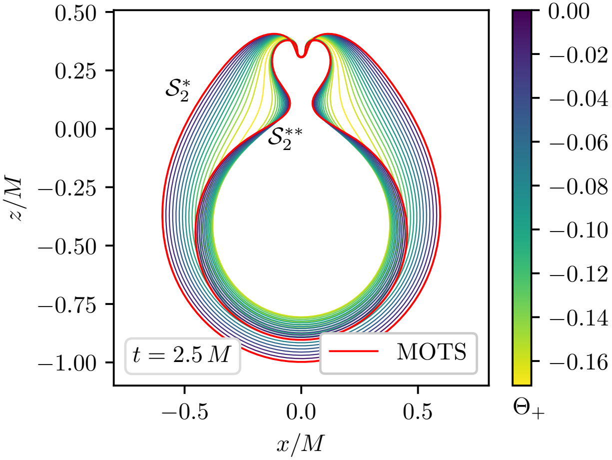

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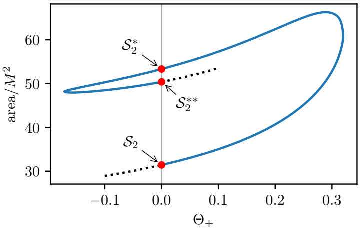

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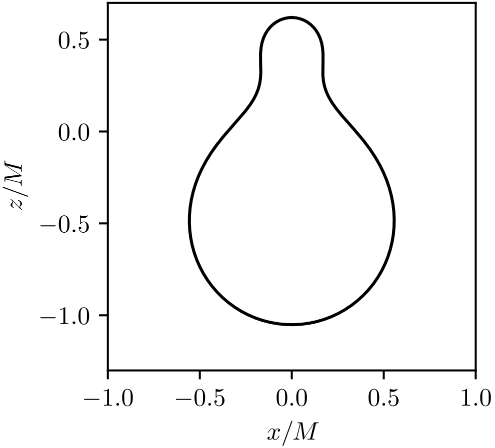
Tracking a MOTS through time

- Simply take the previous MOTS:



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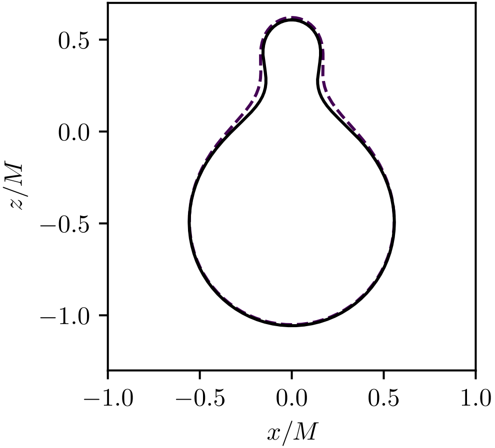

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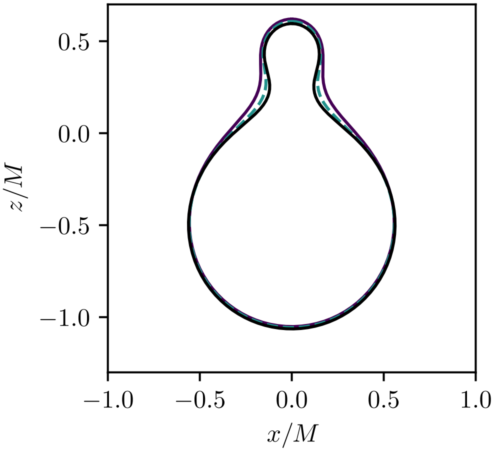

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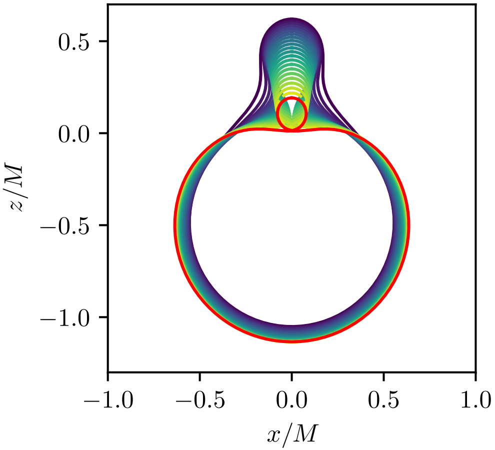
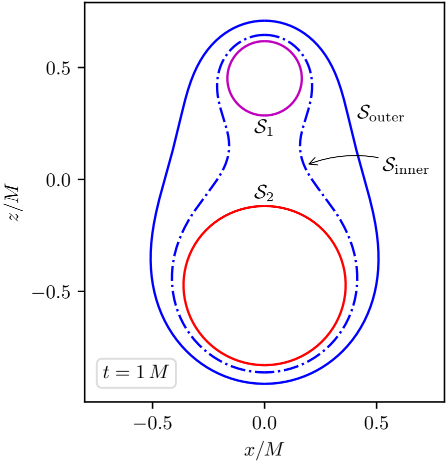
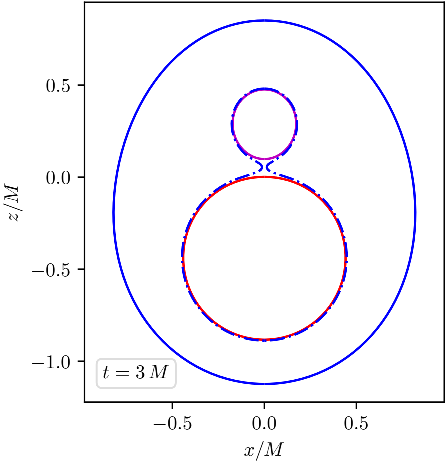


We found . . .





