Symmetric functions in superspace

Luc Lapointe

Banff, January 24, 2019

Symmetric function theory

$$\mathbb{Q}[z_1, \dots, z_N]^{S_N}$$

Bases are indexed by partitions

$(6, 5, 2, 2, 1) \quad \longleftrightarrow$

Simple bases: $m_\lambda, p_\lambda, e_\lambda, h_\lambda$

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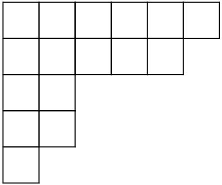

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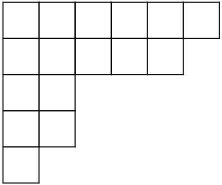

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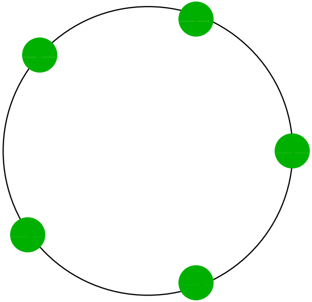
Calogero-Sutherland model

N identical particles on a circle with a pairwise interaction



$$H = \sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} + \beta(\beta-1) \sum_{i<j} \frac{1}{\sin^2(x_i - x_j)}$$

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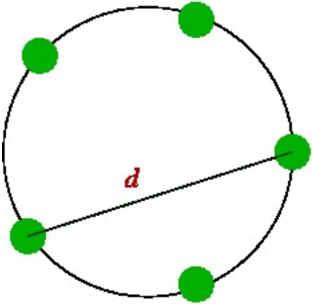

$$H = \sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} + \beta(\beta - 1) \sum_{i<j} \frac{1}{d_{ij}^2}$$

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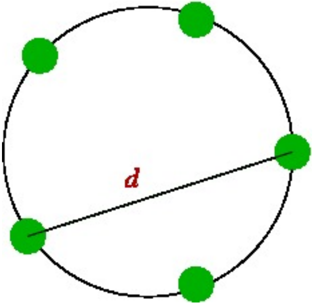

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Calogero-Sutherland model

$$H = \alpha \sum_{i=1}^{N} (z_i \partial_{z_i})^2 + \sum_{1 \le i < j \le N} \left(\frac{z_i + z_j}{z_i - z_j} \right) (z_i \partial_{z_i} - z_j \partial_{z_j})$$

Jack polynomials $J_\lambda^{(\alpha)}$:

$$H\, J_\lambda^{(\alpha)} = \varepsilon_\lambda J_\lambda^{(\alpha)} \qquad \text{and} \qquad J_\lambda^{(\alpha)} = m_\lambda + \text{smaller terms}$$

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Symmetric function theory

$$J_\lambda^{(\alpha)} \xrightarrow{\alpha\to 1} s_\lambda$$

Symmetric function theory

$$\begin{array}{c} M_\lambda^{(q,t)} \\ \Big\downarrow {\scriptstyle q=t^\alpha,\ t\to 1} \\ J_\lambda^{(\alpha)} \\ \Big\downarrow {\scriptstyle \alpha\to 1} \\ s_\lambda \end{array}$$

Symmetric function theory

$M_\lambda^{(q,t)}$ COMBINATORICS

$\Big\downarrow \; {\scriptstyle q=t^\alpha,\ t\to 1}$

$J_\lambda^{(\alpha)}$ PHYSICS

$\Big\downarrow \; {\scriptstyle \alpha\to 1}$

s_λ combinatorics

Symmetric function theory

- Macdonald polynomials: Macdonald positivity, diagonal coinvariants, Catalan combinatorics, Cherednik algebras, Elliptic Hall algebra, torus knots, etc...

- Jack Polynomials: Calogero-Sutherland model, Virasoro algebras, CFT, AGT conjecture, generalized Pauli principle, etc...

