Recovering General Relativity from a Planck scale discrete theory of Quantum Gravity (Work with Jeremy Butterfield 2106.01297)

- Make **two Assumptions** about a (putative) quantum gravity theory X:

  A1. **Recovers GR** and A2. Is **Planck scale discrete**

- Introduce the concepts of **grounding state, Discrete Physical Data** (DPD) and **Discrete-Continuum correspondence** (DCC) for theory X

- State and briefly justify **two Claims:**

  C1. Causal sets can recover GR spacetimes

  C2. There is no other proposal to date for a DPD-set that does the job

- (If there's time, I will explain why "quantum uncertainty" does not invalidate the argument)

- **Conclusion:** no matter what X is fundamentally, if the assumptions hold, then at the point where a GR spacetime needs to be recovered, there is at present no entity other than a causal set that will do the job that Discrete Physical Data in X must do.

Quantum Gravity Theory X

Assumption 1. Theory X recovers GR as an approximation in certain states (physical situations), at macroscopic scales.

Assumption 2. X is physically discrete at the Planck scale.

Comments

1. Analogies: (a) GR recovers Newtonian gravity (b) Molecular dynamics recovers fluid dynamics (continuum is recovered/emergent) (c) Quantum mechanics recovers classical mechanics (either already, or in the future).

2. X must recover a large class of 4-dimensional GR spacetimes including gravitational waves, large portions of Minkowski space, black holes and expanding cosmologies. All assumed to vary slowly on Planckian scales.

3. X has, for each GR spacetime (M,g) to be recovered, a **grounding state** which contains/produces/gives rise to a set of **Discrete Physical Data** (DPD) from which (M,g) can be recovered *essentially uniquely* (i.e. (M,g) is a good approximation to the DPD)

4. The DPD-set contains no geometrical information about the GR spacetime at smaller than Planckian length/time/volume scales.

5. No assumption about the nature of the grounding state (might be a state in a Hilbert space, or a co-event in a quantum measure theory, or …)

6. No assumption about how the state gives rise to the DPD (might be expectation values or eigenvalues of certain self-adjoint operators, or involve some kind of coarse graining, or require more-or-less anthropocentric manoeuvres, or …)

Comments (cont)

1. There must be a **Discrete-Continuum-Correspondence** (DCC-X), DPD <—> GR SPACETIME that says (up to some tolerance) when a GR spacetime is recovered from a DPD-set. c.f. molecular state <—> fluid state

2. Essential uniqueness is necessary for the DCC-X to hold water: If (M,g) and (M', g') are both recovered by the same DPD-set according to DCC-X, then we must have that (M,g) and (M',g') are approximately isometric c.f. if two fluid states can be recovered from the same molecular state, they are approximately equal.

3. "What about superpositions and/or duality"? Our assumption is that X recovers GR and in GR the world is one spacetime (M,g). So the assumption is that the singleness of spacetime can be derived in X and we take the DPD-set in hand *after* this has been done.

4. The argument does not target the use of (a) piecewise flat Lorentzian manifolds as continuum approximations to continuum geometries, nor (b) a discreteness length used as a regulator to be taken to zero in a continuum limit

