Passive Source Localization

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with help from
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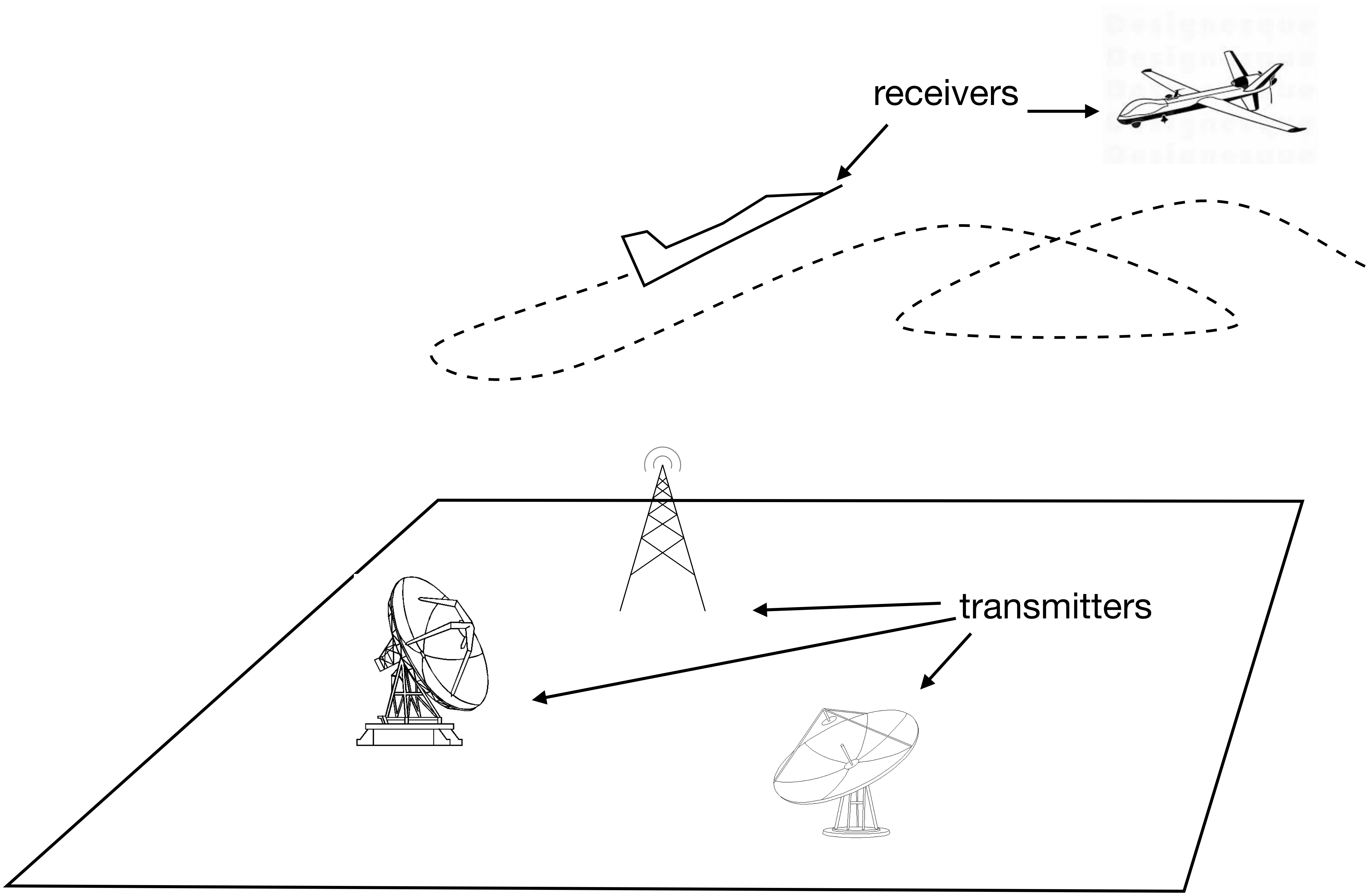
Chad Waddington (AFRL)
Karleigh & Sam Pine (Matrix Research)

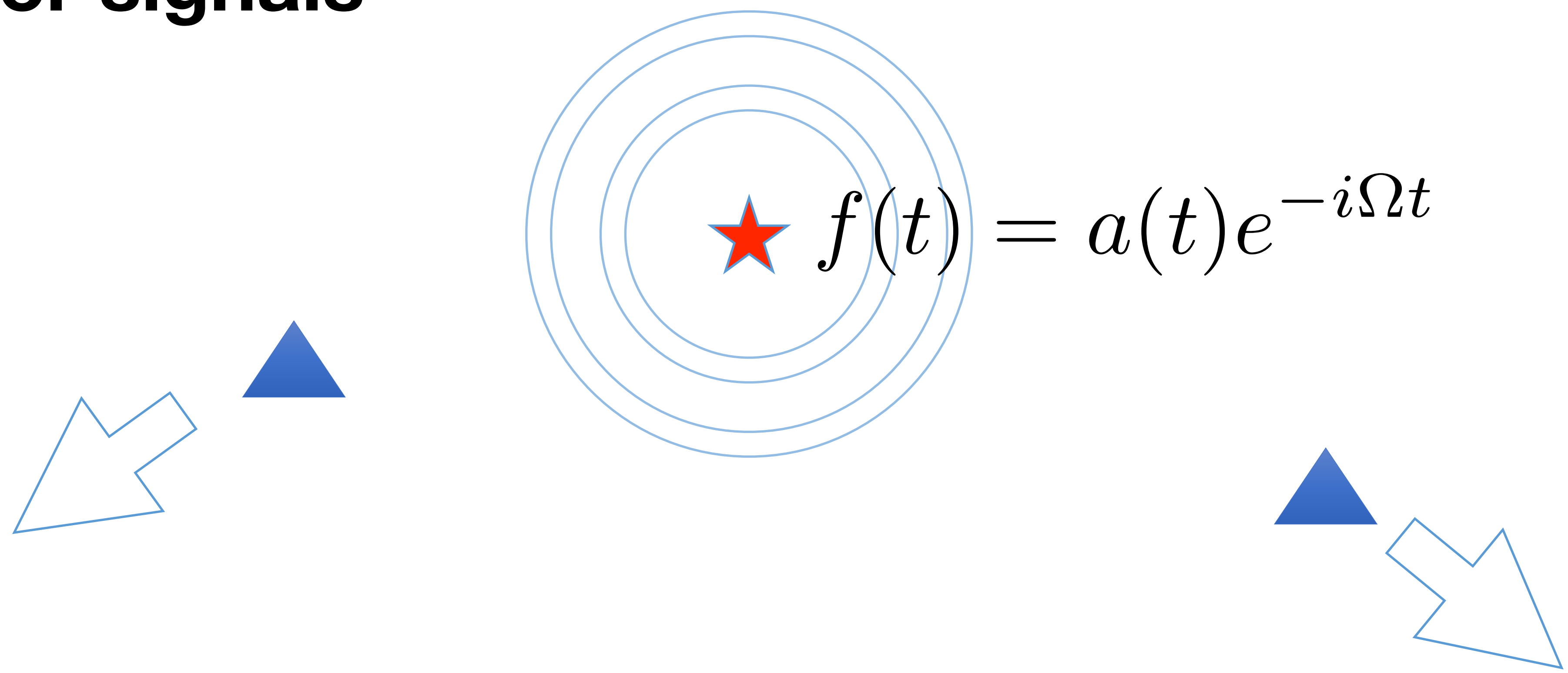
Passive Source Localization



Need for passive source localization

- Locate emergency beacons (downed aircraft, lost hikers, lost drones)
- Find locations where gunshots were fired
- Air and ship traffic management
- Military might want to locate:
  - adversary radars, communications systems, drones and drone controllers, jammers, …..
  - ships, submarines, ….
- Drones make near-field localization systems important

Model for signals



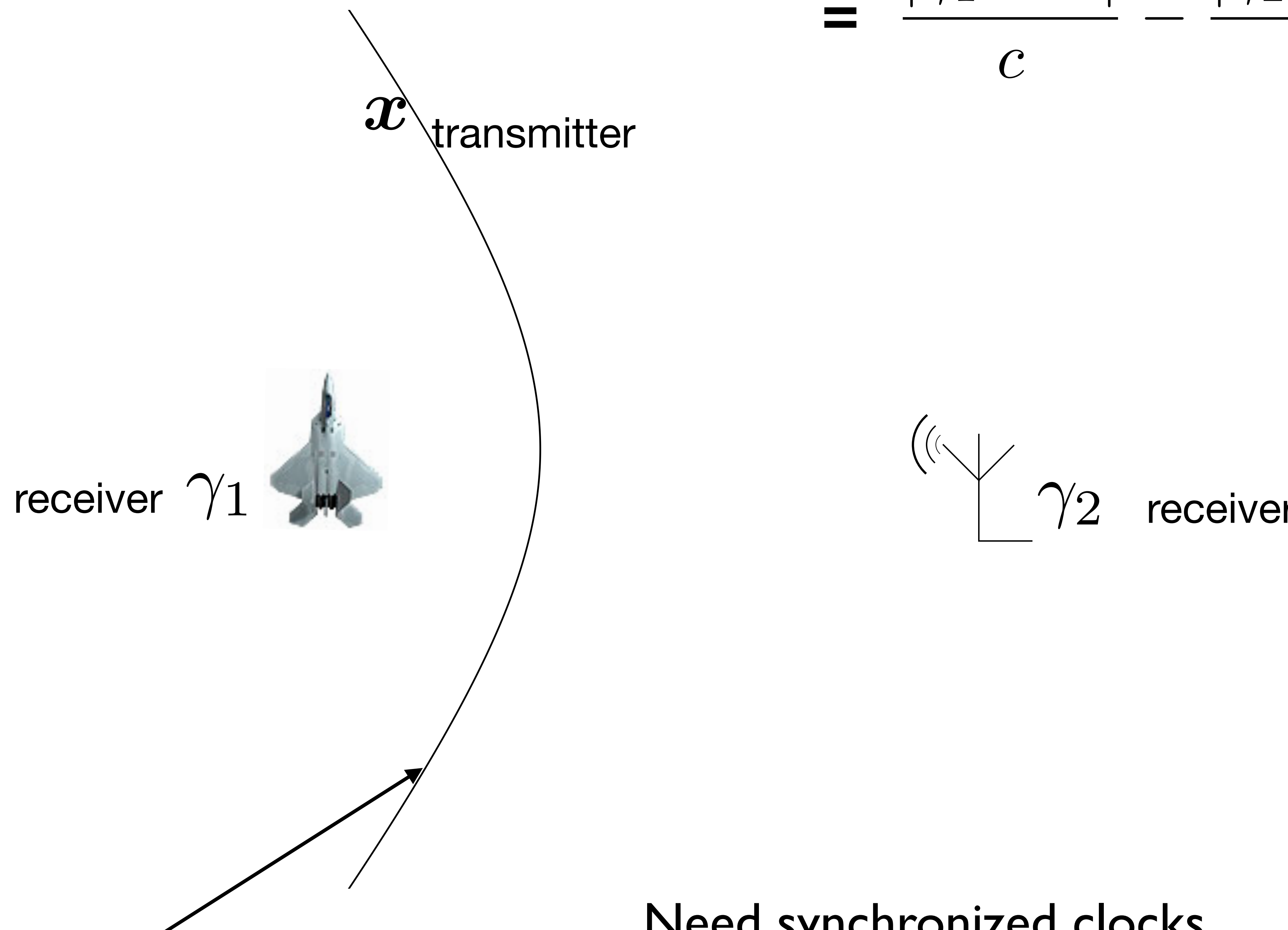
$$f(t) = a(t)e^{-i\Omega t}$$

on j'th receiver: $$d_j(t) = \beta_j a(t - \tau_j) e^{i(\Omega - \nu_j)t} + n_j(t)$$

unknowns: attenuation (β_j), time delay (τ_j), Doppler shift (ν_j)

Time Difference Of Arrival (TDOA) = $\tau_1 - \tau_2$

$$= \frac{|\gamma_1 - \boldsymbol{x}|}{c} - \frac{|\gamma_2 - \boldsymbol{x}|}{c}$$



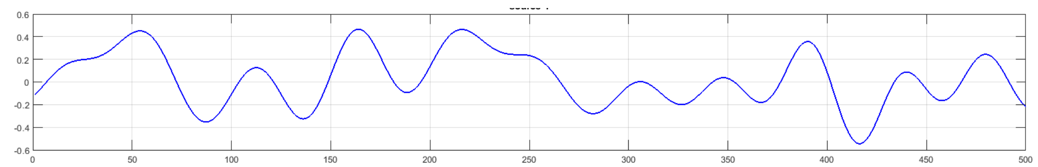
Need synchronized clocks

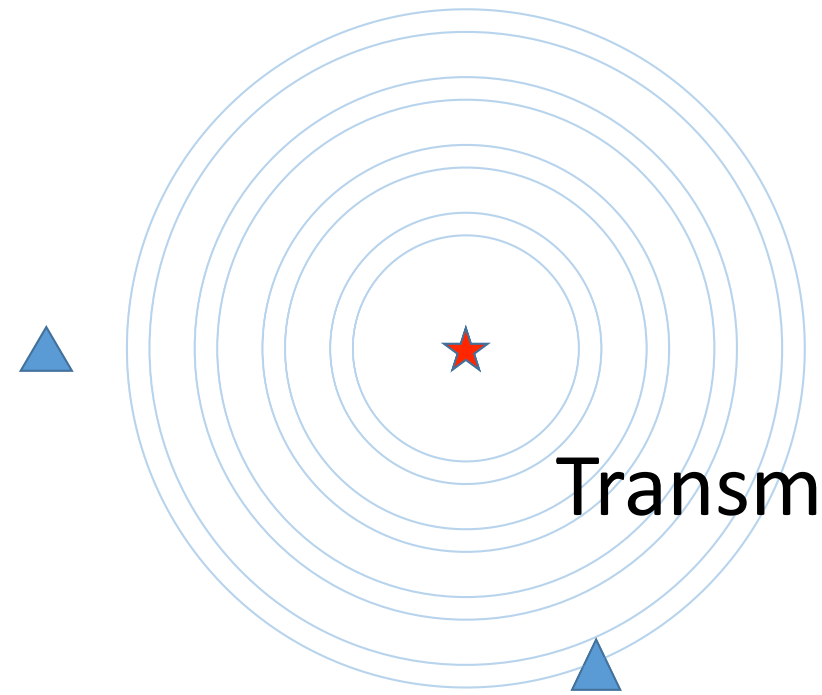
Location of a single source can be found with multiple sensor pairs

How to find TDOA

Receive

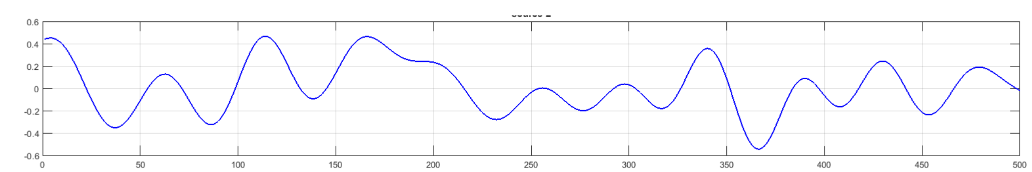
$$s_1(t) = f(t - \tau_1) + n_1(t)$$





Transmits f(t)

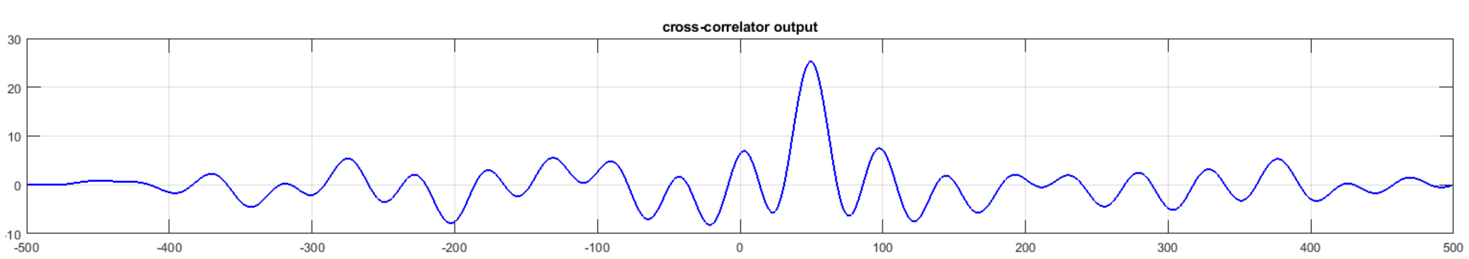
$$s_2(t) = f(t - \tau_2) + n_2(t)$$



$$\mathrm{cor}(s_1, s_2)(\tau) = \int s_1(t) s_2^*(t + \tau) dt$$

$$= \underbrace{\int f(t - \tau_1) f^*(t + \tau - \tau_2) dt}_{\text{max magnitude when } \tau = \tau_2 - \tau_1} + \text{noise}$$

Time Difference Of Arrival (TDOA)