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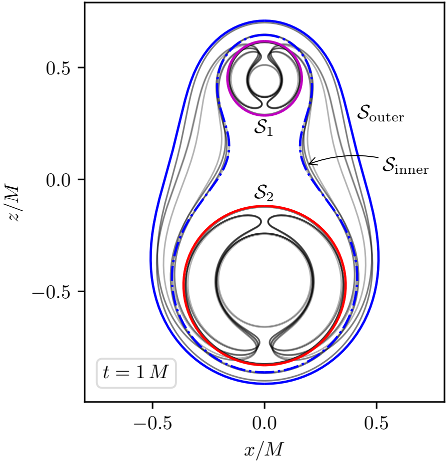

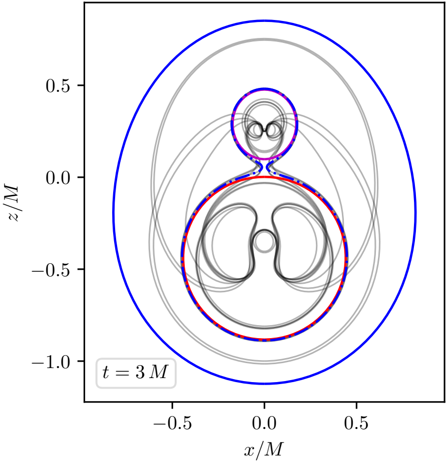

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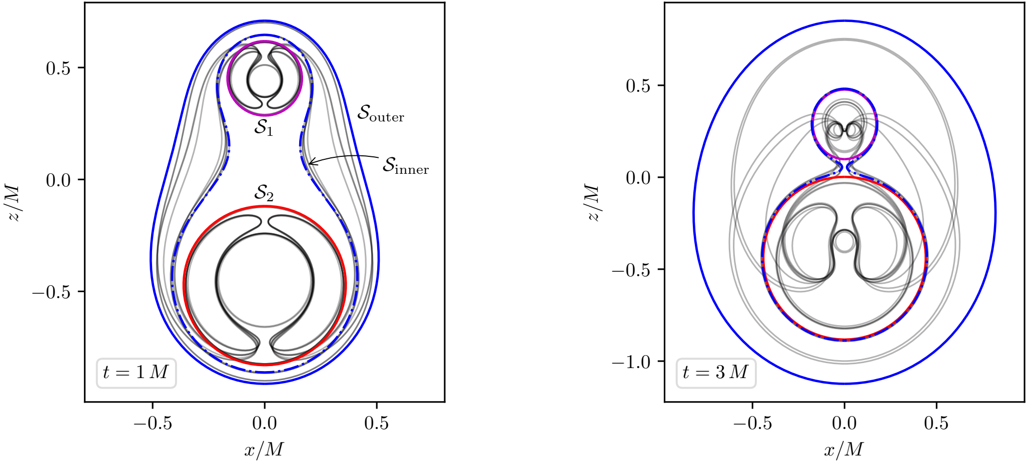
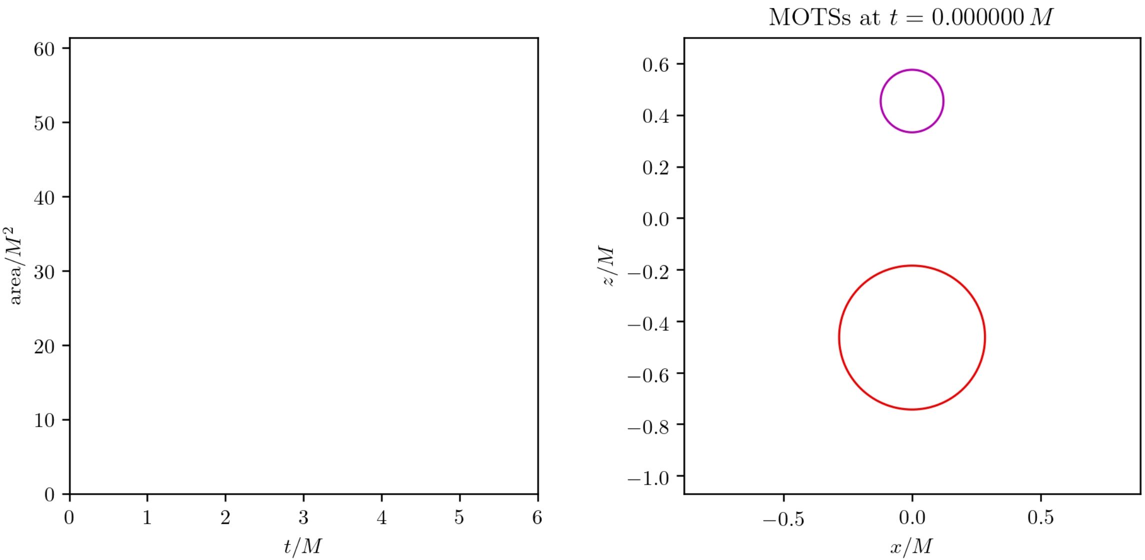
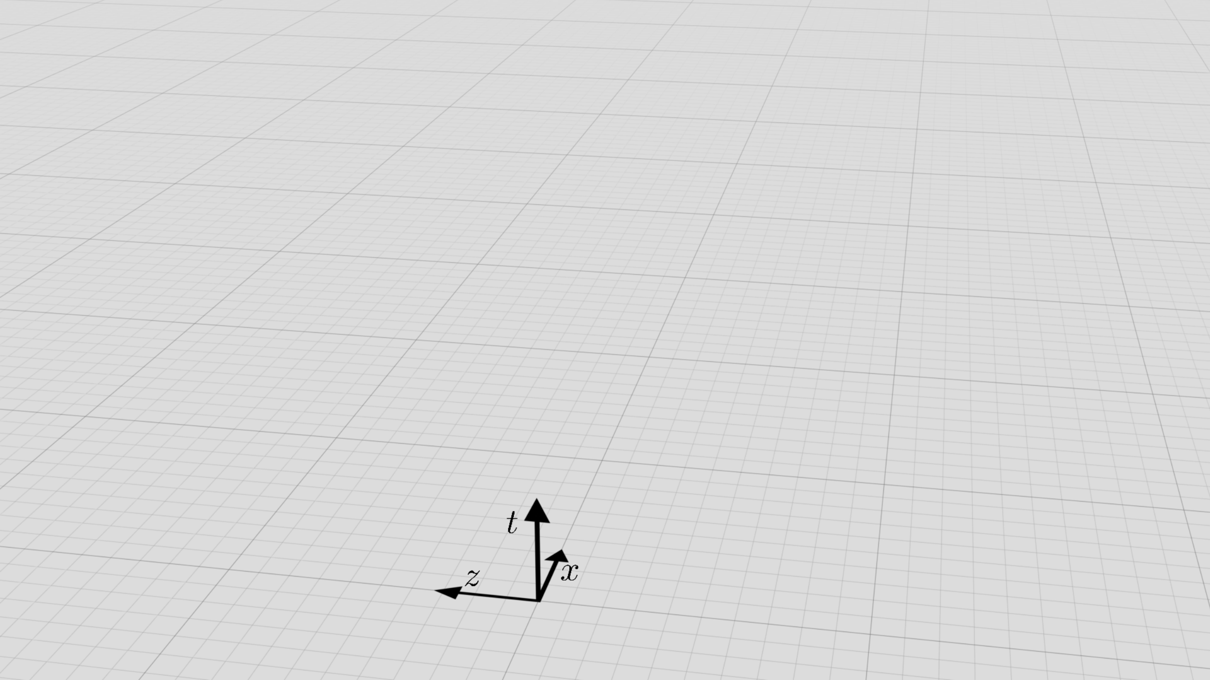


