Pattern formation in gap-junction coupled smooth muscle cells

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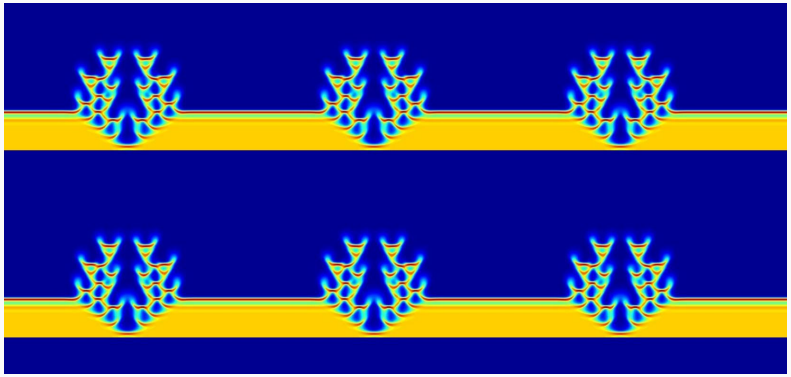
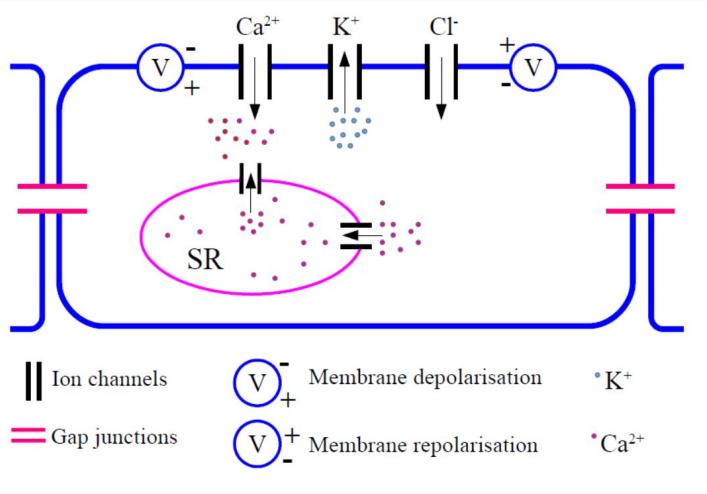


Introduction

- Electro-mechanical coupling (EMC) is the contraction of a smooth muscle cell (SMC) due to its excitation in response to an external stimulation.
- ***Pacemaker dynamics*** in SMCs.

Aims

- To investigate the influence of transmural pressure on EMC activity of SMCs in feline cerebral arteries.
- To study the collective behaviour of a population of coupled the SMCs.



Model Formulation

$$\frac{\partial V}{\partial \tau} = D\frac{\partial^2 V}{\partial X^2} - \bar{g}_L(V - \bar{v}_L) - \bar{g}_K N(V - \bar{v}_K) - \bar{g}_{\mathrm{Ca}} M_\infty(V)(V-1), \quad (1)$$

$$\frac{\partial N}{\partial \tau} = \lambda_N(V)\big(N_\infty(V) - N\big), \quad (2)$$

where V is the membrane potential, N is the fraction of open potassium channels, and

$$M_\infty(V) = 0.5\left(1 + \tanh\left(\frac{V - \bar{v}_1}{\bar{v}_2}\right)\right),$$

$$N_\infty(V) = 0.5\left(1 + \tanh\left(\frac{V - \bar{v}_3}{\bar{v}_4}\right)\right),$$

$$\lambda_N(V) = \psi \cosh\left(\frac{V - \bar{v}_3}{2\bar{v}_4}\right),$$

with no-flux boundary conditions and initial conditions:

$$V(0, X) = V_0(X) \text{ and } N(0, X) = N_0(X), \forall\, X \in \Omega.$$

Dynamics of an Isolated Cell

Modulation of model parameter induces type I and type II excitability.