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Macdonald positivity

$$M^{(q,t)}_{(2,1)} = t\, s_{(3)} + (1 + qt)\, s_{(2,1)} + q\, s_{(1,1,1)}$$

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline \end{array} \qquad \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} \end{array} \quad \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} \end{array} \qquad \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline \end{array}$$

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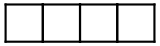
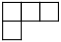
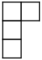
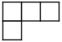
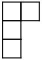
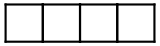
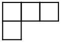
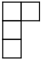
					
	1	$q+q^2+q^3$	q^2+q^4	$q^3+q^4+q^5$	q^6
	t	$1+qt+q^2t$	$q+q^2t$	$q+q^2+q^3t$	q^3
	t^2	$t+qt+qt^2$	$1+q^2t^2$	$q+qt+q^2t$	q^2
	t^3	$t+t^2+qt^3$	$t+qt^2$	$1+qt+qt^2$	q
	t^6	$t^3+t^4+t^5$	t^2+t^4	$t+t^2+t^3$	1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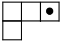	t	$1+qt+q^2t$	$q+q^2t$	$q+q^2+q^3t$	q^3
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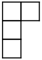	t^3	$t+t^2+qt^3$	$t+qt^2$	$1+qt+qt^2$	q
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	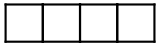	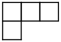		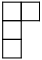	
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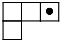	t	$1+qt+q^2t$	$q+q^2t$	$q+q^2+q^3t$	q^3
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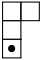	t^3	$t+t^2+qt^3$	$t+qt^2$	$1+qt+qt^2$	q
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L. Butler and L. Butler!!

Symmetric function theory in SUPERSPACE!!!!!

Supersymmetry

2 types of particles in nature

bosons (integer spin: $0, 1, 2, \ldots$)

fermions (half integer spin: $1/2, 3/2, \ldots$)

$\Psi \longrightarrow \Psi$
exchange of two bosons

$\Psi \longrightarrow -\Psi$
exchange of two fermions
(*Pauli's exclusion principle*)

Unification of bosons and fermions in a graded algebra

$$\mathcal{H} = \mathcal{H}_B \oplus \mathcal{H}_F$$

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Donald J. Trump
@realDonaldTrump
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Lapointe is right. Trillions in taxpayers' money wasted on looking for FAKE supersymmetry at CERN. SAD!
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A symmetric function theory in superspace

$$\mathbb{Q}[z_1,\dots,z_N,\theta_1,\dots,\theta_N]^{S_N} \qquad \text{with} \qquad \theta_i\theta_j=-\theta_j\theta_i \quad \text{and} \quad \theta_i^2=0$$

$N=2$: $\qquad (z_1-z_2)\,\theta_1\theta_2=(z_2-z_1)\,\theta_2\theta_1$

Power sums: $p_1,p_2,\dots,\tilde p_0,\tilde p_1,\dots$

$$p_r=z_1^r+z_2^r+\cdots \qquad \text{and} \qquad \tilde p_k=\theta_1 z_1^k+\theta_2 z_2^k+\cdots$$

$$\tilde p_i\tilde p_j=-\tilde p_j\tilde p_i \qquad \text{and} \qquad \tilde p_i^2=0$$

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Superpartitions

$$\Lambda = (\Lambda^a; \Lambda^s) \qquad \begin{cases} \Lambda^s \text{ is a usual partition} \\ \Lambda^a \text{ has no repeated parts} \end{cases}$$

$$(4,2,0;3,2,1,1) \longleftrightarrow (4,3,2,2,1,1,0) \longleftrightarrow$$

$$p_{4,2,0;3,2,1,1} = \tilde{p}_4\, \tilde{p}_2\, \tilde{p}_0\, p_3\, p_2\, p_1\, p_1$$

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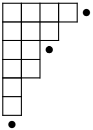