- Do these MOTSs help **explain the fate** of $\mathcal{S}_1$ and $\mathcal{S}_2$?
- Are they all **black hole boundaries**?

(source: Pook-Kolb et al., PRD 104, 084084 (2021), modified)

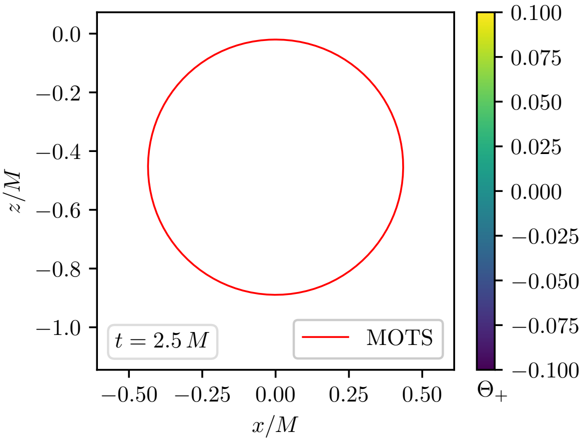
The fate of $\mathcal{S}_2$


MOTSs at $t = 0.000000\,M$
60
50
40
30
20
10
0
area$/M^2$
0
1
2
3
4
5
6
t/M
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
z/M
−0.5
0.0
0.5
x/M


t
z
x

MOTS $\rightarrow$ black hole boundary?

The barrier property



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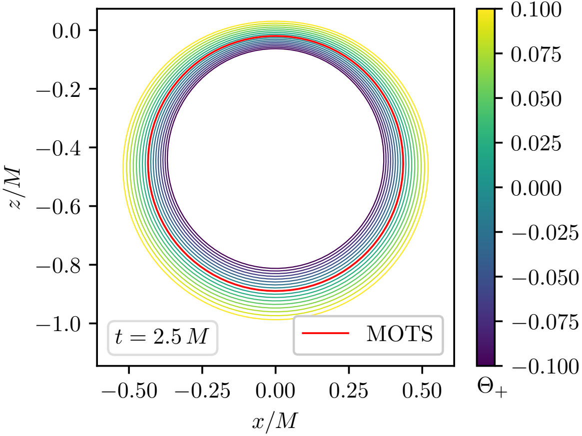

