ICE SHEET FLOW WITH TEMPERATURE-DEPENDENT SLIDING

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MATHEMATICAL MODELLING IN GLACIOLOGY
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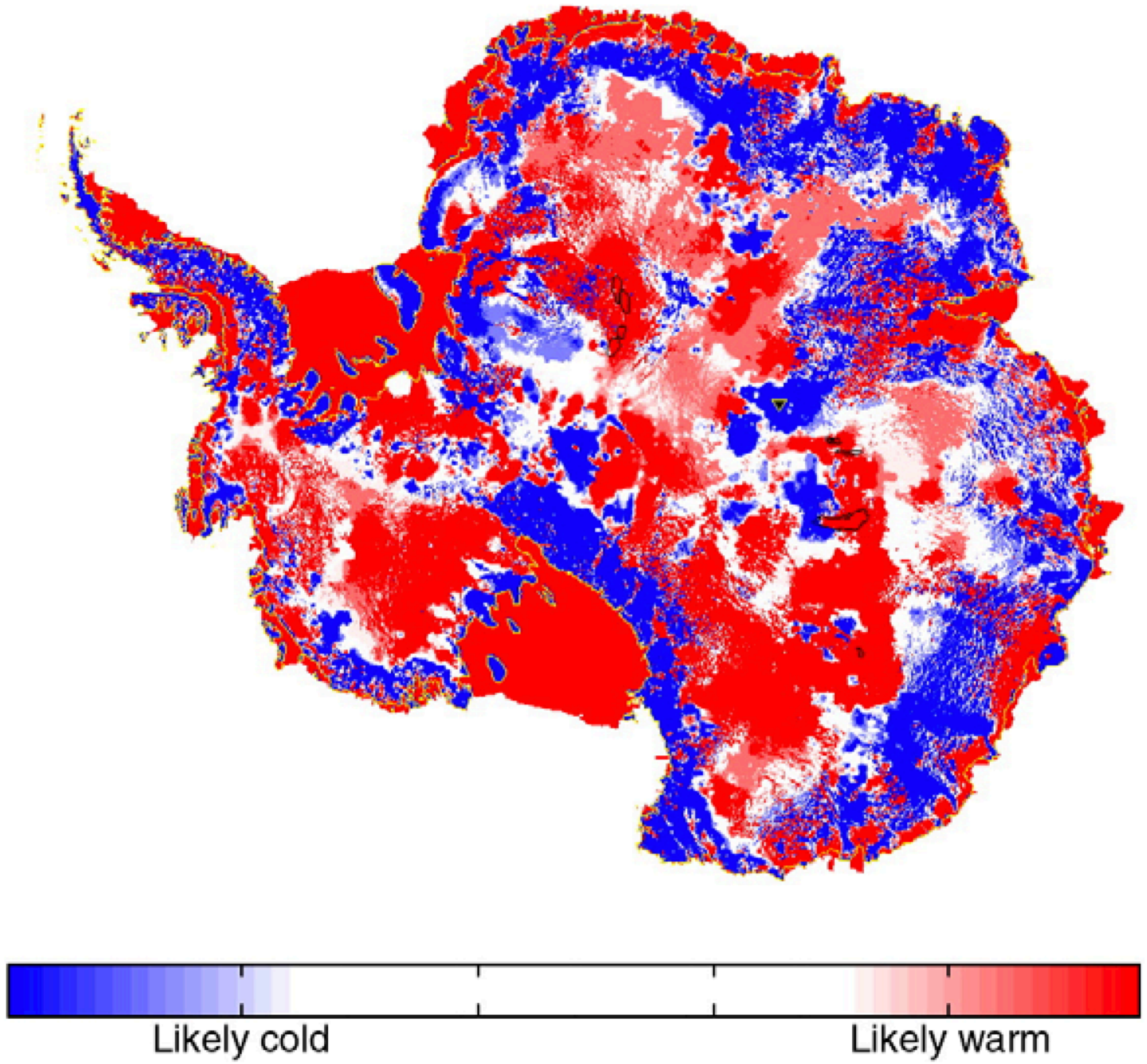
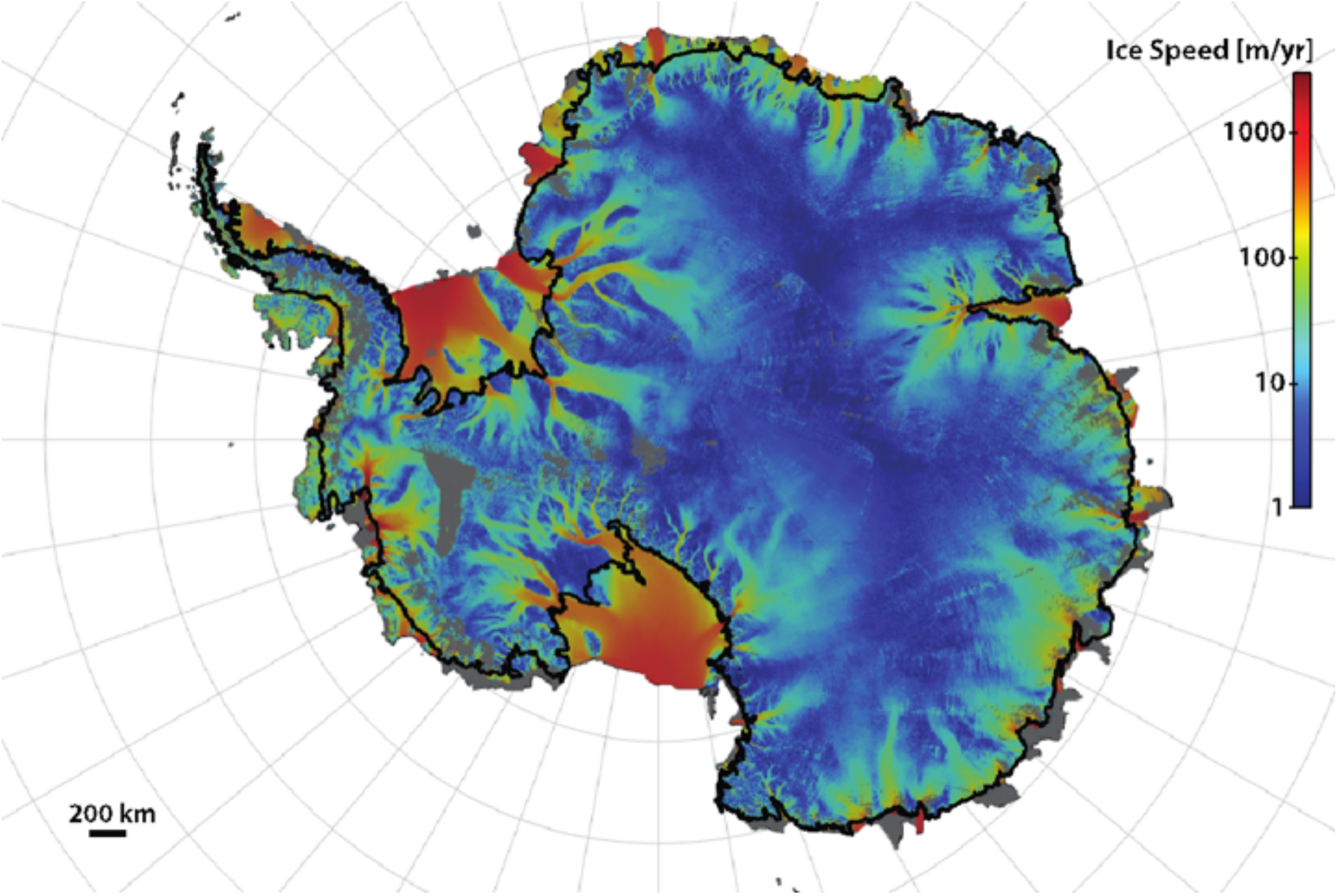
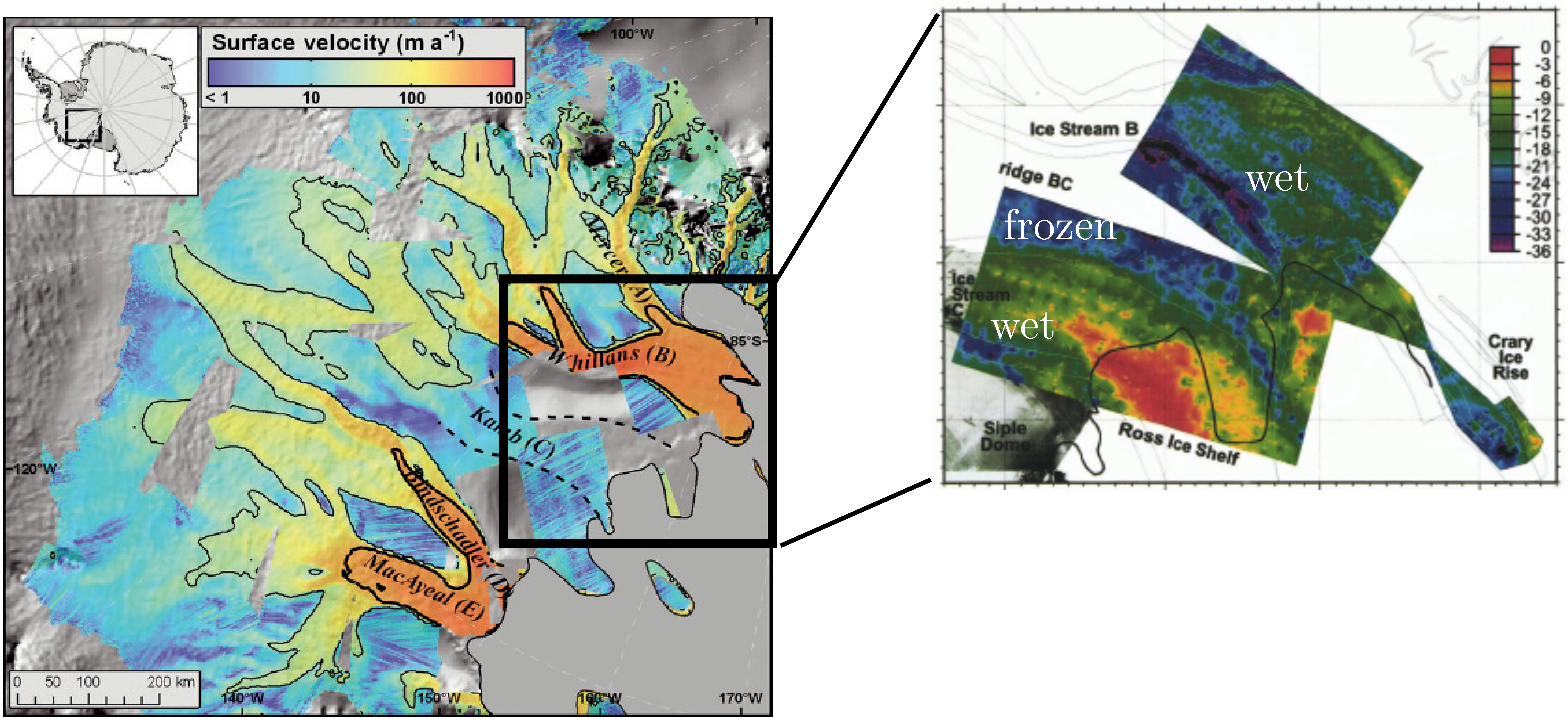
Basal thermal conditions affect ice mechanics





