

Canadian Abstract Harmonic Analysis Symposium (CAHAS) 2022

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1 Abstract Harmonic Analysis

Abstract harmonic analysis is the study of spaces and algebras associated with locally compact groups G , most prominently, but not exclusively, the group algebra $L^1(G)$ and the Fourier algebra $A(G)$. It has evolved out of classical Fourier analysis where, from the abstract point of view, the abelian groups \mathbb{Z} and \mathbb{R}^N are studied. There is strong interplay between abstract harmonic analysis and the theories of Banach algebras, operator algebras, quantum groups, and operator space theory.

2 The Canadian Abstract Harmonic Analysis Symposium

Abstract harmonic analysis has been strongly represented in Canada over the past decades.

The *Canadian Abstract Harmonic Analysis Symposium* (CAHAS) is a series of meetings in the area, which started in 1997 at the University of British Columbia with a meeting in the honor of Edmond Granirer on the occasion of his retirement. Ever since, CAHAS has been ongoing on an annual basis, albeit in varying formats: sometimes it took place as a section at a CMS meetings; sometimes, when other major events in closely related areas took place in Canada, it was subsumed under those events; in 2009, CAHAS was held in the form of a week long international conference at the University of Alberta with almost 80 participants to celebrate the 65th birthday of Anthony To-Ming Lau, who had been a major leader of the field for decades; and in 2014, CAHAS took place as a two-day workshop at BIRS.

There has always been an emphasis at CAHAS meetings to allow junior researchers to get exposure to the community. CAHAS 2022 was no exception.

On June 30, 2020, Anthony To-Ming Lau retired. For this reason, it had been planned to hold a CAHAS meeting at BIRS in May 2020 to bring together his large “extended mathematical family” and celebrate his contributions to abstract harmonic analysis. The Covid-19 pandemic put that plan on hold. CAHAS 2022—the first time a meeting of the series was held in hybrid format—was the attempt to recreate the planned meeting to the extent possible.

3 Scientific Progress and Presentation Highlights

The presentations at the meeting covered a wide range of topics. There were, e.g., several talks on quantum groups and other “quantized” mathematics (Anderson-Sackenev, Crann, Viselter, Lee), wavelets (Hol-

lingsworth, Milad, Potter), and Fourier algebras and their kin (Choi, Sawatzky, Spronk, Thamizhazhagan, Turowska).

The highlights were the presentations by Yemon Choi and Hannes Thiel.

Let G be a locally compact group, let $A(G)$ denote its Fourier algebra, and let $\text{AM}(A(G))$ stand for the so-called amenability constant of $A(G)$, as introduced by the late Barry E. Johnson in [4]. Johnson showed, for finite G , that $\text{AM}(A(G)) = 1$ if G is abelian and that $\text{AM}(A(G)) \geq \frac{3}{2}$ if G is non-abelian. For general locally compact G , it is easy to see that $\text{AM}(A(G)) = 1$ if G is abelian. For (not necessarily finite) non-abelian G , it was shown in [3] that $\text{AM}(A(G)) \geq \frac{2}{\sqrt{3}}$ ([3]). Reporting on his work in [1], Choi showed at the meeting that $\text{AM}(A(G)) \geq \frac{3}{2}$ for any *non-abelian* locally compact group G . What is remarkable about Choi's result is that the proof is very close in spirit to Johnson's original approach.

Let \mathcal{O}_2 denote the Cuntz algebra. Then it is well known—apparently due to George Elliot—that \mathcal{O}_2 is isomorphic to its tensor square $\mathcal{O}_2 \otimes \mathcal{O}_2$ ([6]). For general $p \in [1, \infty)$, generalizations \mathcal{O}_2^p of \mathcal{O}_2 acting on L^p -spaces can be defined ([5]); it is natural to ask if $\mathcal{O}_2^p \cong \mathcal{O}_2^p \otimes \mathcal{O}_2^p$ for $p \neq 2$. In his talk, based on joint work with Choi and Eusebio Gardella, Thiel showed that $\mathcal{O}_2^p \cong \mathcal{O}_2^p \otimes \mathcal{O}_2^p$ if and only if $p = 2$.

4 Impact of the Hybrid Format

One challenge of the 2-day workshops at BIRS is that it can be difficult for researchers outside of Western Canada or the Northwestern United States to justify the time and travel costs associated with getting to Banff for what is, in essence, at most one and a half days of talks. This is especially true for those whose travel plans force them to miss a significant part of the second day.

The hybrid format provided a very elegant—and far less challenging than expected—way around this obstacle. Eventually 17 of the meeting's 40 attendees participated remotely from abroad; six of those—Yemon Choi (University of Lancaster; United Kingdom), Hun Hee Lee (Seoul National University; South Korea), Serap Öztop-Kaptanoğlu (University of Istanbul; Turkey), Hannes Thiel (Kiel University; Germany), Lyudmila Turowska (Chalmers University; Sweden), and Ami Viselter (University of Haifa; Israel)—gave presentations. Thanks to the excellent IT support by BIRS staff, the technical difficulties of handling a hybrid meeting turned out to be surprisingly minor.

In order to give postdocs, graduate students, and untenured faculty a better chance to connect with the community, it was decided that people from those groups should be given preference when it came to in-person participation. Indeed, of 13 in-person participants, there were one untenured assistant professor (Wiersma), three postdocs (Hollingsworth, Thamizhazhagan, Vati), and four graduate students (Anderson-Sackaney, Kazemi, Sawatzky, Vujicic).

Overall, the hybrid format made the meeting a very pleasant experience for both organizers and participants.

5 Outlook

As stated in Section 2, the Canadian Abstract Harmonic Analysis Symposium has been held (mostly) annually in a 2-day format. A 2-day conference is difficult to organize without the backing of, e.g., a large learned society or research institute behind it: few people will travel large distances for a 2-day meeting, funding is difficult to obtain, and the organizational overhead is almost the same as for a longer meeting.

There is currently some discussion in Canada's abstract harmonic analysis community about holding CAHAS bi-annually with BIRS as its base, but in hybrid format: this would increase the circle of potential participants and, at the same, time decrease the burden of legwork on the organizers. BIRS will likely be contacted in this matter very soon.

References

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