CANADIAN MATH KANGAROO CONTEST
4TH WORKSHOP

6-8 MAY 2022, BANFF

- INTRODUCTION
    - OUR AUDIENCE: MATHEMATICALLY PROMISING STUDENTS
    - ROLE AND RESPONSIBILITIES OF TEACHERS
    - ROLE OF PARENTS
    - CURRENT AND FUTURE PLACE AND ROLE OF CMKC

- THE PROBLEMS
    - FAVOURITE TOPICS
    - EXAMPLES OF PROBLEMS

THE INTERNATIONAL CONTEST-GAME "MATH KANGAROO"

- STARTED IN 1991, IN FRANCE
- INTERNATIONAL ASSOCIATION "KANGOUROU SANS FRONTIERES (KSF)" FOUNDED IN 1994
- MAIN GOAL: DE-DEMONIZING MATHEMATICS AND INSPIRING YOUNG PEOPLE
- TARGET: A WIDE RANGE OF STUDENTS (AGE, ABILITIES, INTERESTS)
  - ❑ Math Kangaroo is aiming to attract **big numbers** of students of **various ability levels** and to expose them to good and interesting math challenges (but not necessarily to very difficult ones)
  - ❑ Nonetheless, Math Kangaroo is dedicated to achieving its goals **without compromising the quality and richness of mathematical ideas,** and by strictly following the principles of mathematical sophistication that are respected in the academic math community.

THE START OF MATH KANGAROO IN CANADA

2001 – 2005 THE CONTEST WAS AVAILABLE ONLY IN OTTAWA, AND ONLY UNOFFICIALLY.

2006 PILOT EDITION, THREE CENTRES PARTICIPATED (OTTAWA, TORONTO, EDMONTON)

CANADIAN MATH KANGAROO CONTEST REGISTERED

CANADA WAS ACCEPTED AS A MEMBER OF KSF

41 MEMBER COUNTRIES, ABOUT 4M PARTICIPANTS WORLDWIDE

2007 FIRST OFFICIAL PARTICIPATION OF CANADA; SIX CENTRES IN TOTAL (OTTAWA, TORONTO, EDMONTON, CALGARY, ST. JOHN'S, MONTREAL)

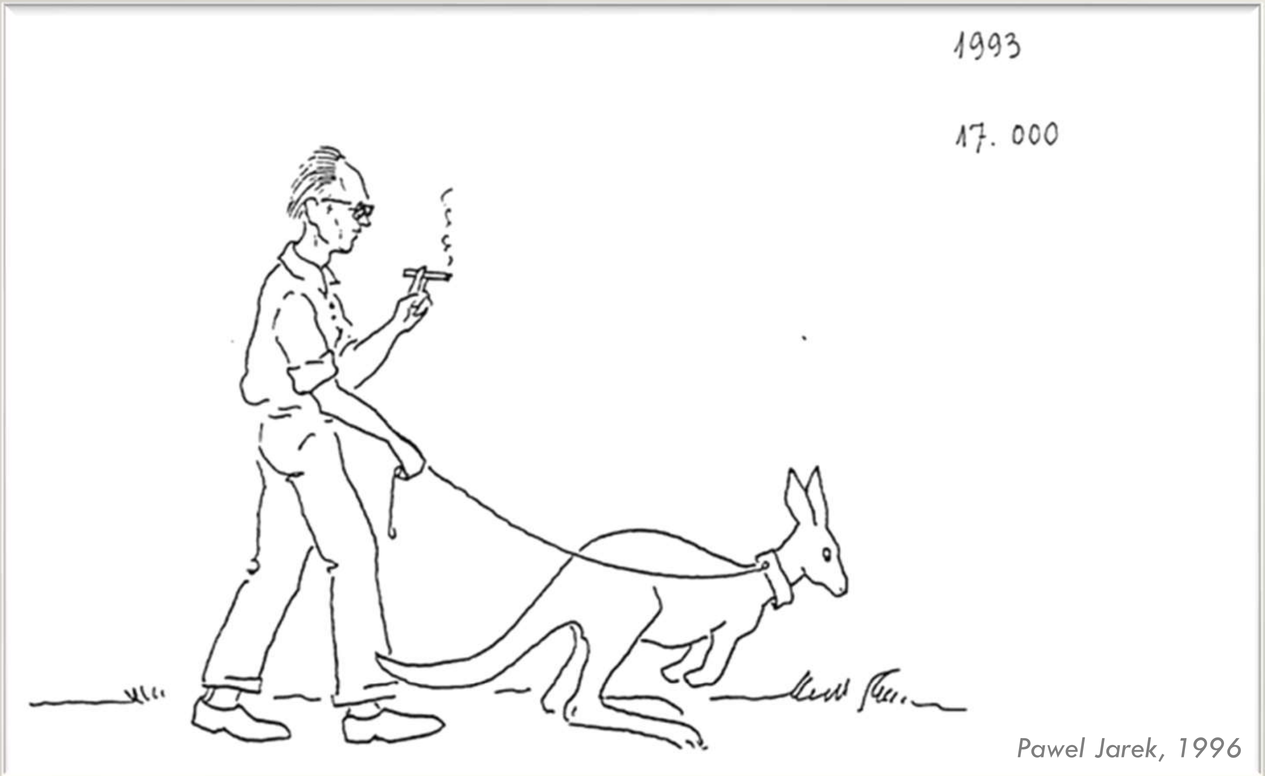
2012 EXPANDED TO 14 CENTRES ACROSS 6 CANADIAN PROVINCES (ON, AB, BC, OC, NFL, NS)

