Optimal Regulated Firm Behaviour in Solar Renewable Energy Certificate (SREC) Markets

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SREC markets

- Conceptually similar to cap-and-trade
- Regulator **sets floor** on SRECs for each power generating firm (proportional to the amount of electricity the firm sells).
- Firms obtain SRECs by generating electricity from **solar** (1 SREC $=$ 1 MWh)
- SRECs submitted to regulator annually - firms face a **monetary penalty** (Solar Alternative Compliance Payment) for each lacking certificate
- Certificates are **tradable** assets

What problem do we address?

- What is the **optimal firm behaviour** to minimize cost / maximize profit?
- Accounting for:
  - **Generation** costs
  - **Trading** costs
  - **Impact** of both on SREC prices

Previous related work

- **Equilibrium models** for carbon spot price & its properties:
  - Seifert, Uhrig-Homburg, and Wagner (2008)
  - Hitzemann and Uhrig-Homburg (2014)
  - Carmona, Fehr, and Hinz (2009)
  - Carmona et al (2010)
- **Structural models** for carbon spot price:
  - Howison and Schwarz (2012)
  - Carmona, Coulon, and Schwarz (2012)
- **Models** – SREC generation is log-linear in integrated "price"
  - Coulon, Khazaei, and Powell (2015)
- **Alternate design**: "bull-spread" rather than step penalty
  - Coulon, Khazaei, and Powell (2017)

Where our work differs

- Focus on **optimal firm behaviour** as opposed to properties of spot price of certificates
- Accounting for **trading and generation impact** on prices
- Modulating **trading speed**
- SREC vs carbon

Simplest possible setup

- Formulate optimal behaviour as a **stochastic control problem**
- Consider a single firm that is regulated for a single compliance period which ends at time T
- Need **controls**, **state variables**, and **performance criterion**
- Formulate in
  - continuous time – develop theory
  - discrete time – to implement numerically

Simplest possible setup cont'd

Some notation

- Control processes:
  - g_t - planned generation rate at t
  - Γ_t - trading rate at t
- System variables:
  - h_t - 'baseline' generation rate at t
  - R - SREC requirement
  - P - monetary penalty per unit of non-compliance for the period
- State processes:
  - b_t - number of banked SRECs at t
  - S_t - SREC spot price at t

Continuous Time Setup

Performance criterion given $g, \Gamma \in \mathcal{A}$ is

$$J^{g,\Gamma}(t,b,S) = \mathbb{E}\Big[-\underbrace{\tfrac{1}{2}\zeta \int_t^T ((g_u - h_u)_+)^2 du}_{\text{Generation Cost}} - \underbrace{\int_t^T \Gamma_u \, S_u^{g,\Gamma} du}_{\text{Trading Cost}} - \underbrace{\tfrac{1}{2}\gamma \int_t^T (\Gamma_u)^2 du}_{\text{Trading Speed Penalty}} - \underbrace{P\,(R - b_T^{g,\Gamma})_+}_{\text{Noncompliance Penalty}} \Big]$$

with state processes satisfying SDEs:

$$dS_t^{g,\Gamma} = (\mu + \eta\,\Gamma_t)dt - \psi\,(\underbrace{g_t dt + \nu dB_t^{(1)}}_{\text{Realized Generation}}) + \sigma\, dB_t^{(2)}$$

$$db_t^{g,\Gamma} = \underbrace{g_t dt + \nu\, dB_t^{(1)}}_{\text{Realized Generation}} + \Gamma_t dt$$

PDE Approach

Value function is

$$V(t,b,S) = \sup_{g,\Gamma \in \mathcal{A}} J^{g,\Gamma}(t,b,S)$$

Resulting **HJB equation** is

$$\partial_t V + \mathcal{L}^{S,b} V + \left(\tfrac{1}{2\zeta} \left(\partial_b V - \psi \partial_s V \right)^2 + h \left(\partial_b V - \psi \partial_s V \right) \right) \mathbb{I}_{\partial_b V \geq \psi \partial_s V} + \tfrac{1}{2\gamma} \left(\partial_b V + \eta \partial_S V - S \right)^2 = 0,$$

$$V(T,b,S) = -P(R-b)_+$$

and **Optimal controls** in feedback form:

$$g^* = (h + \tfrac{1}{\zeta} \left(\partial_b V - \psi \, \partial_S V \right)) \mathbb{I}_{\partial_b V \geq \psi \, \partial_S V}, \qquad \text{and}$$

$$\Gamma^* = \tfrac{1}{\gamma} \left(\partial_b V + \eta \, \partial_S V - S \right).$$