Source: Pattyn 2010, Rignot et al. 2013

Thermal controls on ice streaming, and how sliding is first started

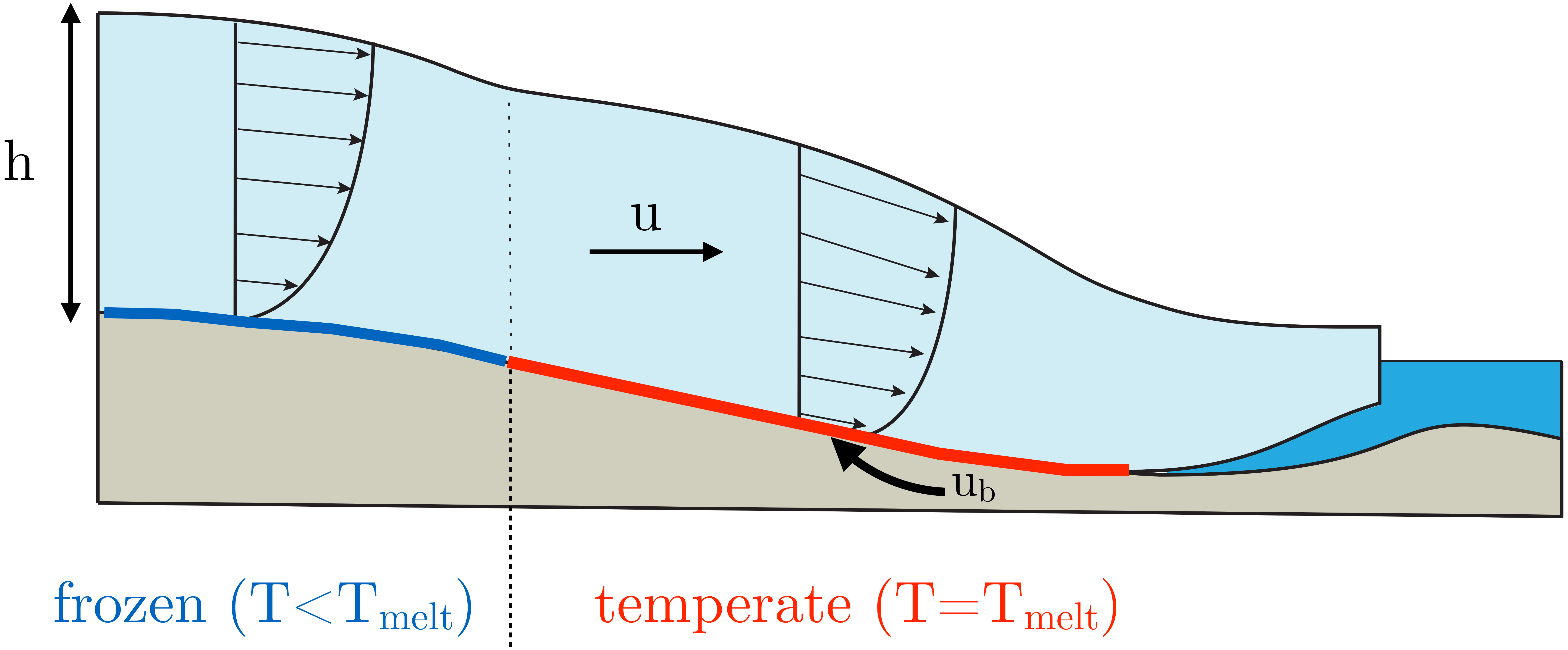


Source: Bentley et al. 1998, LeBroq et al. 2002

The upstream onset of sliding: a contradiction

at the bed: $\begin{cases} u = 0 & \mathit{if}\ T_{bed} < T_{melt} \\ u = u_b & \mathit{if}\ T_{bed} = T_{melt} \end{cases}$

$\mathit{with} \quad u_b = f(\tau_b, \ldots)$



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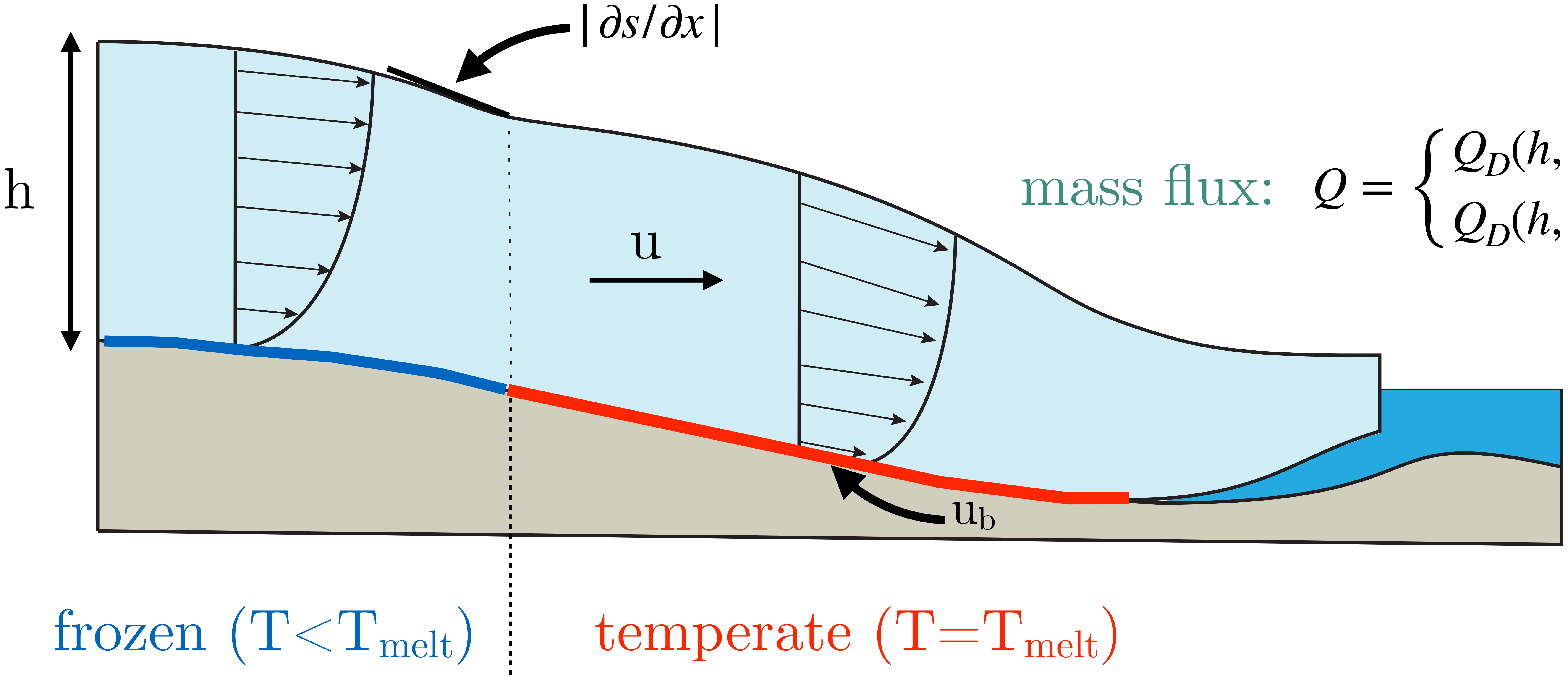
mass flux: $Q = \begin{cases} Q_D(h, |\partial s/\partial x|) & \text{if } T_{bed} < T_{melt} \\ Q_D(h, |\partial s/\partial x|) + u_b h & \text{if } T_{bed} = T_{melt} \end{cases}$



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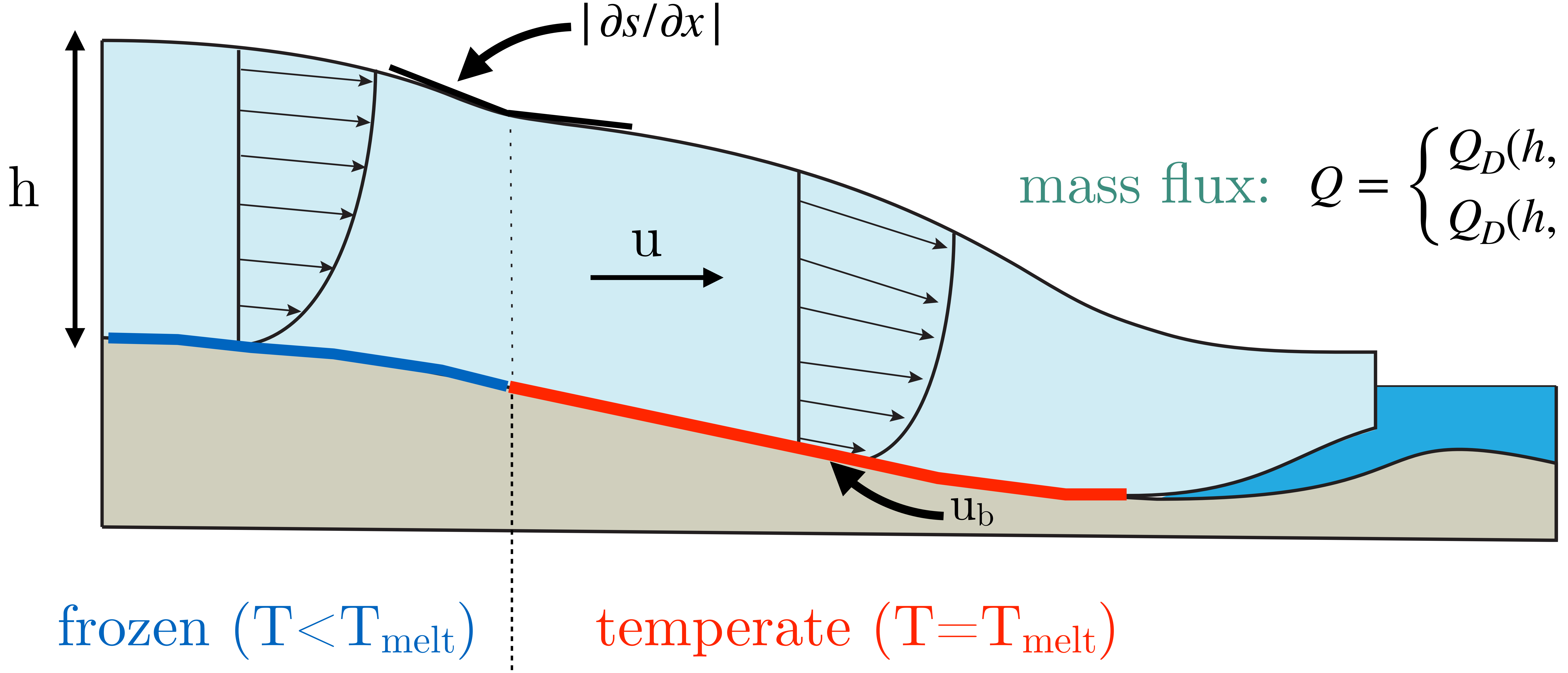