GROWING UP

Pawel Jarek, 1996

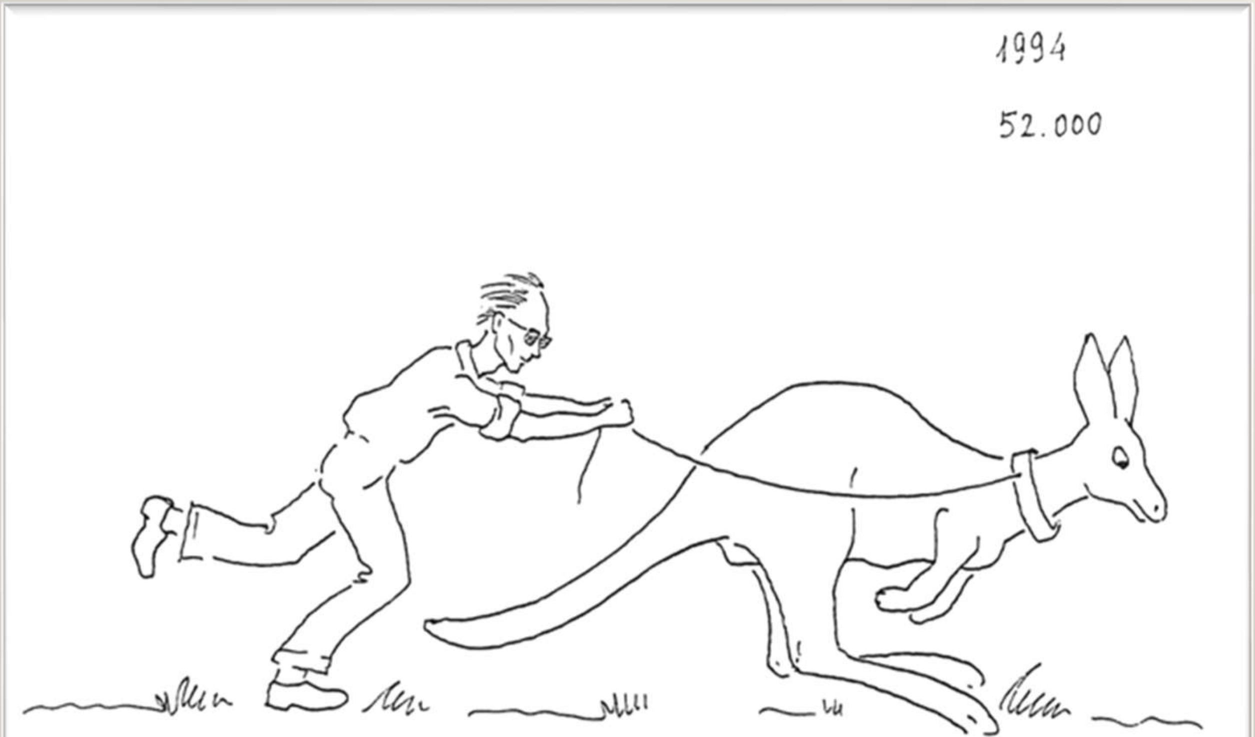
1992

1993



Pawel Jarek, 1996

1994



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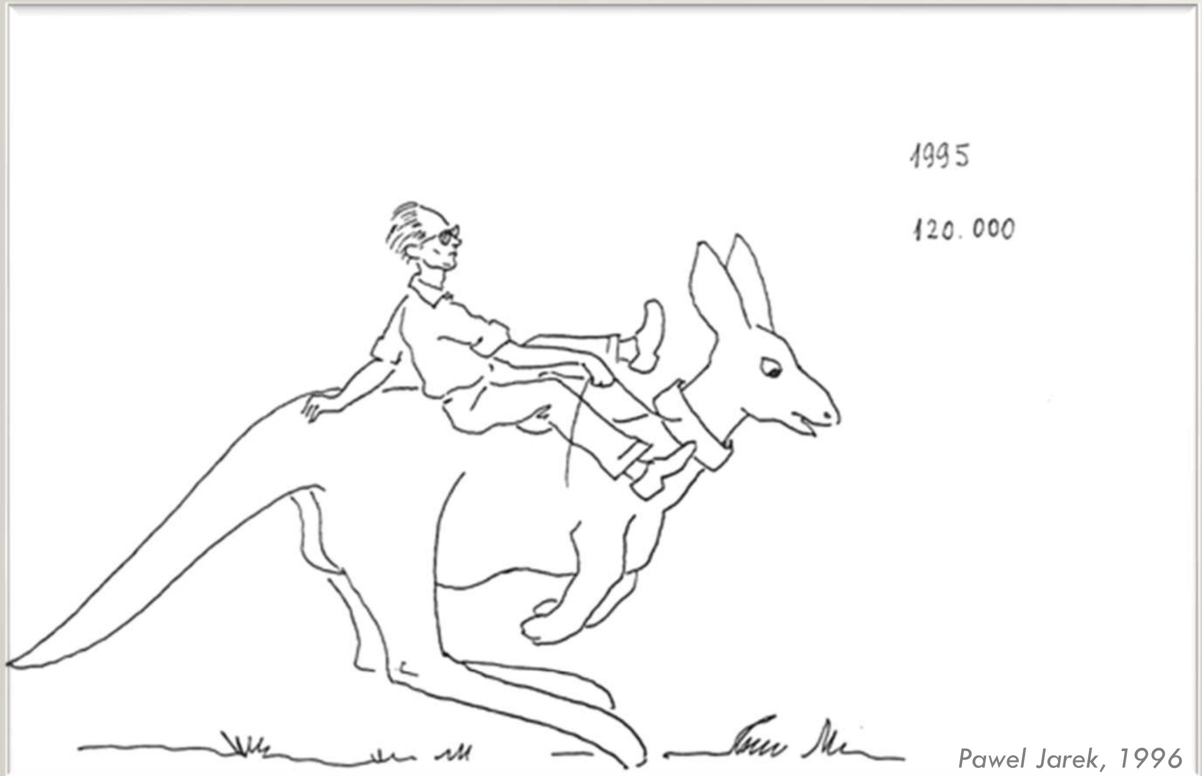
1995



Pawel Jarek, 1996

1996

Pawel Jarek, 1996

CANADIAN MATH KANGAROO CONTEST TODAY

- CMKC MADE HUGE PROGRESS SINCE 2012
- RECENTLY, A SHIFT HAS BEEN OBSERVED, TOWARD RECOGNITION OF DEFICIENCIES IN AND NEEDS FOR STRENGTHENING OF MATH CURRICULUMS

HOWEVER

- RAPIDLY CHANGING EXPECTATIONS OF EDUCATION TO BE ADEQUATE TO NEW REALITIES
- NEW (DIGITAL) GENERATION OF STUDENTS, TEACHERS AND PARENTS

NONETHELESS

- PREVIOUS MYTHS, PERCEPTIONS AND CHALLENGES HAVE NOT CHANGED MUCH

MATHEMATICALLY PROMISING STUDENTS

EDUCATIONAL RESEARCH SHOWS…

HIGHLY MOTIVATED STUDENTS NEED CHALLENGES SO THAT THEY DON'T TURN THEIR ACTIVE MINDS AWAY FROM MATHEMATICS AND TOWARDS ENDEAVOURS THEY FIND MORE APPEALING.

(BARBEAU & TAYLOR, 2005)

IT IS EXTREMELY IMPORTANT TO START CHALLENGING THESE STUDENTS AT A YOUNGER AGE, "WELL BEFORE STUDENTS REACH THE SIXTH OR SEVENTH GRADE". AS RELATED TO THE ADVANTAGES OF STARTING EARLY, LEARNING MATHEMATICS IS SIMILAR TO LEARNING A SECOND LANGUAGE OR TO MUSIC PERFORMANCE.

(USISKIN, 1999)

ALTHOUGH SPECIAL CARE IS USUALLY ASSIGNED TO LOWER ABILITY STUDENTS SO THAT THEY MAY COVER THE BASIC STANDARDS, ALMOST NO ATTENTION IS PAID TO MATHEMATICALLY TALENTED STUDENTS.