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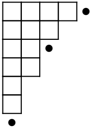
Superpartitions

$$\Lambda = ({\color{red}\Lambda^a}; {\color{blue}\Lambda^s})$$

$$\begin{cases} {\color{blue}\Lambda^s} \text{ is a usual partition} \\ {\color{red}\Lambda^a} \text{ has no repeated parts} \end{cases}$$

$$({\color{red}4}, {\color{red}2}, {\color{red}0}; {\color{blue}3}, {\color{blue}2}, {\color{blue}1}, {\color{blue}1}) \longleftrightarrow ({\color{red}4}, {\color{blue}3}, {\color{red}2}, {\color{blue}2}, {\color{blue}1}, {\color{blue}1}, {\color{red}0}) \longleftrightarrow$$



$$p_{{\color{red}4,2,0};{\color{blue}3,2,1,1}} = {\color{red}\tilde{p}_4\, \tilde{p}_2\, \tilde{p}_0}\, {\color{blue}p_3\, p_2\, p_1\, p_1}$$

Supersymmetric Calogero-Sutherland model

$$H_{susy} = H - 2 \sum_{1\leq i<j\leq N} \frac{z_i z_j}{(z_i - z_j)^2} (\theta_i - \theta_j)(\partial_{\theta_i} - \partial_{\theta_j})$$

Extra operator:

$$I_{susy} = \alpha \sum_{i=1}^{N} z_i \theta_i \partial_{z_i} \partial_{\theta_i} + \sum_{1\leq i<j\leq N} \frac{z_i\theta_j + z_j\theta_i}{z_i - z_j} (\partial_{\theta_i} - \partial_{\theta_j})$$

Common eigenfunctions are Jack polynomials in superspace

$$J_\Lambda^{(\alpha)}(z,\theta)$$

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Symmetric function theory in superspace

$$J_\Lambda^{(\alpha)}$$

$$\Big\downarrow \alpha \to 1$$

$$s_\Lambda^1$$

Symmetric function theory in superspace

$$\begin{array}{c} J_\Lambda^{(\alpha)} \\ \Big\downarrow \scriptstyle{\alpha\to 1} \\ s_\Lambda^1 \end{array}$$

PHYSICS

NO combinatorics

Symmetric function theory in superspace

- Jack polynomials in superspace: super Virasoro, super CFT, AGT, generalized Pauli principle

Symmetric function theory in superspace

$$J_\Lambda^{(\alpha)} \quad \text{PHYSICS}$$

$$\Big\downarrow \alpha \to 1$$

$$s_\Lambda^1 \quad \text{NO combinatorics}$$

Symmetric function theory in superspace

$$M_\Lambda^{(q,t)}$$

$$\Big\downarrow \; {\scriptstyle q=t^\alpha,\ t\to 1}$$

$$J_\Lambda^{(\alpha)}$$

PHYSICS

$$\Big\downarrow \; {\scriptstyle \alpha\to 1}$$

$$s_\Lambda^1$$

NO combinatorics

Symmetric function theory in superspace

$$M_\Lambda^{(q,t)} \qquad \text{COMBINATORICS}$$

$$\Big\downarrow \; {\scriptstyle q=t^\alpha,\ t\to 1}$$

$$J_\Lambda^{(\alpha)} \qquad \text{PHYSICS}$$

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$$s_\Lambda^1 \qquad \text{NO combinatorics}$$

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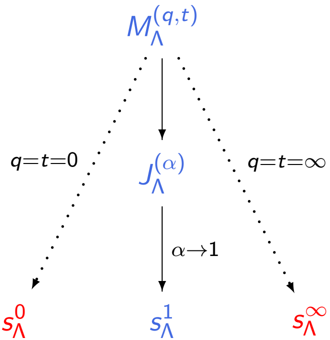

Macdonald positivity conjecture in superspace

$$M_\Lambda^{(q,t)} = \sum_\Omega K_{\Omega\Lambda}(q,t)\, s_\Omega^0 \qquad \text{with} \qquad K_{\Omega\Lambda}(q,t) \in \mathbb{N}[q,t]???$$

$K_{\Omega\Lambda}(1,1)$ = dimension of ????