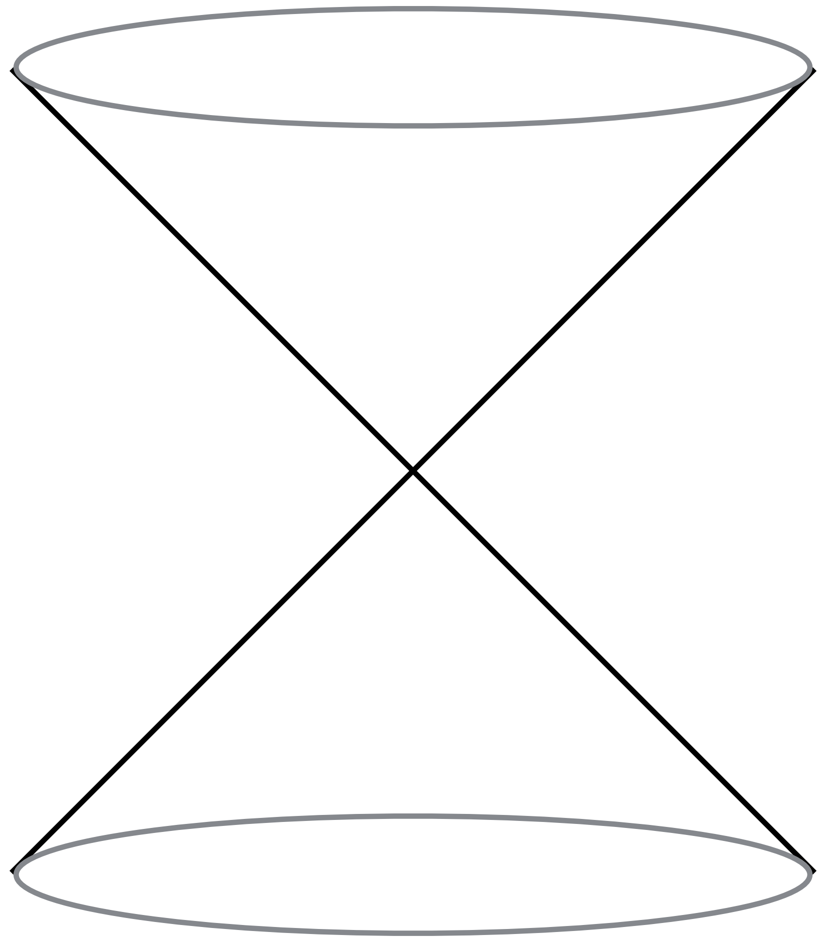
Two Claims

Claim 1: A causal set—a locally finite partial order—is a set of DPD that, taken as being discrete on the Planck scale, can recover a GR spacetime as a continuum approximation. (Order + Number = Lorentzian Geometry)

Claim 2: There is in the current literature no other proposal for a set of Planck scale DPD that can recover a GR spacetime as a continuum approximation.

Recall what X recovers

Causal order is central to the **physics** of GR.