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heating rate: $Q \cdot |\partial s/\partial x|$

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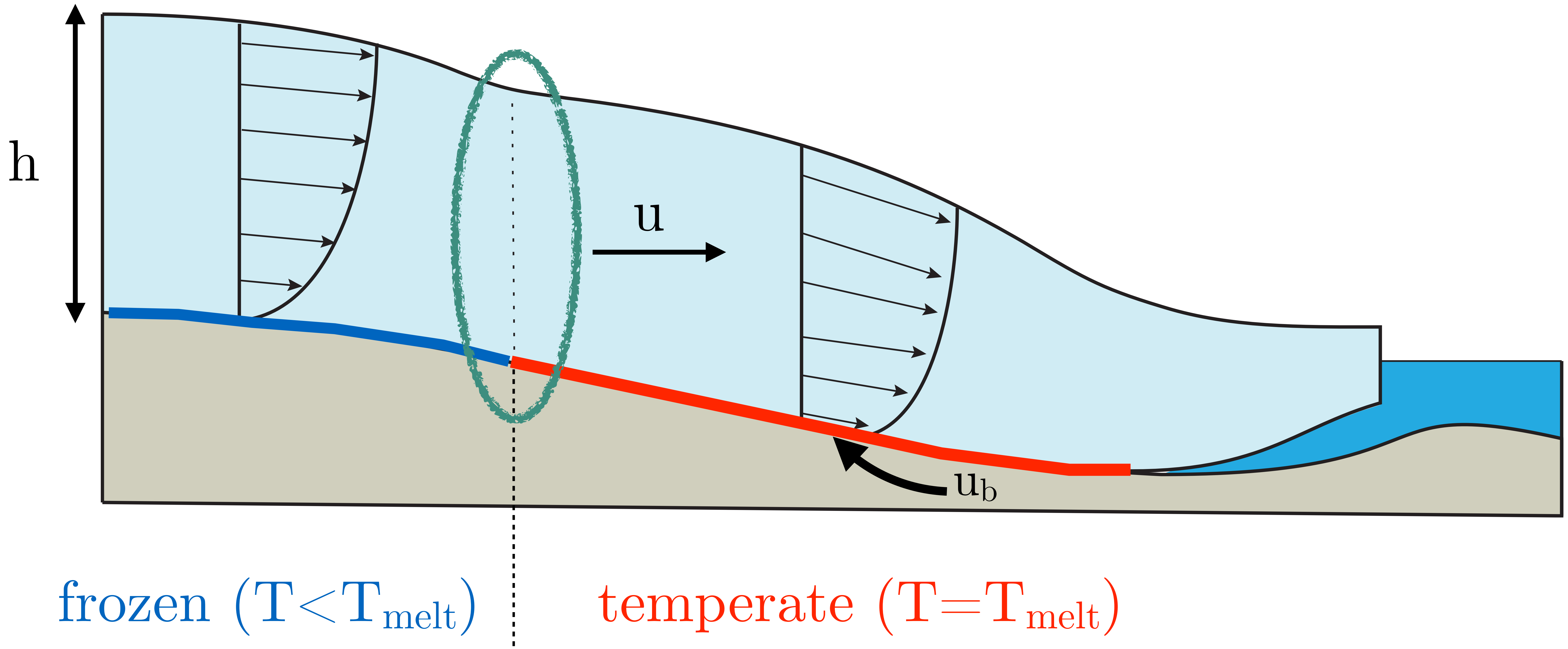
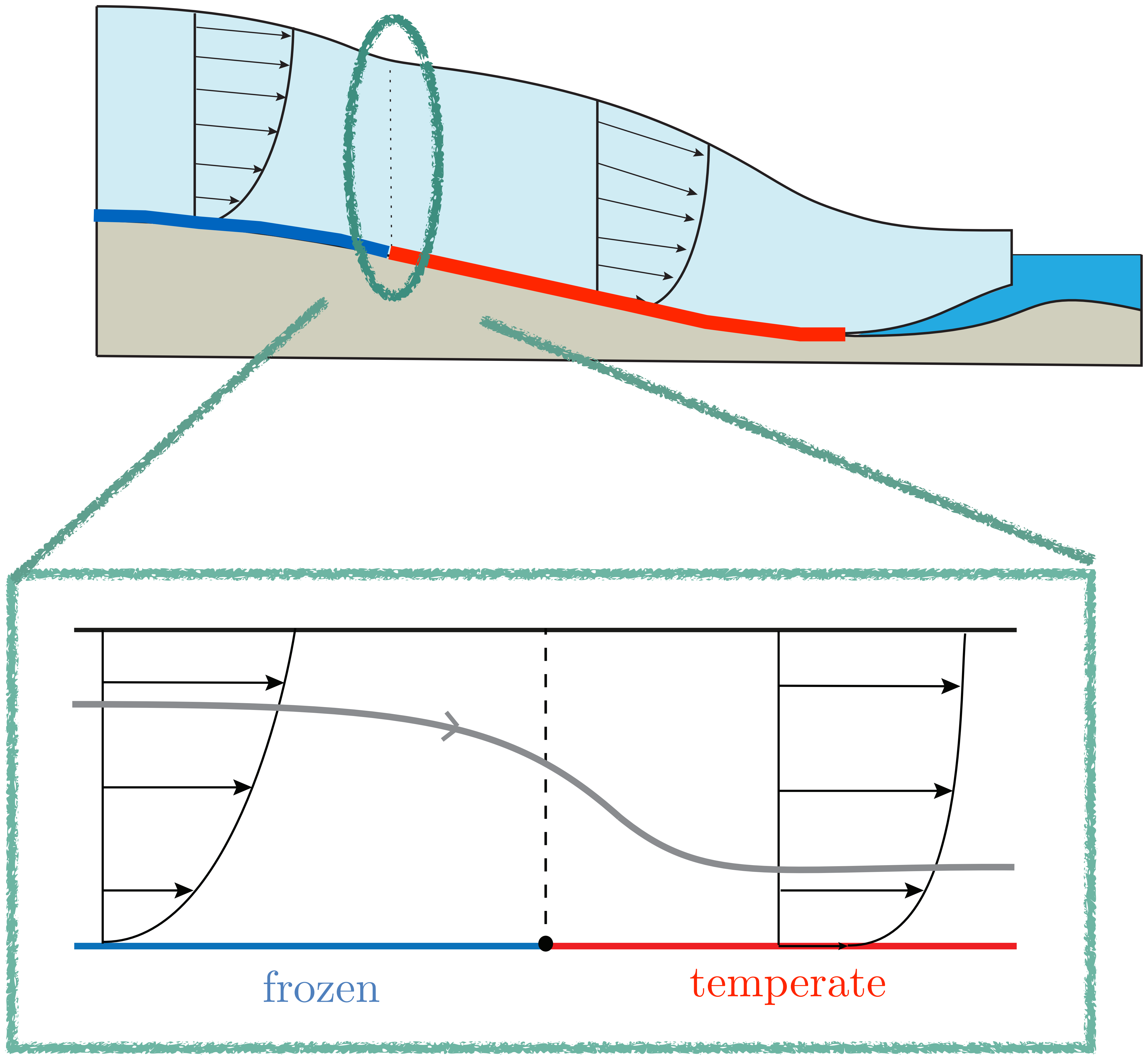