(KENDEROV & MAKRIDES,2006)

THE STUDENT MOST NEGLECTED IN TERMS OF REALISING FULL POTENTIAL, IS THE GIFTED STUDENT OF MATHEMATICS.

(NCTM, 1980, AS CITED BY SHEFFIELD, 1999))

THE NEEDS OF MATHEMATICALLY PROMISING STUDENTS, ESPECIALLY IN ELEMENTARY AND MIDDLE GRADES, ARE NOT KNOWN, NOT UNDERSTOOD, OR NEGLECTED IN CANADIAN SCHOOLS.

MYTHS

1. GIFTED STUDENTS NEED LITTLE TEACHER SUPPORT.

Gifted students need little support on routine tasks. However, to maximise learning they need support and guidance on challenging tasks.

2. MATHEMATICS COMPETITIONS [ALONE] CATER FOR THE NEEDS OF MATHEMATICALLY GIFTED STUDENTS.

While the competitions are very important as part of the process, more important for developing their potential are the ongoing opportunities to practice and prepare for them.

3. ADDITIONAL EXERCISES OR ACTIVITIES FOR "FAST FINISHERS" CATER FOR THE NEEDS OF MATHEMATICALLY GIFTED STUDENTS.

If these tasks are only at the same level as the class work it is merely "busy work" and lacks the opportunity for math learning

4. GIFTED STUDENTS DO NOT NEED OPPORTUNITIES TO WORK WITH OTHER GIFTED STUDENTS.

Their interests and abilities can vary substantially from peers their age, hence, without the company of other gifted students they can be very isolated and have limited learning opportunities.