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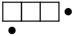
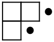
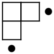
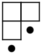
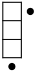
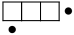
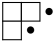
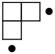
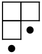
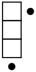
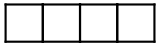
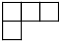
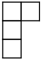
$$M_\Lambda^{(q,t)} = \sum_\Omega K_{\Omega\Lambda}(q,t)\, s_\Omega^0 \qquad \text{with} \qquad K_{\Omega\Lambda}(q,t) \in \mathbb{N}[q,t]???$$

$K_{\Omega\Lambda}(1,1) =$ dimension of ????

					
	1	q	$q+q^2$	q^2	q^3
	qt	1	$q+q^2t$	q^3t	q^2
	t	t	$1+qt$	q	q
	t^2	qt^3	$t+qt^2$	1	qt
	t^3	t^2	$t+t^2$	t	1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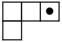	t	$1+qt+q^2t$	$q+q^2t$	$q+q^2+q^3t$	q^3
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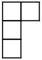
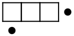
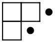
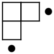
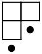
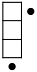
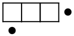
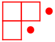
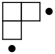
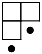
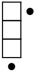
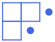
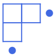
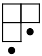
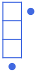
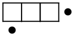
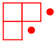
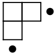
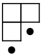
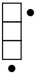
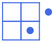
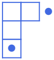
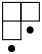
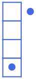
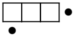
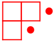
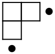
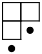
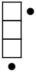
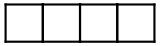
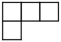
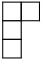
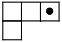
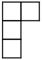
	t^3	$t+t^2+qt^3$	$t+qt^2$	$1+qt+qt^2$	q
	t^6	$t^3+t^4+t^5$	t^2+t^4	$t+t^2+t^3$	1

					
	1	q	$q+q^2$	q^2	q^3
	qt	1	$q+q^2t$	q^3t	q^2
	t	t	$1+qt$	q	q
	t^2	qt^3	$t+qt^2$	1	qt
	t^3	t^2	$t+t^2$	t	1

					
	1	q	$q+q^2$	q^2	q^3
	qt	1	$q+q^2t$	q^3t	q^2
	t	t	$1+qt$	q	q
	t^2	qt^3	$t+qt^2$	1	qt
	t^3	t^2	$t+t^2$	t	1

					
	1	q	$q+q^2$	q^2	q^3
	qt	1	$q+q^2t$	q^3t	q^2
	t	t	$1+qt$	q	q
	t^2	qt^3	$t+qt^2$	1	qt
	t^3	t^2	$t+t^2$	t	1

					
	1	$q+q^2+q^3$	q^2+q^4	$q^3+q^4+q^5$	q^6
	t	$1+qt+q^2t$	$q+q^2t$	$q+q^2+q^3t$	q^3
	t^2	$t+qt+qt^2$	$1+q^2t^2$	$q+qt+q^2t$	q^2
	t^3	$t+t^2+qt^3$	$t+qt^2$	$1+qt+qt^2$	q
	t^6	$t^3+t^4+t^5$	t^2+t^4	$t+t^2+t^3$	1

Macdonald positivity conjecture in superspace

$$M^{(q,t)}_{\substack{\square\,\bullet\\ \square}} = t\, s_{\square\square\,\bullet} + s_{\substack{\square\,\bullet\\ \square}} + qt\, s_{\substack{\square\square\\ \bullet}} + q\, s_{\substack{\square\\ \square\\ \bullet}}$$

$$M^{(q,t)}_{\substack{\square\square\\ \square}} = t\, s_{\square\square\square} + (1+qt)s_{\substack{\square\square\\ \square}} + q\, s_{\substack{\square\\ \square\\ \square}}$$

Refinement of the original problem!!