Lorentzian geometry is **very different** from Riemannian geometry

Lorentzian geometry is (bordering on) non-local and the notion of "physically close" is not captured well in any picture: the chosen frame is a huge impediment to understanding

e.g. the points one Planck time in the future of P in Minkowski space lie on an infinite spatial hyperboloid that asymptotes to the future light cone.

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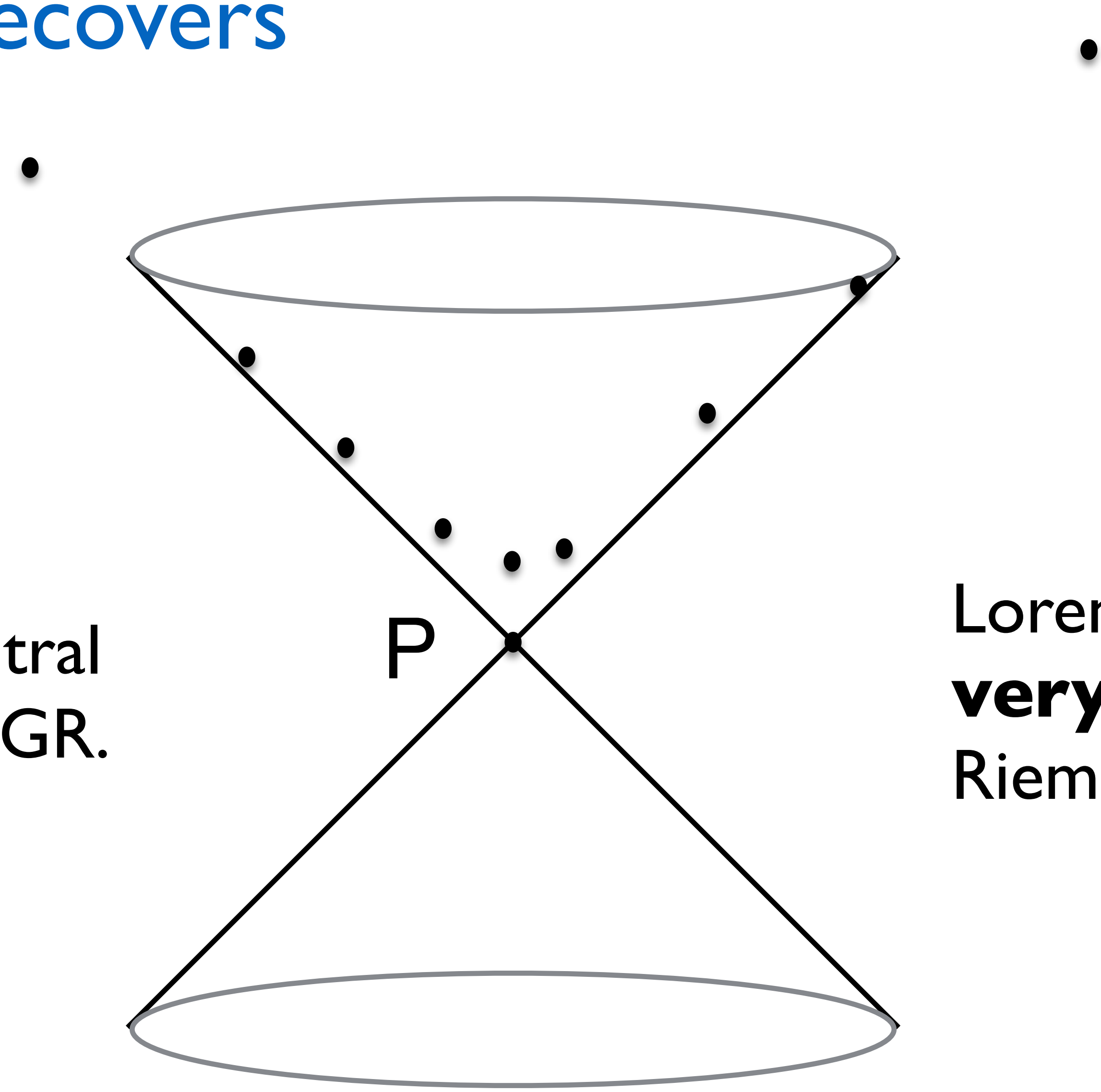