PDE Approach

$$g^* = \left(h + \tfrac{1}{\zeta}\left(\partial_b V - \psi\,\partial_S V\right)\right)\mathbb{I}_{\partial_b V \geq \psi\,\partial_S V}\,, \qquad \text{and}$$
$$\Gamma^* = \tfrac{1}{\gamma}\left(\partial_b V + \eta\,\partial_S V - S\right)\,.$$

- Generate **above baseline or not at all**, based on trade-off of generation
- **Purchasing generally negatively correlated** to S
- if $\text{sgn}(\partial_S V) > 0$, speed up purchasing, slow down generation
- if $\text{sgn}(\partial_b V) > 0$, speed up purchasing, speed up generation

Discrete Time Setup

Performance criterion given $g, \Gamma \in \mathcal{A}$ is

$$J^{g,\Gamma}(t,b,S) = \mathbb{E}\Big[\underbrace{\tfrac{1}{2}\zeta \sum_{i=1}^{n} ((\boldsymbol{g_{t_i}} - \boldsymbol{h_{t_i}})_+)^2 \Delta t}_{\text{Cost of Generation}} + \underbrace{\sum_{i=1}^{n} \boldsymbol{\Gamma_{t_i}} \boldsymbol{S_{t_i}^{g,\Gamma}} \Delta t}_{\text{Cost of Trading}}$$

$$+ \underbrace{\frac{\gamma}{2} \sum_{i=0}^{n} \boldsymbol{\Gamma_{t_i}}^2 \Delta t}_{\text{Trading Speed Penalty}} + \underbrace{\boldsymbol{P_T}(\boldsymbol{R_T} - \boldsymbol{b_T^{g,\Gamma}})_+}_{\text{Noncompliance Penalty}} \Big]$$

Goal is to find

$$(g^*, \Gamma^*) = \underset{g,\Gamma \in \mathcal{A}}{\text{argmin}}\, J^{g,\Gamma}(t,b,S)$$

Discrete Time Setup

State variable dynamics:

$$\tilde{S}_{t_i}^{g,\Gamma} = S_{t_{i-1}}^{g,\Gamma} + \left(\mu + \eta\,\Gamma_{t_{i-1}}\right) - \psi\,\underbrace{\left(g_{t_{i-1}}\Delta t + \psi\nu\sqrt{\Delta t}\epsilon_{t_i}\right)}_{\text{Realized Generation}} + \sigma\sqrt{\Delta t}\,Z_{t_i},$$

$$b_{t_i}^{g,\Gamma} = b_{t_{i-1}}^{g,\Gamma} + \Gamma_{t_{i-1}}\Delta t + \underbrace{g_{t_{i-1}}\Delta t + \nu\sqrt{\Delta t}\epsilon_{t_i}}_{\text{Realized Generation}}$$

$$Z_i \sim N(0,1)\ (iid)$$
$$\epsilon_i \sim N(0,1)\ (iid)$$

We truncate S so that $S_t \in [0, P]$, $\forall\ t \in [0, T]$ by

$$S_{t_i}^{g,\Gamma} = \min(\max(\tilde{S}_{t_i}^{g,\Gamma}, 0), P)$$

Discrete Time Setup

Use dynamic programming to solve for

$$V(t, b, S) = \inf_{g,\Gamma} J^{g,\Gamma}(t, b, S)$$

Applying Bellman Principle gives

$$V(t_i, b_{t_i}, S_{t_i}) = \inf_{g_{t_i},\Gamma_{t_i}} \Big\{ \left(\tfrac{1}{2}\zeta((g_{t_i} - h_{t_i})_+)^2 + \Gamma_{t_i} S_{t_i}^{g,\Gamma} + \tfrac{\gamma}{2}\Gamma_{t_i}^2 \right) \Delta t + \mathbb{E}[V(t_{i+1}, b_{t_{i+1}}^{g,\Gamma}, S_{t_{i+1}}^{g,\Gamma})] \Big\}$$

$$V(T, b_T, S_T) = P(R - b_T)_+$$

Discrete Time Setup

We solve this by

- Introducing a space grid $\mathcal{G} = \mathcal{S} \times \mathcal{B}$
- At each time t_i simulate 100 paths of S and b by reusing the same Z_i, ϵ_i for all grid points
- Estimate $\mathbb{E}[V(t_{i+1}, b^{g,\Gamma}_{t_{i+1}}, S^{g,\Gamma}_{t_{i+1}})]$ using Monte Carlo and interpolation of $V(t_{i+1}, \cdot, \cdot)$
- maximize RHS of Bellman Equation over g and Γ at each point in $\mathcal{G}$
- iterate backwards