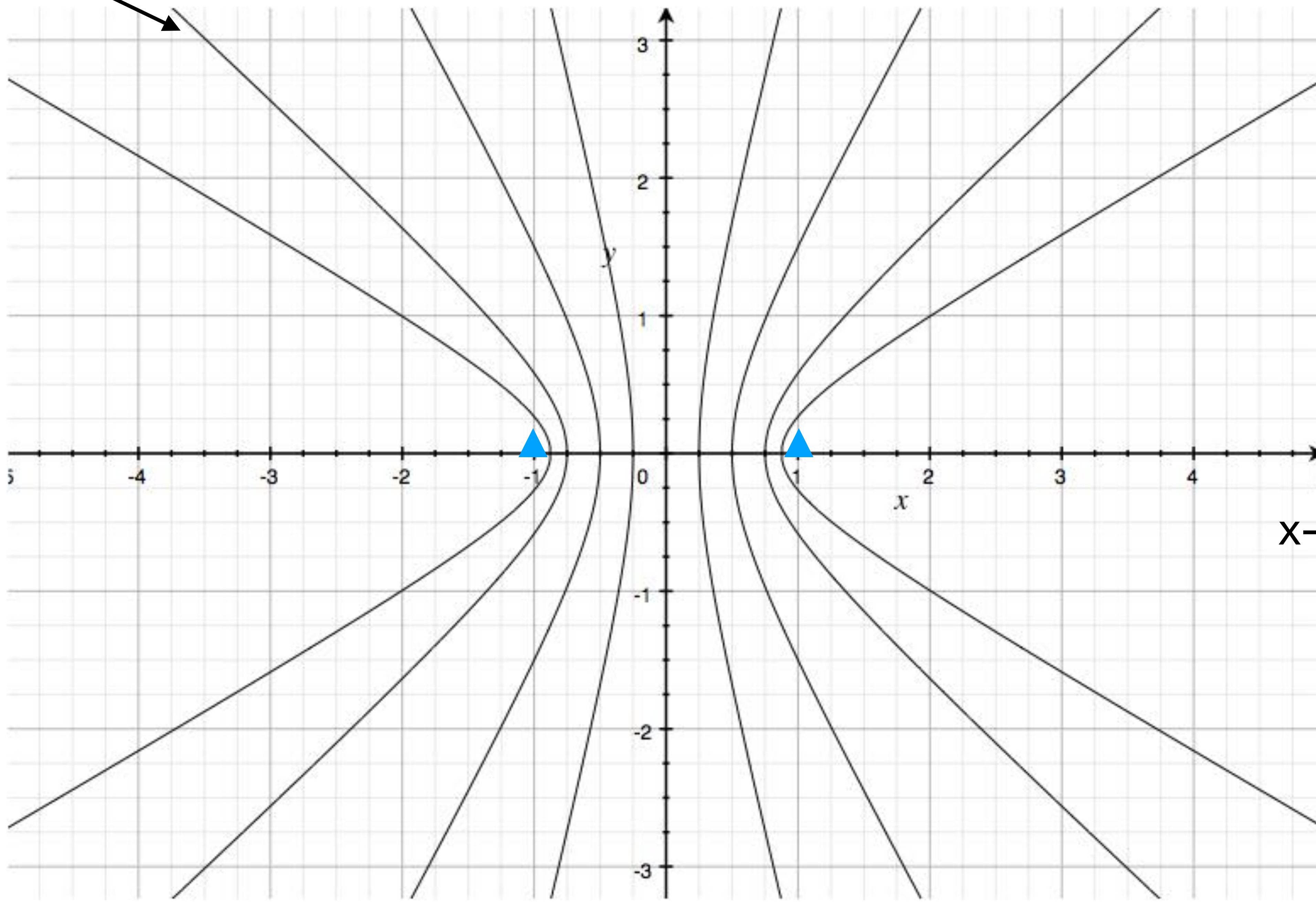


TDOA curves

sensors at (1,0) and (-1,0)

set of points where a transmitter could be for a given TDOA value

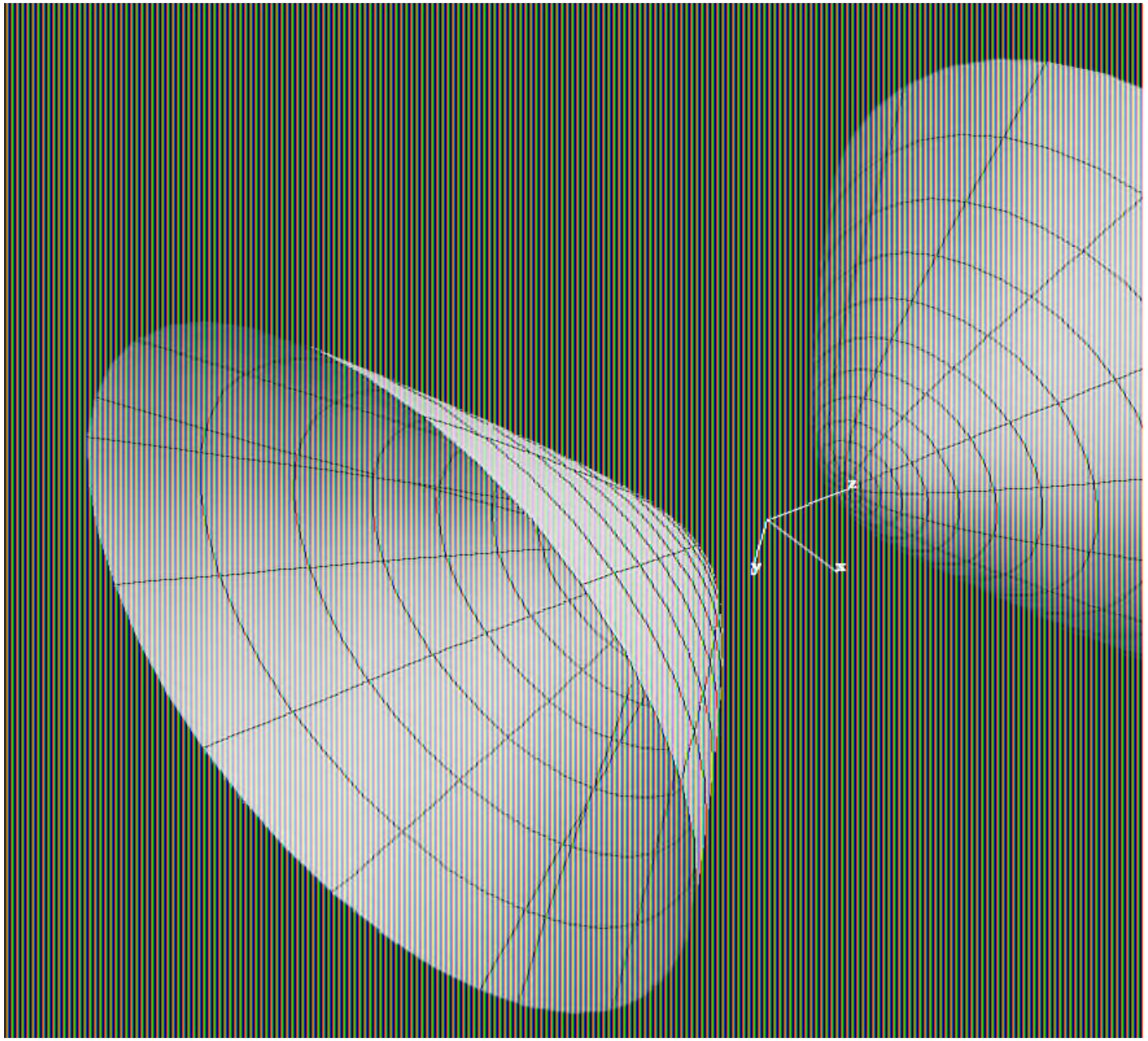
different curves correspond to different TDOA values



x-position

y-position

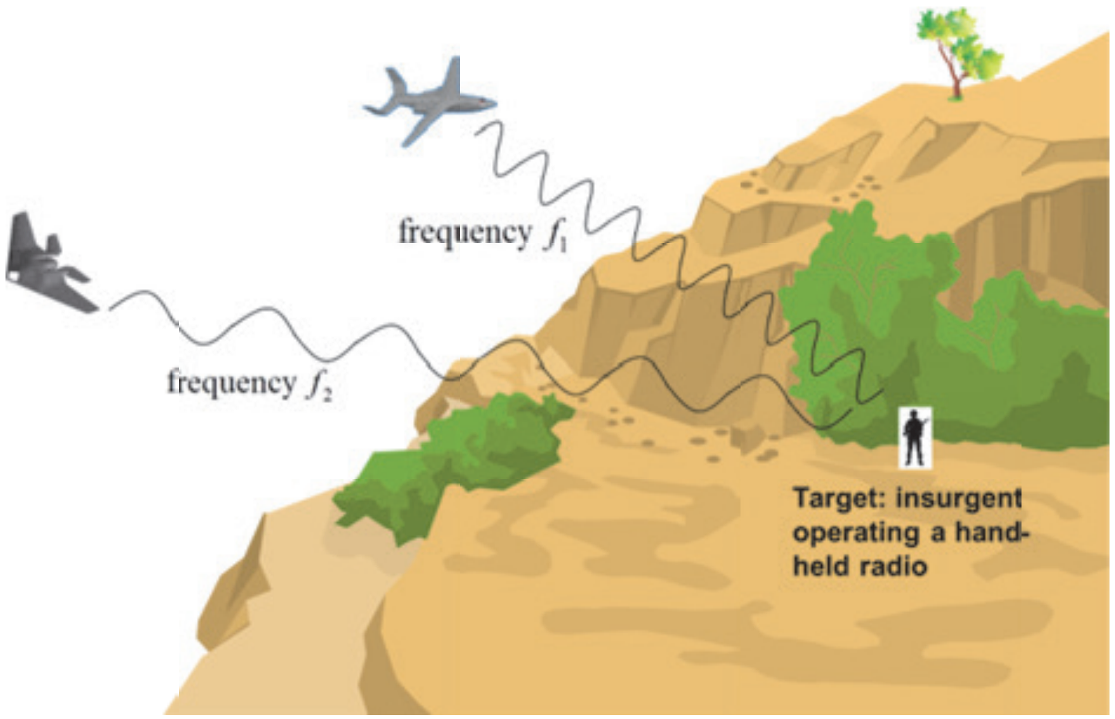
TDOA Hyperboloids revolve 2D hyperbola around x axis



http://virtualmathmuseum.org/Surface/hyperboloid2/hyperboloid2.html

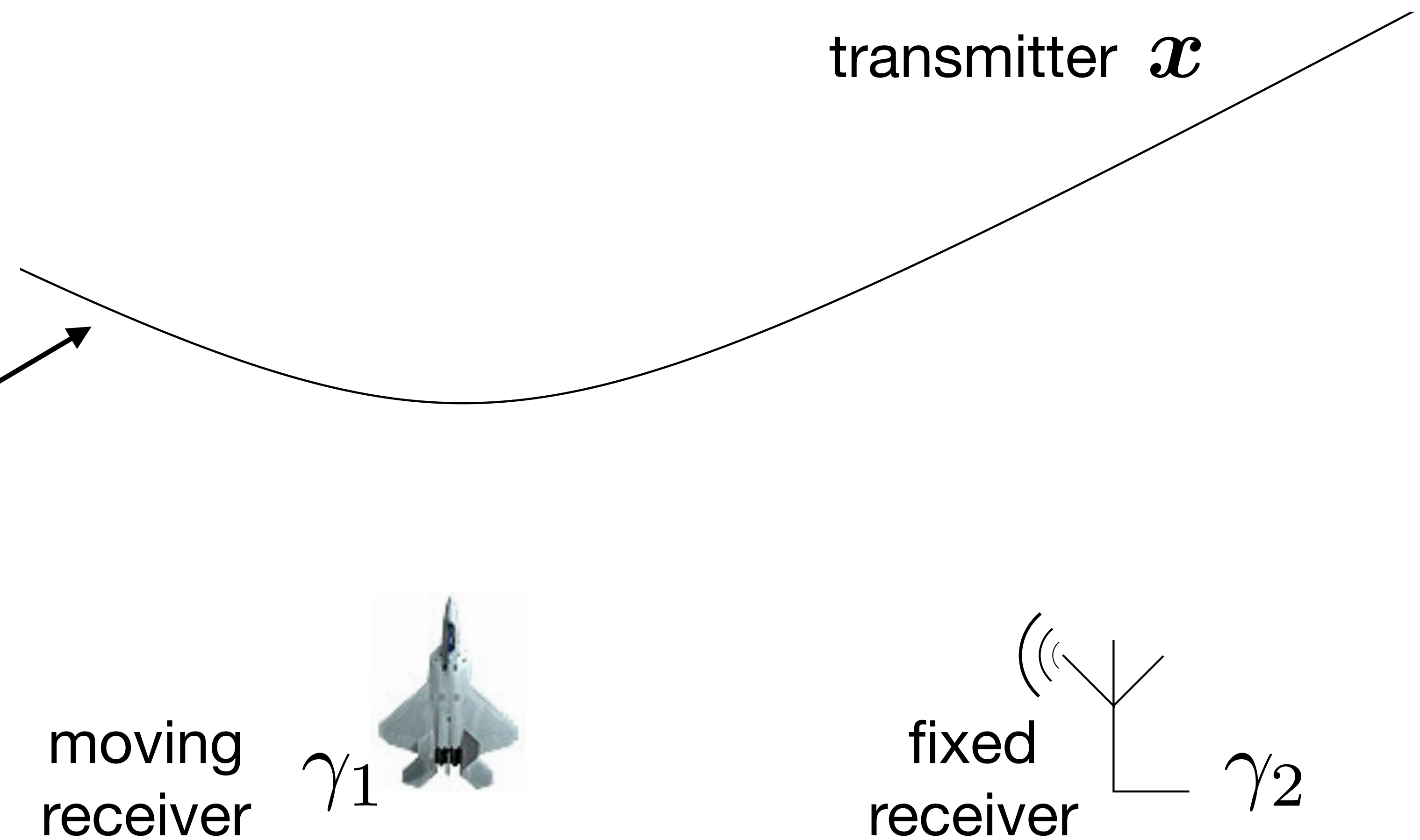
Frequency Difference Of Arrival (FDOA) = $\nu_1 - \nu_2$

$$= \omega \left(\widehat{\gamma_1 - \boldsymbol{x}} \cdot \frac{\dot{\gamma}_1}{c} - \widehat{\gamma_2 - \boldsymbol{x}} \cdot \frac{\dot{\gamma}_2}{c} \right)$$



Kimberly Hale 2012 dissertation
Pardee Rand Graduate School

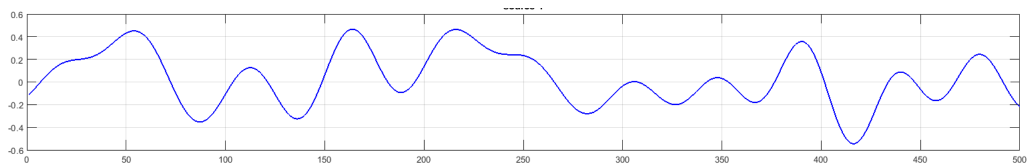
set of points where transmitter could be located, for a given value of FDOA

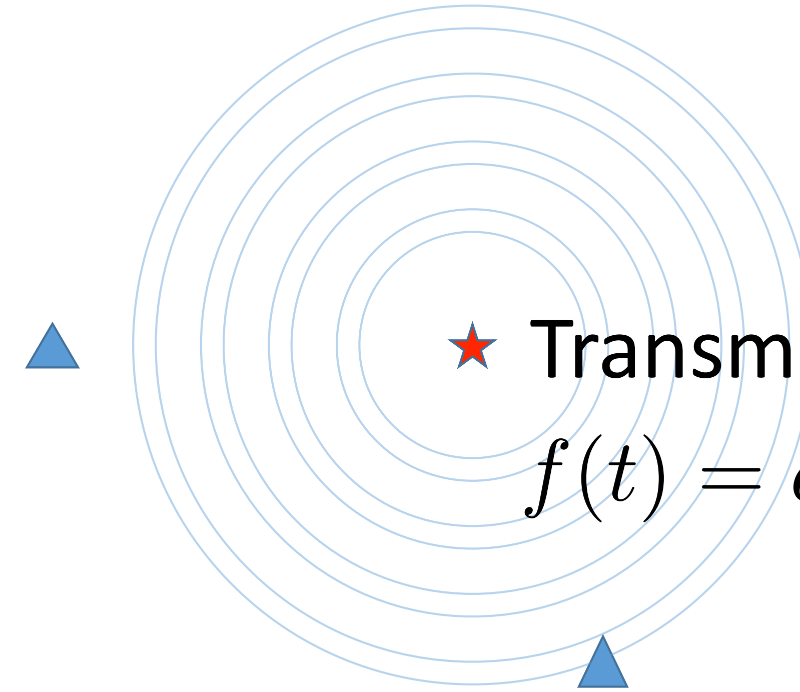


How to find both TDOA and FDOA

Receive

$$s_1(t) = f(t-\tau_1)e^{i\nu_1 t} + n_1(t)$$

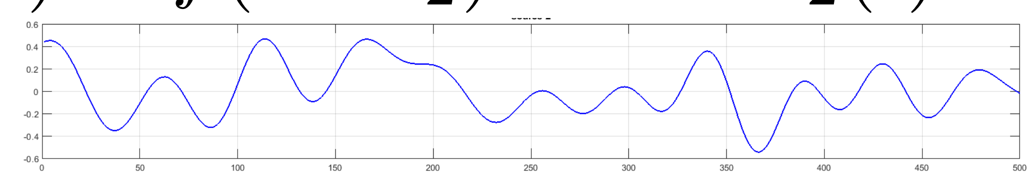




Transmits

$$f(t) = a(t)e^{i\Omega t}$$

$$s_2(t) = f(t-\tau_2)e^{i\nu_2 t} + n_2(t)$$

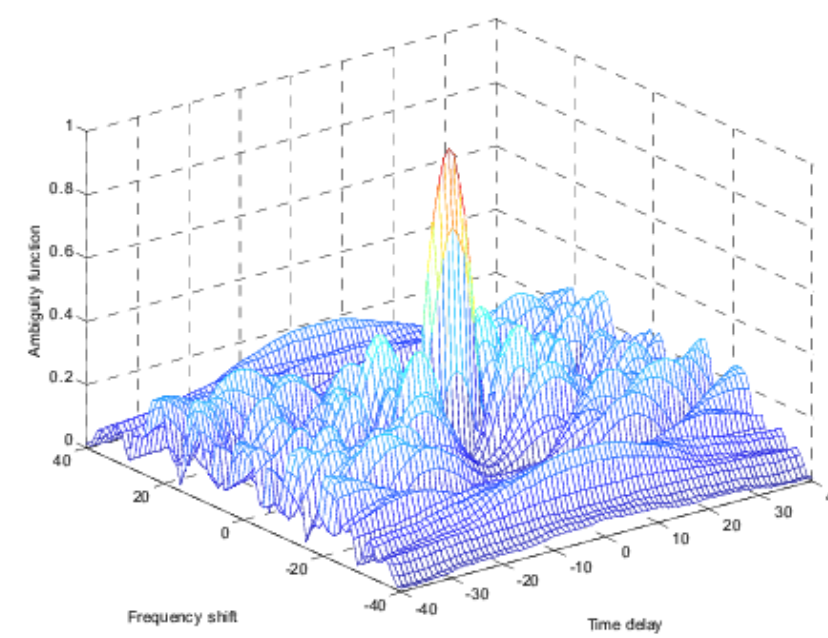


$$\text{ambiguity}(\tau,\nu) = \int s_1(t)s_2^*(t+\tau)e^{i\nu t}dt$$

$$= \int f(t-\tau_1)e^{i\nu_1 t}f^*(t+\tau-\tau_2)e^{-i\nu_2(t+\tau)}e^{i\nu t}dt$$