Near sliding onset: a boundary layer model



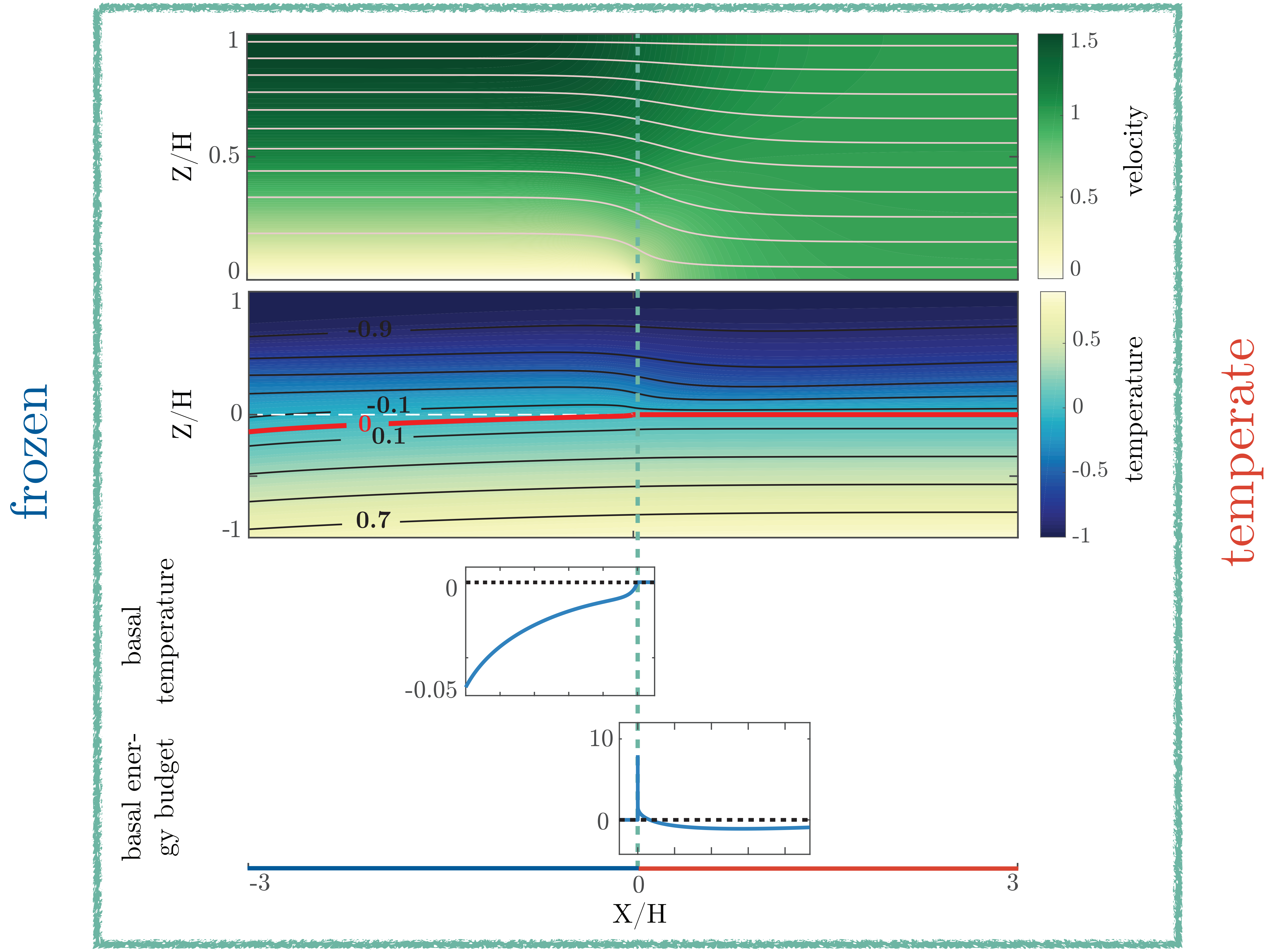
cold bed:
$$\begin{cases} u_b = 0, \\ \left[-\kappa \partial T/\partial z\right]_-^+ = 0 \end{cases}$$

temperate bed:
$$\begin{cases} \tau_b = f(u_b), \\ T = T_{melt}, \end{cases}$$

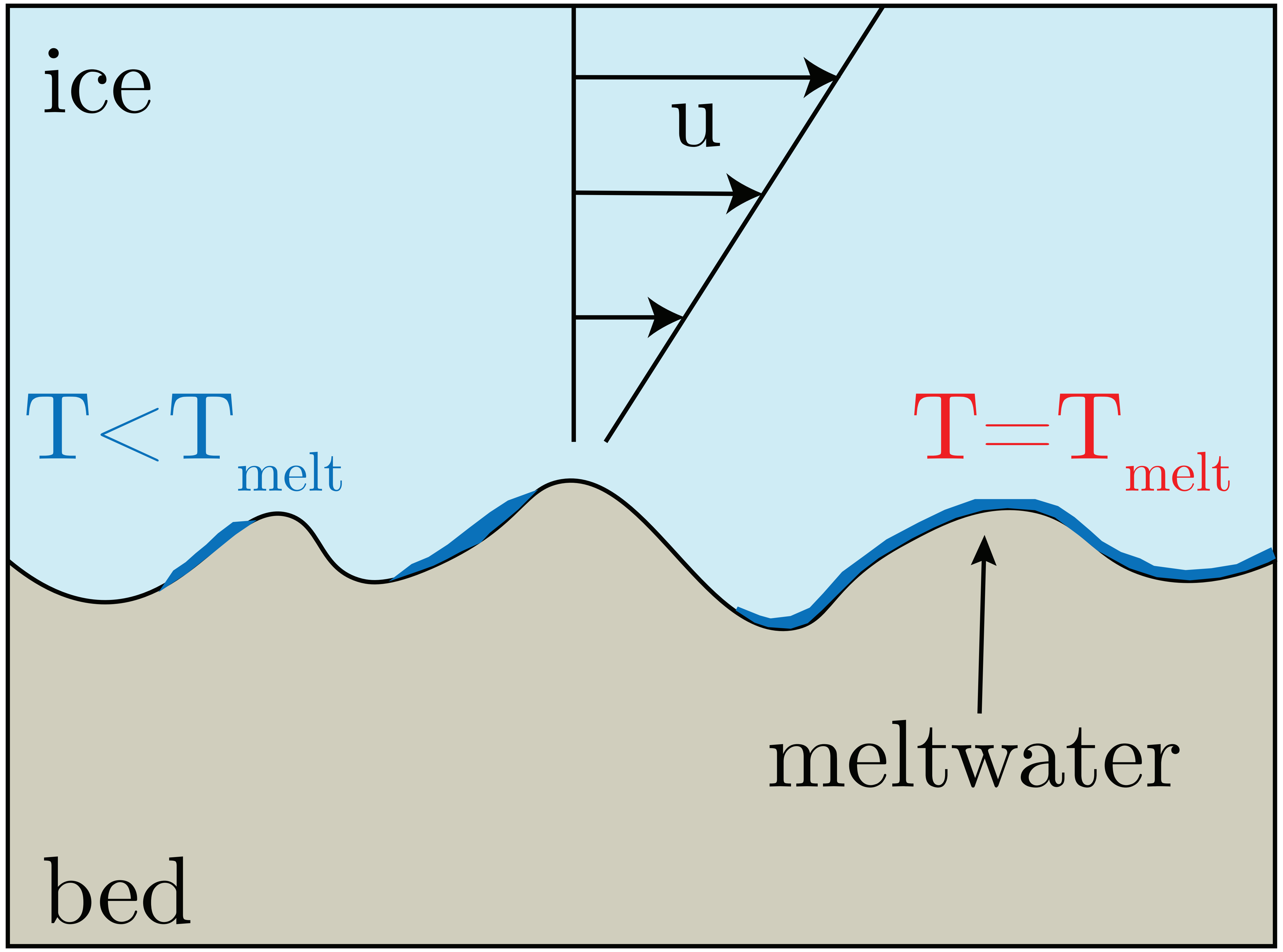
onset:
$$\begin{cases} T < T_{melt} \quad \mathit{if} \quad u_b = 0, \\ m > 0 \quad \mathit{if} \quad T = T_{melt}, \end{cases}$$

$$m = [-\kappa \partial T/\partial z]_-^+ + \tau_b u_b,$$

Strong advection causes refreezing



Sliding below the melting point



Pre-melting and regelation lubricate the bed below the melting point.

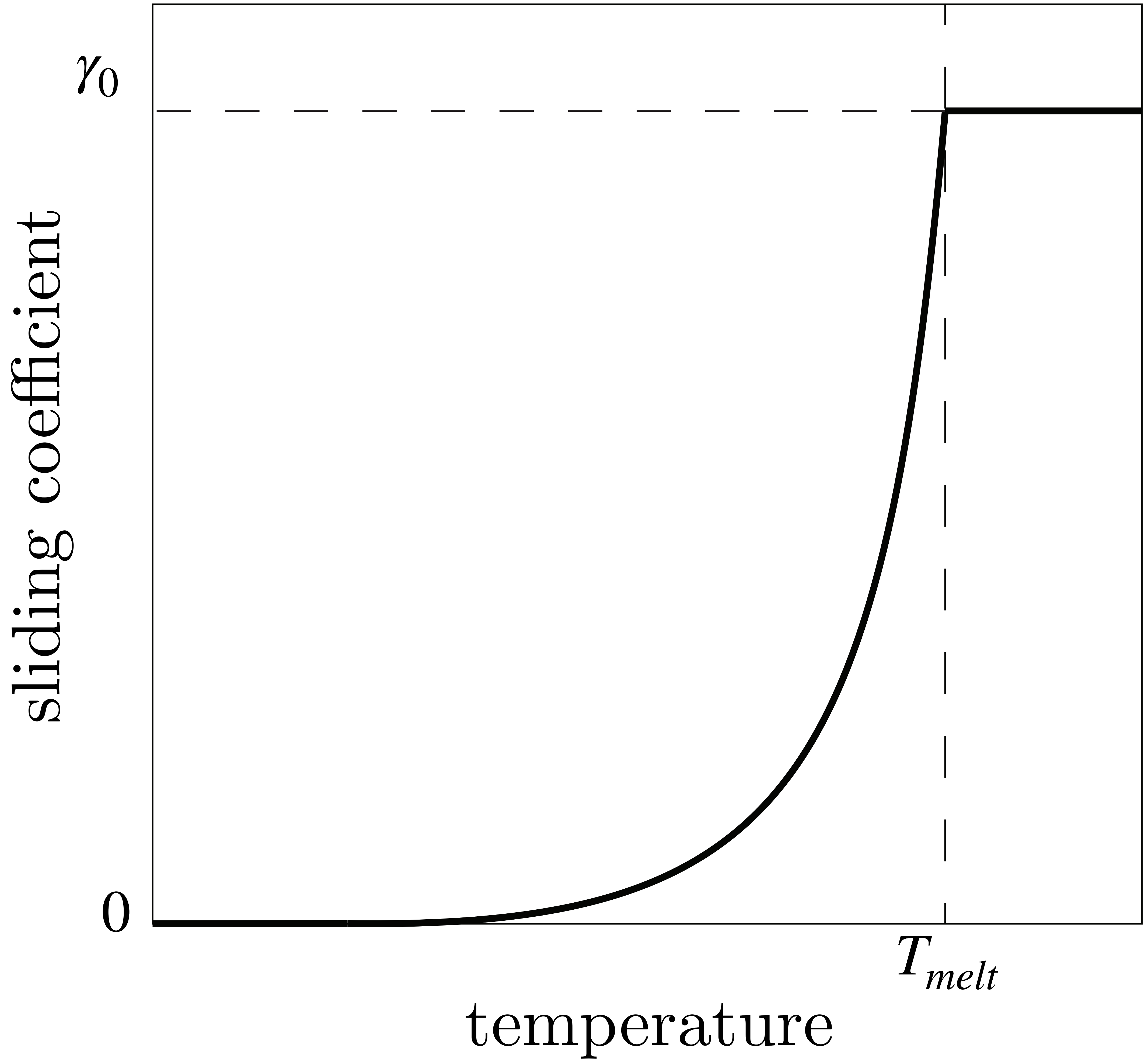
More so the closer to melting.