Macdonald positivity conjecture in superspace

$$M^{(q,t)}_{(1;1)} = t\, s_{(2;)} + s_{(1;1)} + qt\, s_{(0;2)} + q\, s_{(0;1,1)}$$

$$M^{(q,t)}_{(2,1)} = t\, s_{(3)} + (1+qt) s_{(2,1)} + q\, s_{(1,1,1)}$$

Refinement of the original problem!!

Macdonald positivity conjecture in superspace

$$M^{(q,t)}_{\ytableaushort{{}{\bullet},{}}} = t\, s_{\ytableaushort{{}{}{\bullet}}} + 1 \cdot s_{\ytableaushort{{}{\bullet},{}}} + qt\, s_{\ytableaushort{{}{},{\bullet}}} + q\, s_{\ytableaushort{{},{},\none[\bullet]}}$$

$$M^{(q,t)}_{\ydiagram{2,1}} = t\, s_{\ydiagram{3}} + (1 + qt) s_{\ydiagram{2,1}} + q\, s_{\ydiagram{1,1,1}}$$

Refinement of the original problem!!

Macdonald positivity conjecture in superspace

$$M^{(q,t)}_{\begin{smallmatrix}\square\square\\ \square\bullet\end{smallmatrix}} = t^2\, s_{\begin{smallmatrix}\square\square\square\bullet\end{smallmatrix}} + qt\, s_{\begin{smallmatrix}\square\square\square\\ \bullet\end{smallmatrix}} + {\color{red}(t+qt^2)}\, s_{\begin{smallmatrix}\square\square\bullet\\ \square\end{smallmatrix}}$$

$$+ {\color{red}(1+q^2t^2)}\, s_{\begin{smallmatrix}\square\square\\ \square\bullet\end{smallmatrix}} + {\color{red}(q+q^2t)}\, s_{\begin{smallmatrix}\square\square\\ \square\\ \bullet\end{smallmatrix}} + qt\, s_{\begin{smallmatrix}\square\bullet\\ \square\\ \square\end{smallmatrix}} + q^2\, s_{\begin{smallmatrix}\square\\ \square\\ \square\\ \bullet\end{smallmatrix}}$$

Not refined enough...

More supersymmetries????

Macdonald positivity conjecture in superspace

$$M^{(q,t)}_{\substack{\square\square\\ \square\bullet}} = t^2\, s_{\square\square\square\bullet} + qt\, s_{\substack{\square\square\square\\ \bullet}} + (t+qt^2)\, s_{\substack{\square\square\bullet\\ \square}} + (1+q^2t^2)\, s_{\substack{\square\square\\ \square\bullet}} + (q+q^2t)\, s_{\substack{\square\square\\ \square\\ \bullet}} + qt\, s_{\substack{\square\bullet\\ \square\\ \square}} + q^2\, s_{\substack{\square\\ \square\\ \square\\ \bullet}}$$

Not refined enough...

More supersymmetries????

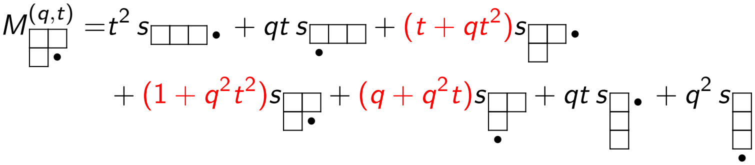
Macdonald positivity conjecture in superspace

$$M^{(q,t)}_{(1;2)} = t^2\, s_{(3;)} + qt\, s_{(0;3)} + (t+qt^2)\, s_{(2;1)} + (1+q^2t^2)\, s_{(1;2)} + (q+q^2t)\, s_{(0;2,1)} + qt\, s_{(1;1,1)} + q^2\, s_{(0;1,1,1)}$$

Not refined enough...