Numerical implementation - parameter choice

n	T	P ($/ lacking SRECs)	R (SRECs)	$h(t)$ (SREC/y)
50	1	300	500	500

Table: Compliance parameters.

μ	σ	ν	ψ	η	ζ	γ
0	10	10	0	0	0.6	0.6

Table: Model Parameters.

These parameters chosen for illustrative purposes.

Single firm, single period - optimal behaviour

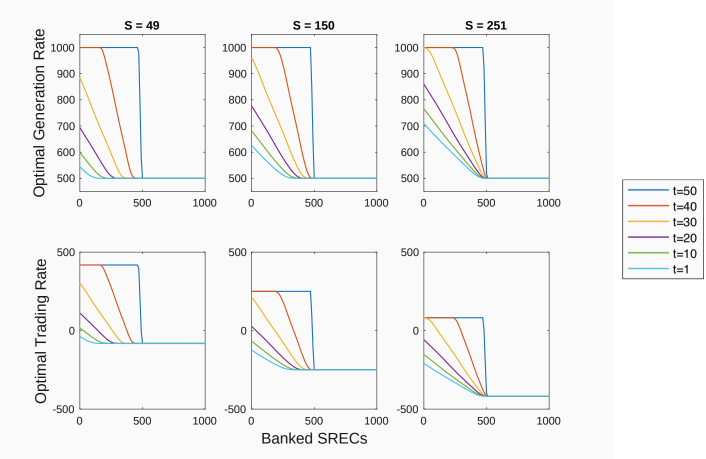

Figure: Optimal firm behaviour (top panel: generation rate, bottom panel: trading rate).

Intuition of optimal behaviour

Three regimes (heuristically)

- Marginal benefit of additional SREC is P
- Marginal benefit of additional SREC is between 0 and P
- Marginal benefit of additional SREC is 0

Sample path

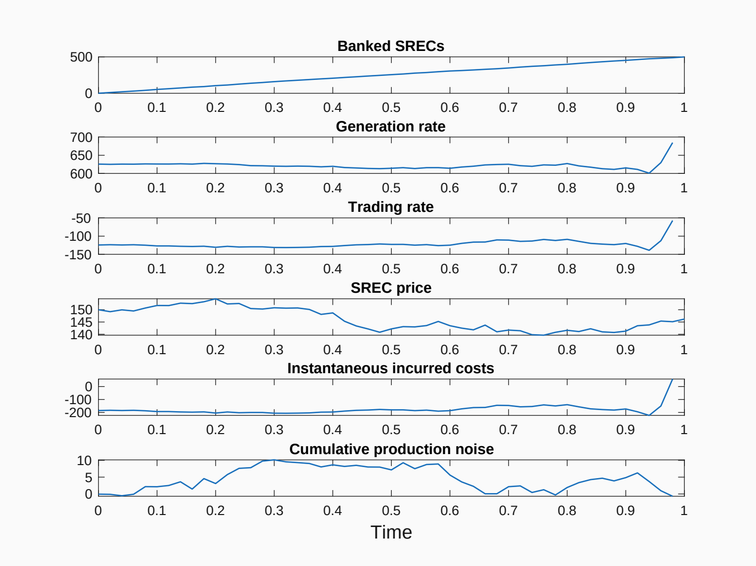

Figure: Sample path with $b_0 = 0$, $S_0 = 150$.