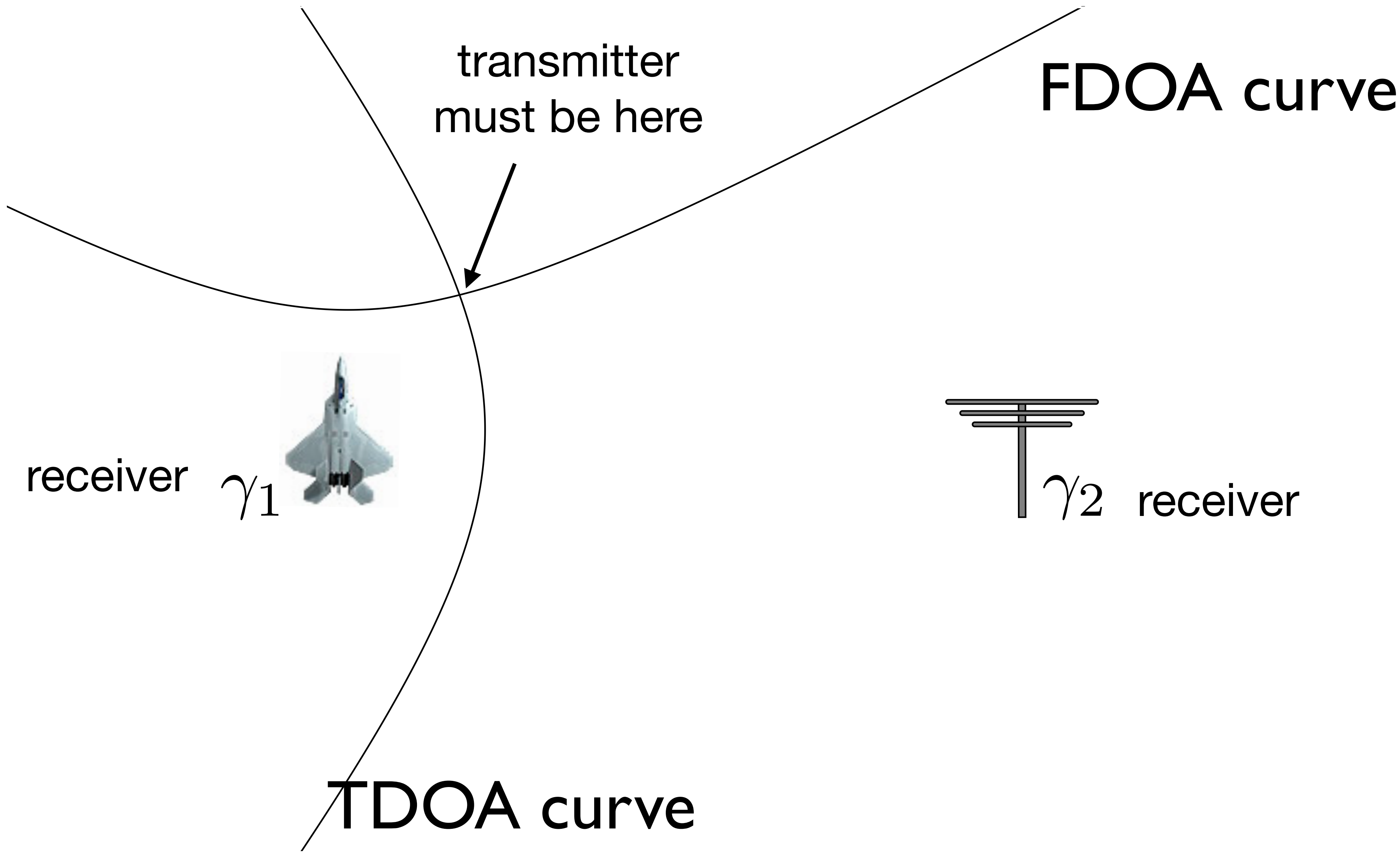
$$\text{has max when } \tau = \tau_2 - \tau_1 \text{ and } \nu = \nu_2 - \nu_1$$

TDOA FDOA

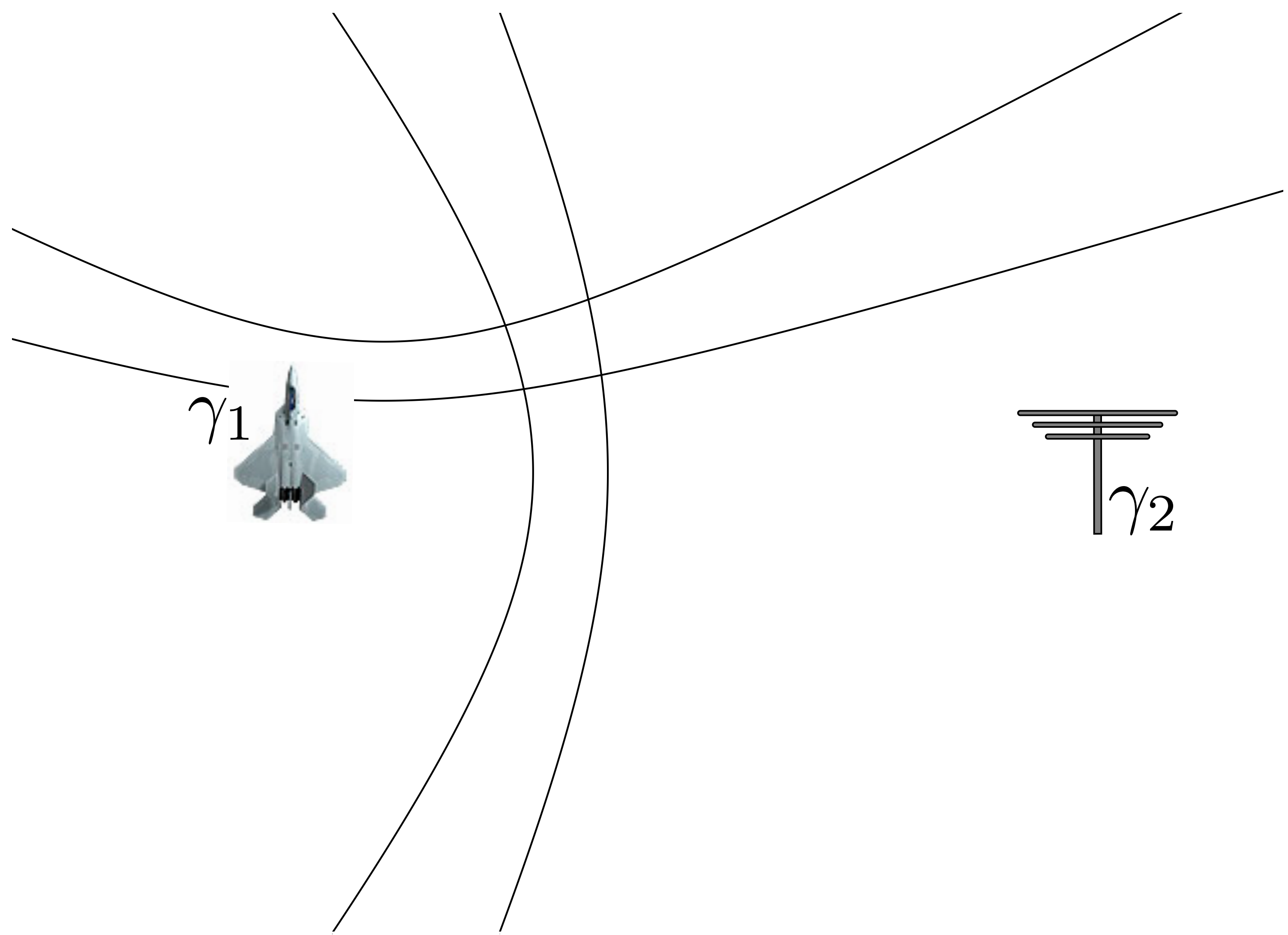


Jarvensivu, P., Marja Matinmikko, and A. Mammela. "Signal design for LS and MMSE channel estimators." In *The 13th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 2, pp. 956-960. IEEE, 2002.

TDOA/FDOA geolocation



TDOA/FDOA with two emitters

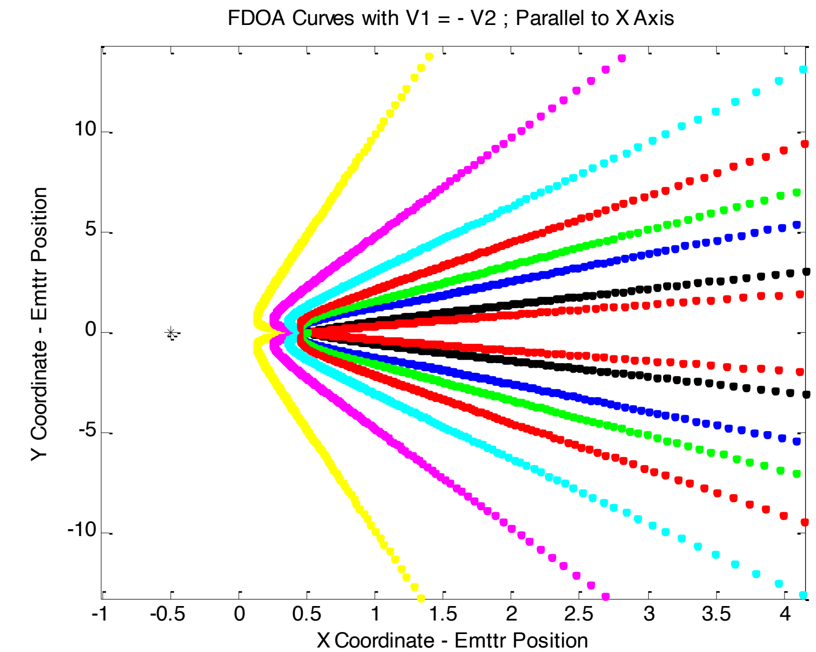


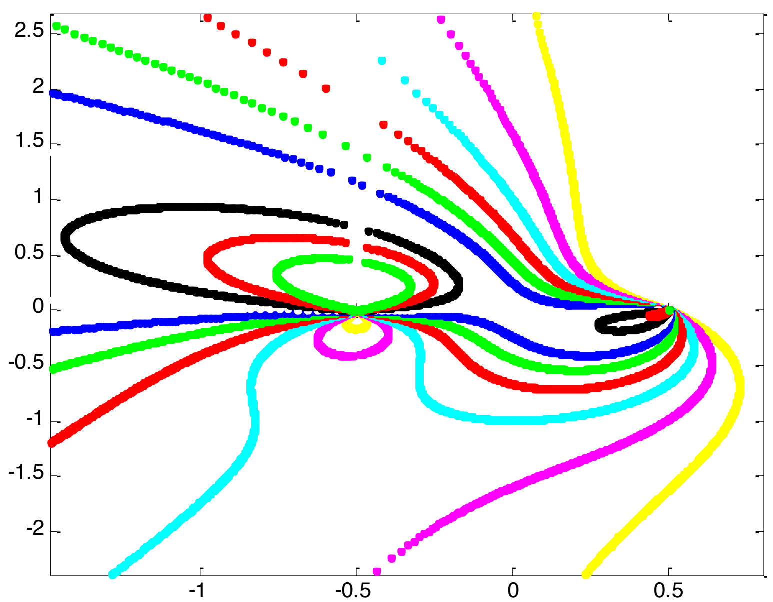
instead try to form SAR-like image

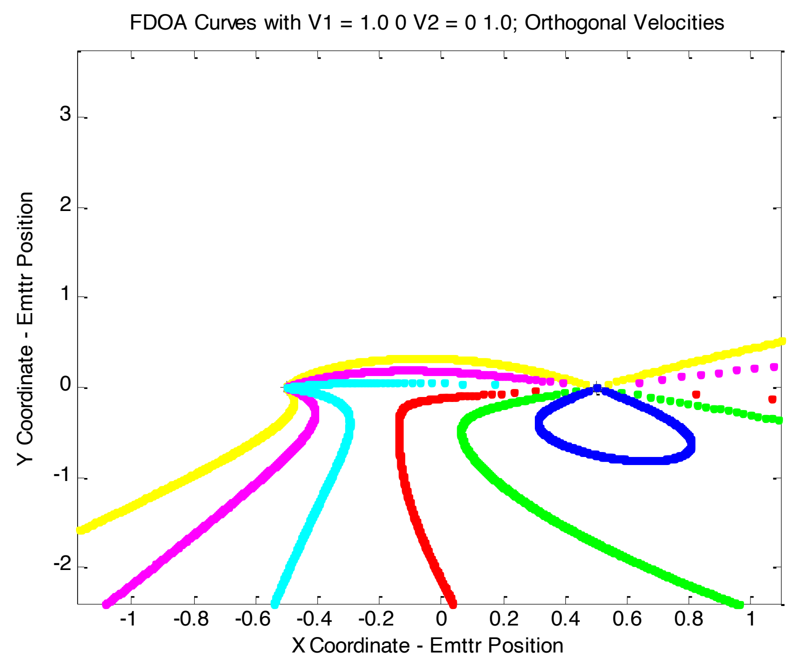
when both velocities are nonzero:

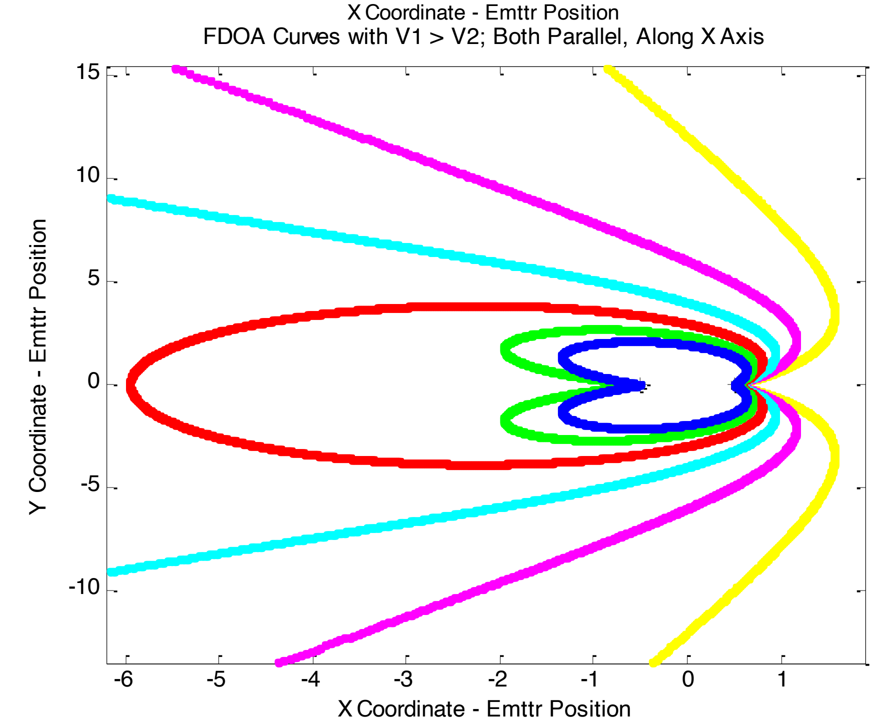
FDOA curves

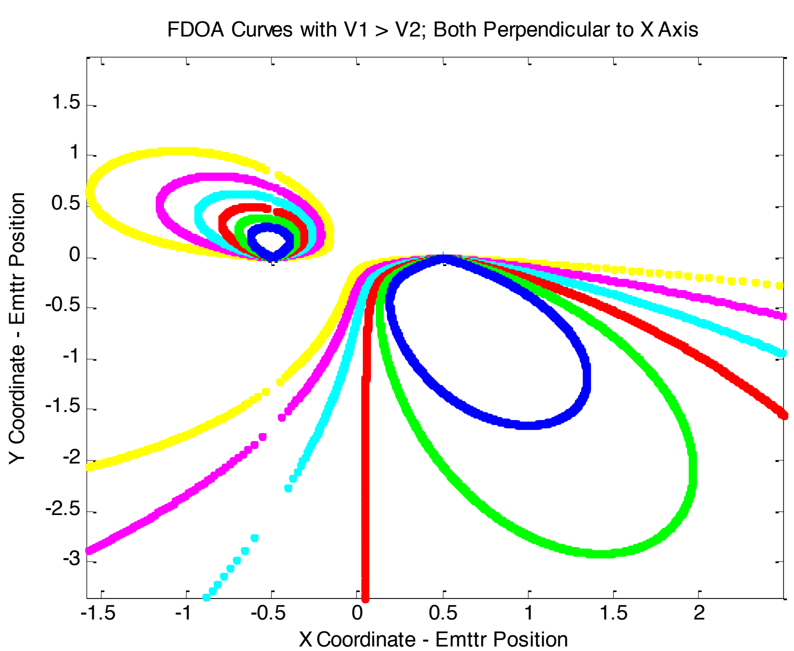
(from Jim Given, NRL)

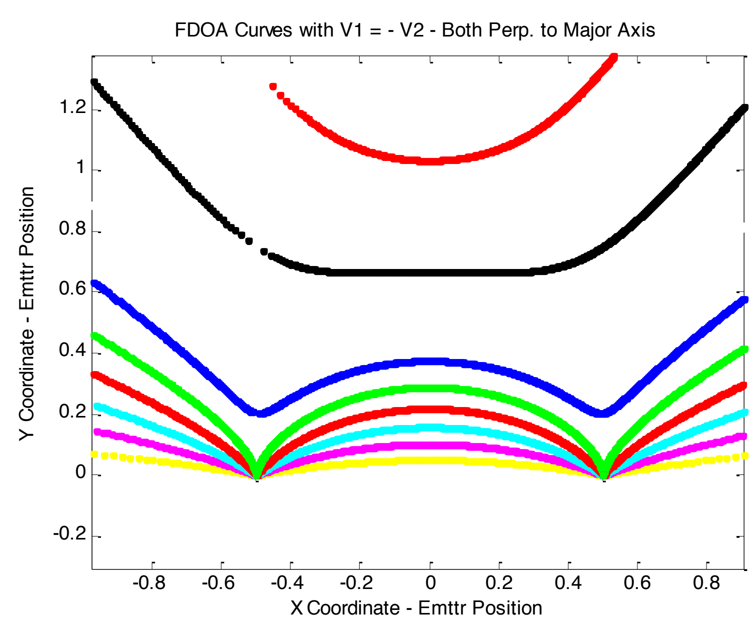


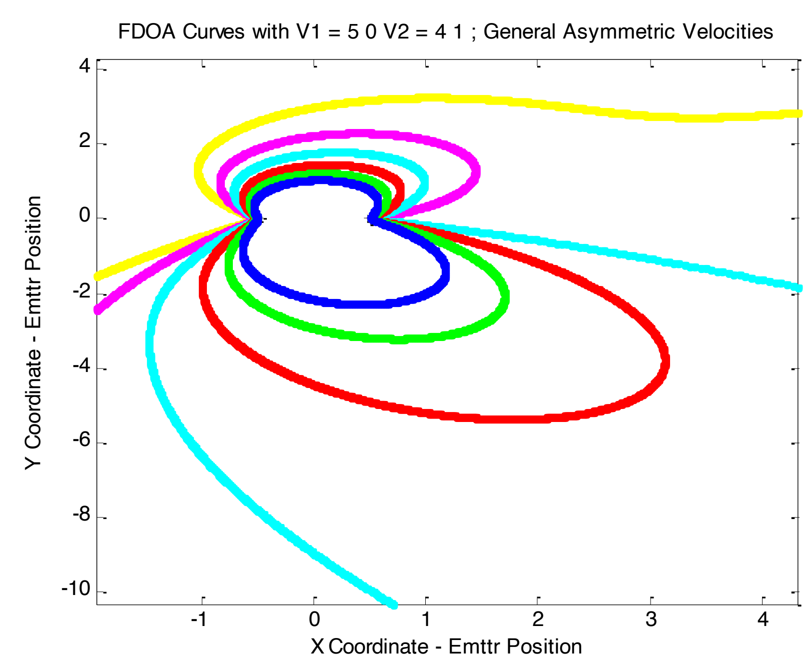






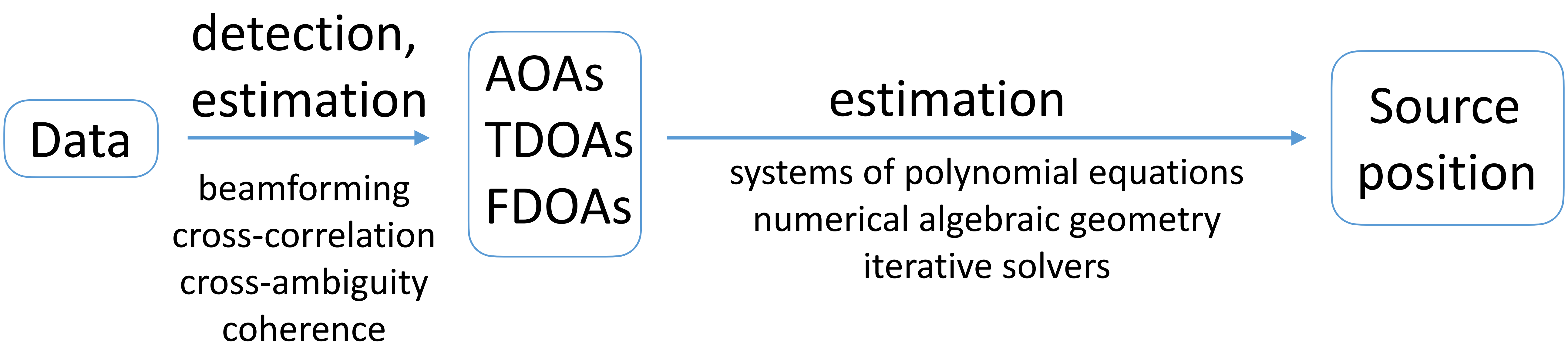






Geolocation Approaches

- current use: two-step methods



- one-step methods: use all the data together
  - backproject cross-correlation or cross-ambiguity, sum coherently (tomographic imaging)
    - focus is determined by TDOA and FDOA