e.g., Fowler 1986; Barnes et al. 1971; McCarthy et al 2017

Temperature-dependent sliding

sliding law

$$\begin{cases} u_b = \overbrace{\gamma_0\,\Gamma(T)}^{\text{sliding coeff.}}\,\tau_b \\ \Gamma = \max\left\{\exp\left(\dfrac{T - T_{melt}}{\delta}\right), 1\right\} \end{cases}$$



e.g., Fowler 1986

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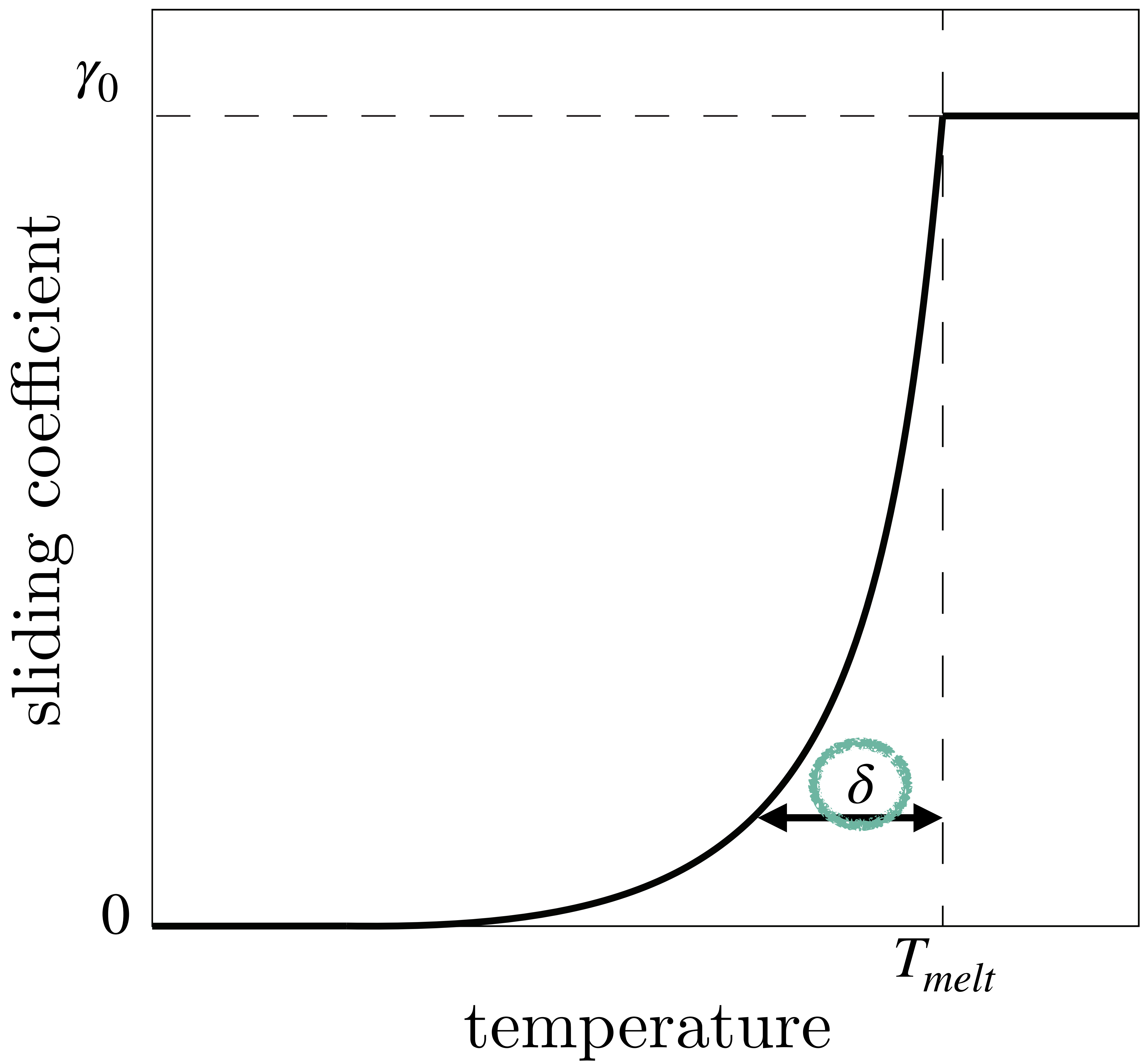

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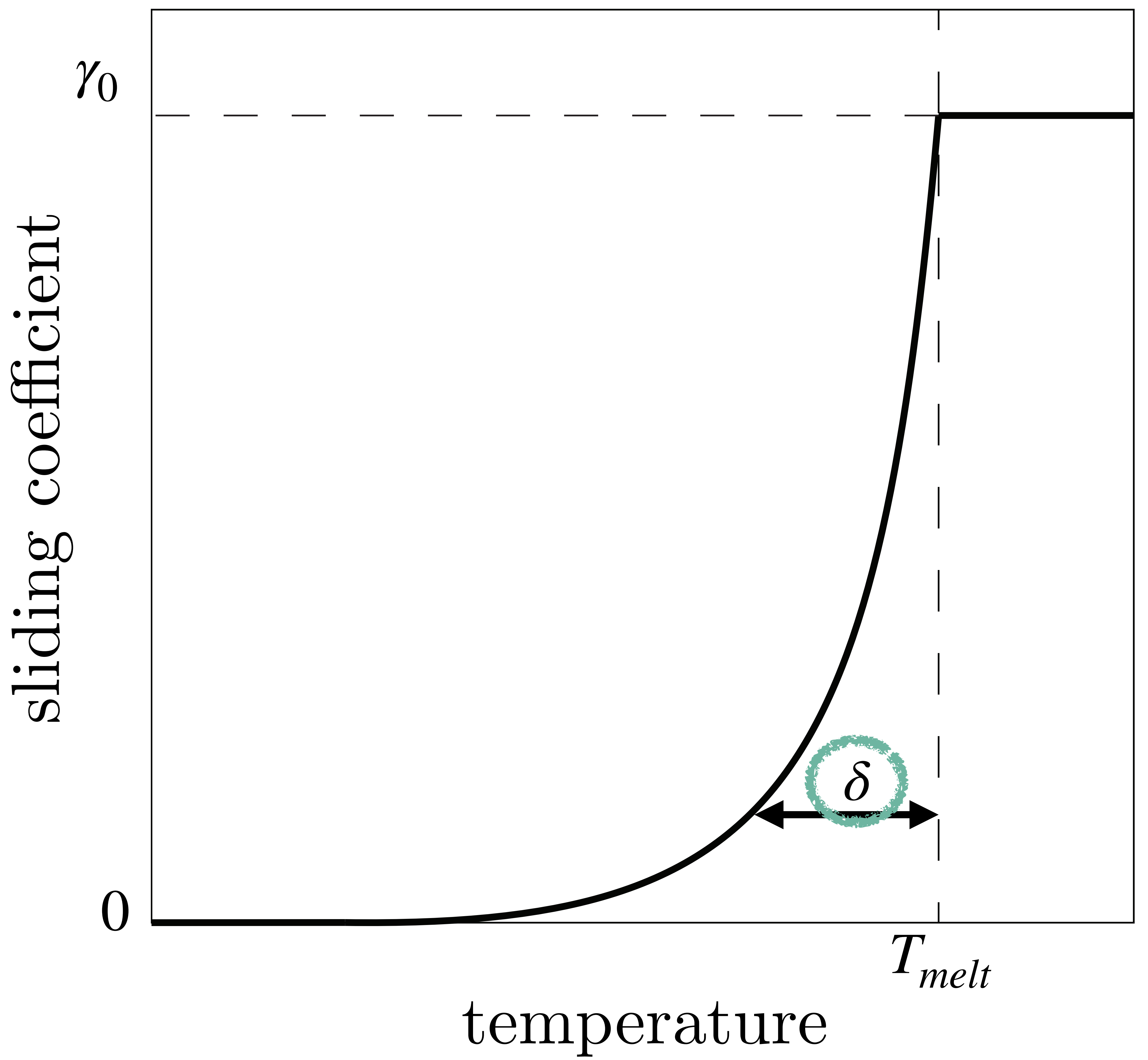
bed boundary conditions

$$\begin{cases} u = u_b \\ e = 0 \ \mathit{if} \ T \leq 0 \\ T = 0 \ \mathit{if} \ e > 0 \end{cases}$$