Summary Statistics

Statistic	Mean	SD	$P_{0.05}$	$P_{0.25}$	$P_{0.75}$	$P_{0.95}$
b_T	501.01	1.50	498.55	500.02	502.00	503.47
$\int_0^T g_u du$	625.55	6.64	614.53	621.12	630.08	636.99
$\int_0^T \Gamma_u du$	-124.56	6.79	-135.78	-129.44	-119.84	-113.58
Profit	9,200.00	1,050.00	7,440.00	8,500.00	9,900.00	10,970.00

Table: Summary statistics using 1,000 sample paths of S, b for a firm following the optimal strategy with initial condition $S_0 = 150$, $b_0 = 0$

Summary Statistics

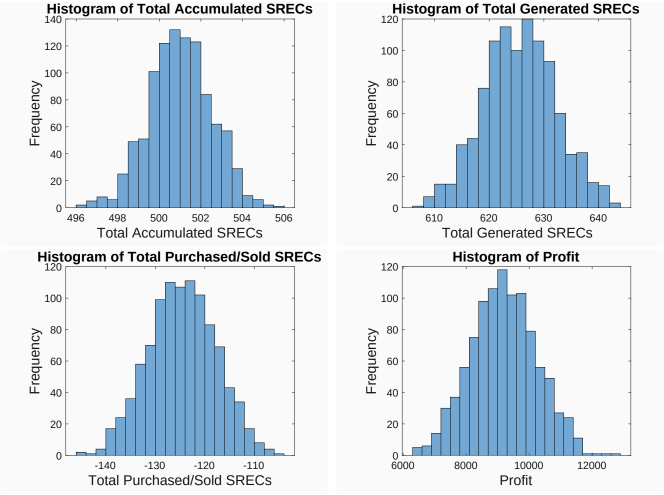

Figure: Histograms of statistics

Comparison to other strategies

Natural to compare optimal strategy to others

- No trade (NT): $g_t = 500, \Gamma_t = 0$ for all t yields 0 profit in best case scenario
- Naive Optimal Constant (NOC): $g_t = 625, \Gamma_t = -125$ for all t (for $S_0 = 150$)

Strategy	Mean Profit	SD of Profit	Q1 Profit	Q3 Profit
NT	-1,250	1,800	-2,210	0
Optimal	9,210	1,045	8,490	9,960
NOC	8,150	1,913	7,140	9,520

Table: Summary statistics of the three strategies: No-Trade, Optimal, and Naive Optimal Constant. Initial condition $S_0 = 150$, $b_0 = 0$.

Comparison of Strategies

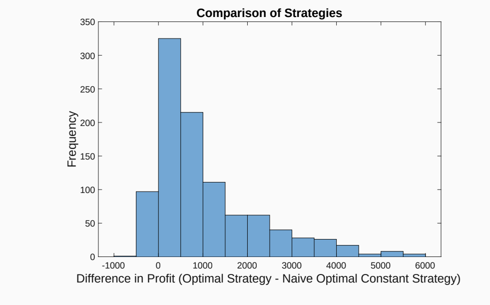

Figure: Comparison of the Optimal and Naive Optimal Constant Strategies.

Optimal behaviour (price impacts)

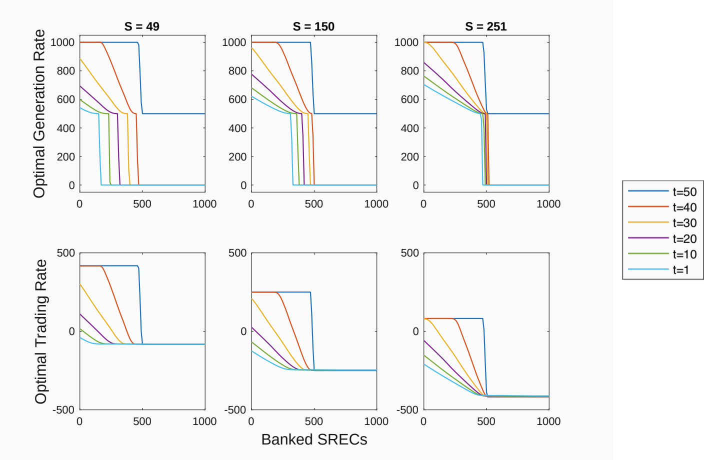

Figure: Optimal firm behaviour with price impact parameters $\eta = 0.01$, $\psi = 0.005$ (top panel: generation rate, bottom panel: trading rate)