5. THERE ARE INSUFFICIENT FUNDS AND TIME TO DEDICATE TO THE EDUCATION OF MATHEMATICALLY GIFTED STUDENTS.

This view is short-sighted and overlooks the long-term return that a small investment in the education of mathematically gifted students should yield.

Reference: Diezmann & Watters (2002)

THE ROLE AND RESPONSIBILITY OF THE TEACHERS

- SELECTING TASKS THAT ARE APPROPRIATELY CHALLENGING.
- PROACTIVELY AND CONSISTENTLY SUPPORTING STUDENTS' COGNITIVE ACTIVITY WITHOUT REDUCING THE COMPLEXITY AND COGNITIVE DEMAND OF THE TASK.
- FACILITATING COGNITIVE APPRENTICESHIP THROUGH EFFECTIVE AND COLLABORATIVE STUDENT-TEACHER INTERACTION.
- MONITORING AND RESPONDING TO THE STUDENT'S THINKING.
- DEALING WITH ISSUES AND PROBLEMS THAT ARISE FROM THE SPECIFIC SOCIAL CHARACTERISTICS OF THIS GROUP OF STUDENTS.
- BEING INSPIRATIONAL AND ABLE TO PASS THE ADMIRATION TOWARD MATH TO STUDENTS.

MOST TEACHERS, ESPECIALLY IN ELEMENTARY AND MIDDLE GRADES, ARE NOT INTERESTED, NOT MOTIVATED, NOT CONFIDENT, AND NOT SUPPORTED TO PLAY SUCH ROLE.

Reference: Diezmann & Watters (2002)

THE INVOLVEMENT AND ROLE OF THE PARENTS

DEPENDS ON:

- PROFESSIONAL/EDUCATION BACKGROUND.
- OWN SCHOOL EXPERIENCE.
- HOW REALISTIC THEIR INFORMATION IS REGARDING THEIR CHILDREN'S MATH ABILITIES AND LEARNING.
- OVERALL SOCIO-ECONOMIC SITUATION.
- AVAILABILITY AND MOTIVATION.

EVEN IN THE IDEAL SITUATION AND CIRCUMSTANCES, PARENTS' ROLE AND INFLUENCE DECREASES AS THEIR CHILDREN GROW

THUS, EXTRACURRICULAR OPPORTUNITIES ARE CRITICALLY NEEDED FOR MATHEMATICALLY PROMISING AND TALENTED STUDENTS

- WHILE THERE ARE MORE SUCH OPPORTUNITIES FOR HIGH SCHOOL STUDENTS, THE CANADIAN MATH KANGAROO CONTEST IS STILL ONE OF THE FEW AVAILABLE FOR YOUNGER STUDENTS

POSSIBLE FUTURE DIRECTIONS

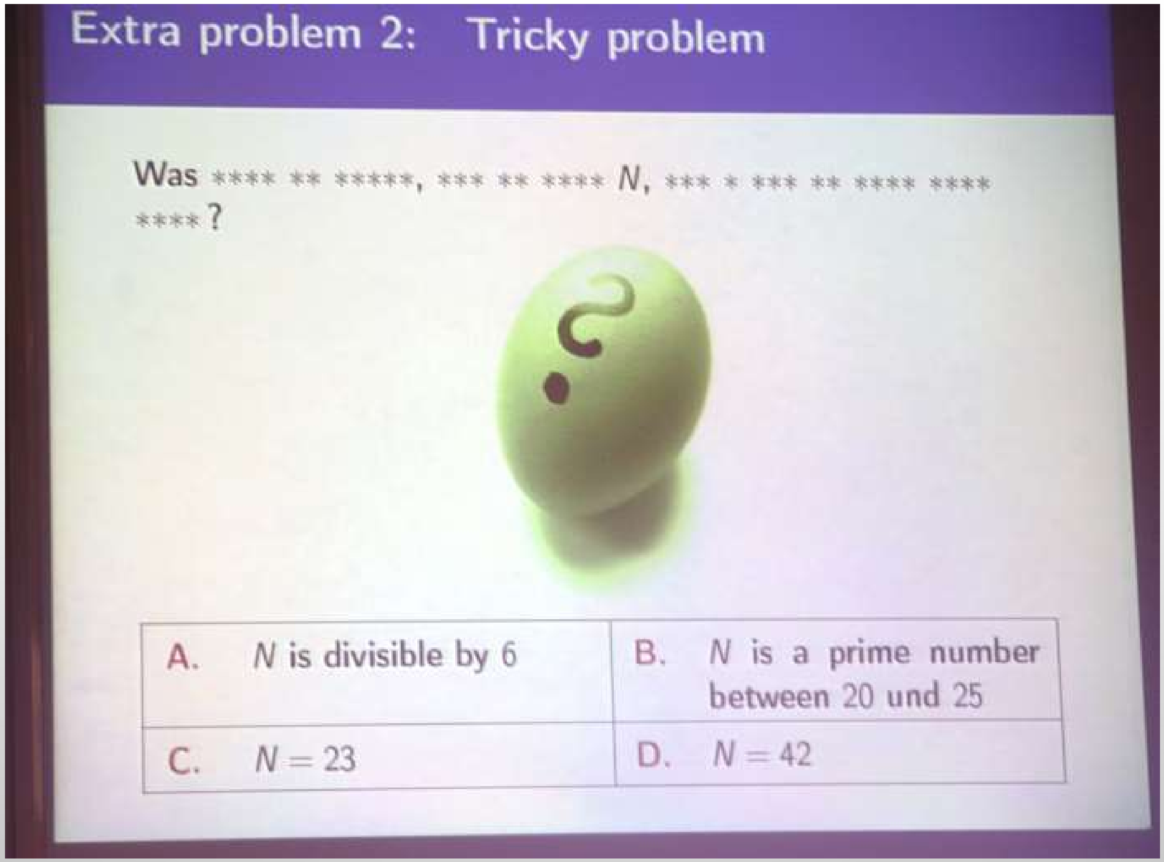
- OFFERING DIFFERENT OPPORTUNITIES FOR MATH ENRICHMENT LEARNING, FOR A WIDE AUDIENCE OF STUDENTS, IS AS IMPORTANT AS THE CONTEST ITSELF