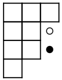
More supersymmetries????

Macdonald positivity conjecture in superspace



Not refined enough...

More supersymmetries????



Symmetric function theory in superspace

- What is known: eigenoperators, norm, evaluation, duality, Pieri rules (Jack, Schur)
- What is conjectured: symmetry, Pieri rules (Macdonald)
- What is unknown: A lot!!

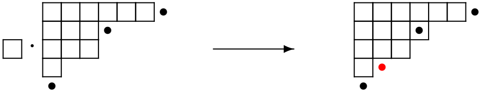
Pieri rules for Jack polynomials in superspace

Expansion coefficients of $e_1 J_\Lambda^{(\alpha)}$ are quotients of linear factors in α

$$\frac{3\alpha(5\alpha+2)}{(3\alpha+2)^2(5\alpha+3)}$$

Pieri rules for Jack polynomials in superspace

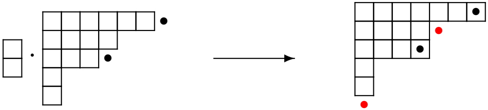
Expansion coefficients of $e_1 J_\Lambda^{(\alpha)}$ are quotients of linear factors in α



$$\frac{3\alpha(5\alpha+2)}{(3\alpha+2)^2(5\alpha+3)}$$

Pieri rules for Jack polynomials in superspace

Expansion coefficients of $e_2 J_\Lambda^{(\alpha)}$ are quotients of linear factors in α.
Sometimes there is a quadratic factor

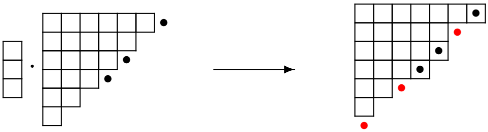


$$-\frac{2\alpha^3(3\alpha^2+\alpha-1)}{(6\alpha+5)(7\alpha+5)(\alpha+1)(\alpha+2)(3\alpha+1)(2\alpha+1)}$$

Sum of 2 terms

Pieri rules for Jack polynomials in superspace

Expansion coefficients of $e_3 J_\Lambda^{(\alpha)}$ are quotients of linear factors in α.
Sometimes there is a degree 6 factor!!!!!!!!

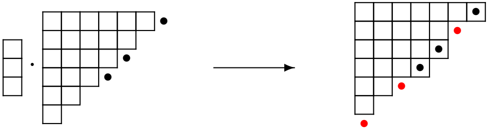


$$\frac{1}{1152}\,\frac{\alpha^4(2\alpha+3)(3\alpha+4)(416\alpha^6+2000\alpha^5+3484\alpha^4+2608\alpha^3+559\alpha^2-256\alpha-108)}{(4\alpha+3)(5\alpha+4)(7\alpha+6)(2\alpha+1)(\alpha+1)^{10}}$$

Sum of 6 terms???????

Pieri rules for Jack polynomials in superspace

Expansion coefficients of $e_3 J_\Lambda^{(\alpha)}$ are quotients of linear factors in α.
Sometimes there is a degree 6 factor!!!!!!!!



$$\frac{1}{1152}\,\frac{\alpha^4(2\alpha+3)(3\alpha+4)(416\alpha^6+2000\alpha^5+3484\alpha^4+2608\alpha^3+559\alpha^2-256\alpha-108)}{(4\alpha+3)(5\alpha+4)(7\alpha+6)(2\alpha+1)(\alpha+1)^{10}}$$

Sum of 7 terms!!!!!!!!!!!!!

Why 7???