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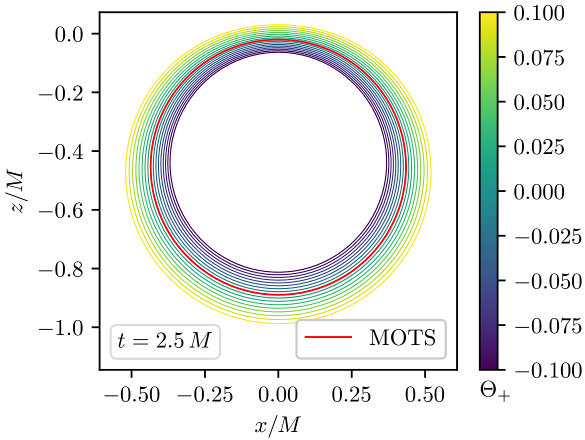
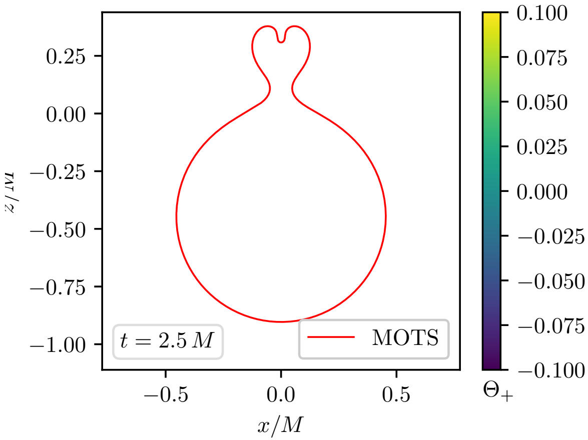


- *Every* close-by untrapped surface lies outside
- *Every* close-by trapped surface lies inside

⇒ MOTS has "barrier" property!

The barrier property



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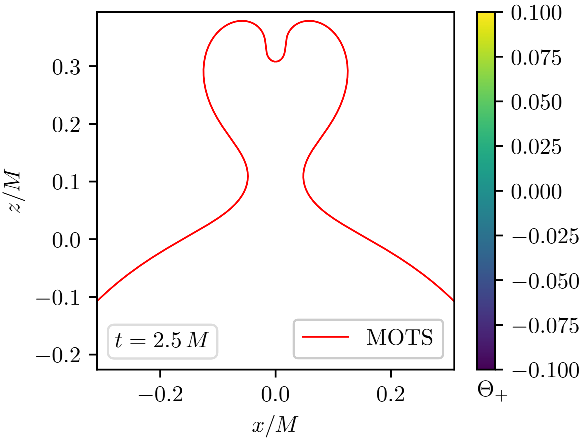

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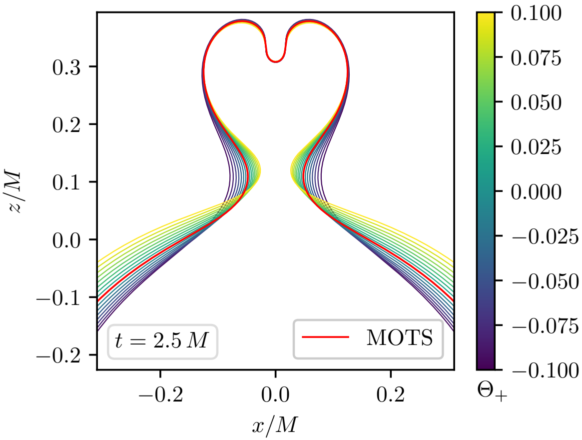

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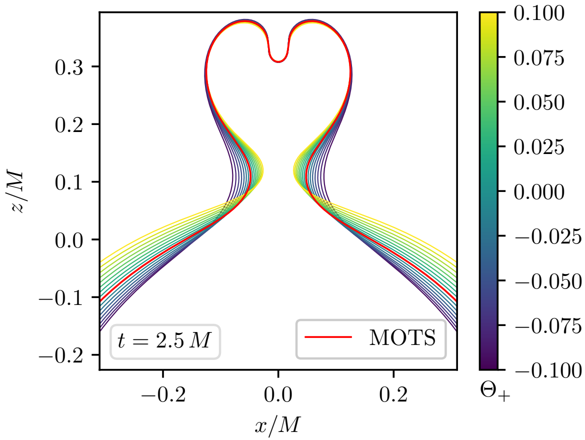


- ▶ Close-by untrapped surfaces cross the MOTS
- ▶ Close-by trapped surfaces cross the MOTS

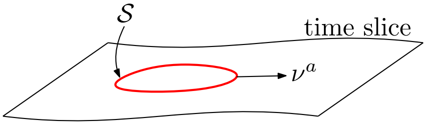
$\Rightarrow$ No "barrier" property!

“Barrier” $\longleftrightarrow$ “Stability”

- **Definition:** “Stability operator” (Vacuum)

  linear, 2nd order, elliptic, generally not self-adjoint

  (Andersson, Mars, Simon, PRL 95, 111102 (2005))



$$L\Psi = \delta_{2\Psi\nu}\Theta_+ = \frac{d}{ds}\bigg|_{s=0}\Theta_+^s$$

$$L\Psi = -\triangle\Psi + \left(\frac{1}{2}\mathcal{R} - 2|\sigma_+|^2\right)\Psi$$

$$L\Psi = \lambda\Psi$$

$\Psi : \mathcal{S} \to \mathbb{R}$ describes deformation, $\mathcal{R} =$ Ricci scalar of $\mathcal{S}$, $\sigma_+ =$ shear of $\mathcal{S}$

$\triangle = (\mathcal{D}_A - \omega_A)(\mathcal{D}^A - \omega^A)$, $\omega_A = \ell_\alpha^- \nabla_A \ell_+^\alpha$, axisymmetry + no spin $\Rightarrow \triangle = \Delta_{\mathcal{S}} =$ Laplacian on $\mathcal{S}$