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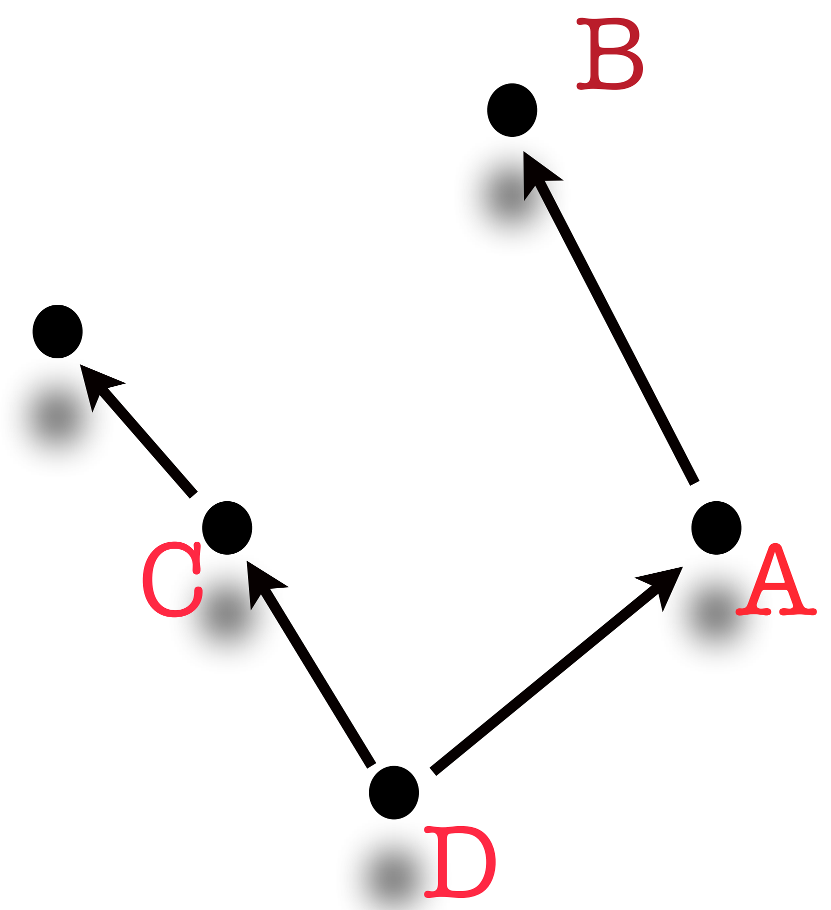
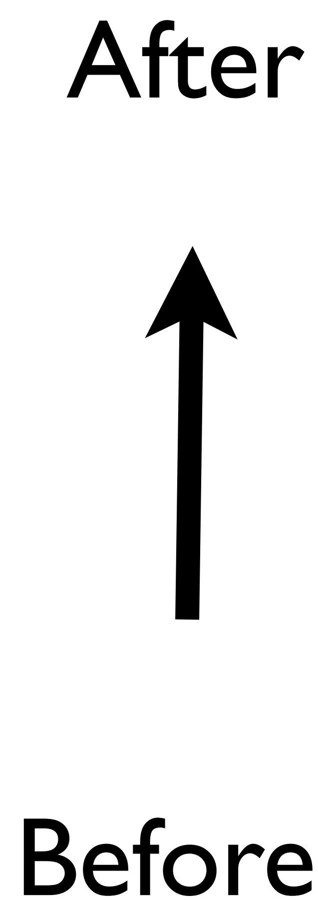
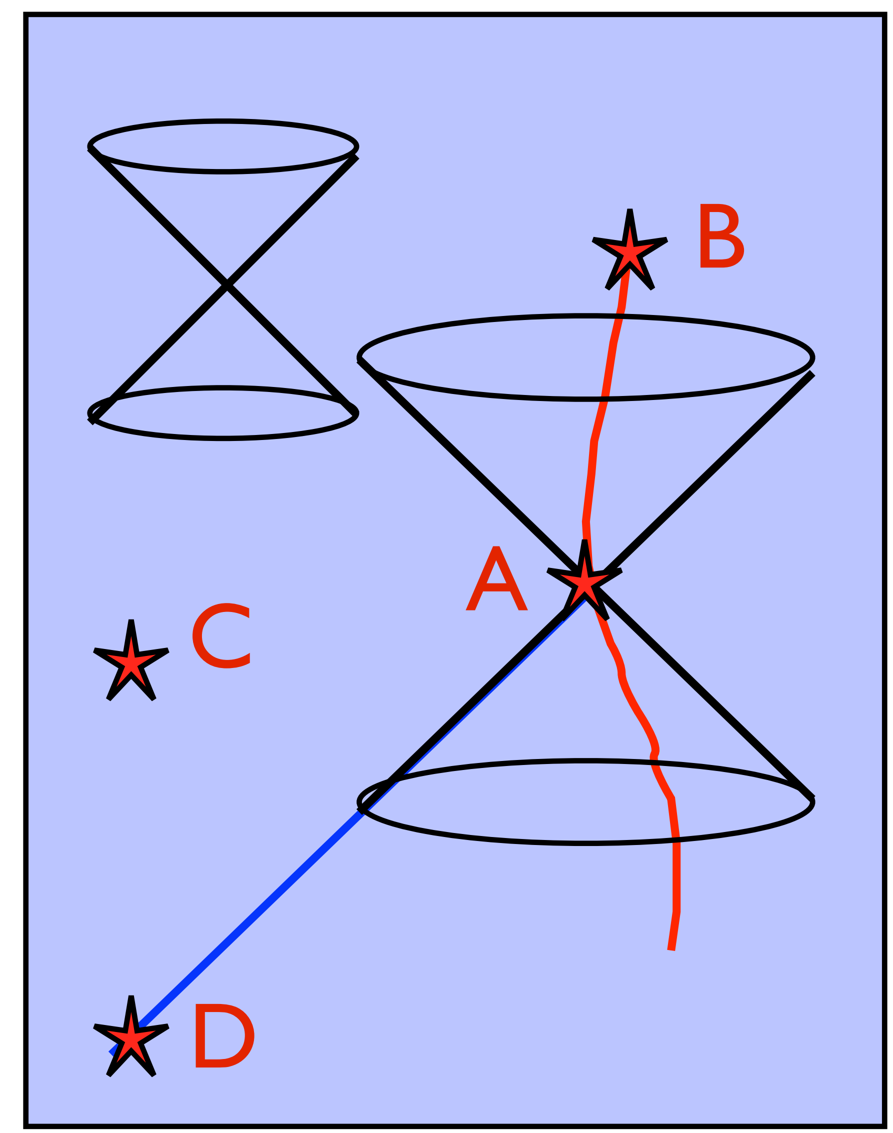
Claim 1: Discrete Order = causal set