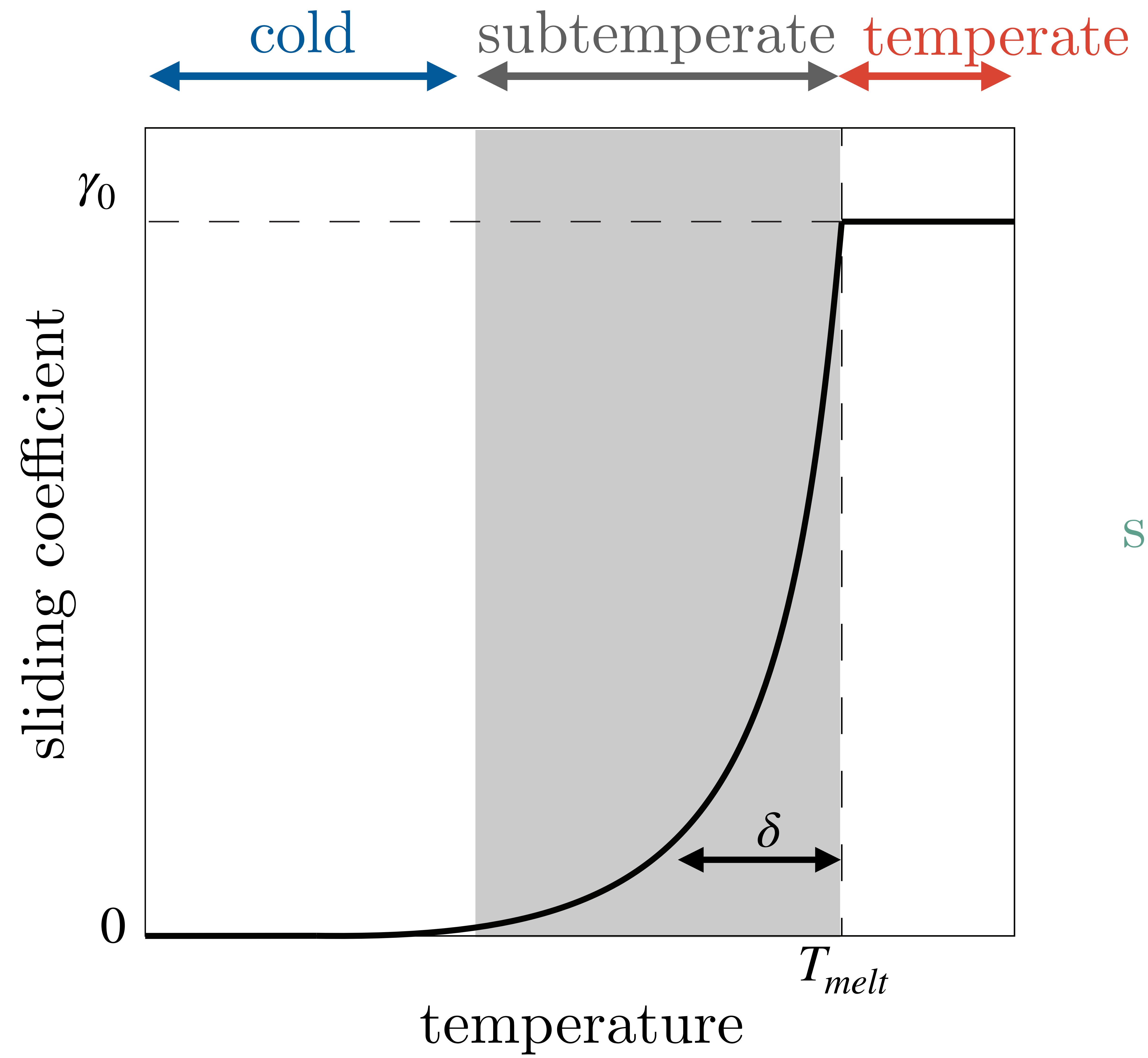
bed enthalpy

$$\frac{\partial e}{\partial t} = \left[\kappa \nabla T \cdot \vec{n}\right]_{z \to b^-}^{z \to b^+} + \tau_b u_b$$



e.g., Fowler 1986

The limit $\delta \to 0$



cold bed: $(T < T_{melt})$

$$\begin{cases} u_b = 0, \\ \left[-\kappa \partial T/\partial z\right]_-^+ = 0 \end{cases}$$

subtemperate bed: $(u_b < C_0 \tau_b^m)$

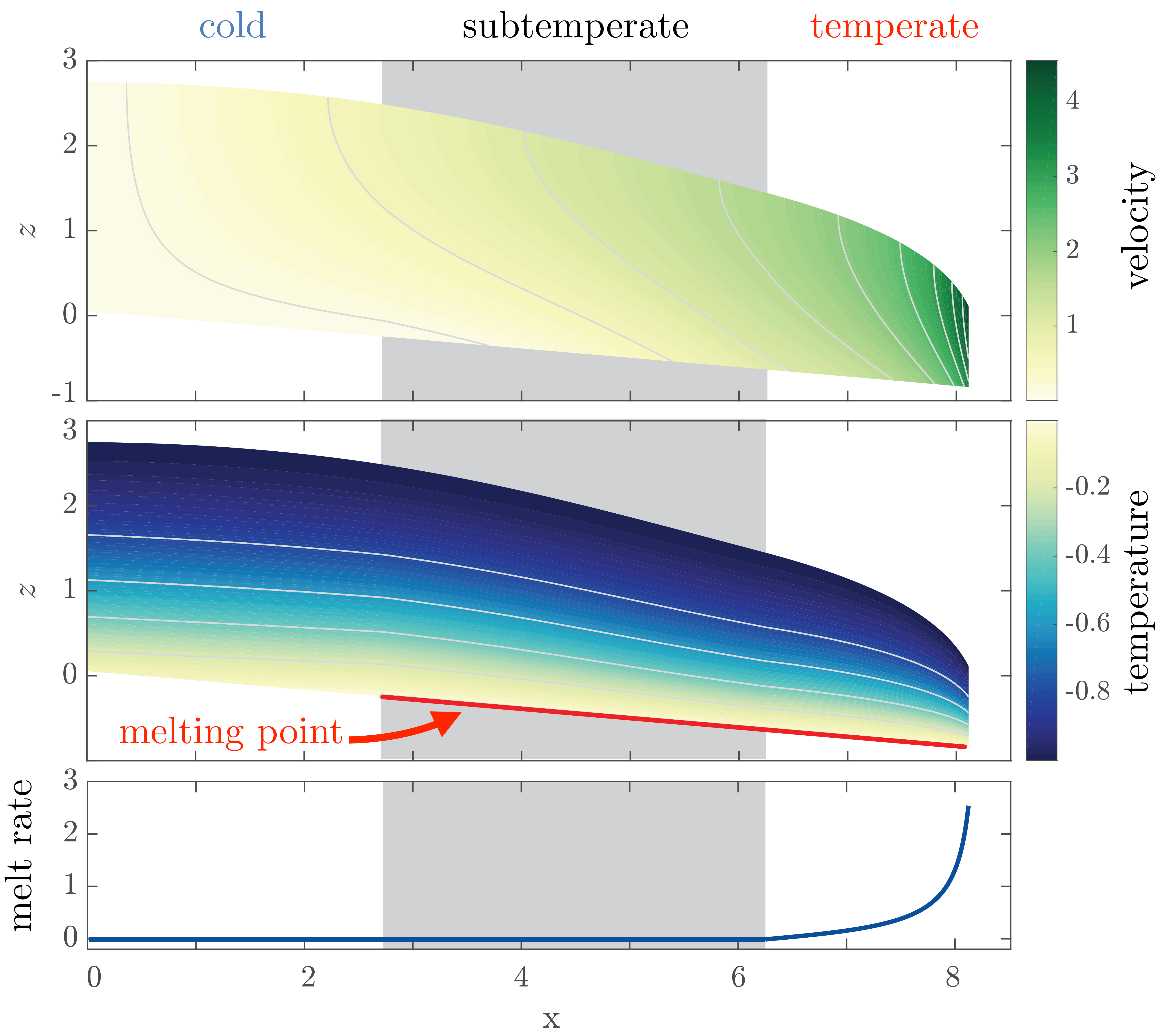
$$\begin{cases} T = T_{melt}, \\ [\kappa \partial T/\partial z]_-^+ + \tau_b u_b = 0 \end{cases}$$

temperate bed: $(m > 0)$

$$\begin{cases} T = T_{melt} \\ u_b = \gamma_0 \tau_b \end{cases}$$

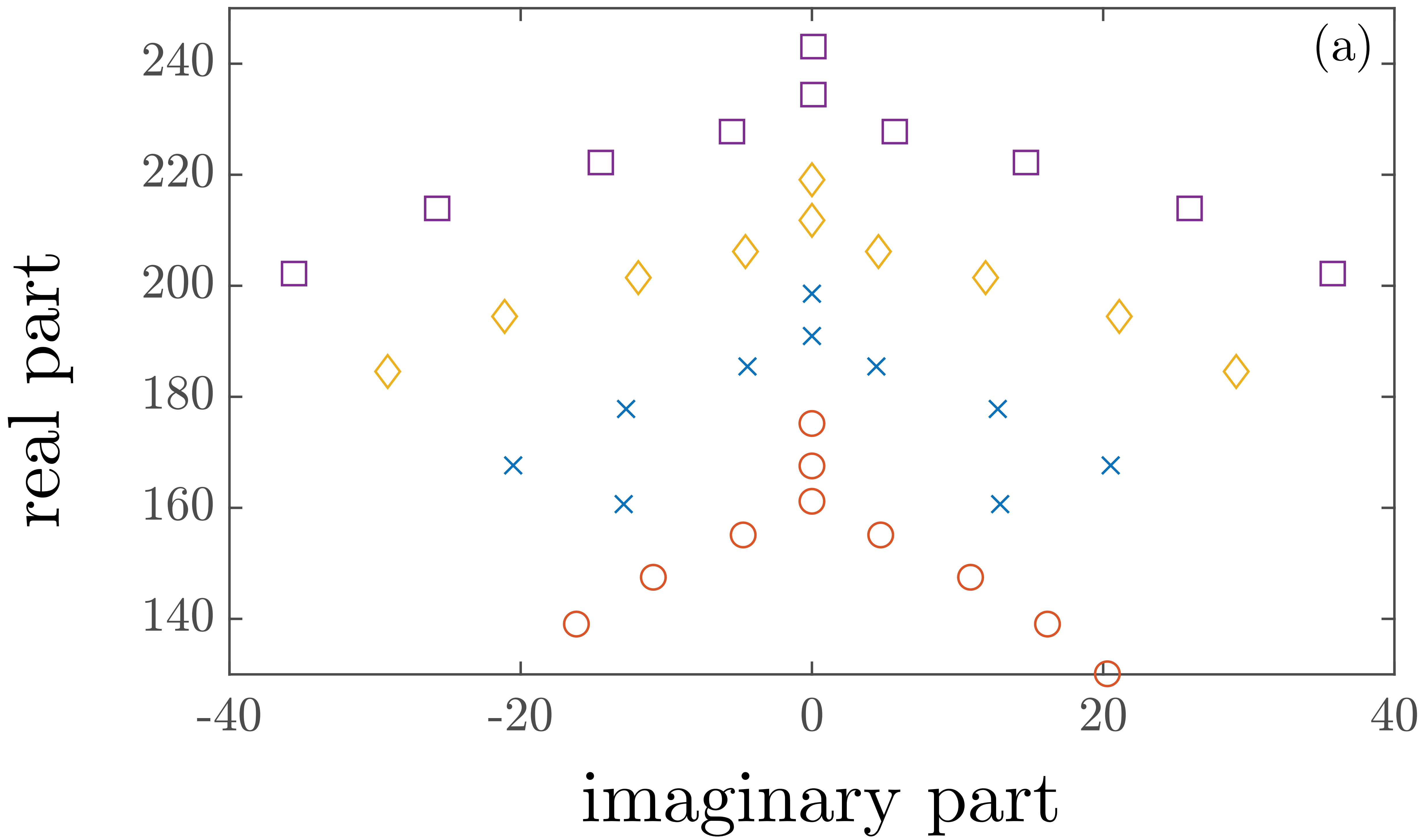
Fowler 2001, Fowler & Larson 1978

We can now construct a steady state, but ...