Q: How to make the contest more appealing for students?

Q: Is it worth/feasible organizing a second round of the contest, for the top 5-10%?
(e.g., full solutions required; summer Math camps for the winners)

- OFFERING PROFESSIONAL DEVELOPMENT FOR TEACHERS

Q: What would be required so that such professional development be recognized?


Extra problem 2: Tricky problem
Was **** ** *****, *** ** **** N, *** * *** ** **** ****
**** ?
A. N is divisible by 6
B. N is a prime number between 20 und 25
C. N = 23
D. N = 42

THE PROBLEMS

THEY DO NOT REQUIRE ADVANCED LEVEL KNOWLEDGE.

MOST PROBLEMS REQUIRE:

- CREATIVE WAY OF USING THE KNOWLEDGE ACQUIRED IN THE REGULAR CURRICULUM FOR THE RESPECTIVE GRADE;
- TO SEE THINGS OUTSIDE THE BOX;
- TO GO BEYOND STUDIED PROCEDURES;
- ATTENTION TO DETAIL.

SOME PROBLEMS REQUIRE CONTEST-SPECIFIC KNOWLEDGE APPROPRIATE FOR THE RESPECTIVE GRADE LEVEL.

FAVOURITE TOPICS

Topic	Gr.1-2	Gr.3-4	Gr.5-6	Gr.7-8	Gr.9-10	Gr.11-12
ARITHMETIC, NUMBERS, SMART CALCULATIONS	■	■	■	■	■	■
DIVISIBILITY			■	■	■	■
PROBLEMS WITH THE CURRENT YEAR		■	■	■	■	■
COMBINATORICS		■	■	■	■	■
GEOMETRY, SPATIAL SENSE, MEASUREMENT	■	■	■	■	■	■
GAMES, DICE	■	■	■	■	■	■
LOGICAL PROBLEMS	■	■	■	■	■	■
CLOCK / CALENDAR	■	■	■	■	■	■
SEQUENCES, PATTERNS	■	■	■	■	■	■
CIRCLE MEASUREMENT				■	■	■
CRYPTARITHMETICS	■	■	■	■	■	■
PROPORTIONS, PERCENT			■	■	■	■
EXPONENTS			■	■	■	■
ALGEBRA				■	■	■
TRIGONOMETRY						■
PROBABILITY				■	■	■

EXAMPLES

EXAMPLE 1:

Logic, arithmetic

(2008, 3-4C, 5-6B, 9-10A)

A BOX CONTAINS SEVEN CARDS.

THE CARDS ARE NUMBERED FROM 1 TO 7.

MARY PICKS, AT RANDOM, THREE CARDS FROM THE BOX. THEN JOHN PICKS TWO CARDS, AND TWO CARDS ARE LEFT IN THE BOX.

MARY SAYS TO JOHN: “I KNOW THAT THE SUM OF THE NUMBERS OF YOUR CARDS IS EVEN.”

WHAT IS THE SUM OF THE NUMBERS ON MARY’S CARDS?

A) 10 B) 12 C) 6 D) 9 E) 15

EXAMPLE 1 SOLUTION:

1 2 3 4 5 6 7

MARY IS 100% CERTAIN THAT THE SUM OF THE TWO CARDS OF JOHN IS EVEN

(E.G., BOTH NUMBERS ODD OR BOTH NUMBERS EVEN)

THIS IS POSSIBLE ONLY IF SHE HOLDS ALL EVEN NUMBERS, THUS, LEAVING ONLY ODD NUMBERS FOR JOHN. THEREFORE, THE SUM OF THE NUMBERS ON HER CARDS IS 2+4+6=12.