- Principal eigenvalue $\lambda_0 > 0 \to$ MOTS is **strictly stable** $\Rightarrow$ *barrier*
- MOTS is a *barrier* $\Rightarrow \lambda_0 \geq 0$

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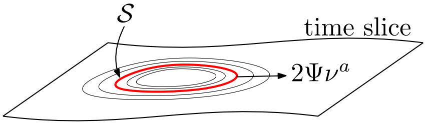

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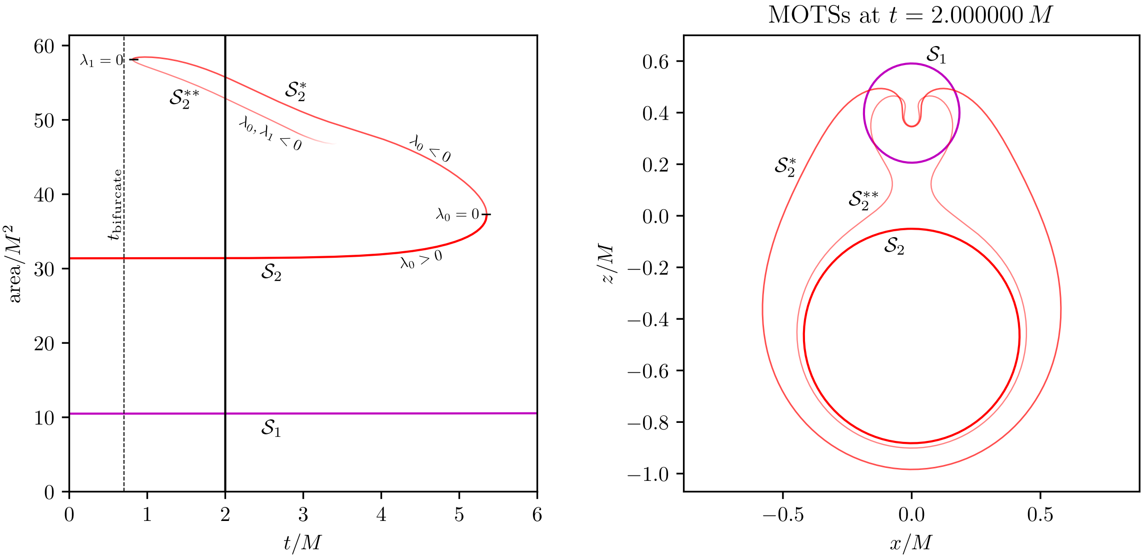
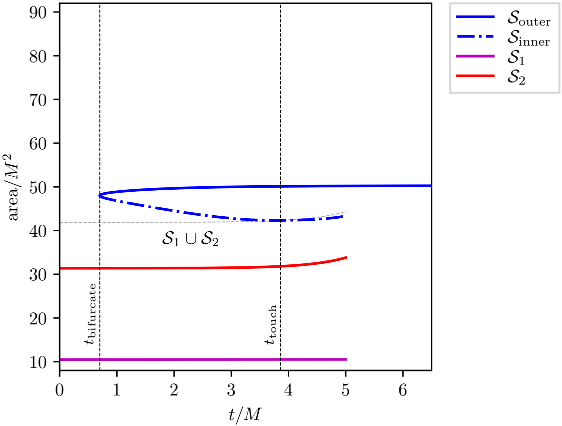


MOTSs at $t = 2.000000\,M$
$\lambda_1 = 0$
$\mathcal{S}_2^{**}$
$\mathcal{S}_2^{*}$
$\lambda_0, \lambda_1 < 0$
$\lambda_0 < 0$
$\lambda_0 = 0$
$\lambda_0 > 0$
$t_{\mathrm{bifurcate}}$
$\mathcal{S}_2$
$\mathcal{S}_1$
area$/M^2$
t/M
z/M
x/M

A new picture of the full merger

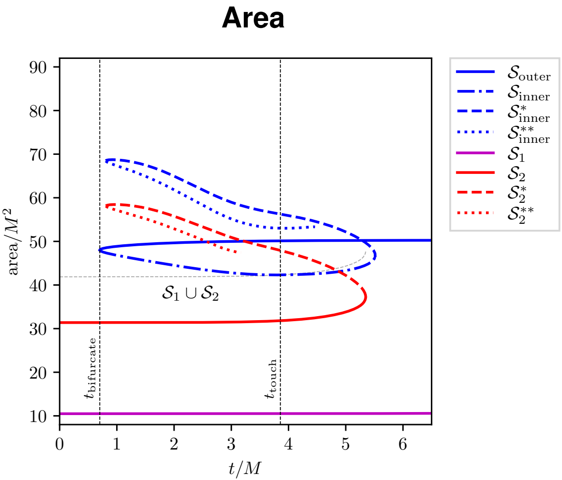
The new picture

Area



(source: Pook-Kolb et al., PRL 127, 181101 (2021) and Pook-Kolb et al., PRD 104, 084084 (2021), modified)

The new picture



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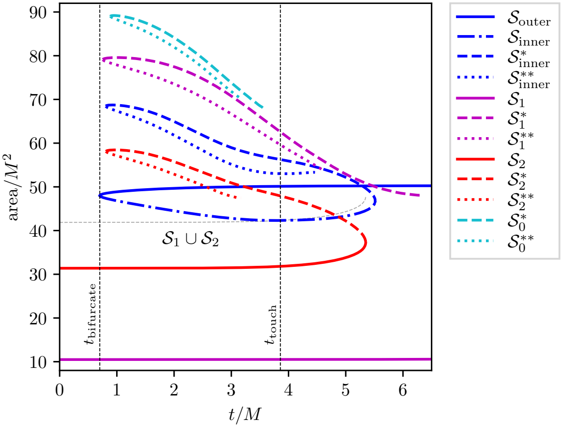
The new picture