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Alternating Sign Matrices

$$1, 2, 7, 42, 429, \ldots$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

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Square Ice

Square Ice

$$
\begin{array}{ccccccccccccccccc}
H & - & O &   & H & - & O & - & H &   & O & - & H &   & O & - & H \\
  &   & | &   &   &   &   &   &   &   & | &   &   &   & | &   &   \\
  &   & H &   &   &   & H &   &   &   & H &   &   &   & H &   &   \\
  &   &   &   &   &   & | &   &   &   &   &   &   &   &   &   &   \\
H & - & O & - & H &   & O &   & H & - & O & - & H &   & O & - & H \\
  &   &   &   &   &   & | &   &   &   &   &   &   &   & | &   &   \\
  &   & H &   &   &   & H &   &   &   & H &   &   &   & H &   &   \\
  &   & | &   &   &   &   &   &   &   & | &   &   &   &   &   &   \\
H & - & O &   & H & - & O &   & H & - & O &   & H & - & O & - & H \\
  &   &   &   &   &   & | &   &   &   &   &   &   &   &   &   &   \\
  &   & H &   &   &   & H &   &   &   & H &   &   &   & H &   &   \\
  &   & | &   &   &   &   &   &   &   & | &   &   &   & | &   &   \\
H & - & O &   & H & - & O & - & H &   & O & - & H &   & O & - & H
\end{array}
$$

Square Ice

```
H − O   H − O − H   O − H   O − H
    |               |       |
    H       H       H       H
            |
H − O − H   O   H − O − H   O − H
            |               |
    H       H       H       H
    |               |
H − O   H − O   H − O   H − O − H
            |
    H       H       H       H
    |               |       |
H − O   H − O − H   O − H   O − H
```

$$\mathrm{H} - \mathrm{O} - \mathrm{H} \qquad\qquad z$$

$$\begin{array}{c}\mathrm{H}\\ |\\ \mathrm{O}\\ |\\ \mathrm{H}\end{array} \qquad\qquad z^{-1}$$

$$\begin{array}{ccc}\mathrm{H} & - & \mathrm{O}\\ & & |\\ & & \mathrm{H}\end{array} \quad \text{or} \quad \begin{array}{ccc} & & \mathrm{H}\\ & & |\\ \mathrm{H} & - & \mathrm{O}\end{array} \qquad\qquad \frac{az-(az)^{-1}}{a-a^{-1}} = [az]$$

$$\begin{array}{ccc}\mathrm{O} & - & \mathrm{H}\\ | & & \\ \mathrm{H} & & \end{array} \quad \text{or} \quad \begin{array}{ccc}\mathrm{H} & & \\ | & & \\ \mathrm{O} & - & \mathrm{H}\end{array} \qquad\qquad \frac{z-z^{-1}}{a-a^{-1}} = [z]$$

Partition function of the square ice model

$$z \to z_{ij} = x_i/y_j$$

There is a closed form formula for the partition function:

$$Z_n(x,y;a) = \frac{\prod_i x_i/y_i \prod_{i,j}[x_i/y_j]\,[ax_i/y_j]}{\prod_{i,j}[x_i/x_j]\,[y_i/y_j]} \det\left(\frac{1}{[x_i/y_j]\,[ax_i/y_j]}\right)_{i,j}$$

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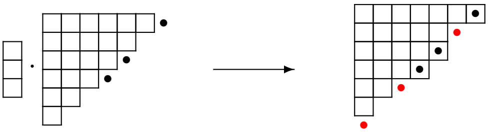
Pieri rules for Jack polynomials in superspace

Expansion coefficients of $e_n J_\Lambda^{(\alpha)}$ are quotients of linear factors in α times the partition function of the Square Ice model



$$\frac{1}{1152}\,\frac{\alpha^4(2\alpha+3)(3\alpha+4)(416\alpha^6+2000\alpha^5+3484\alpha^4+2608\alpha^3+559\alpha^2-256\alpha-108)}{(4\alpha+3)(5\alpha+4)(7\alpha+6)(2\alpha+1)(\alpha+1)^{10}}$$

How to go beyond??