Forward Problem
(multiple sources)

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathcal{E}(t,\boldsymbol{x}) = p(t,\boldsymbol{x})$$ = waveform transmitted from location $\boldsymbol{x}$

Ideal measured data: $d_1(t) = \mathcal{E}_1(t,\gamma_1(t)), \quad d_2(t) = \mathcal{E}_2(t,\gamma_2(t))$

for known flight paths γ_1, γ_2

plus noise

Inverse Problem

Given data: $$d_1(t) = \mathcal{E}_1(t, \gamma_1(t)), \quad d_2(t) = \mathcal{E}_2(t, \gamma_2(t))$$

for known flight paths γ_1, γ_2

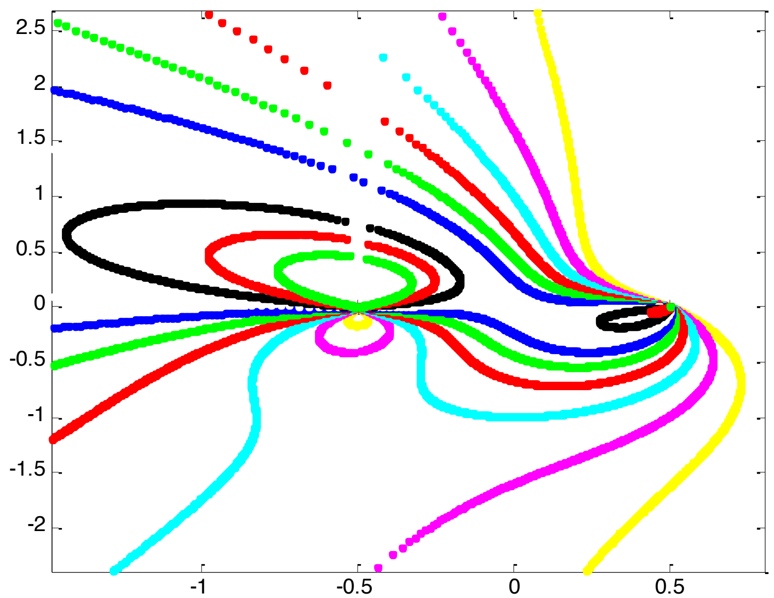
Find: $p(t, \boldsymbol{x})$? underdetermined

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathcal{E}(t, \boldsymbol{x}) = p(t, \boldsymbol{x})$$

$$\rho(\boldsymbol{x}) = \int |p(t, \boldsymbol{x})|^2 dt?$$

Today: Pieces of the problem

- Geometry of two-step localization from fixed positions: What are FDOA iso-surfaces?



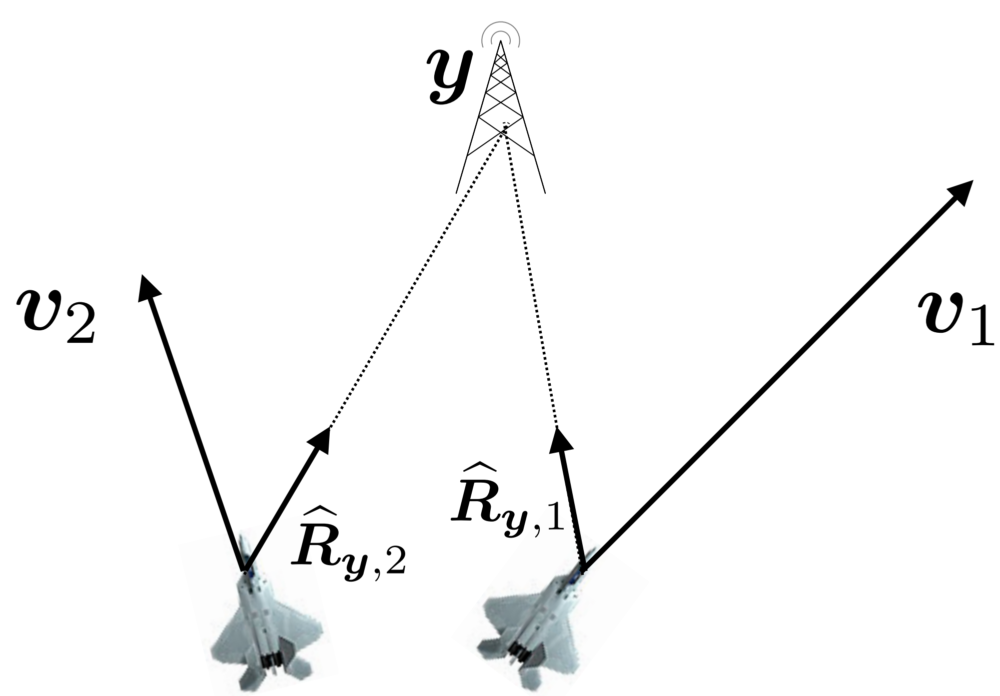
- Synthetic-aperture approaches (for a distribution of sources)

Basic Facts about FDOA

- Obtain FDOA from coherence detector, cross-ambiguity functions, or frequency ratios

$$\frac{\omega_2}{\omega_1} = \frac{\omega_0\left(1 + \widehat{\boldsymbol{R}}_{\boldsymbol{y},2} \cdot \frac{\boldsymbol{v}_2}{c}\right)}{\omega_0\left(1 + \widehat{\boldsymbol{R}}_{\boldsymbol{y},1} \cdot \frac{\boldsymbol{v}_1}{c}\right)} = 1 + \frac{1}{c} \cdot \left(\underbrace{\widehat{\boldsymbol{R}}_{\boldsymbol{y},2} \cdot \boldsymbol{v}_2 - \widehat{\boldsymbol{R}}_{\boldsymbol{y},1} \cdot \boldsymbol{v}_1}_{\text{FDOA}} + \cdots \right)$$

- Notation:



$$\text{FDOA} = \widehat{\boldsymbol{R}}_{\boldsymbol{y},2} \cdot \boldsymbol{v}_2 - \widehat{\boldsymbol{R}}_{\boldsymbol{y},1} \cdot \boldsymbol{v}_1$$

- sensitivity

$$\nabla_{\boldsymbol{y}}\text{FDOA} = \frac{I - \widehat{\boldsymbol{R}}_{\boldsymbol{y},2}\widehat{\boldsymbol{R}}^T_{\boldsymbol{y},2}}{|\boldsymbol{R}_{\boldsymbol{y},2}|} \cdot \boldsymbol{v}_2 - \frac{I - \widehat{\boldsymbol{R}}_{\boldsymbol{y},1}\widehat{\boldsymbol{R}}^T_{\boldsymbol{y},1}}{|\boldsymbol{R}_{\boldsymbol{y},1}|} \cdot \boldsymbol{v}_1$$

In far-field, |R| is large, so changes in source position have small effect on FDOA

- far-field analysis

$$\text{FDOA} \approx (\boldsymbol{v}_1 - \boldsymbol{v}_2) \cdot \widehat{\boldsymbol{R}}_{\boldsymbol{y}}$$

cone!

K.J. Cameron & S.J. Pine, A novel method for determining DOA from far-field FDOA or FDOA
2018, arXiv:1808.04741v1

when $\boldsymbol{v}_1 = \boldsymbol{v}_2 \doteq \boldsymbol{v}$

$$\text{FDOA} = \frac{\boldsymbol{v} \cdot \left[I - \widehat{\boldsymbol{R}_{\boldsymbol{y}}}\widehat{\boldsymbol{R}_{\boldsymbol{y}}}^T\right] \cdot \Delta \boldsymbol{s}}{|\boldsymbol{R}_{\boldsymbol{y}}|}$$

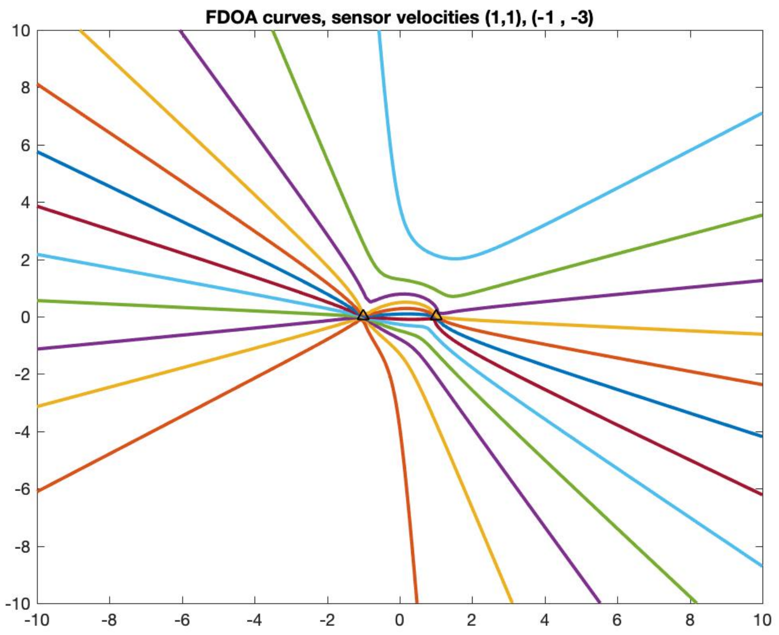
vector difference between sensors

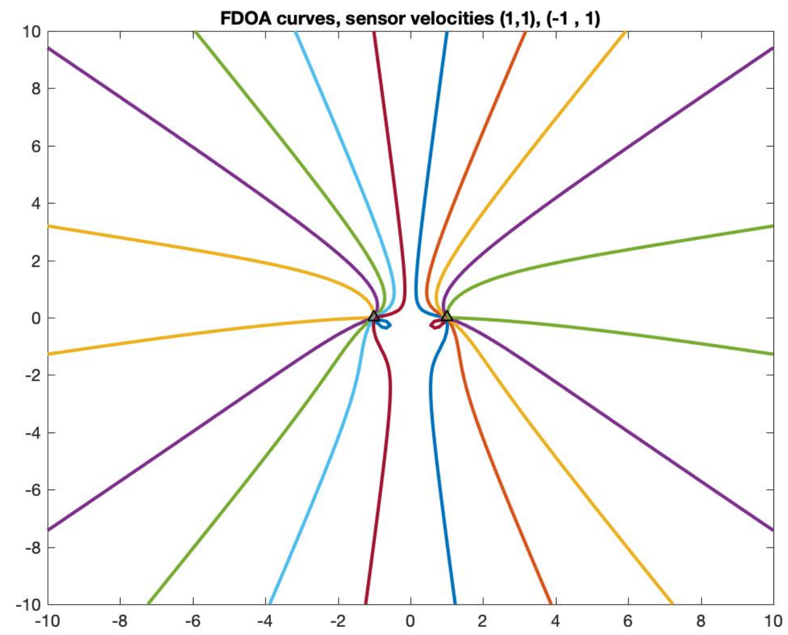
or

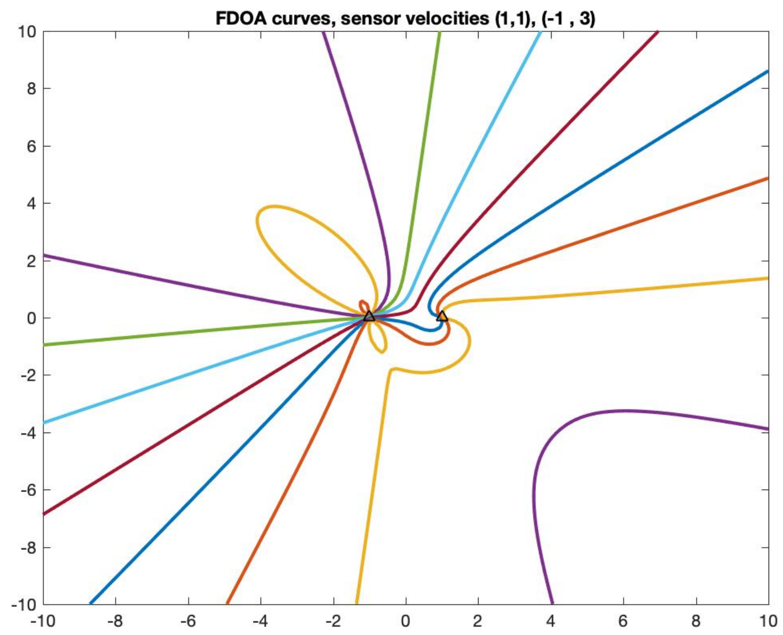
$$|\boldsymbol{R}_{\boldsymbol{y}}| = \frac{\boldsymbol{v} \cdot \left[I - \widehat{\boldsymbol{R}_{\boldsymbol{y}}}\widehat{\boldsymbol{R}_{\boldsymbol{y}}}^T\right] \cdot \Delta \boldsymbol{s}}{\text{FDOA}}$$

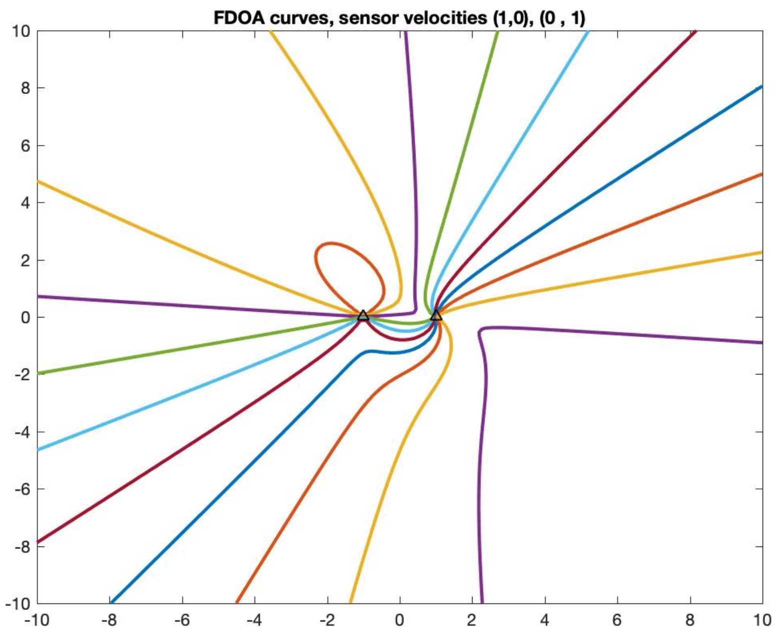
can use to plot approximate far-field FDOA curves when velocities are the same