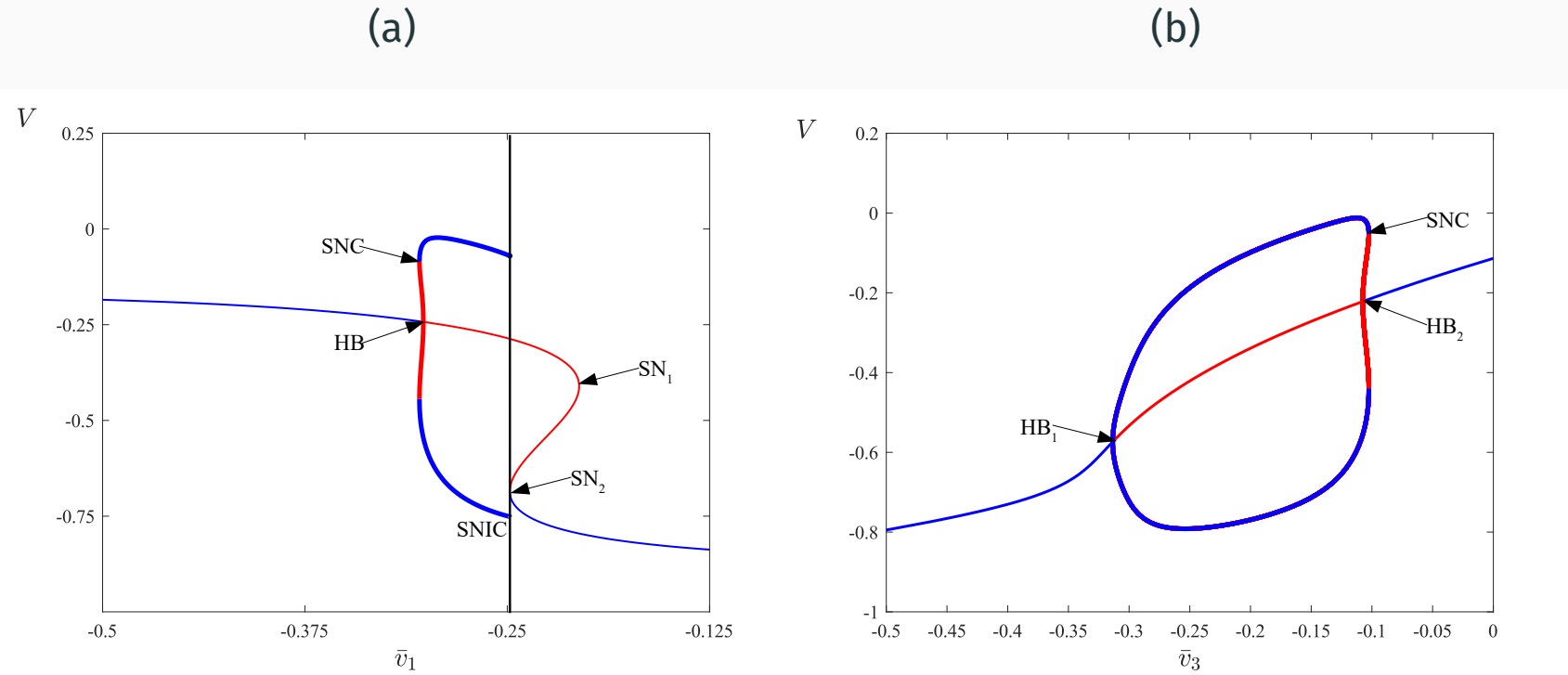


Figure 2: Bifurcation diagram of the membrane potential V with $\bar{v}_1$ and $\bar{v}_3$ as the bifurcation parameters.