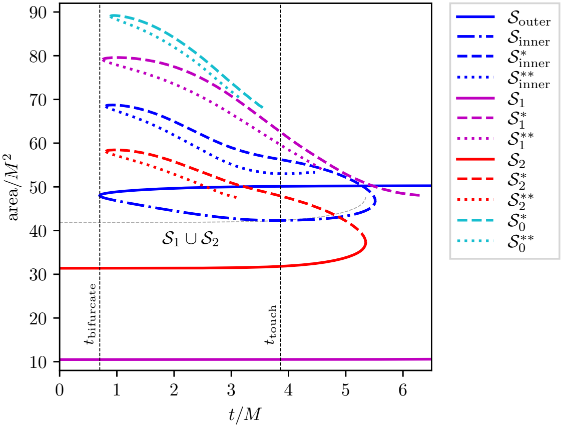
Area



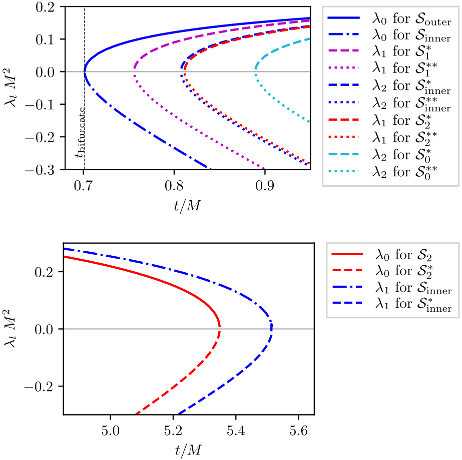
(source: Pook-Kolb et al., PRL 127, 181101 (2021) and Pook-Kolb et al., PRD 104, 084084 (2021), modified)

The new picture

Area



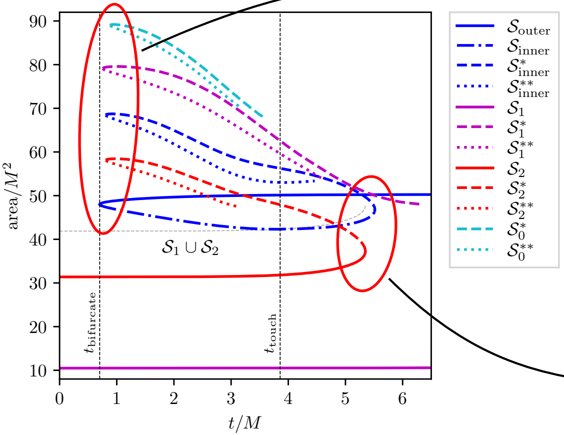
Stability



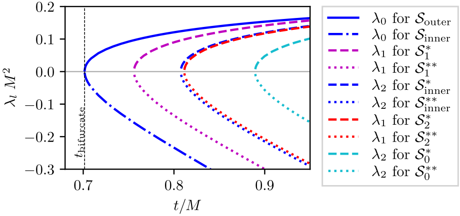
(source: Pook-Kolb et al., PRL 127, 181101 (2021) and Pook-Kolb et al., PRD 104, 084084 (2021), modified)

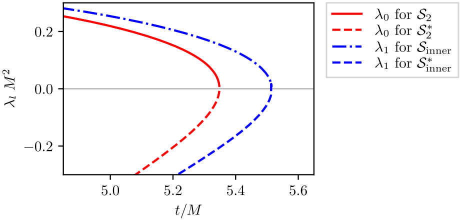
The new picture

Area



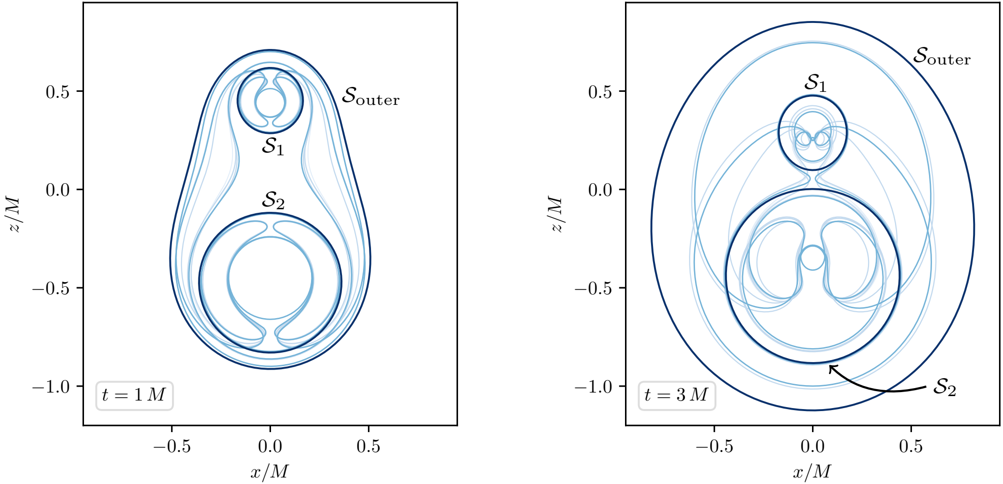
Stability





(source: Pook-Kolb et al., PRL 127, 181101 (2021) and Pook-Kolb et al., PRD 104, 084084 (2021), modified)