ANSWER: B) 12

EXAMPLES

Ratios

EXAMPLE 2:
(2006, 7-8C, 9-10C)

TWO FRIENDS, ALEX AND BEN, WERE CAMPING AND STARTED A FIRE TO COOK THEIR DINNER. THEY USED 15 IDENTICAL WOODEN BRICKS: ALEX BROUGHT 8 OF THEM AND BEN BROUGHT 7 OF THEM.

CHARLIE, WHO WAS CAMPING NEARBY BUT HAD NOT BROUGHT ANY WOOD, ASKED TO USE THEIR FIRE TO COOK HIS FOOD.

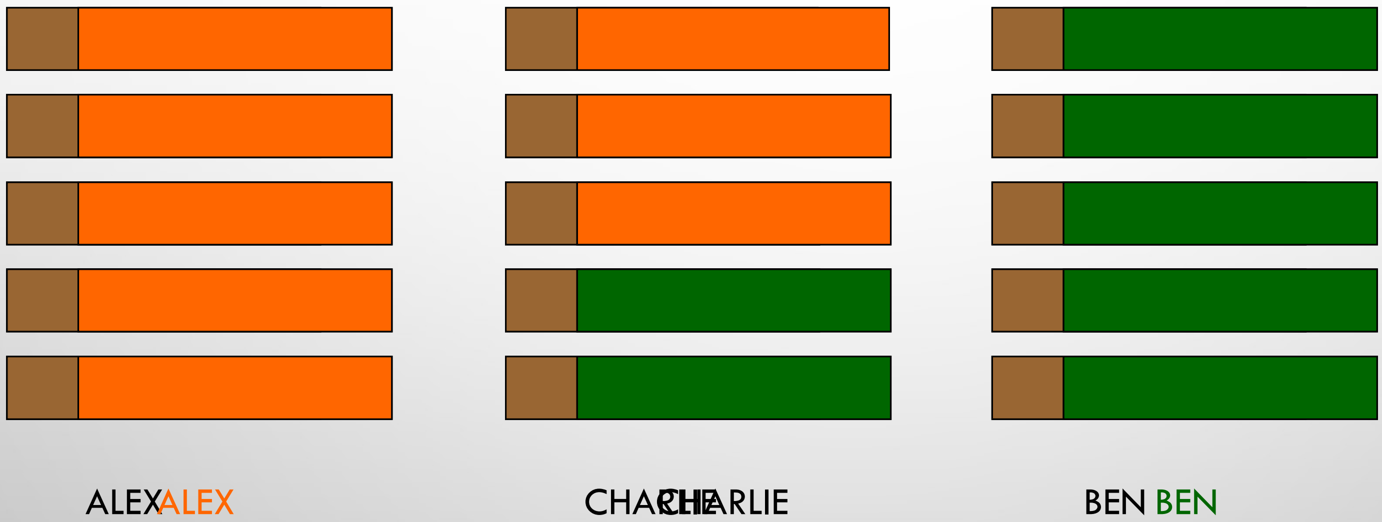
IN RETURN FOR THE FAVOUR, CHARLIE OFFERED ALEX AND BEN 30 CANDIES.

WHAT IS THE FAIR WAY TO SPLIT THE CANDIES BETWEEN ALEX AND BEN?

A) 22 TO ALEX AND 8 TO BEN
B) 20 TO ALEX AND 10 TO BEN
C) 15 TO ALEX AND 15 TO BEN
D) 16 TO ALEX AND 14 TO BEN
E) 18 TO ALEX AND 12 TO BEN

EXAMPLE 2 SOLUTION:

WE SHOULD ASSUME THAT THE THREE BOYS SHARED THE FIRE EQUALLY, E.G., EACH OF THEM USED THE ENERGY FROM FIVE WOODEN BRICKS.



THE DIAGRAM SHOWS THAT CHARLIE USED 3 OF ALEX’S AND 2 OF BEN’S BRICKS, THUS, THE CANDIES MUST BE SHARED IN A RATIO 3:2.

ANSWER: E) 18 TO ALEX AND 12 TO BEN

EXAMPLES

Arithmetic

EXAMPLE 3:

(2006, 9-10B)

ONE PACK OF *CHOCOFRUIT* CANDIES COSTS 10 CROWNS.

(THE OFFICIAL CURRENCY IN THE CZECH REPUBLIC IS CALLED CROWN).

THERE IS A COUPON INSIDE EVERY PACK.

FOR THREE COUPONS YOU CAN HAVE ANOTHER PACK OF *CHOCOFRUIT* CANDIES.

AT MOST HOW MANY PACKS OF *CHOCOFRUIT* CANDIES CAN YOU HAVE FOR 150 CROWNS?

A) 15 B) 17 C) 20 D) 21 E) 22

EXAMPLE 3 SOLUTION:

FIRST, YOU CAN BUY 15 PACKS (150÷10=15).

AFTER THIS, YOU HAVE 15 COUPONS, AND YOU CAN BUY 5 MORE PACKS (15 ÷3=5).