Transition between Type I and Type II Excitability

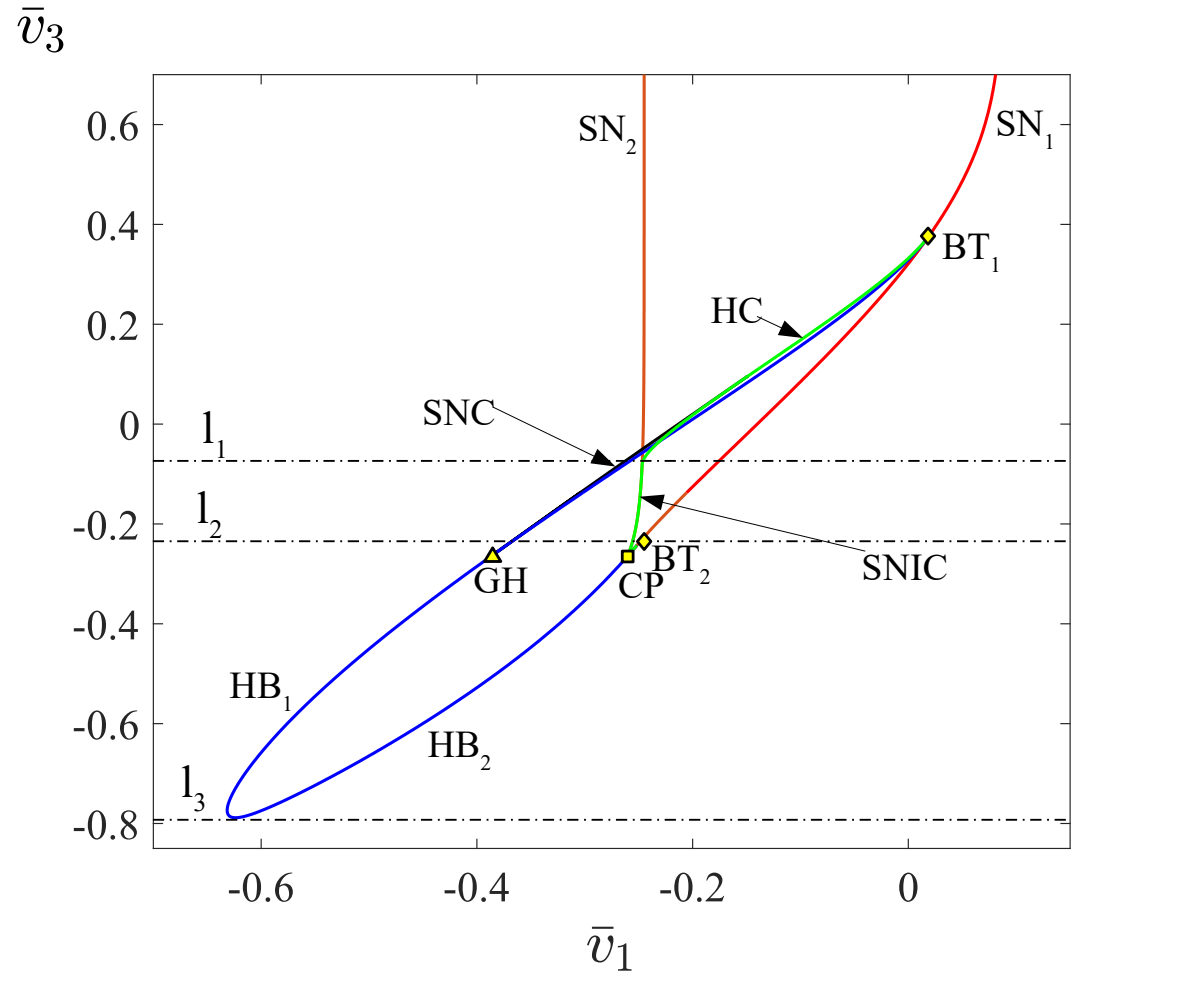


Figure 3: Codimension-2 bifurcation diagram in $(\bar{v}_1, \bar{v}_3)$-plane. The codimension-2 bifurcations are: Bogdanov-Takens-BT, Generalised Hopf-GH, Cusp point-CP