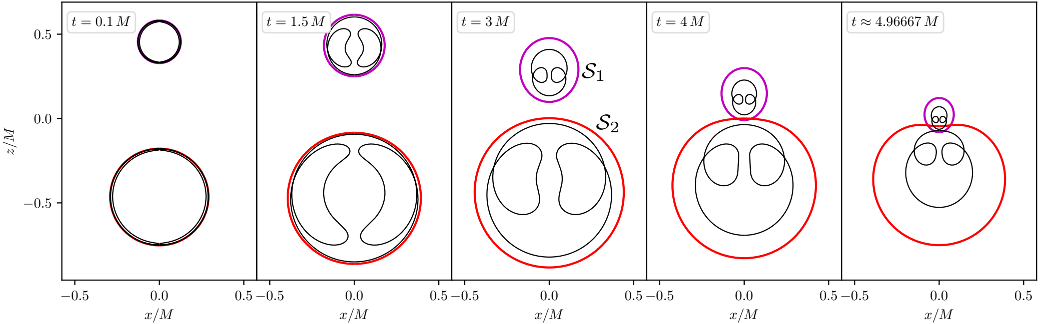
Stability $\longrightarrow$ Black hole boundaries



- lighter = “more unstable” = larger number of negative stability eigenvalues

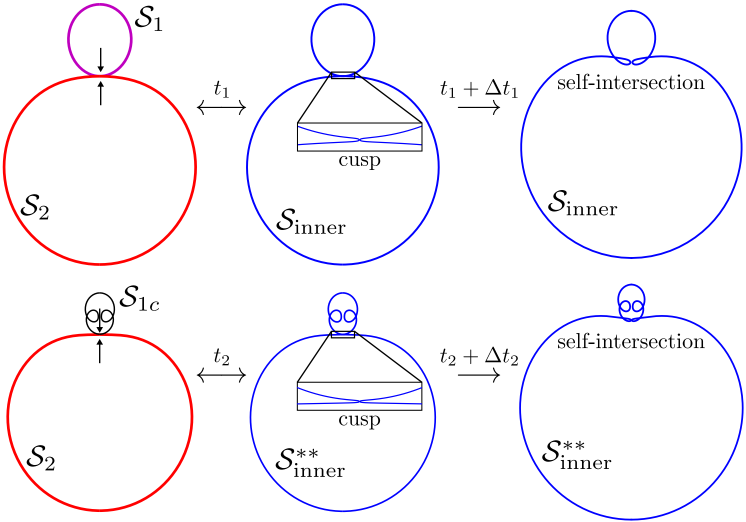
(Pook-Kolb et al., PRD 104, 084084 (2021))

Cusps and self-intersections



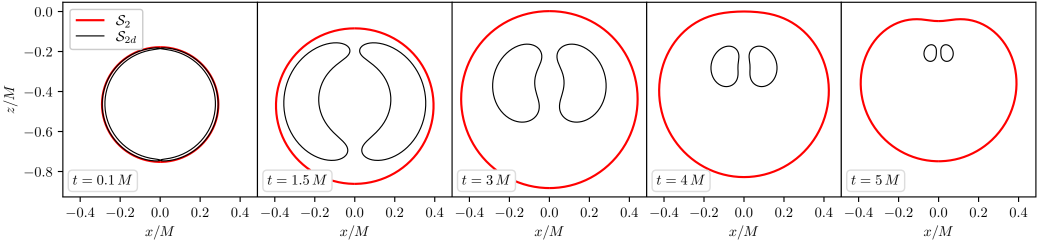
- MOTSs with self-intersections inside $\mathcal{S}_1$ and $\mathcal{S}_2$
- They touch pairwise and then intersect

Cusps and self-intersections



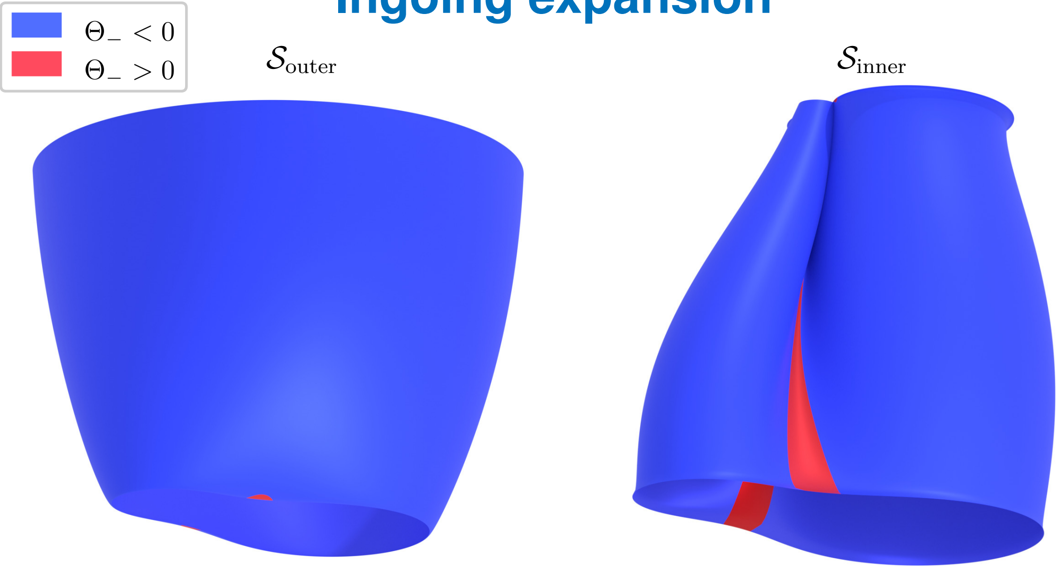
- $\mathcal{S}'$ and $\mathcal{S}''$ touch $\leftrightarrow \mathcal{S}' \cup \mathcal{S}'' = \mathcal{S}$
- $\mathcal{S}$ has a cusp, later self-intersection
- Empirically:
  $$N' + N'' + 1 = N$$
  $N =$ number of $\lambda < 0$