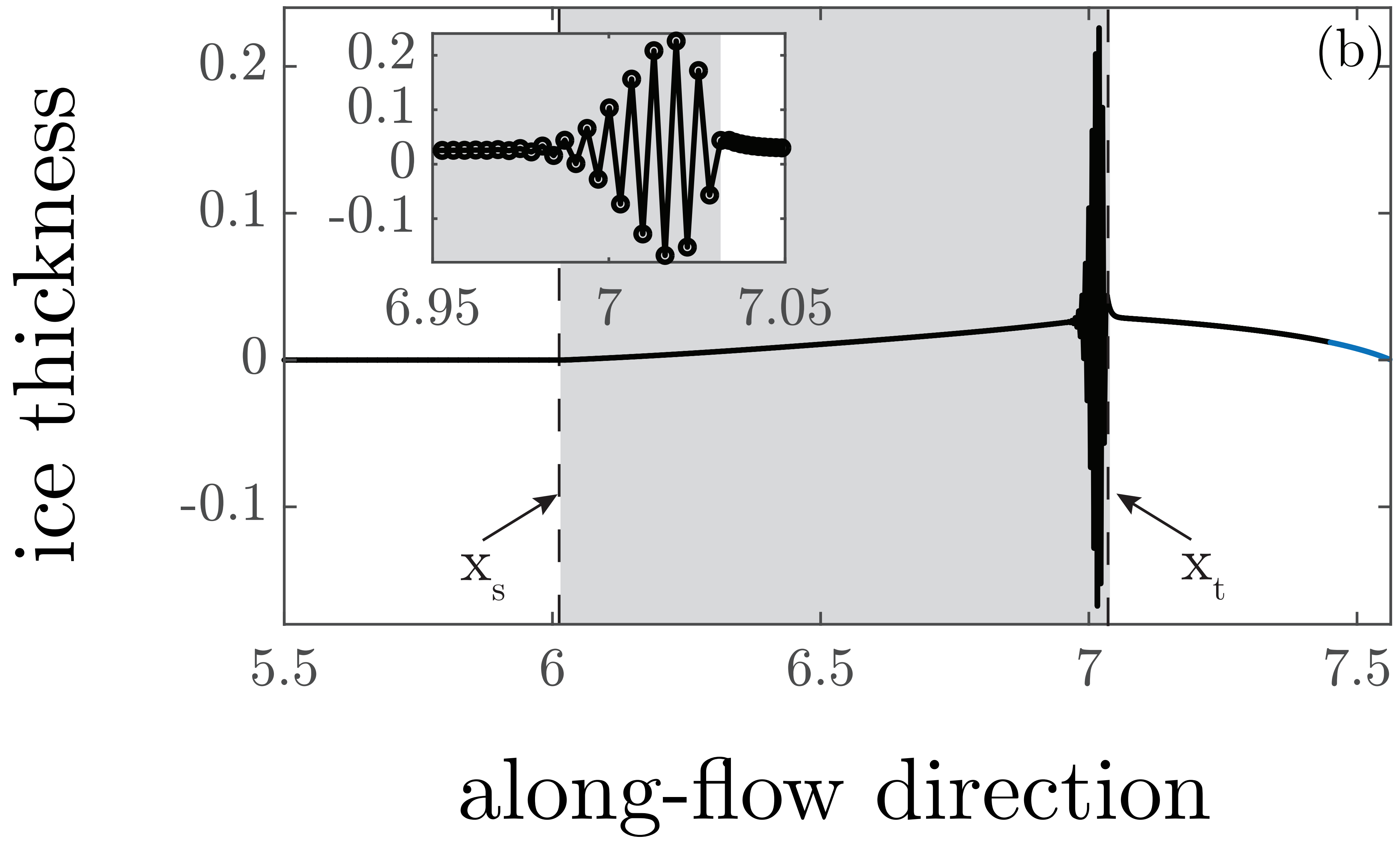


… this steady state is always unstable

eigenvalue



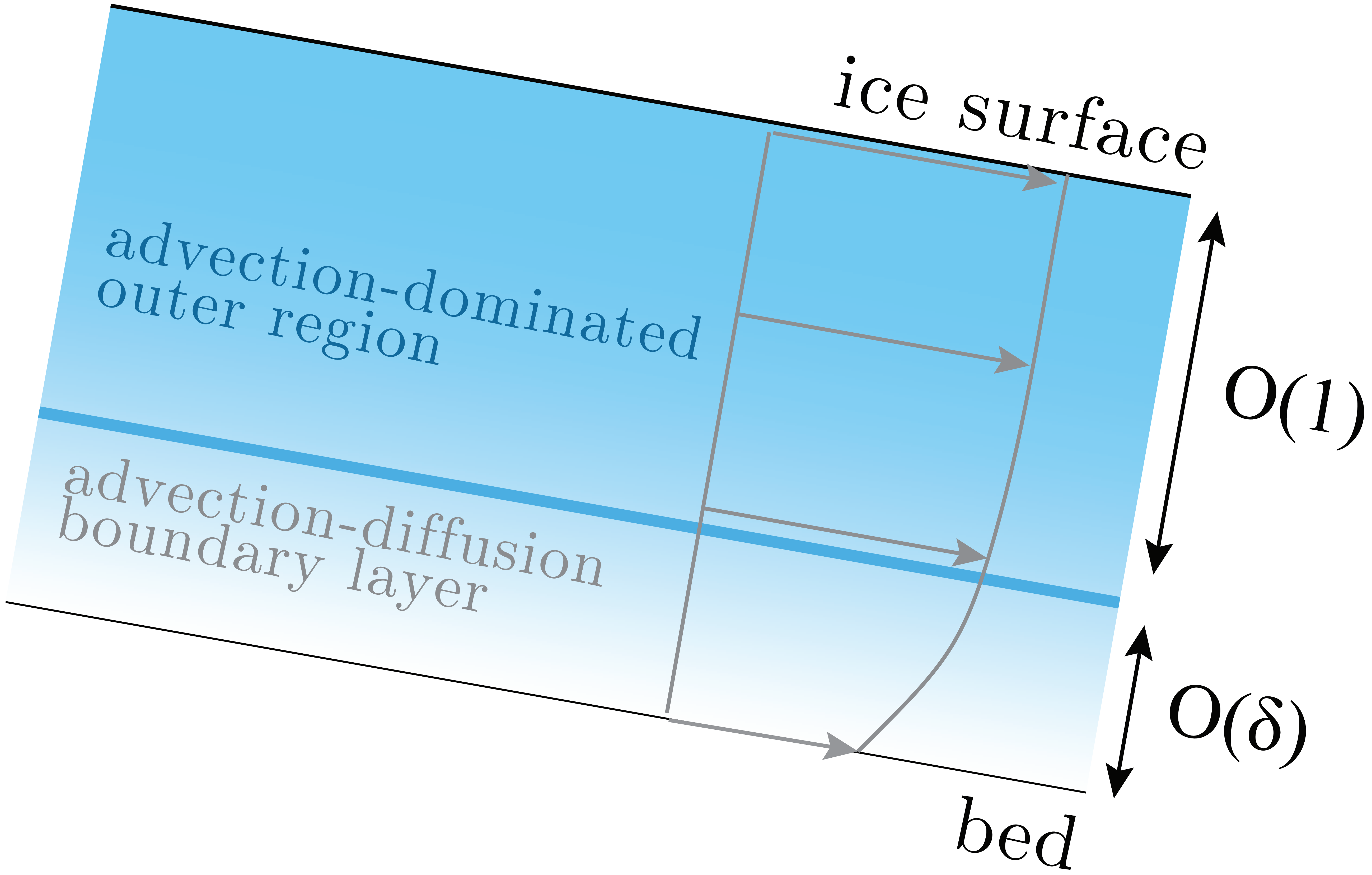
eigenfunction



velocity-weakening friction:

$$\tau_b = \frac{(Q_{ice} - Q_{geo})}{u_b}$$

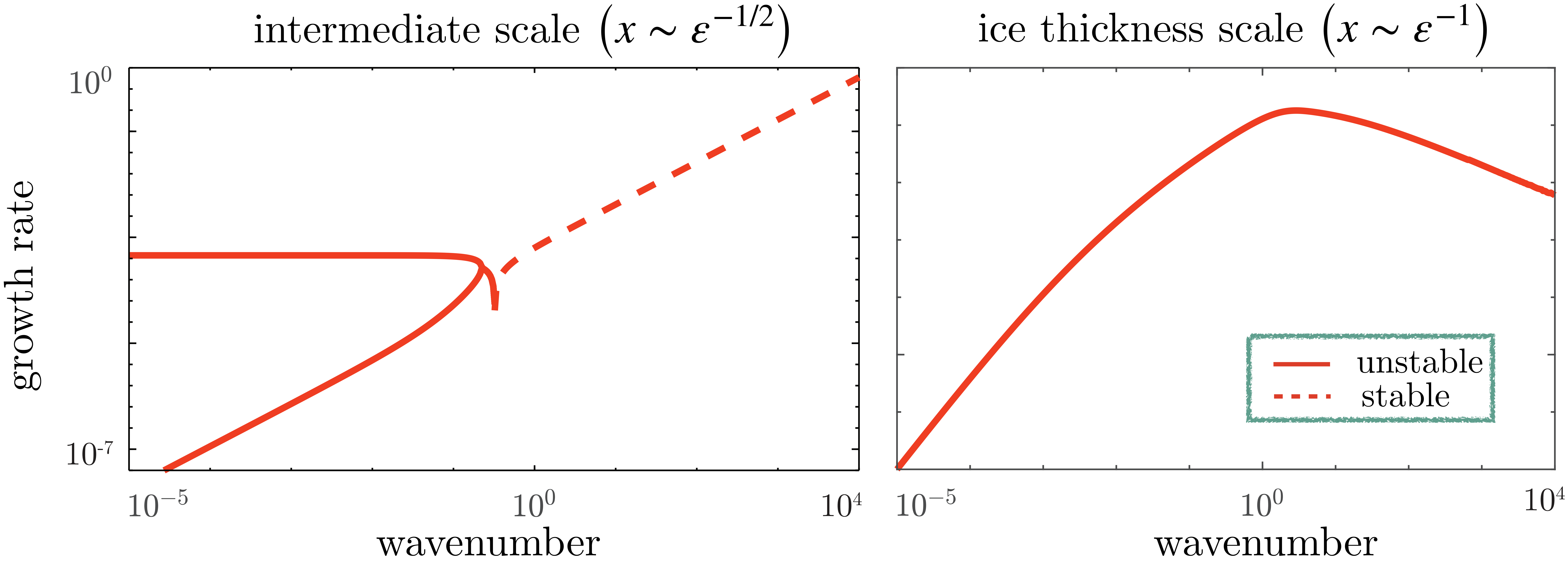
A fix for the ill-posedness: finite δ $\left(\sim \varepsilon^{1/2}\right)$ and the ice thickness scale