Spatiotemporal patterns: varying $\bar{v}_1$

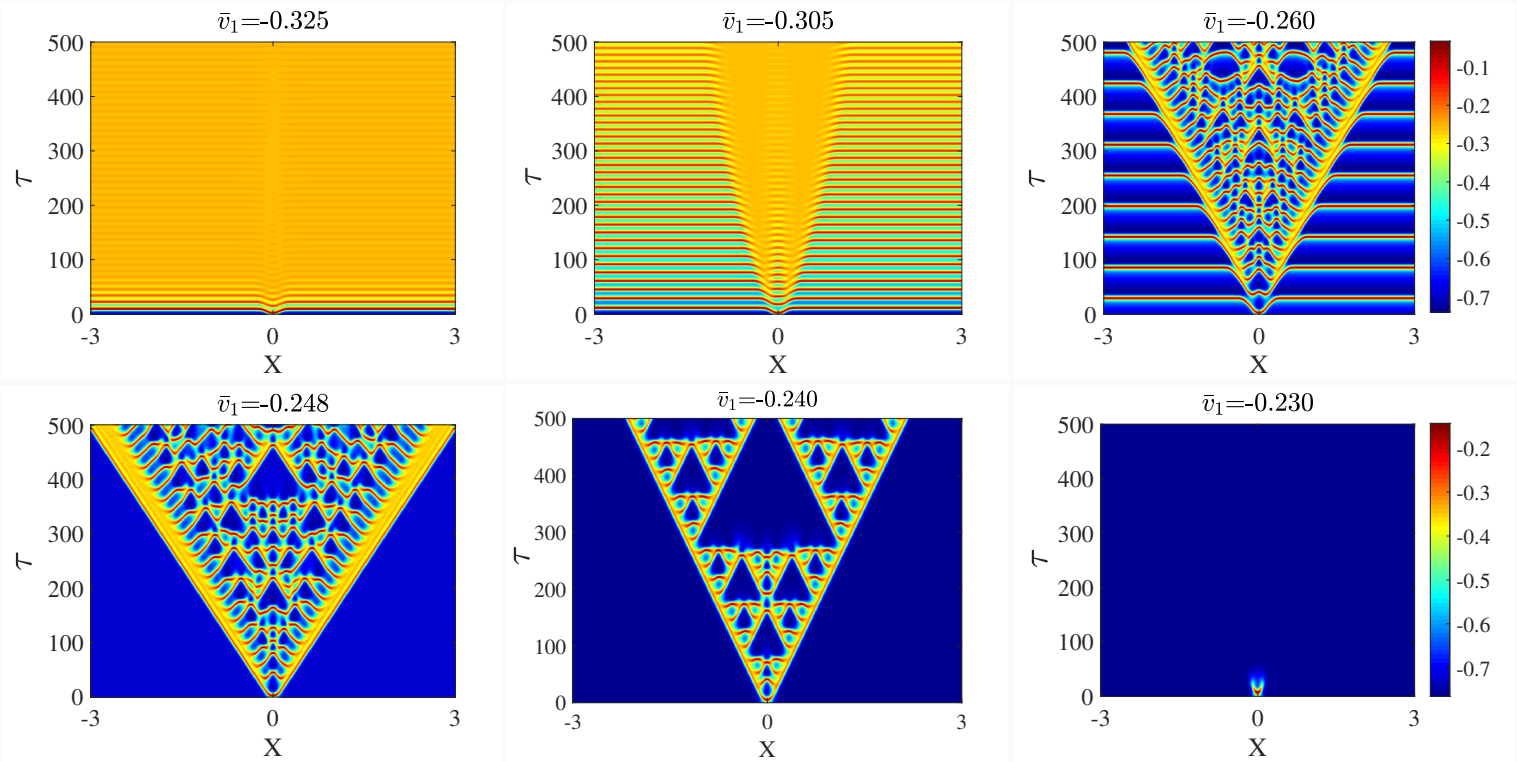


Figure 4: Space-time plot of the membrane potential V for selected values of parameter $\bar{v}_1$