A MOTS with *toroidal* topology



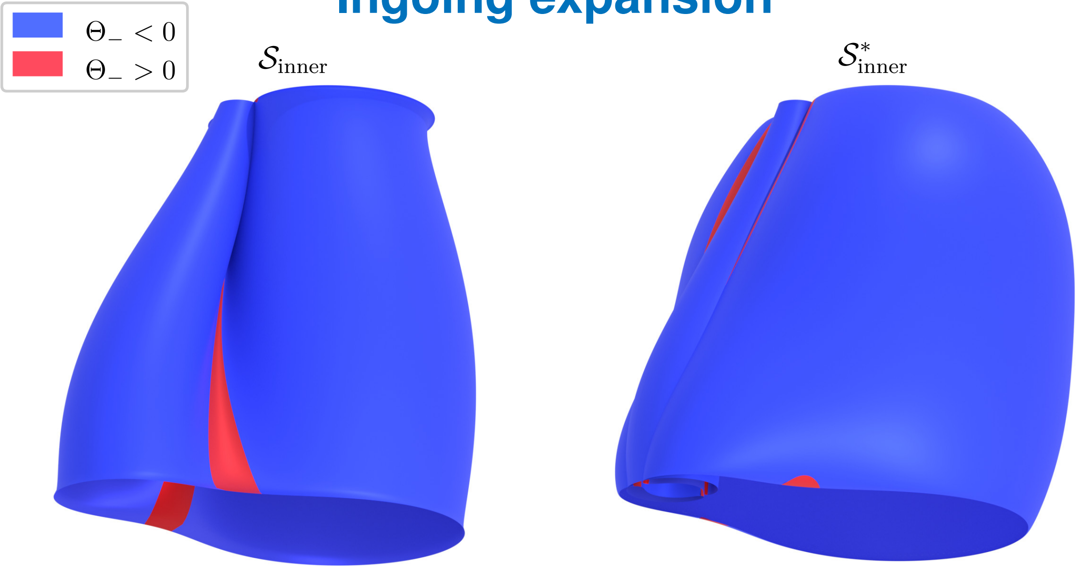
- Multiple negative stability eigenvalues

Ingoing expansion



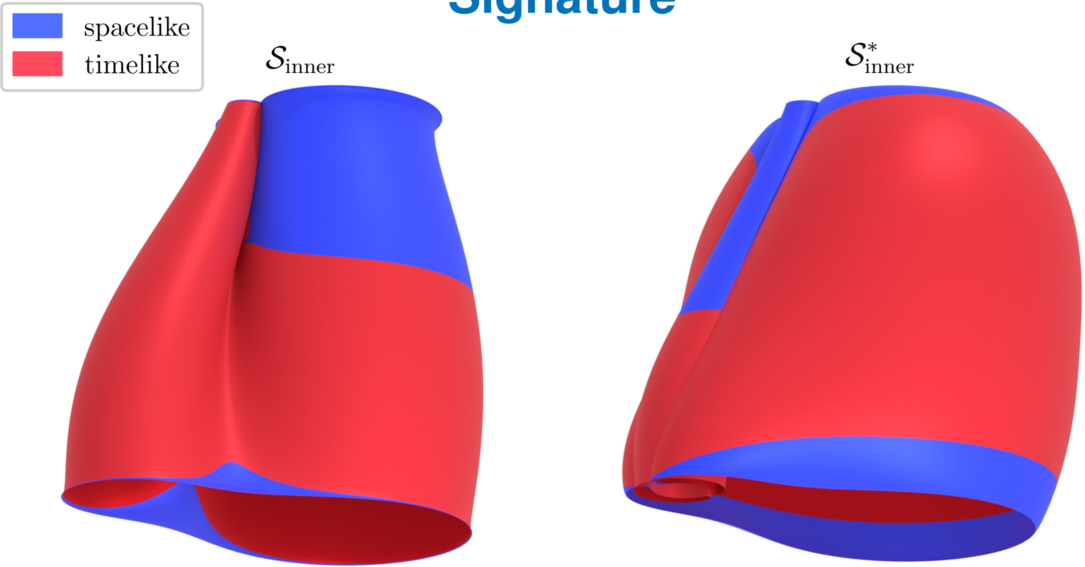
(source: Pook-Kolb et al., PRD 104, 084084 (2021))

Ingoing expansion



(source: Pook-Kolb et al., PRD 104, 084084 (2021))

Signature



(source: Pook-Kolb et al., PRD 104, 084084 (2021))

Summary

- Two **new methods** for finding unexpected MOTSs
- Many **bifurcations** and **annihilations**
- Individual horizons **annihilate** independently
- **Only three** MOTSs are strictly stable: $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_{\text{outer}} \to$ black hole **boundaries**
- We now know what to look for in **fully generic** cases