Sample path

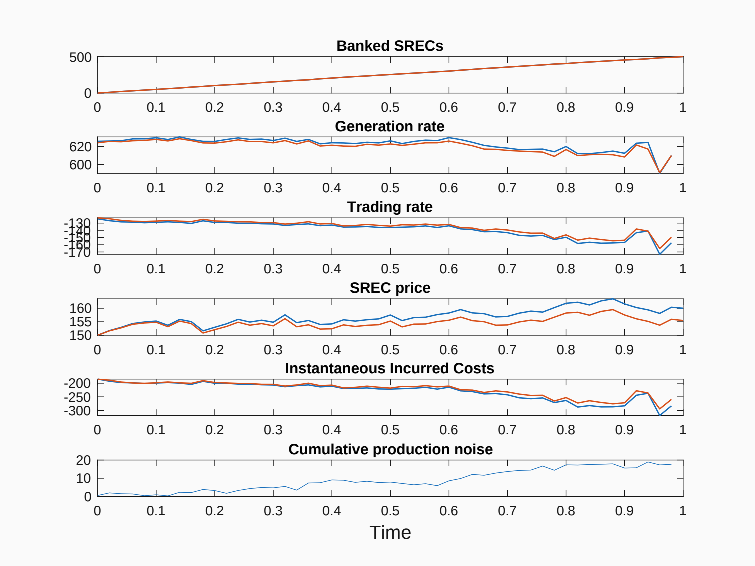

Figure: Sample path with $b_0 = 0$, $S_0 = 150$. Blue $\eta = \psi = 0$. Red $\eta = 0.01$, $\psi = 0.005$.

Multiple periods - 5 period example

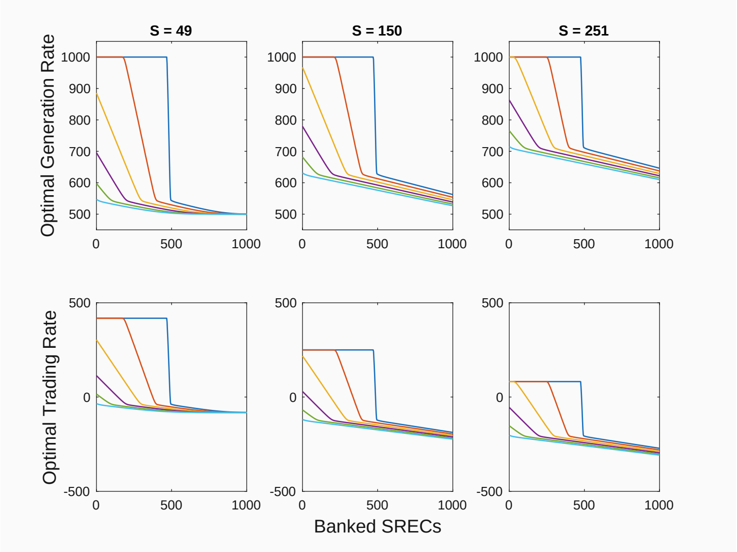

Figure: Optimal firm behaviour in the first period of a 5-period model

Multiple periods - 5 period example

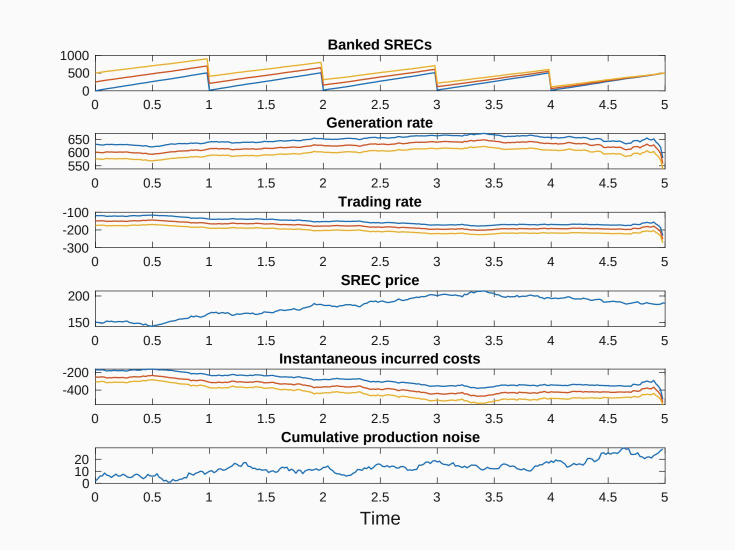

Figure: Paths of three optimally behaving firms in a 5-period compliance system with $S_0 = 150$, $b_0 = 0$ (blue), $b_0 = 250$ (red), $b_0 = 500$ (yellow)

Next steps

- Parameter estimation from data
- More realistic SREC price process
- Reinforcement learning (work in progress)
- Multiple firms (work in progress)

Thank you for your attention!