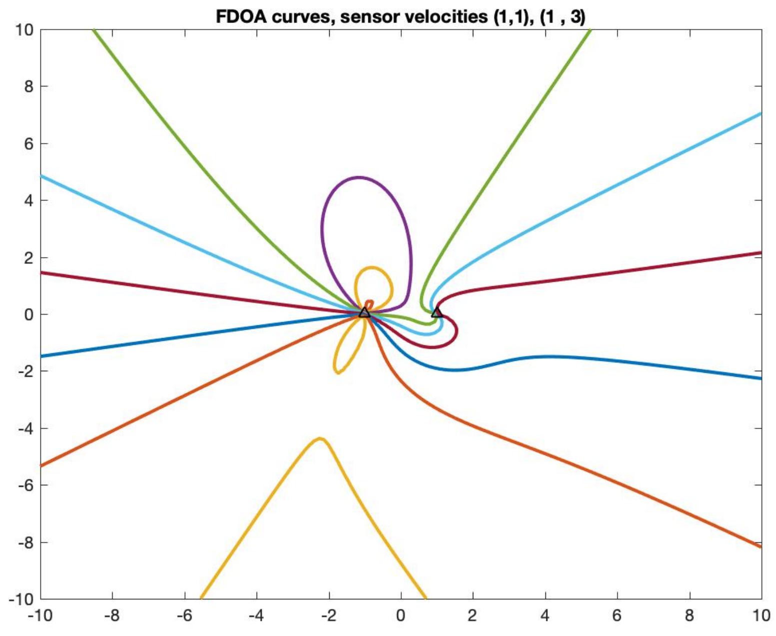
FDOA curves with unequal velocities

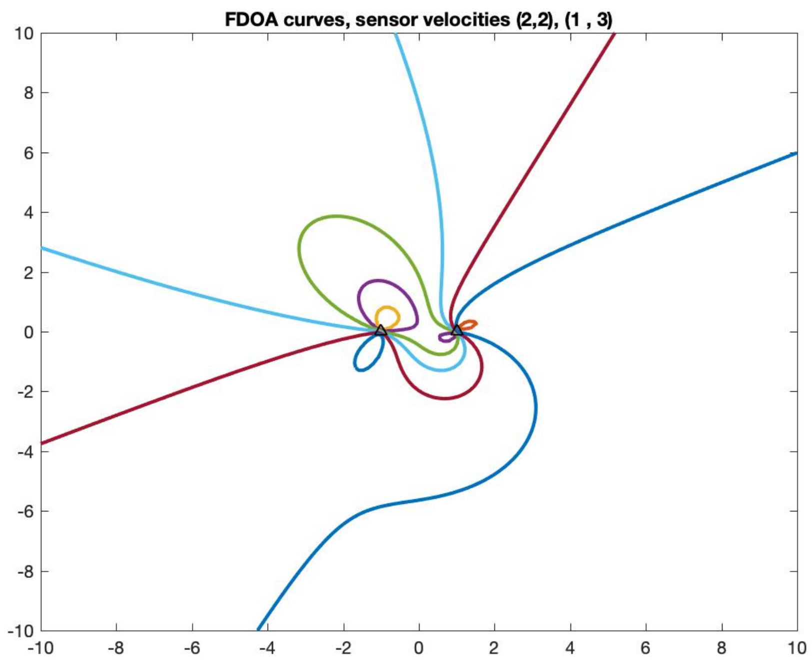








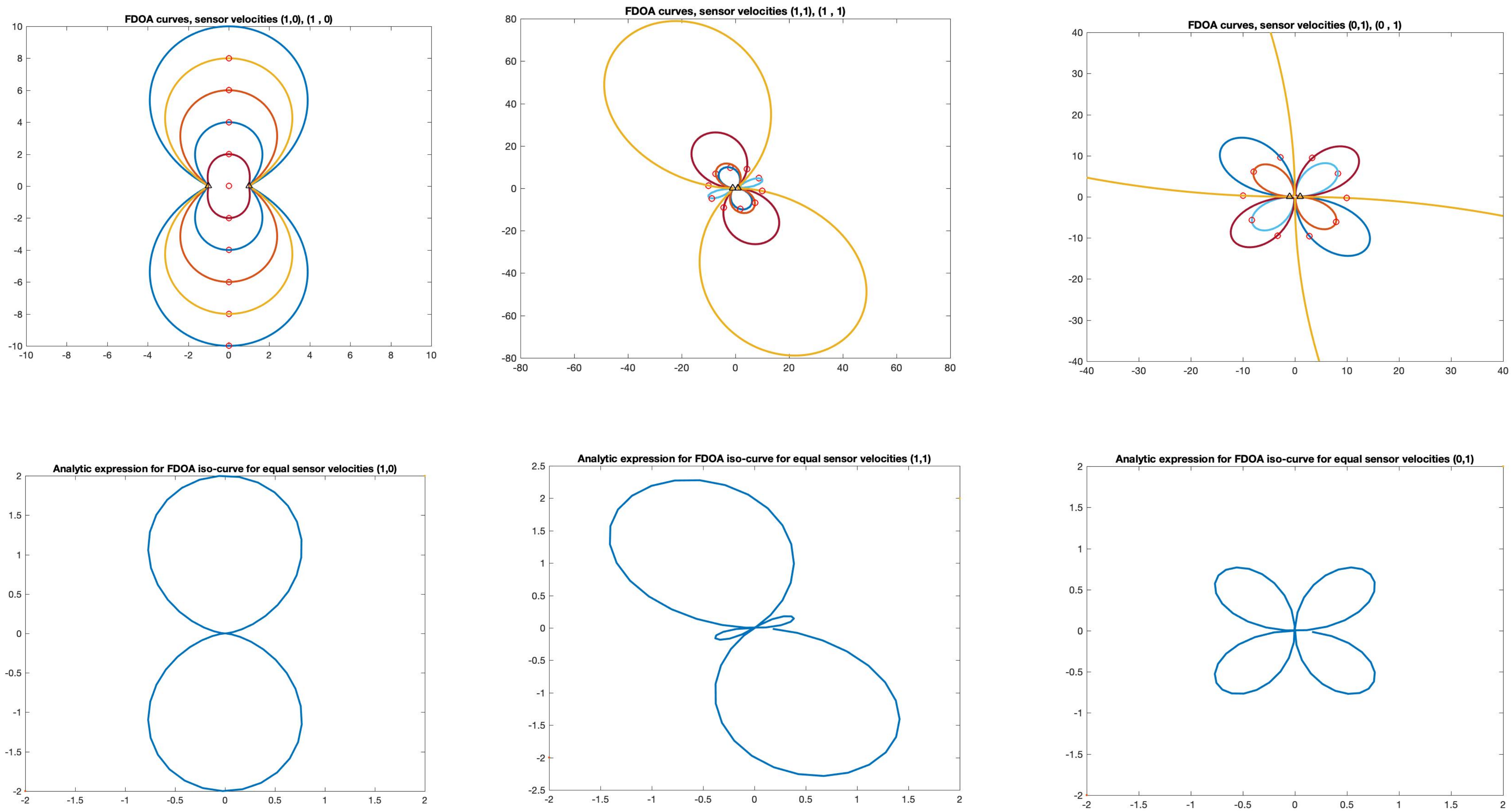




Note linearity in far field as predicted by Cameron-Pine theory

Far-field, case of equal sensor velocities

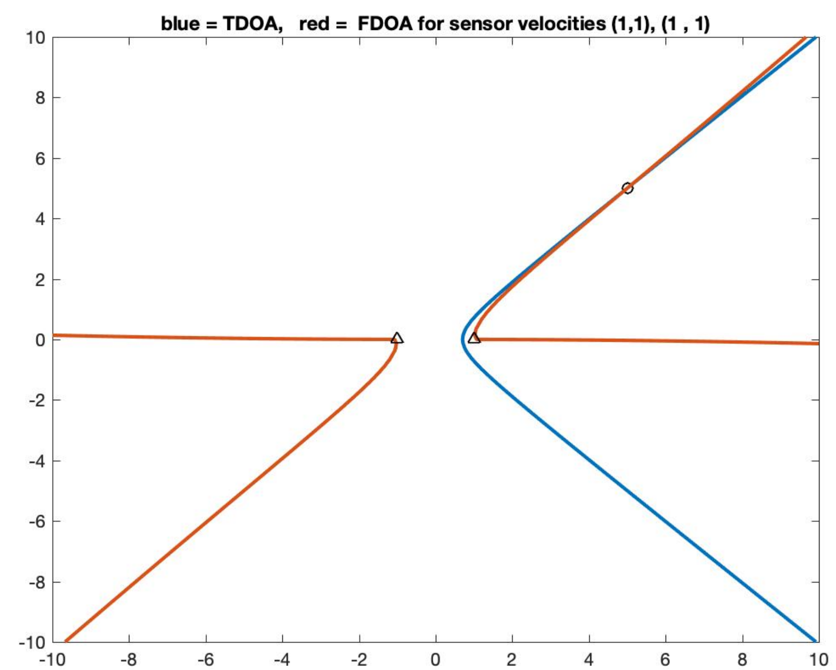
top: true FDOA curves passing through small red circles

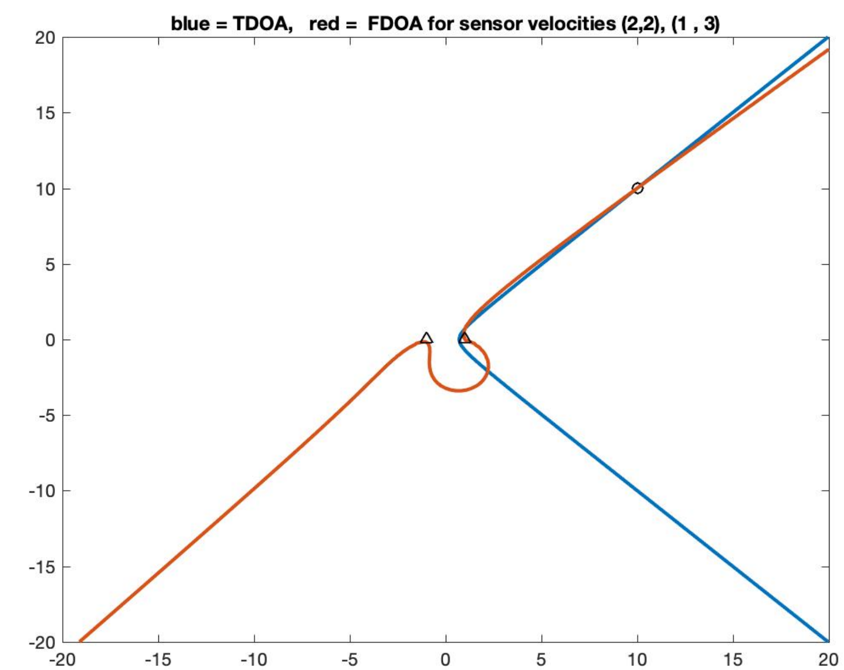


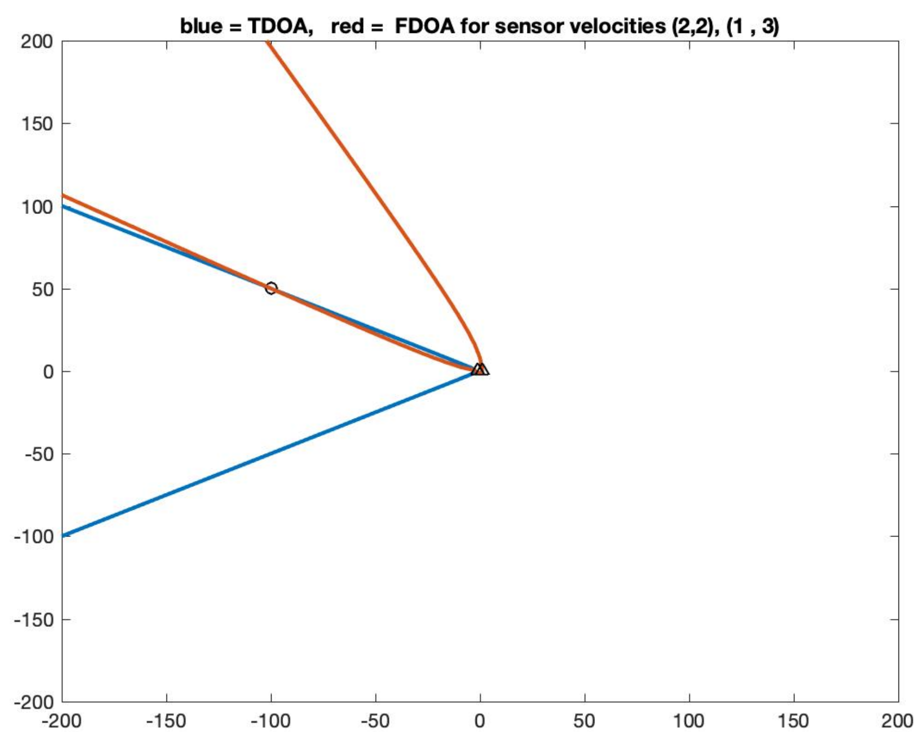
bottom: approximate far-field formula

nulls are along velocity directions
and along sensor axis

TDOA-FDOA far-field 2-sensor geolocation, unequal velocities

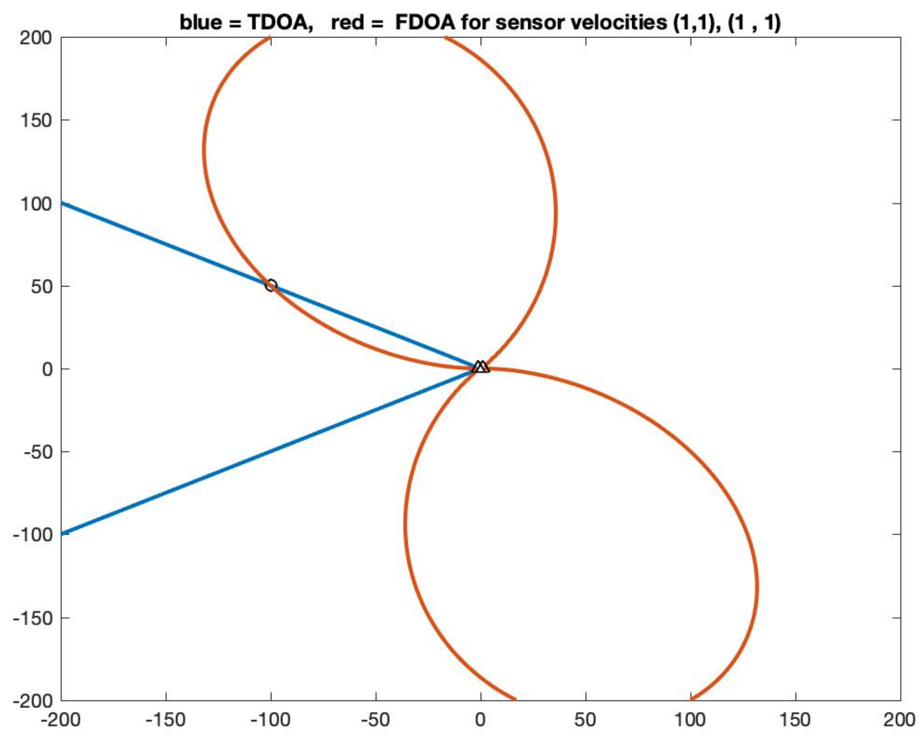


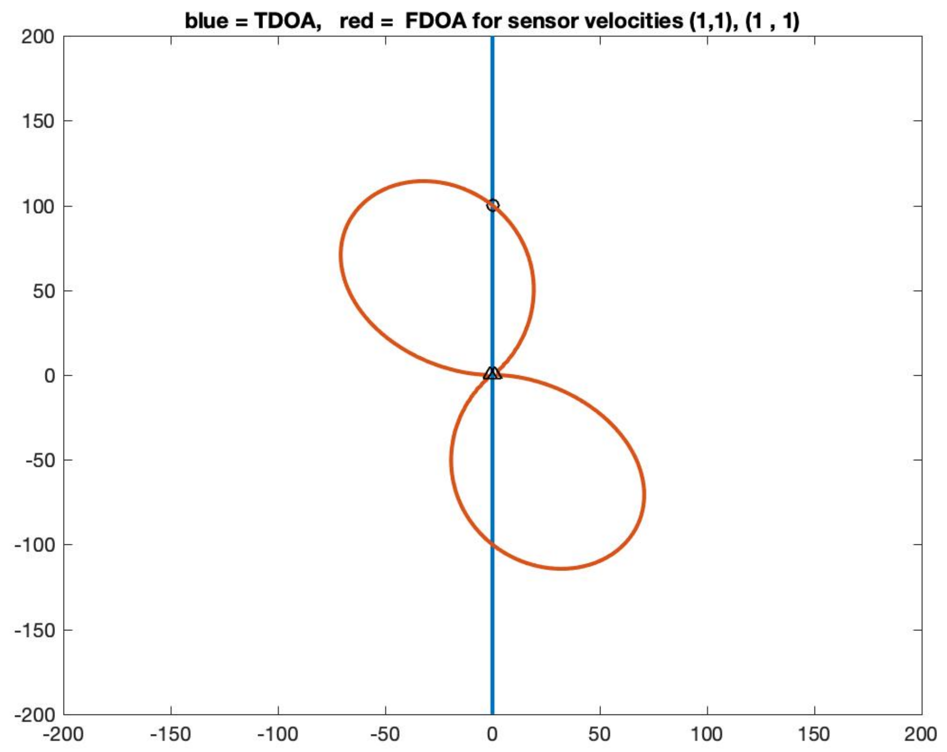


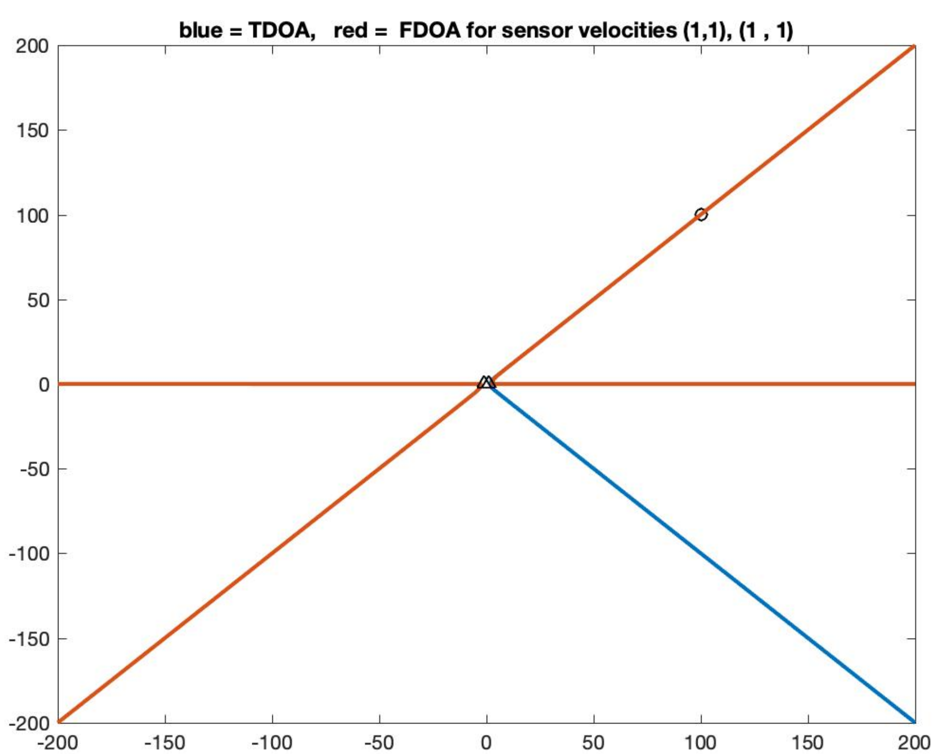


TDOA and FDOA curves
are tangent in the far field

TDOA-FDOA far-field 2-sensor geolocation, equal velocities







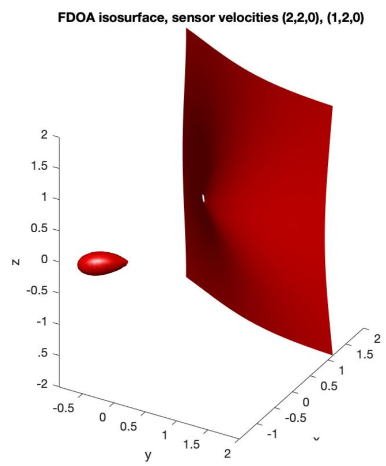
degenerate case:
source lies along velocity axis or
source lies along sensor axis
(FDOA = 0)

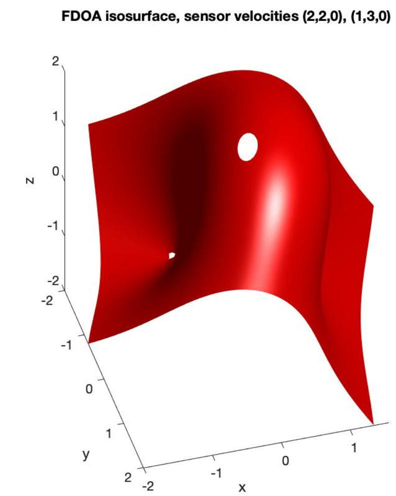
$$|\boldsymbol{R_y}| = \frac{\boldsymbol{v} \cdot \left[I - \widehat{\boldsymbol{R_y}}\widehat{\boldsymbol{R_y}}^T \right] \cdot \Delta \boldsymbol{s}}{\mathrm{FDOA}}$$

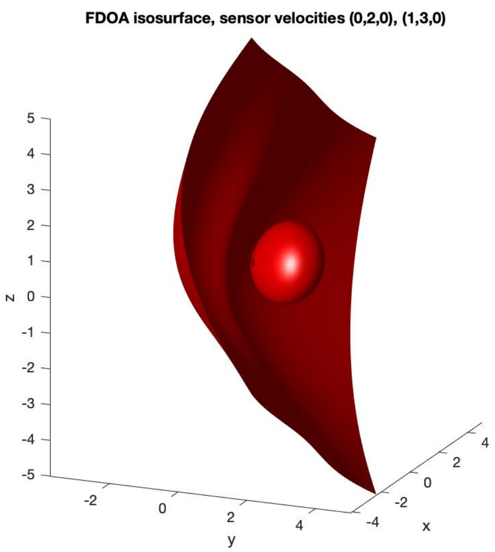
FDOA Iso-surfaces: Unequal velocities in the sensor plane

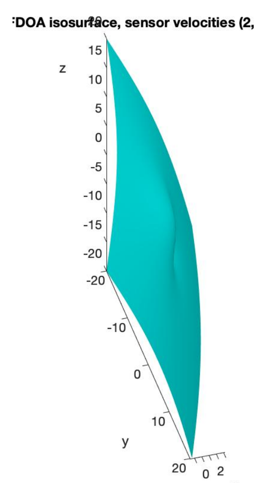
sensors are always at (1,0,0) and (-1,0,0)

