Low-Degree Hardness of Maximum Independent Set

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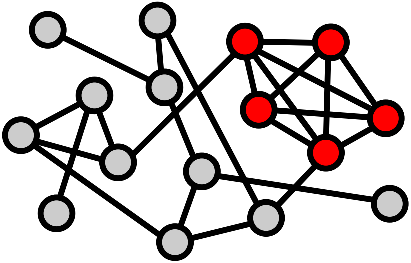

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Q: What is the typical value of the optimum (OPT)?

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Q: In cases where it seems hard to reach a particular objective value, can we understand why? In a unified way?

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Local algorithms achieve value ALG **and no better**

[Gamarnik, Sudan '13; Rahman, Virág '14]

Spherical Spin Glass

Example (spherical p-spin model): for $Y \in (\mathbb{R}^n)^{\otimes p}$ i.i.d. $\mathcal{N}(0,1)$,

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Approximate message passing (AMP) algorithms achieve value ALG_p **and no better** [El Alaoui, Montanari, Sellke '20]

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Solution: lower bounds against a larger class of algorithms (low-degree polynomials) that contains both local and AMP algorithms

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- Power iteration: $Y^k \mathbf{1}$
- Approximate message passing
- Local algorithms on sparse graphs
- Or any of the above applied to $\tilde{Y} = g(Y)$

Planted Problems

Low-degree algorithms are already well-studied for problems with a planted signal

[Barak, Hopkins, Kelner, Kothari, Moitra, Potechin '16]

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<u>This work</u>: extend low-degree framework to non-planted setting

Results: Max Independent Set

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Theorem [Gamarnik, Jagannath, W. '20; W. '20]
No polynomial $f : \{0,1\}^{\binom{n}{2}} \to \mathbb{R}^n$ of degree $\mathrm{polylog}(n)$ achieves both of the following with probability $1 - \exp(-n^{\Omega(1)})$:

- $f_i(Y) \in [0, 1/3] \cup [2/3, 1]$ for most i
- $\{i : f_i(Y) \in [2/3, 1]\}$ is a near-indep set of size $(1+\epsilon)\frac{\log d}{d} n$

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For random optimization problems, need different approach:

- Stability of low-degree polynomials
- Overlap gap property (OGP)
  [Gamarnik, Sudan '13]
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Theorem [Gamarnik, Jagannath, W. '20]

$$\Pr_{Y^{(0)},\ldots,Y^{(m)}} [\nexists\, c\text{-bad } i] \geq p^{4D/c}$$

With non-trivial probability (over path), f's output is "smooth"

Overlap Gap Property [Gamarnik, Sudan '13]

Overlap gap property (OGP): with high probability over $Y \sim G(n, d/n)$, there does not exist $S, T \subseteq [n]$ such that

- S, T independent sets
- $|S|, |T| \geq (1 + \frac{1}{\sqrt{2}})\Phi$ $\qquad \Phi := \frac{\log d}{d} n$
- $|S \cap T| \approx \Phi$

Proof: first moment method [Gamarnik, Sudan '13]

Ensemble OGP [CGPR'17, GJ'19]

Ensemble OGP: with high probability over

$$Y^{(0)} \quad Y^{(1)} \quad Y^{(2)} \quad \dots \quad Y^{(m-1)} \quad Y^{(m)}$$

there does not exist $S, T \subseteq [n]$ such that

- S independent set in some $Y^{(i)}$
- T independent set in some $Y^{(j)}$
- $|S|, |T| \geq (1 + \frac{1}{\sqrt{2}})\Phi \qquad \Phi := \frac{\log d}{d} n$
- $|S \cap T| \approx \Phi$

Proof [Gamarnik, Jagannath, W. '20]

Proof that low-degree polynomials cannot reach $(1 + \frac{1}{\sqrt{2}})\Phi$:

Suppose $f(Y)$ outputs independent sets of size $(1 + \frac{1}{\sqrt{2}})\Phi$

$$Y^{(0)} \quad Y^{(1)} \quad Y^{(2)} \quad \ldots \quad Y^{(m-1)} \quad Y^{(m)}$$

Separation: $f(Y^{(0)})$ and $f(Y^{(m)})$ are "far apart"

Stability: with probability $\gtrsim n^{-D}$, there are no big "jumps"

$$f(Y^{(i)}) \to f(Y^{(i+1)})$$

Contradicts OGP

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