mechanical problem: $u_b = \gamma_0 \Gamma\left(T_{bed}\right)\, \tau_b$

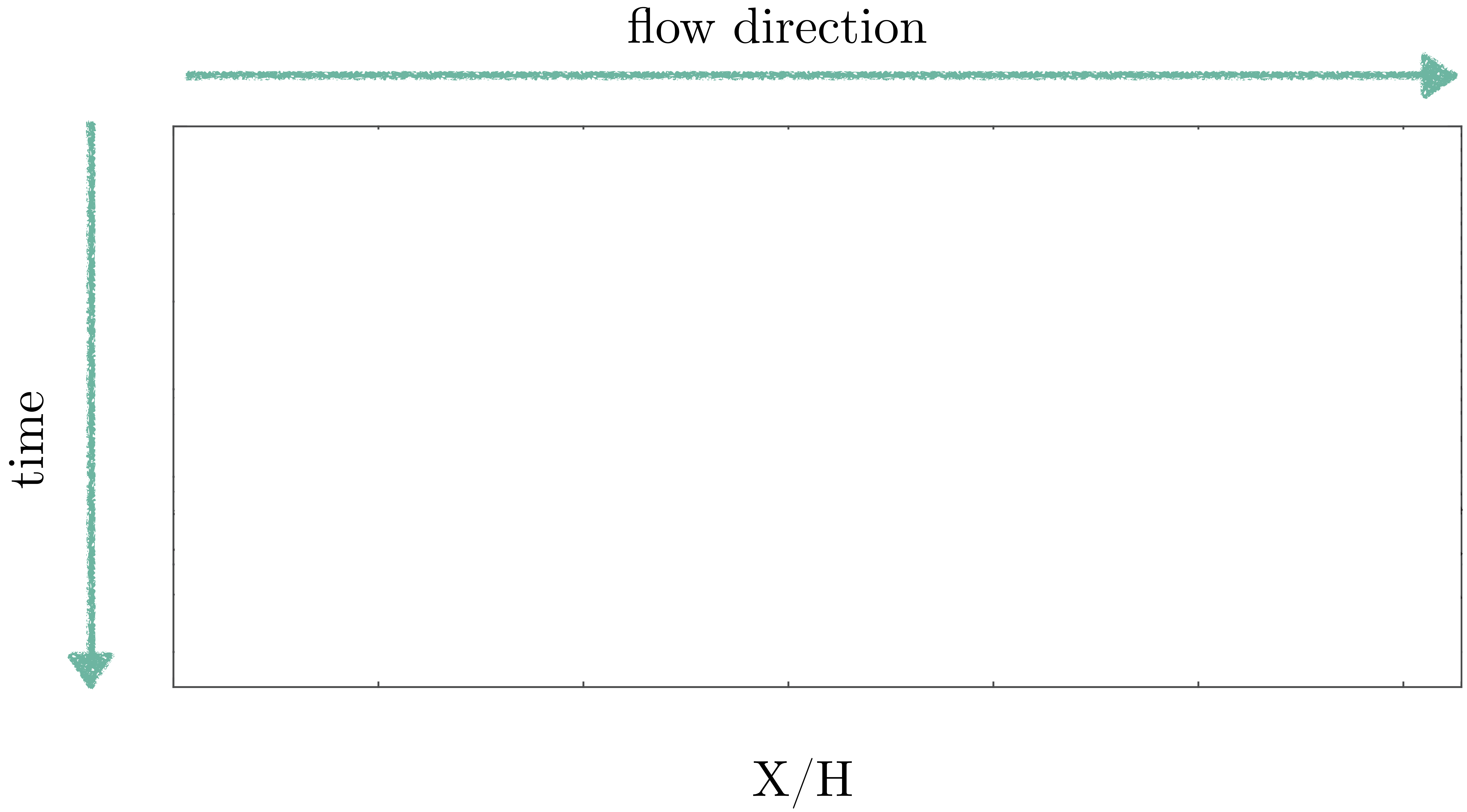
thermal problem: $[\kappa \partial T/\partial z]_-^+ + \tau_b u_b = 0$

The subtemperate region is unstable over a wide range of length scales


intermediate scale $\left(x \sim \varepsilon^{-1/2}\right)$

ice thickness scale $\left(x \sim \varepsilon^{-1}\right)$

Small-scale structure grows unboundedly and cannot be parameterized



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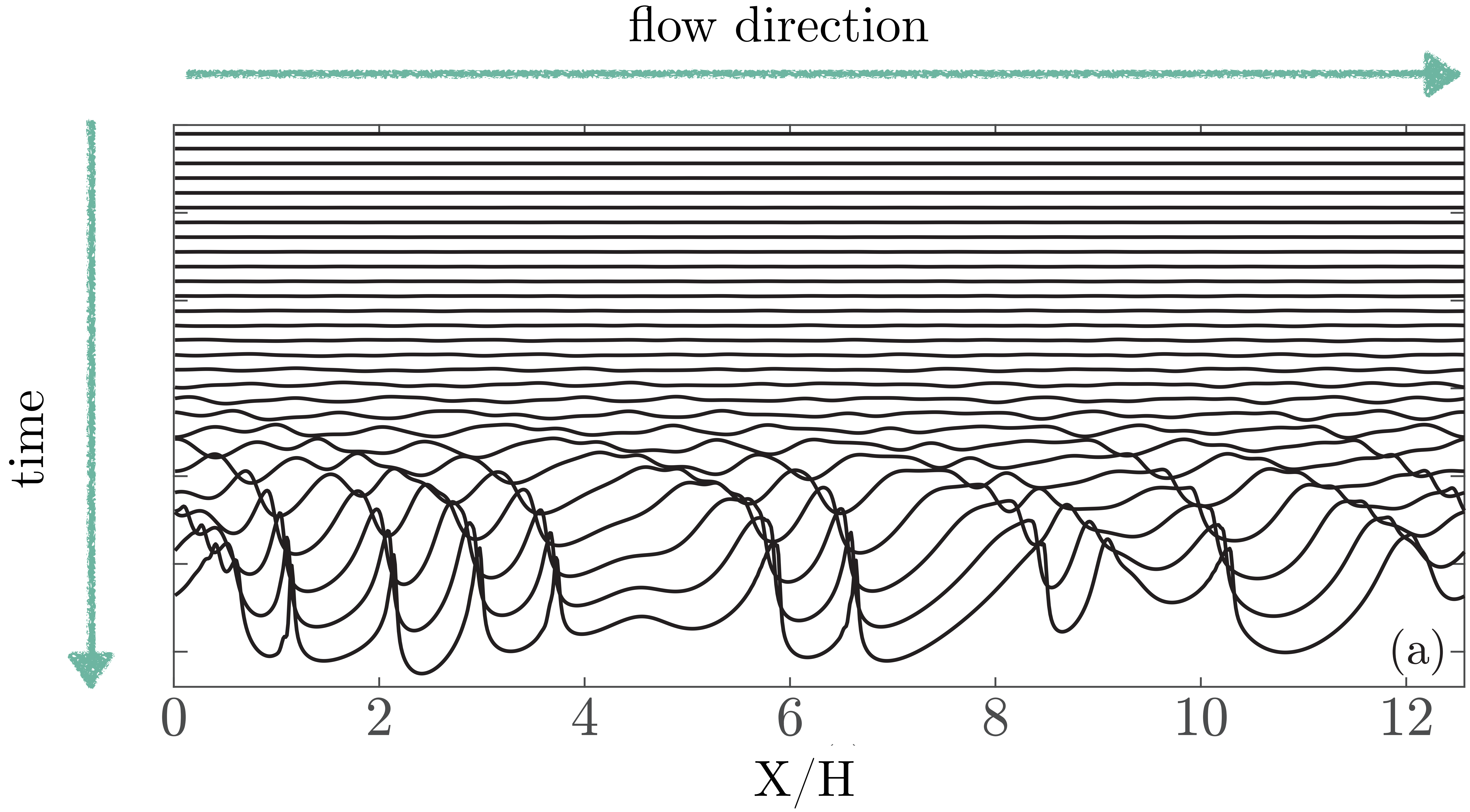
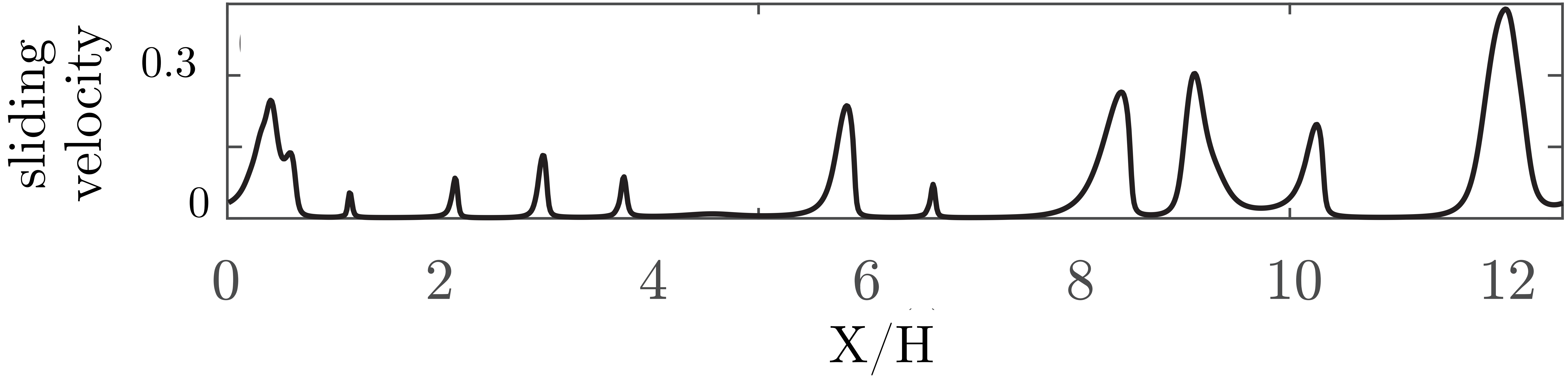


Small-scale structure grows unboundedly and cannot be parameterized

flow direction



Summary & outlook

- Along a flow-line, sliding onset at the melting point leads to no solution
- We can construct a solution if we allow sliding to depend explicitly on temperature, but the ice sheet doesn't want to settle in a steady state
- The continent-scale evolution of such a laterally uniform ice sheet remains unclear
- Most of these difficulties might be resolved in a three-dimensional model