FOR THREE OF THE 5 NEW COUPONS, YOU CAN BUY ONE MORE PACK. THE TWO REMAINING COUPONS, PLUS THE COUPON FROM THE LAST PACK, ALLOWS BUYING ONE MORE PACK.

IN TOTAL, WE CAN BUY 15+5+1+1=22 PACKS OF *CHOCOFRUIT* CANDIES.

ANSWER: E) 22

EXAMPLES

Divisibility

EXAMPLE 4:

(2006, 9-10A, MODIFIED)

PETER SAYS THAT A THIRD OF HIS BOOKS ARE ENGLISH NOVELS, A QUARTER OF THE BOOKS ARE FRENCH NOVELS, AND 30% OF THEM ARE POETRY.

HOW MANY BOOKS DOES HE HAVE?

A)90 B)100 C)120 D) 140 E)144

Hint: As the three categories of books must be represented by an integer number, it is enough to notice that the total number of books must be divisible by 3, 4, and 10. Among the proposed answers, the only such number is 120.

ANSWER: C) 120

EXAMPLES

Divisibility

EXAMPLE 5:

THE TOTAL NUMBER OF STUDENTS IN A SCHOOL IS BETWEEN 500 AND 1000.

THE SCHOOL ADMINISTRATION NEEDED TO PRINT A LIST OF ALL STUDENTS. THE SECRETARY TRIED TO USE TEMPLATES WITH 18, OR 20, OR 24 NAMES PER PAGE, AND NOTICED THAT THERE WERE ALWAYS 7 NAMES LEFT ON THE LAST PAGE.

HOW MANY STUDENTS ARE THERE IN THE SCHOOL?

A)609　　B)848　　C)727　　D) 707　　E) NOT ENOUGH INFORMATION

Hint: The total number of students is a number that leaves a remainder of 7 when divided by 18, or 20, or 24. Therefore, it is 7 more than a common multiple of 18, 20, and 24. Since LCM (18, 20, 24)=360, the total number of students is 7 more than a multiple of 360. In the range 500 to 1000, only 727 is such a number.

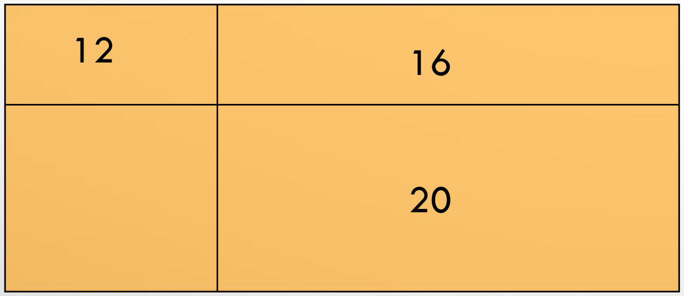
ANSWER: C) 727

EXAMPLES

Geometry

EXAMPLE 6: (GRADE 3-4 OR 5-6)

A RECTANGLE IS DIVIDED INTO 4 RECTANGLES, AS SHOWN ON THE FIGURE.

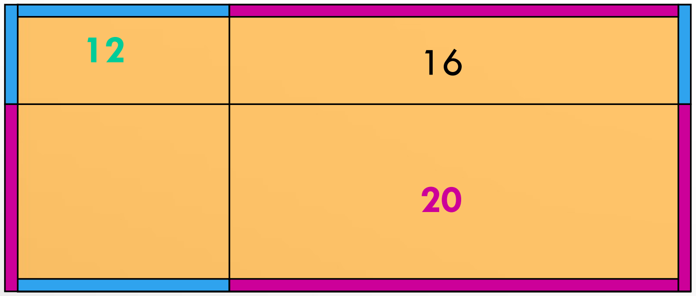


THE PERIMETERS OF THREE OF THEM ARE RESPECTIVELY 12, 16, AND 20.

WHAT IS THE PERIMETER OF THE FOURTH RECTANGLE?

A) 48 B) 32 C) 28 D) 16 E) NONE OF THESE

SOLUTION EXAMPLE 6:



FROM THE DIAGRAM, THE PERIMETER OF THE ORIGINAL RECTANGLE IS EQUAL TO THE SUM OF THE PERIMETERS OF THE TWO RECTANGLES THAT SHARE A VERTEX ONLY.

ONE THE ONE HAND, IT IS 12+20=32. ON THE OTHER HAND, IT IS THE SUM OF 16 AND THE UNKNOWN PERIMETER. SO, THE FOURTH, UNKNOWN PERIMETER, IS 32-16=16.

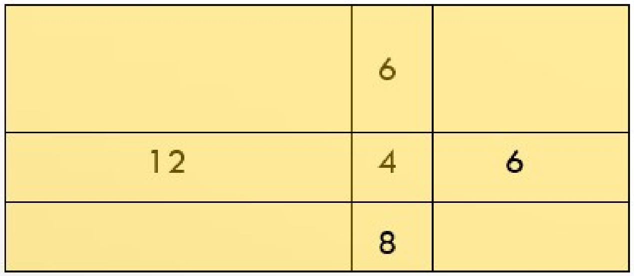
ANSWER: D) 16

EXAMPLES

EXAMPLE 7 (GRADE 7-8, 9-10)

Geometry

A RECTANGLE IS DIVIDED INTO 9 RECTANGLES, AS SHOWN ON THE FIGURE.