A transitive, directed, acyclic graph





Continuum approximation (fluid)



Evidence for Claim 1 (Order + Number = Geometry)

The Discrete-Continuum Correspondence for causal sets :

A causal set C recovers (M,g) if C **faithfully embeds** in (M,g) at Planck density: the embedding respects

(i) Number-Volume correspondence in large, physically nice regions and

(ii) The order

As far as we know (i) means that C must be a **random** sample of (M,g)

A. Kronheimer-Penrose-Hawking-Malament theorem: Order + Volume = Lorentzian Geometry. A causal set is a random sample of the Order and furnishes the Volume for free, by counting (c.f. Riemann)

B. At infinite density, a faithfully embedded causal set —> (M,g)

C. Direct evidence: e.g. dimension, geodesic proper time

Claim 2: Combinatorial Lorentzian Regge Complex

Proposal for alternative DPD-set: A combinatorial 4d Lorentzian Regge complex (CLRC)

Concept: a geodesic dome with only **combinatorial** connectivity information plus Lorentzian edge-length (including, if appropriate, direction) labels on the 1-d edges which are no more than a few in Planck units. No geometrical information in the interior of the simplices — that would be continuum information.

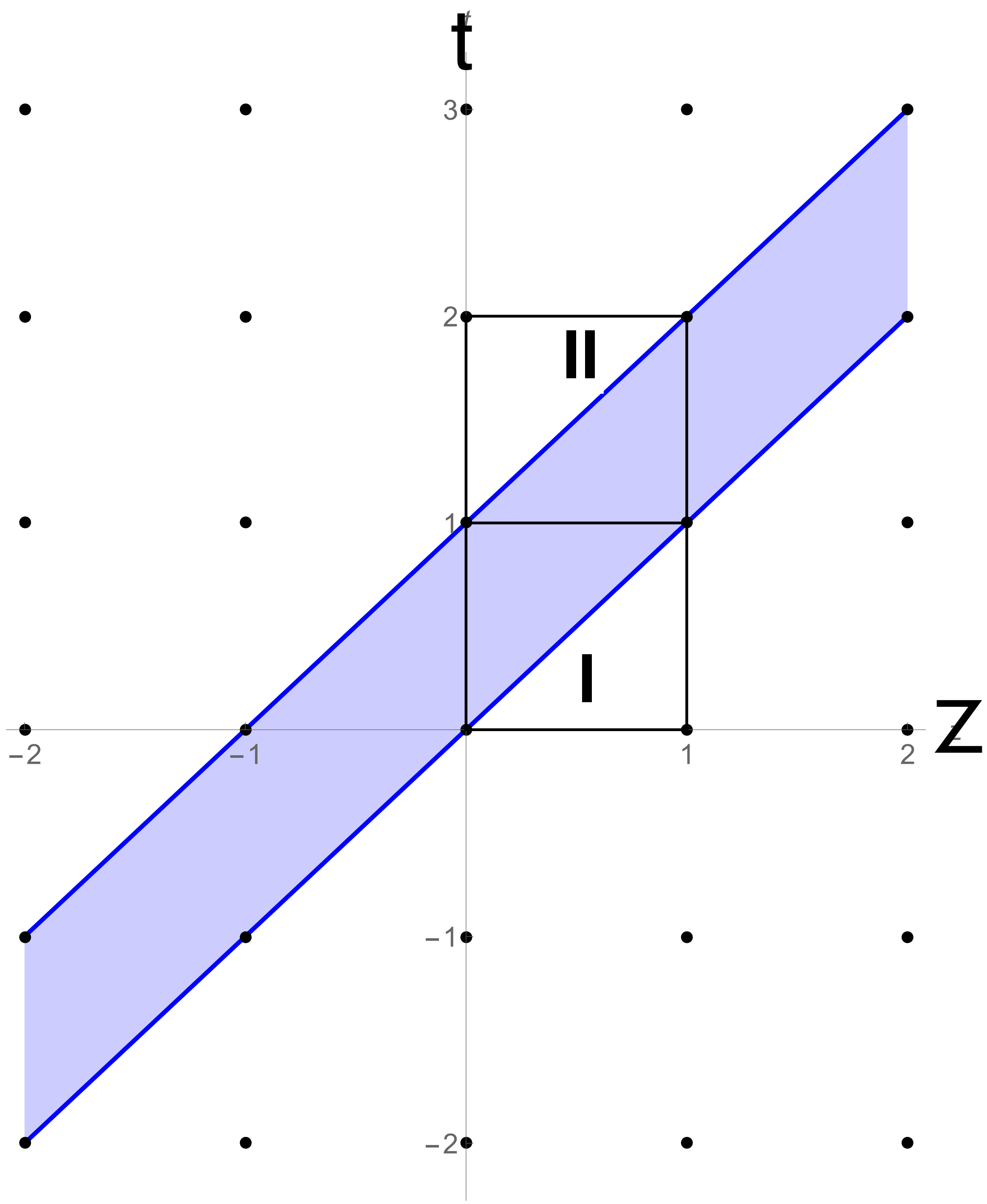
Evidence for Claim 2

The Discrete-Continuum Correspondence for CLRC :

A CLRC recovers (M,g) if the CLRC (i) is the combinatorial information in a triangulation of manifold M (ii) can be embedded in (M,g) such that the geodesics between the embedded vertices have lengths = edge-length labels in Planck units (approximately)

This DCC-CLRC fails. There exists a CLRC that (according to this DCC) "recovers" Minkowski space and **also** "recovers" a spacetime that is a perturbation of Minkowski space with a physical, plane fronted gravitational wave burst.

Example based on integer lattice in 3+1 dims



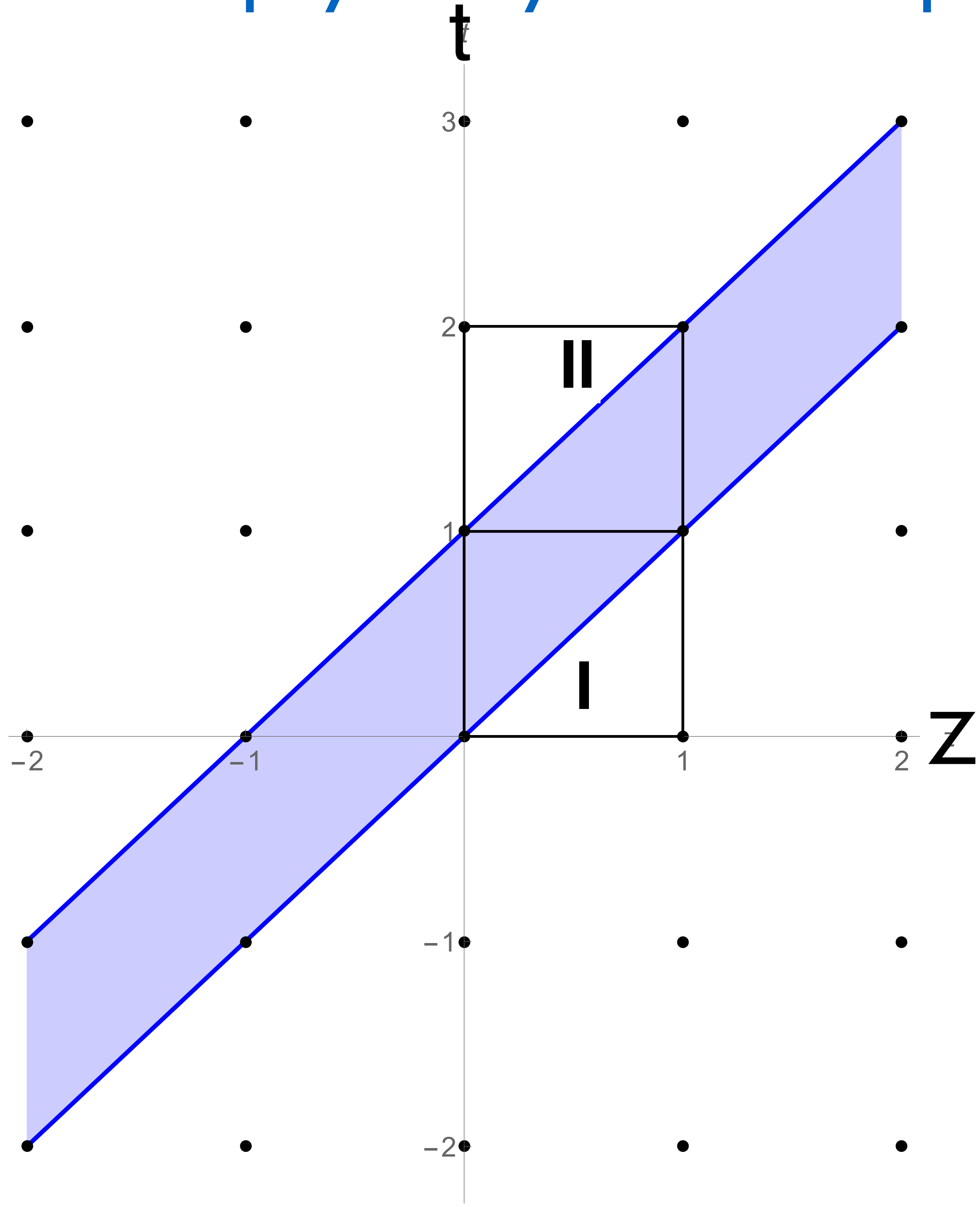
Define a CLRC:

- Each hypercube is triangulated into simplexes
- Each edge is labelled by the Lorentzian length of the edge in the corresponding triangulation of Minkowski space
- This recovers GW burst spacetime (1) for any *h(u) including h(u) = 0*
- Key point: there are no vertices embedded in the shaded region: it's a **void**

$$ds^2 = -dt^2 + dz^2 + (1 - h(u))dx^2 + (1 + h(u))dy^2 \quad (1)$$

$$h(u) \neq 0 \quad in \quad 0 < u < 1 \quad and \quad \int_0^1 du\, h(u) = 0$$

This is a physically nice GR spacetime



- The void is a large, physically nice region of spacetime.
- It contains approximately flat causal diamonds of height 1 second
- **Lorentz invariance** is the key to this counterexample
- To see this: do a Lorentz boost in the z direction with gamma factor of 10^{44}

$$\{t, x, y, z\} \to \{t', x, y, z'\}$$

$$ds^2 = -dt'^2 + dz'^2 + (1 - H(u'))dx^2 + (1 + H(u'))dy^2 \qquad (2)$$

$$u' \approx 10^{44}u, \quad H(u') = h(u) \neq 0, 0 < u' < 10^{44}$$

What about other alternative DPD-sets?

- Whatever the DPD-set is, without a Number-Volume correspondence in the DCC-X, there will be large, physically nice regions voids, without enough data
- When there are large, physically nice voids, the DCC-X will not work because the DPD cannot recover the Lorentzian geometry in the voids
- If one demands the Number-Volume correspondence in the DCC-X, there is only one proposal for DPD in the literature: causal sets

Quantum uncertainty does not invalidate this conclusion

1. Causal sets: There is quantum uncertainty in a grounding state that recovers (M,g) if the DPD-set is {causal set C **or** C' **or** any causal set that faithfully embeds in (M,g)]}. Such a DPD-set is a **coarse graining** of the full detailed information in any particular faithfully embedded C. (M,g) is a **common** approximation to each of them and (M,g) can be recovered from **any one** of them.

2. Combinatorial Lorentzian Regge Complexes: There is quantum uncertainty in a grounding state that recovers (M,g) if the DPD-set is {CLRC S **or** S' **or** any CLRC that consistently embeds in (M,g)}. Such a DPD-set is a **coarse graining** of the full detailed information in any particular consistently embeddable S. **But, (**M,g) cannot be recovered from any one of them: (M,g) is not an approximation to any one of them. Quantum uncertainty makes the failure of CLRCs worse because each CLRC lacks information in the voids and coarse graining **throws away** information.

Conclusion

No matter what X is fundamentally, if the assumptions hold, then at the point where a GR spacetime needs to be recovered, there is at present no entity in the literature other than a causal set that can do the job of recovery that Discrete Physical Data in a grounding state in X must do.