Spatiotemporal patterns: varying ψ

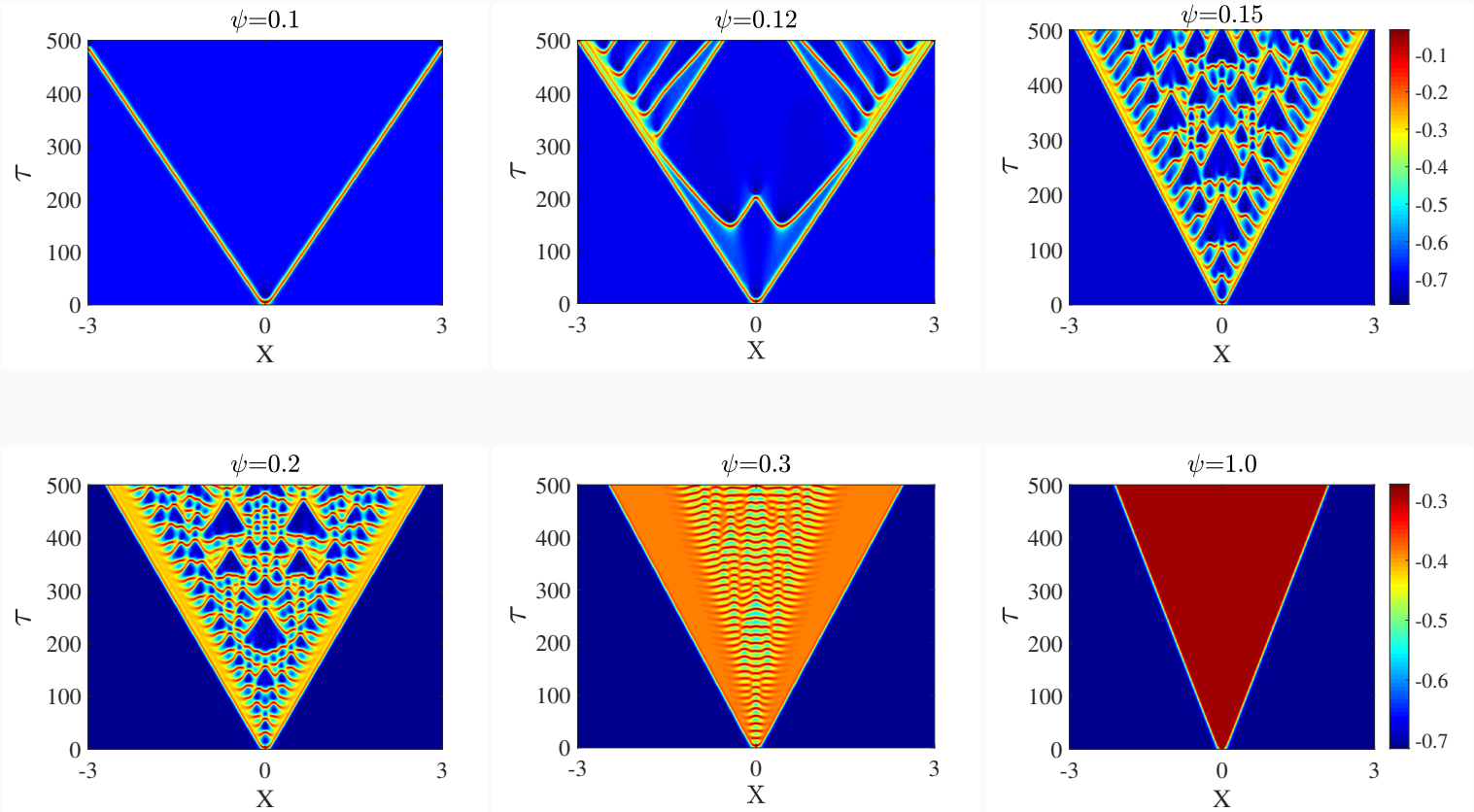


Figure 5: Space-time plot of the membrane potential *V* for selected values of parameter ψ

Conclusion

- It is found that the EMC is regulated by model parameters not external sources.
- The results indicate that in some parameter regimes the coupled cells exhibit spatiotemporal chaos.
- These results could be useful in improving the understanding of physiological responses and disorders in smooth muscle cells.

Ongoing work

- Spectral stability analysis of the travelling wave solutions observed in the model.

Questions?