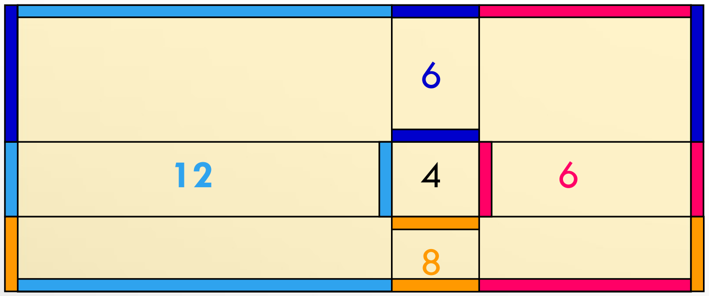


THE PERIMETERS OF FIVE OF THEM ARE RESPECTIVELY 12, 6, 6, 4, AND 8.

WHAT IS THE PERIMETER OF THE ORIGINAL RECTANGLE?

A) 26 B) 28 C) 36 D) 40 E) 32

SOLUTION EXAMPLE 7:



FROM THE COLOURING ON THE DIAGRAM, THE PERIMETER, P, OF THE ORIGINAL RECTANGLE EQUALS

$$P=12+8+6+6-4=28$$

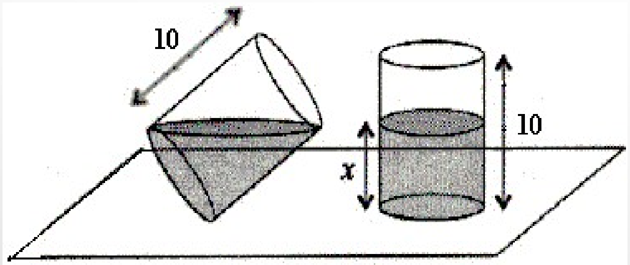
ANSWER: B)28

EXAMPLES

Measurement

EXAMPLE 8: (2003, GR. 5-6 OR HIGHER)

A CYLINDRICAL GLASS, 10CM HIGH, IS PARTIALLY FILLED WITH WATER. YOU SEE THE GLASS IN TWO POSITIONS.



WHAT IS THE HEIGHT OF THE WATER WHEN THE GLASS IS UPRIGHT?

A) 3CM B) 4CM C) 5CM D) 6CM E) 7CM

ANSWER: C) 5CM

EXAMPLES

EXAMPLE 9:

Combinatorics

ADAM HAS A DRAWER FULL OF SOCKS NOT ARRANGED IN PAIRS.

THERE ARE EITHER BLACK OR WHITE, EITHER LONG OR SHORT, EITHER SYNTHETIC OR COTTON SOCKS.

IT IS DARK AND ADAM IS IN A HURRY.

AT LEAST HOW MANY SOCKS MUST HE PULL OUT OF THE DRAWER TO BE CERTAIN THAT HE HAS A PAIR OF SOCKS (SAME COLOUR, LENGTH AND MATERIAL)?

A)6 B)7 C)8 D)9 E)10

Hint:
There are 8 various types of socks. If Adam pulls out 8+1=9 socks, there must be at least two of the same type.

ANSWER: D) 9

EXAMPLES

EXAMPLE 10:

Combinatorics, Divisibility

ADAM HAS A STACK OF 1000 CARDS. ON EACH CARD, A NATURAL NUMBER IS WRITTEN. AT LEAST HOW MANY CARDS MUST ADAM RANDOMLY SELECT FROM THE STACK TO BE CERTAIN THAT, AMONG HIS SELECTION, THERE WILL BE TWO CARDS SUCH THAT THE DIFFERENCE OF THEIR NUMBERS IS A MULTIPLE OF 8?

A)6 B)7 C)8 D)9 E)10

Hint: There are only 8 possible remainders when natural numbers are divided by 8. If Adam selects randomly 8+1=9 cards, at least two of the numbers on them must produce the same remainder (8 holes and 9 pigeons). Their difference is a multiple of 8.

ANSWER: D) 9

EXAMPLES

EXAMPLE 11:

Probability, Logic

WHAT IS THE PROBABILITY THAT IN A SOCCER CLUB WITH 25 MEMBERS (23 PLAYERS AND TWO COACHES) THERE WILL BE AT LEAST THREE PEOPLE CELEBRATING THEIR BIRTHDAYS IN THE SAME MONTH?

A)1 B) 7/12 C) 3/25 D)1/4 E) 0

Hint: There are only 12 possible months. Assuming that there are at most two people born in each month, we can only have at most $2\times12=24$ people. As the team has $2\times12+1=25$ members, there must be at least three people born in the same month.
Therefore, the required probability is 1.

ANSWER: A) 1