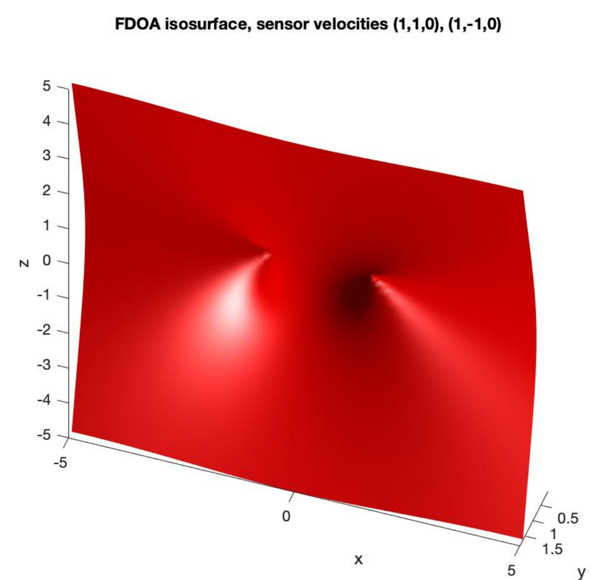
red: surface passes through (1,1,1)
cyan: surface passes through (1,10,0)

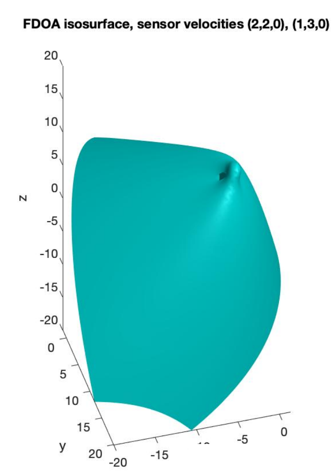


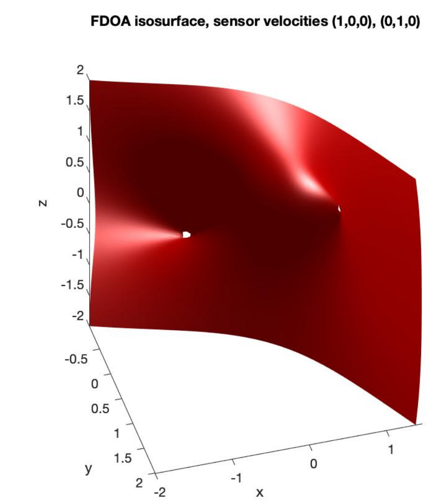


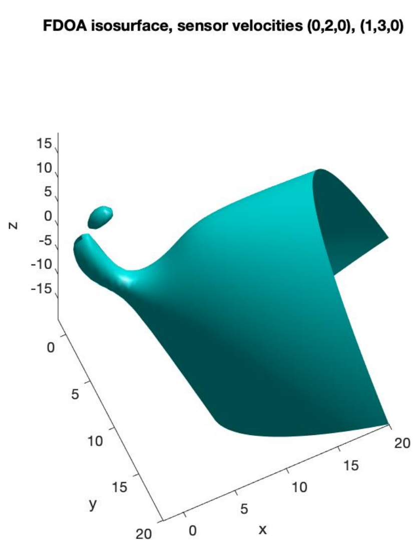




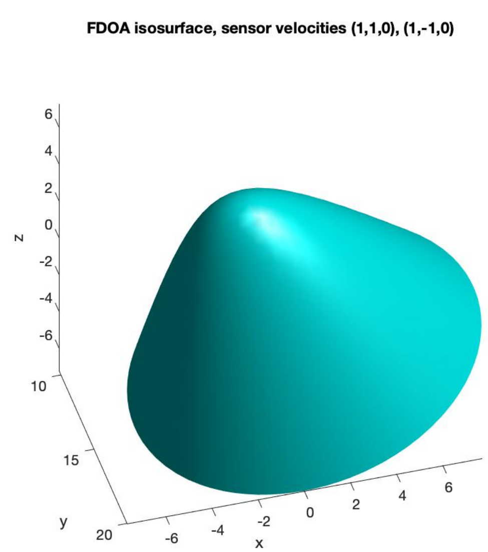


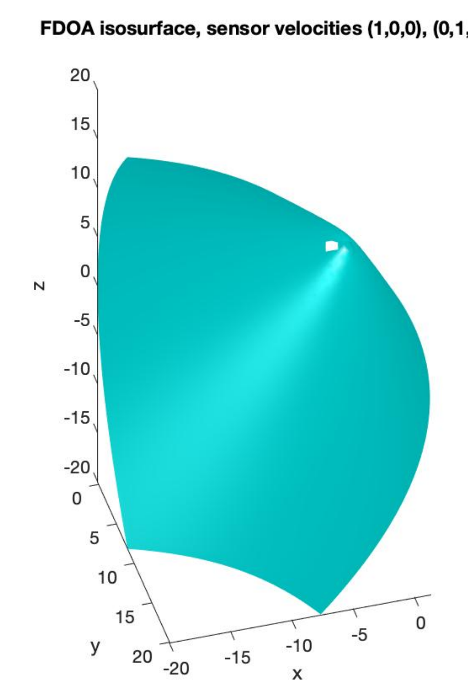




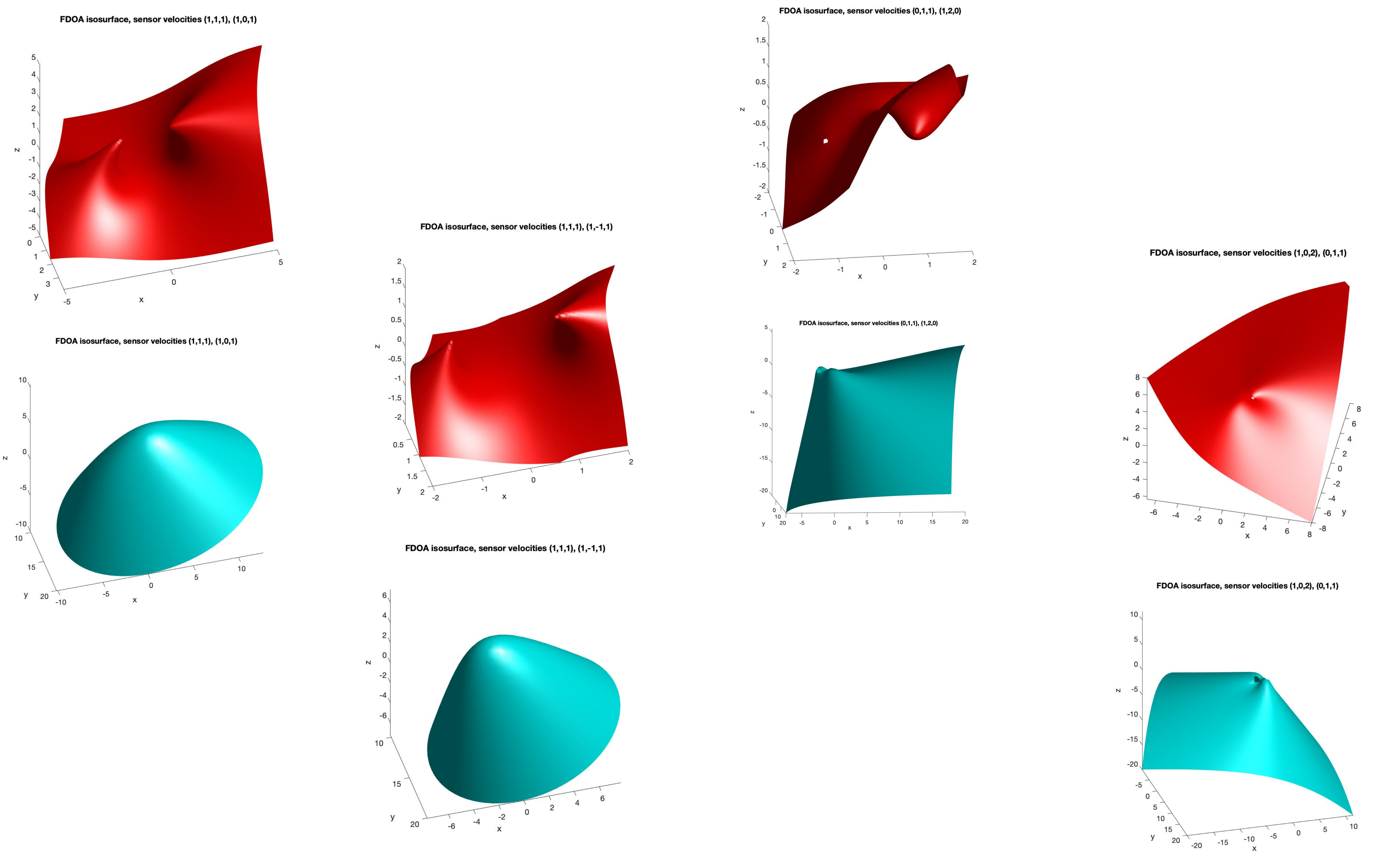


recall: far-field is cone with axis = velocity difference





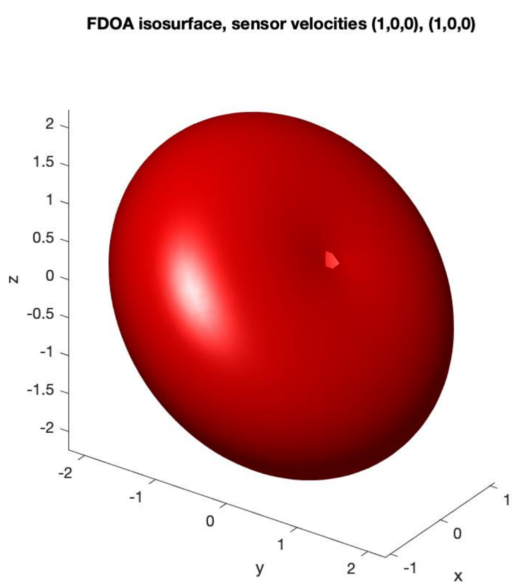
Unequal velocities out of the sensor plane

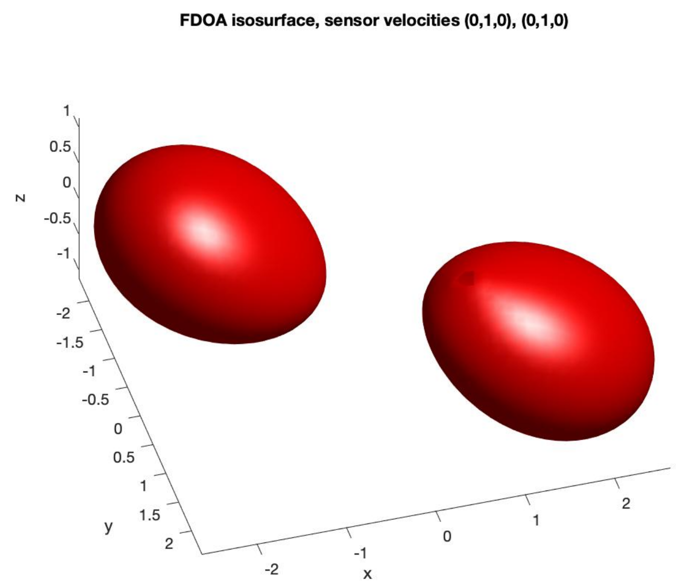


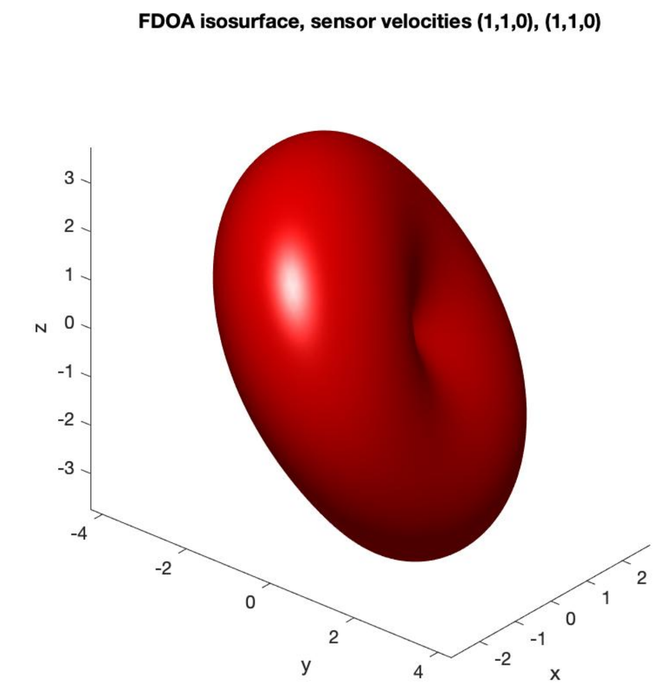
Equal velocities

sensors are always at (1,0,0) and (-1,0,0)

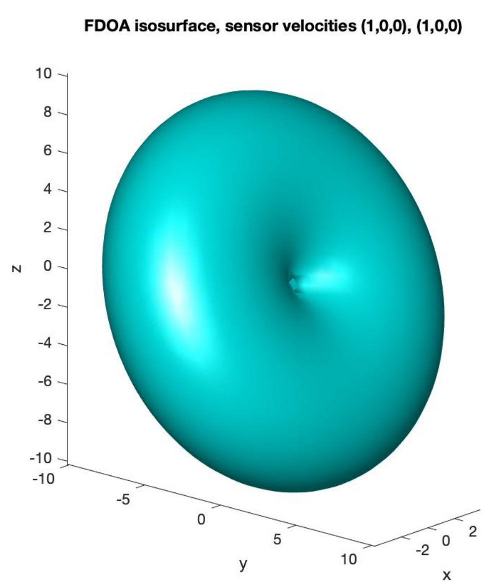
red: surface passes through (1,1,1)
cyan: surface passes through (1,10,0)

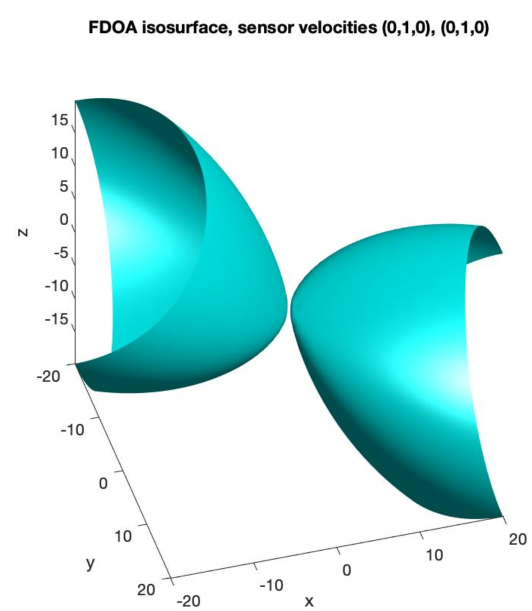


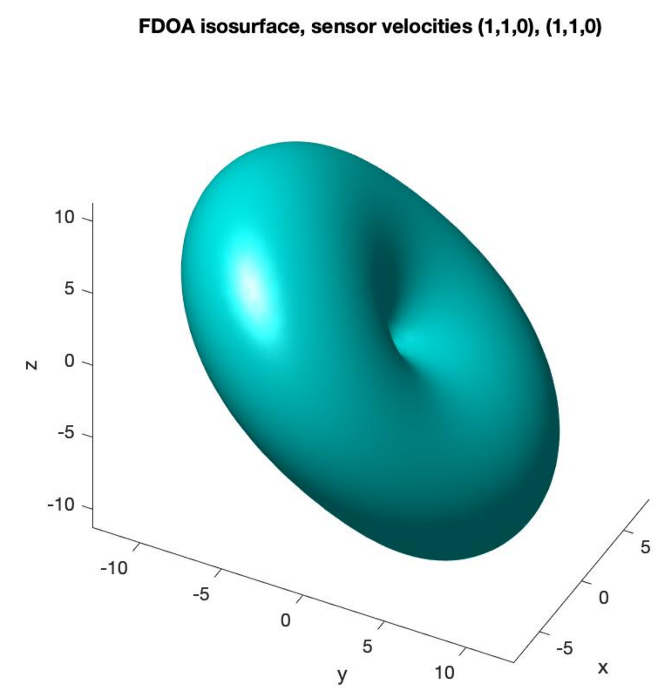




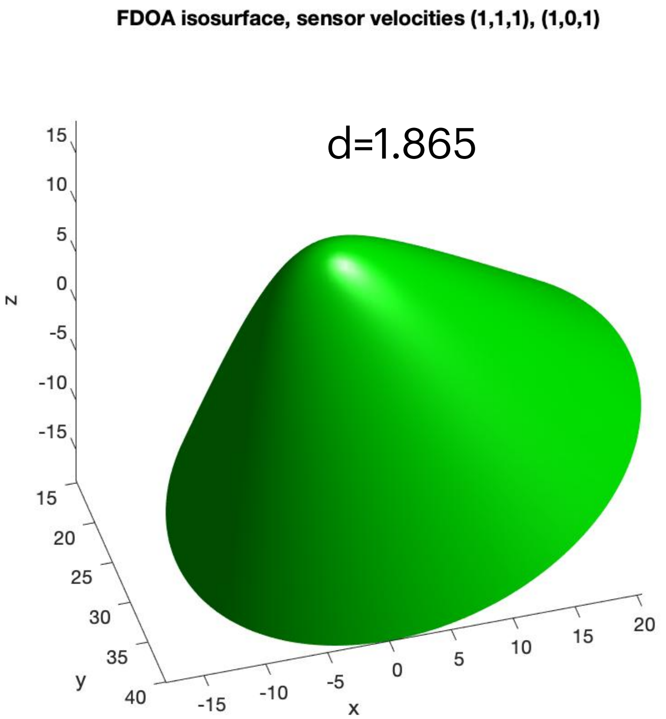
extra symmetry

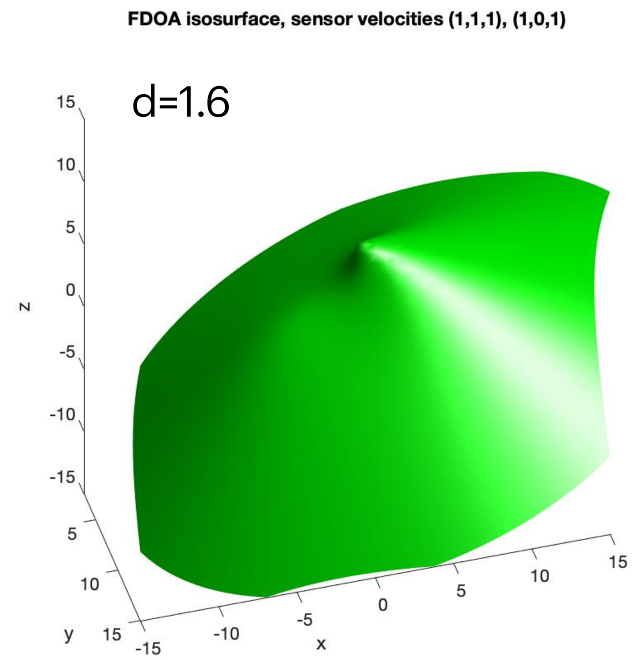


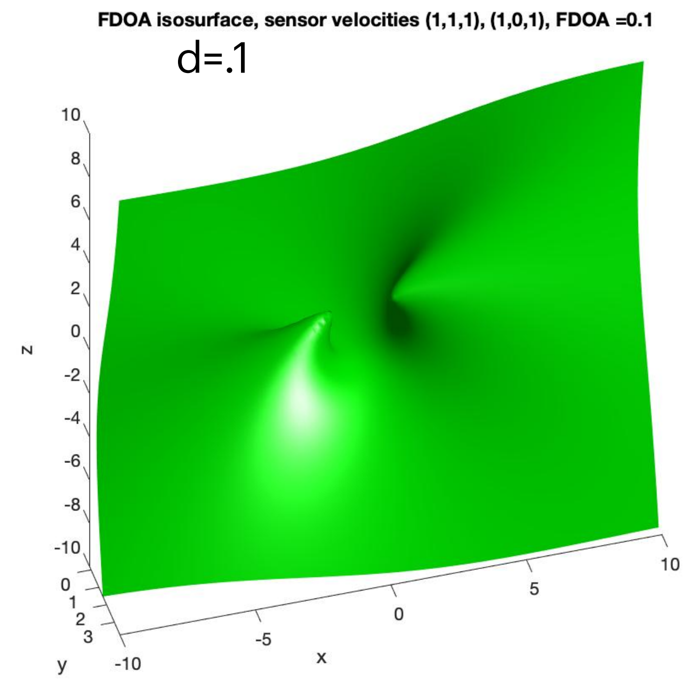


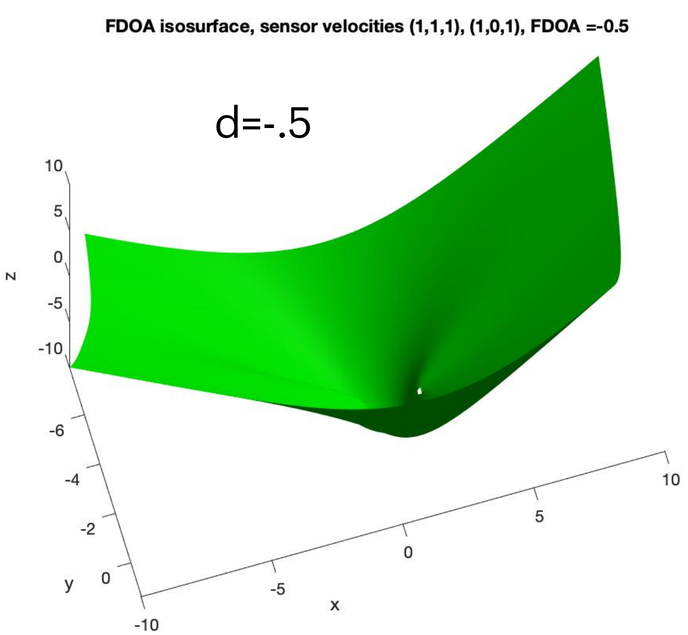


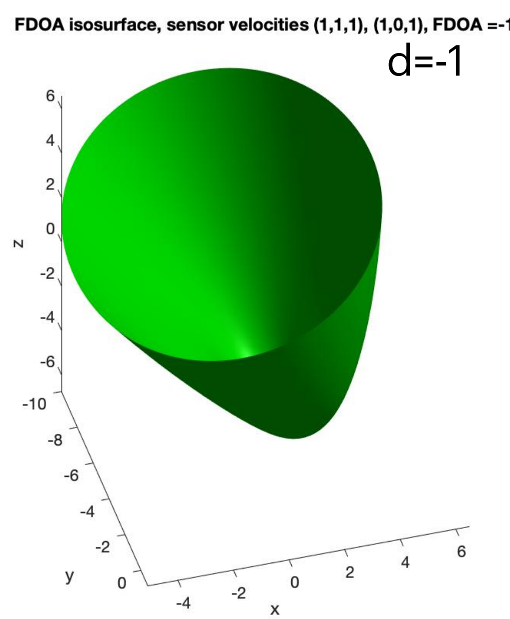
Sequence of FDOA isosurfaces for velocity pair (1,1,1), (1,0,1)

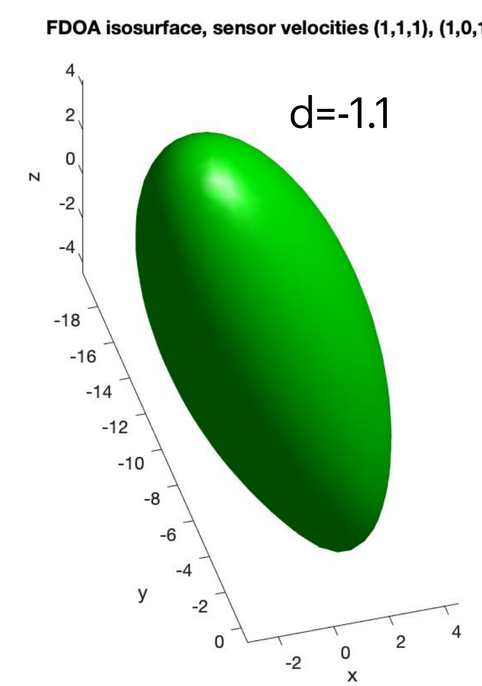


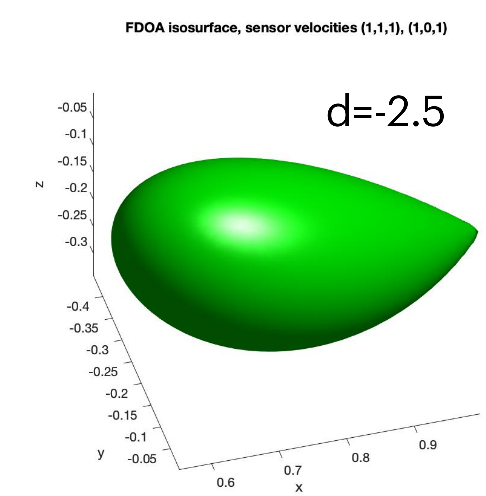




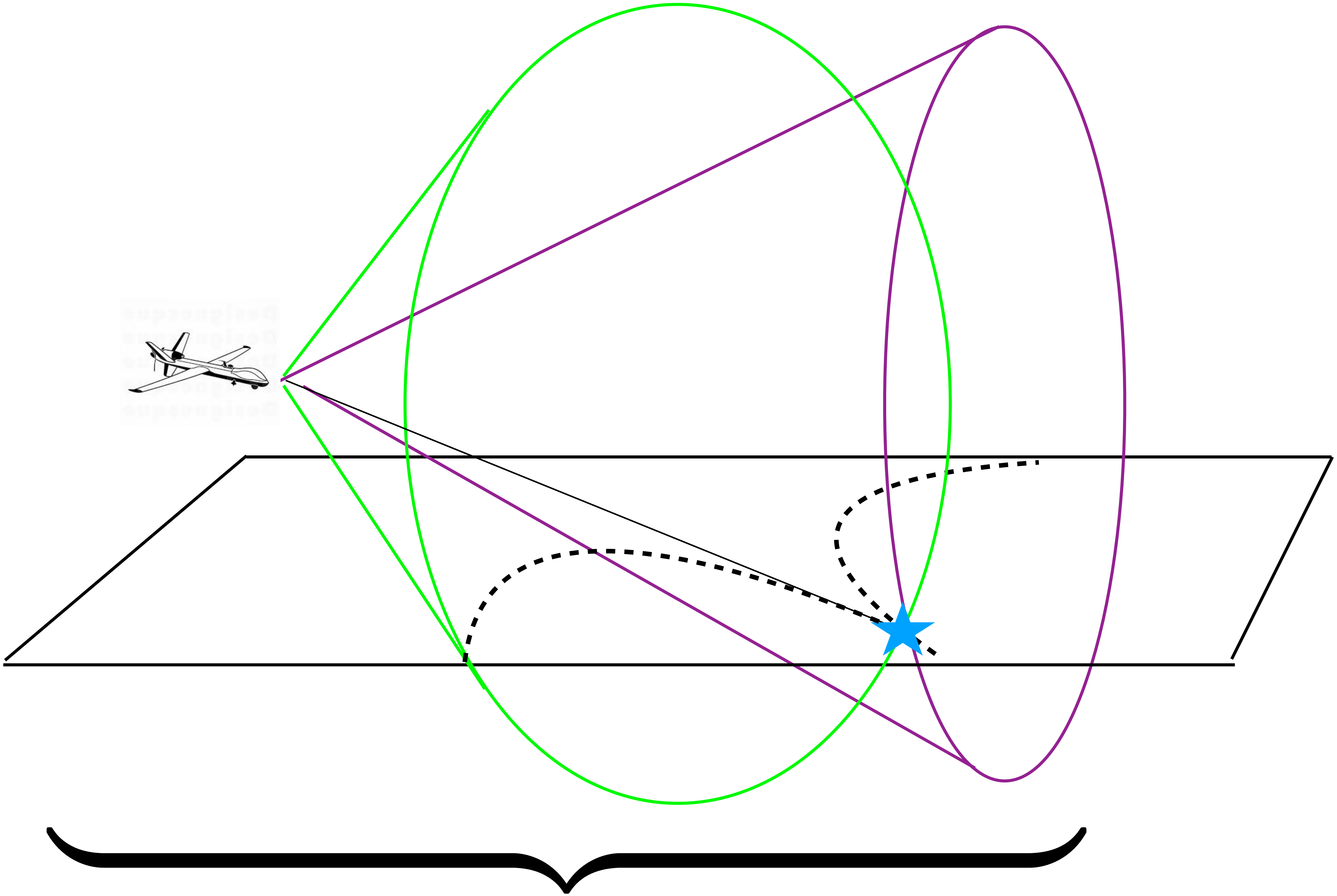






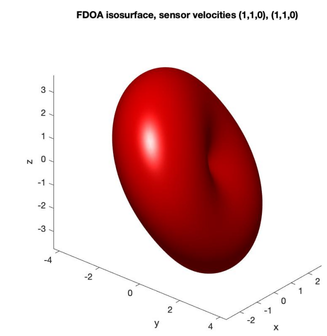


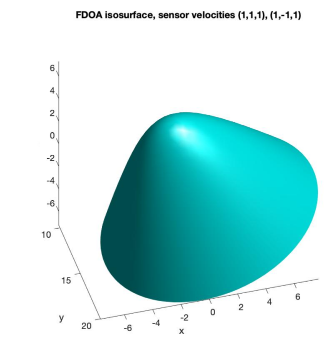
note scale!
This shrinks to nothing
as d becomes more negative



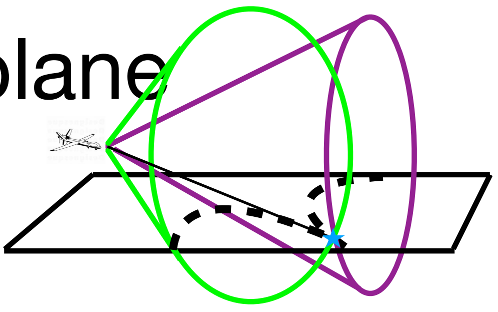
quasi-near-field region
good for TDOA-FDOA and FDOA-FDOA localization with unequal velocities

Conclusions for (single-look) FDOA localization





- Far-field is now reasonably well-understood
- TDOA/FDOA far-field localization:
  - with different velocities: far-field TDOA and FDOA lines are tangent (bad)
  - with equal velocities: OK away from sensor axis and velocity axis
  - quasi-near-field region for source on a known plane
- Near-field ??????

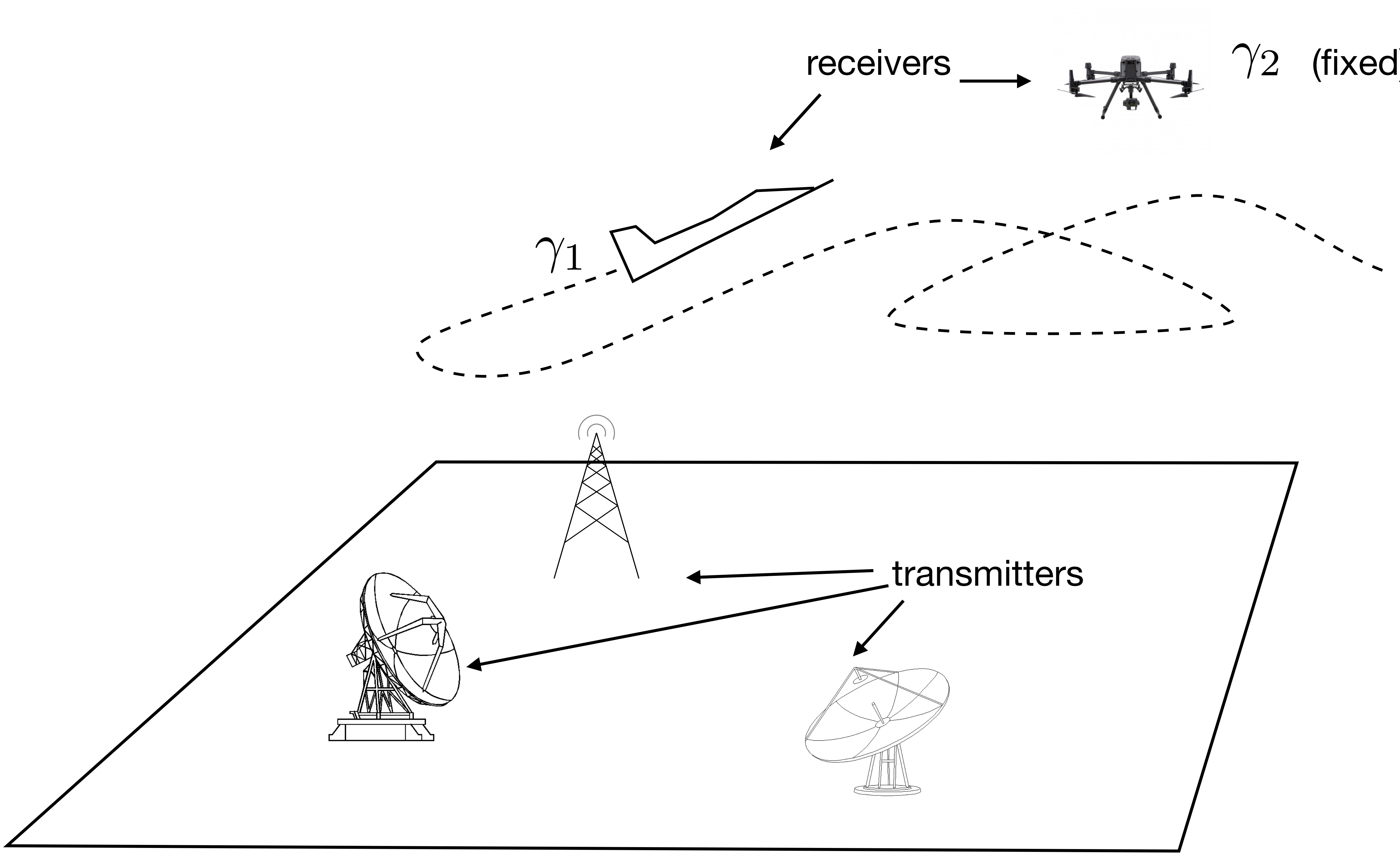


Pine, K.C., Pine, S. and Cheney, M., 2021.
The Geometry of Far-Field Passive Source Localization with TDOA and FDOA.
IEEE Trans. on Aerospace and Electronic Systems.

One-step approach:

Synthetic Aperture Source Localization (SASL)

PhD dissertation of Chad Waddington



$$\nabla^2 \mathcal{E} - \frac{1}{c^2}\frac{\partial^2 \mathcal{E}}{\partial t^2} = \sum_n p_n(t)\delta(\boldsymbol{x} - \boldsymbol{e}_n)$$

source locations

slow time variable

field at sensor i

$$\mathcal{E}_i(s,t) = \sum_n \frac{p_n(t - |\gamma_i(s) - \boldsymbol{e}_n|/c)}{4\pi|\gamma_i(s) - \boldsymbol{e}_n|}$$

cross-correlate

$$d(s,t) = \int \mathcal{E}_1^*(s, t+\tau)\mathcal{E}_2(s,\tau)d\tau$$

diagonal terms

cross terms

Diagonal terms

Assume: $\sum |P_n(\omega)|^2 \delta(\boldsymbol{y} - \boldsymbol{e}_n) \approx B(\omega) V(\boldsymbol{y})$

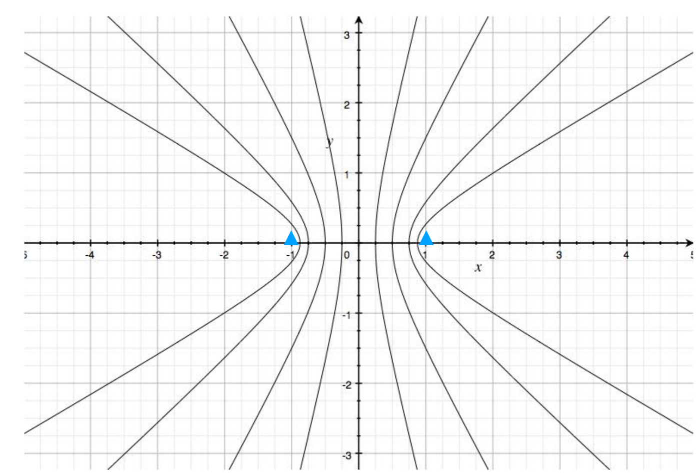
Fourier transform of waveform ($|P_n(\omega)|$)

source density function ($V(\boldsymbol{y})$)

$$d_D(s,t) = \int e^{-i\omega(t - r(s,\boldsymbol{y})/c)} A(s,\boldsymbol{y}) B(\omega) d\omega \; V(\boldsymbol{y}) d\boldsymbol{y}$$

TDOA $r(s,\boldsymbol{y}) = |\gamma_2(s) - \boldsymbol{y}| - |\gamma_1(s) - \boldsymbol{y}|$

Fourier integral operator

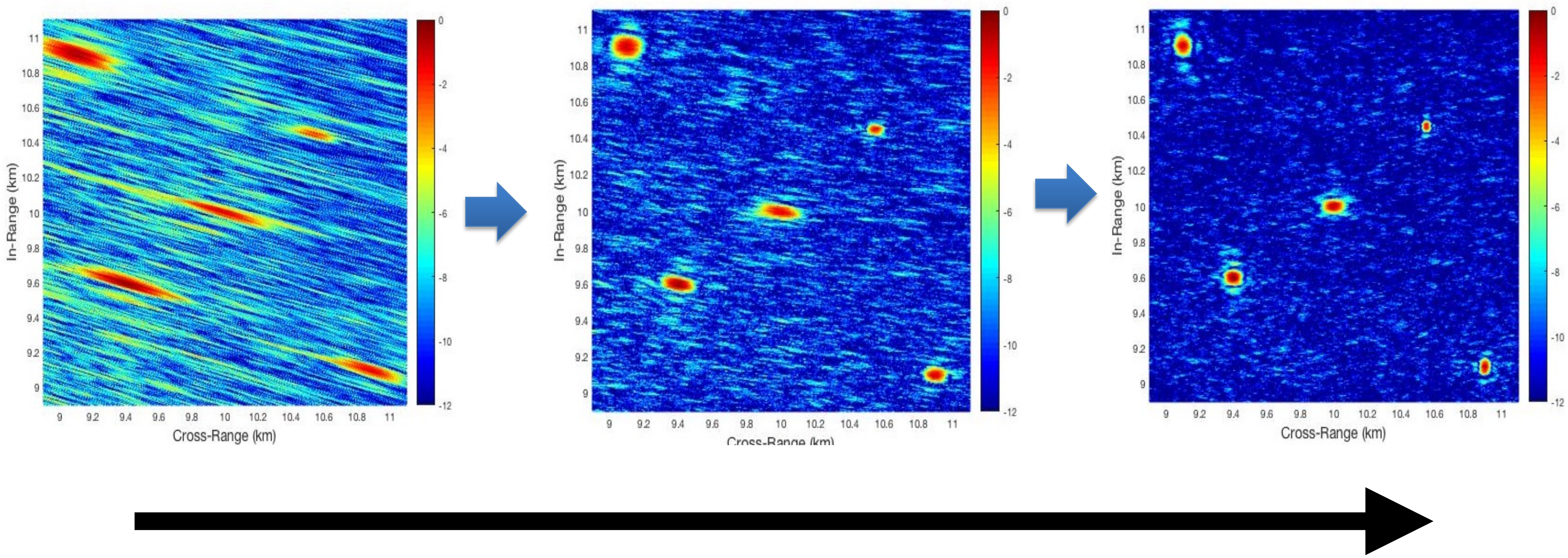


Form image based on diagonal terms.

$$I(\boldsymbol{z}) = \int e^{i\omega(t - r(s,\boldsymbol{z})/c)} Q(s,\omega,\boldsymbol{z}) d\omega \; d(s,t) ds dt$$

What happens to cross terms?

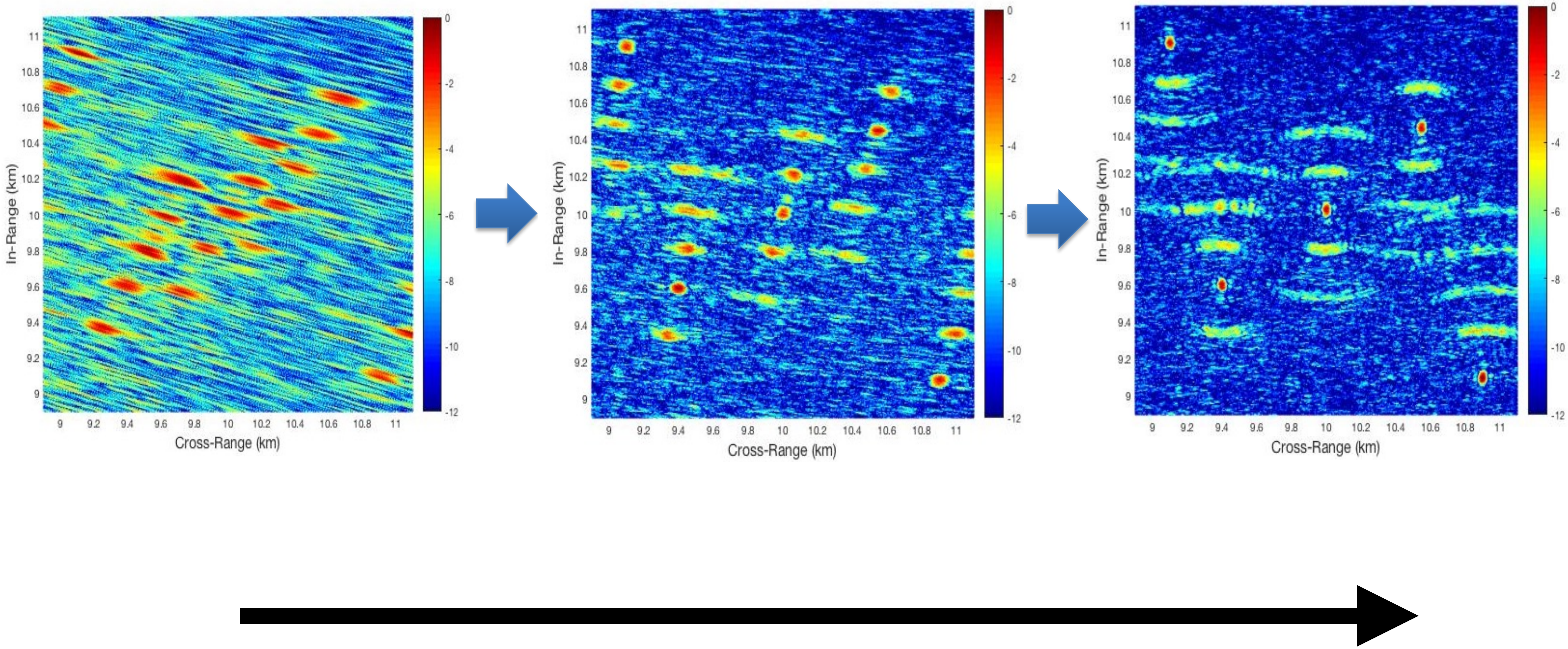
- When waveforms are uncorrelated, cross term artifacts are negligible



Longer synthetic aperture

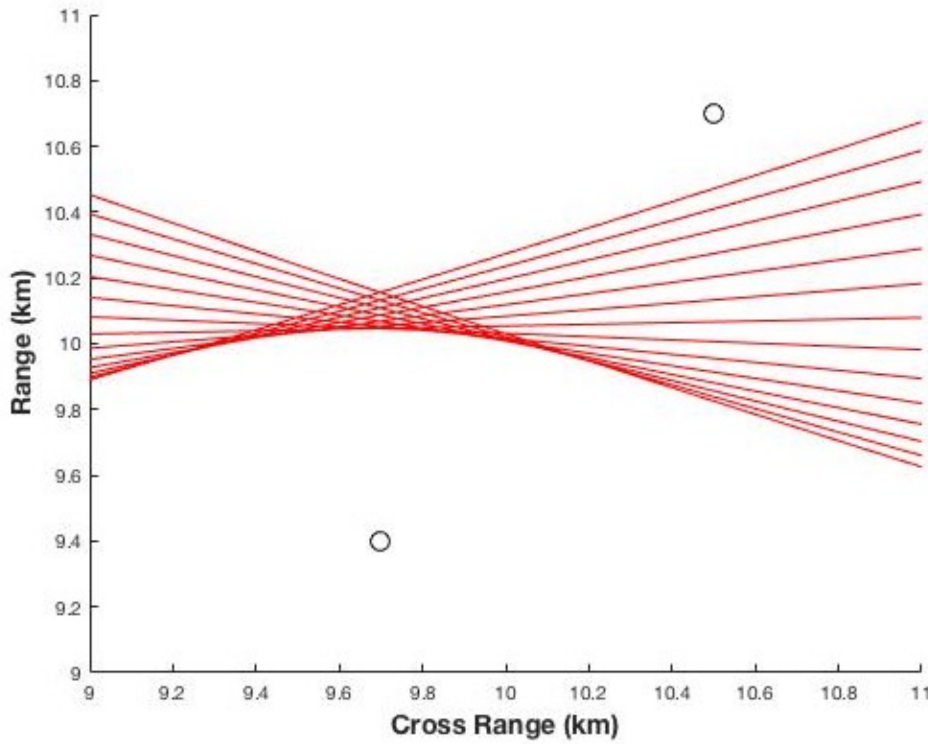
- When sources are transmitting the SAME waveform, cross terms still focus only at correct source positions!!!

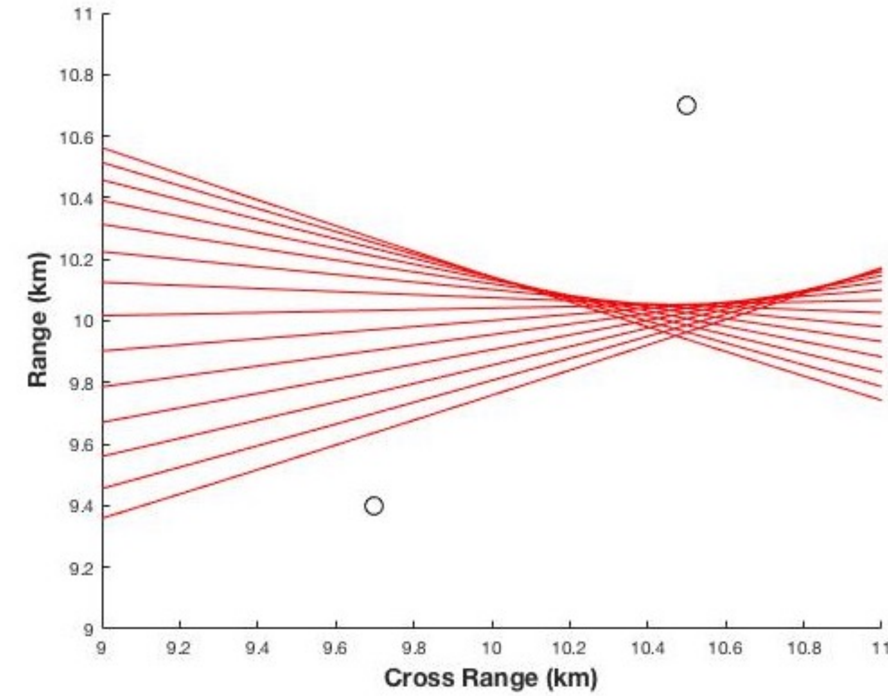
5 sources transmitting the same waveform



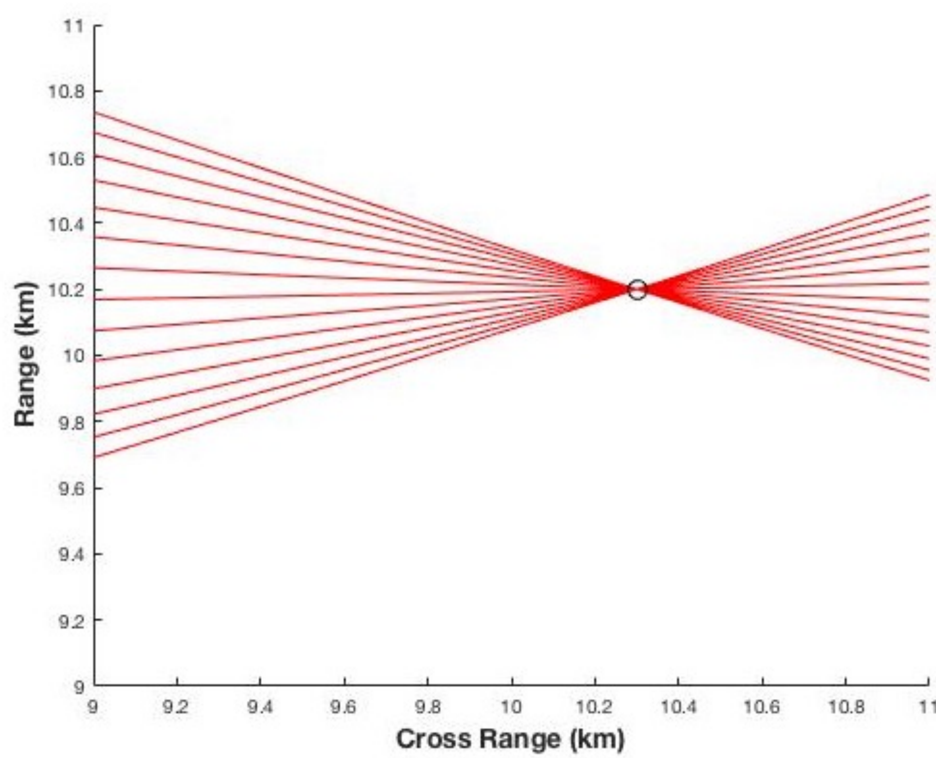
Longer synthetic aperture

cross terms:





diagonal terms:



Analysis of Image

$$I(\boldsymbol{z}) = \int e^{i\omega(t - r(s,\boldsymbol{z})/c)} Q(s,\omega,\boldsymbol{z}) d\omega \; d(s,t) ds dt$$

plug in ↑

$$d_D(s,t) = \int e^{-i\omega(t - r(s,\boldsymbol{y})/c)} A(s,\boldsymbol{y}) B(\omega) d\omega \; V(\boldsymbol{y}) d\boldsymbol{y}$$

stationary phase calculation: main contribution comes from

$$r(s,\boldsymbol{y}) = r(s,\boldsymbol{z})$$ TDOAs match $$r(s,\boldsymbol{y}) = |\gamma_2(s) - \boldsymbol{y}| - |\gamma_1(s) - \boldsymbol{y}|$$

$$\frac{\partial r(s,\boldsymbol{y})}{\partial s} = \frac{\partial r(s,\boldsymbol{z})}{\partial s}$$ FDOAs match

analysis of cross terms proceeds similarly

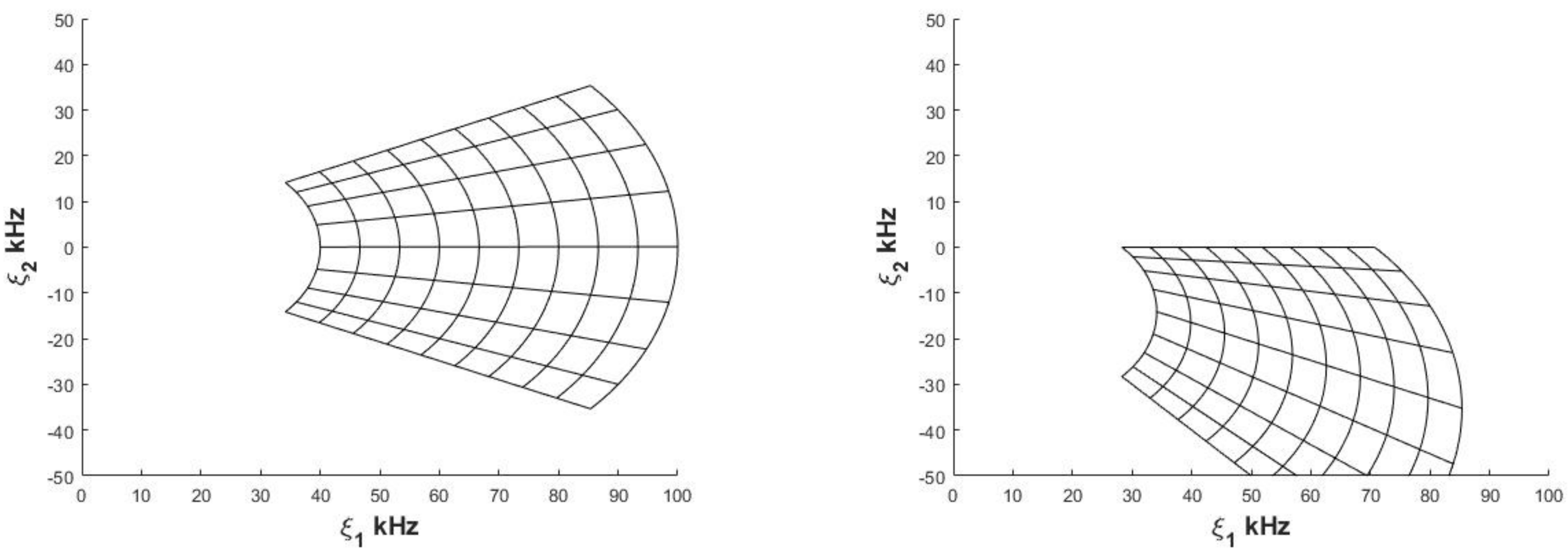
Resolution (analysis of diagonal terms)

Point-spread function

$$K(\boldsymbol{y}, \boldsymbol{z}) \approx \int e^{i(\boldsymbol{y}-\boldsymbol{z})\cdot\xi} d\xi$$

$$\left\{\xi = \frac{\omega}{c}\left(\widehat{\boldsymbol{R}}_{2,s,\boldsymbol{z}} - \widehat{\boldsymbol{R}}_{1,s,\boldsymbol{z}}\right)\right\}$$

desirable vs. undesirable geometry



(a) The DCM when the stationary receiver is located at (10km,20km).

(b) The DCM when the stationary receiver is located at (0km, 20km).

Figure 1: The Data Collection Manifold(DCM) for the scene center for two locations of the stationary receiver

Summary

- Cross-correlation + synthetic aperture imaging
  - Deterministic sources $\Rightarrow$ cross terms
    (no incoherent source approximation)
- Analysis of cross terms
- Resolution

Focusing occurs only in correct source locations!!!

"Synthetic Aperture Source Localization",
C. Waddington, M. Cheney, J.A. Given,
Inverse Problems 36 (2020) 015007

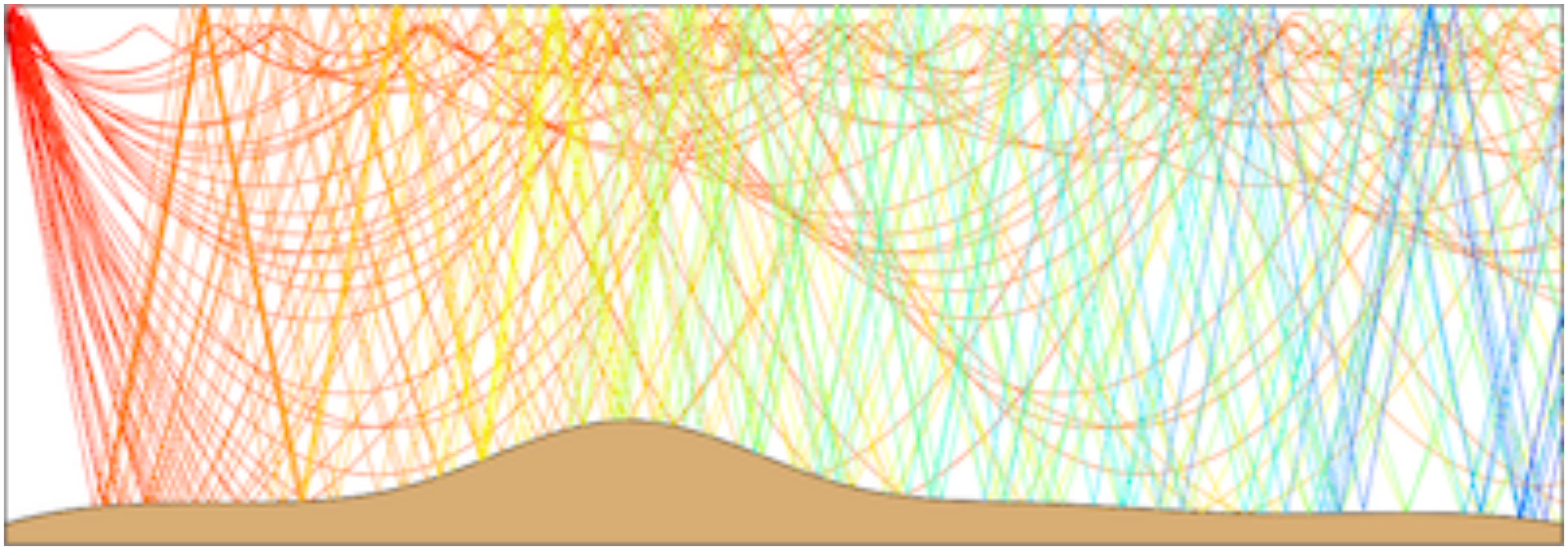
Open questions (SASL)

- Avoid assumption about waveforms
- Interaction of different waveforms
- Use cross-correlation function or cross-ambiguity function?
- Artifacts?
- Both receivers moving
- Desirable and undesirable flight paths
- Positioning uncertainty & noise

Open questions (other methods)

- Analysis of FDOA curves & surfaces (algebraic geometry)
- Exploit partial information about waveform
- Treat other waveforms as noise?

Want to do the same for sonar!



https://www.comsol.com/model/underwater-ray-tracing-tutorial-in-a-2d-axisymmetric-geometry-44711

Thanks!