Can the Macdonald positivity conjectures be further refined?

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Extra supersymmetries

$$\mathbb{Q}[z_1,\dots,z_N,\theta_1,\dots,\theta_N,\phi_1,\dots,\phi_N]^{S_N}$$

with

$$\theta_i\theta_j=-\theta_j\theta_i,\qquad \phi_i\phi_j=-\phi_j\phi_i \qquad \theta_i\phi_j=-\phi_j\theta_i$$

Power sums: $p_1,p_2,\dots,\tilde p_0,\tilde p_1,\dots,\bar p_0,\bar p_1,\dots,\hat p_0,\hat p_1,\dots$

$$p_r=z_1^r+z_2^r+\cdots \qquad \tilde p_k=\theta_1 z_1^k+\theta_2 z_2^k+\cdots$$

$$\bar p_r=\phi_1 z_1^r+\phi_2 z_2^r+\cdots \qquad \hat p_k=\theta_1\phi_1 z_1^k+\theta_2\phi_2 z_2^k+\cdots$$

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$J_\Lambda^{(\alpha)}$ are eigenfuntions of a $\mathcal{N} = 2$ supersymmetric model

Combinatorics seems to be much more mysterious...

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One circle of each type

$$M_\Lambda^{(q,t)} \longleftrightarrow \mathcal{S}^t_{\{m+1,m+2,\dots\}}\, E_\eta^{(q,t)}$$

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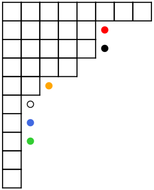

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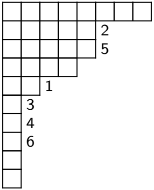
$$M_\Lambda^{(q,t)} \longleftrightarrow \mathcal{S}^t_{\{m+1,m+2,\dots\}} E_\eta^{(q,t)}$$



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- After some serious tweaking we get Macdonald positivity conjectures!!
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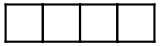
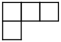
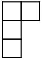
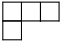
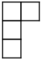
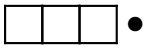
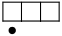
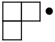
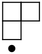
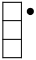
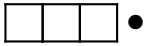
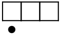
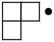
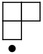
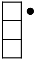
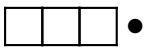
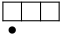
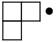
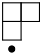
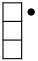
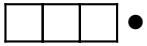
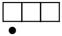
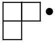
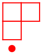
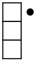
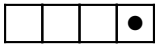
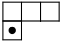
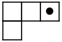
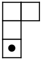
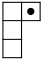
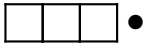
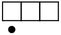
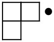
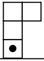
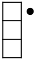
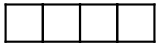
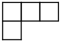
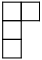
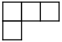
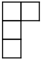
					
	1	$q+q^2+q^3$	q^2+q^4	$q^3+q^4+q^5$	q^6
	t	$1+qt+q^2t$	$q+q^2t$	$q+q^2+q^3t$	q^3
	t^2	$t+qt+qt^2$	$1+q^2t^2$	$q+qt+q^2t$	q^2
	t^3	$t+t^2+qt^3$	$t+qt^2$	$1+qt+qt^2$	q
	t^6	$t^3+t^4+t^5$	t^2+t^4	$t+t^2+t^3$	1

							
	1	q^3	$q+q^2$	q^2+q^4	q^4+q^5	q^3	q^6
	t	1	$qt+q^2t$	$q+q^2t$	$q+q^2$	q^3t	q^3
	t	q^2t	$1+qt$	$q+q^2t$	q^2+q^3t	q	q^3
	t^2	qt	$t+qt^2$	$1+q^2t^2$	$q+q^2t$	qt	q^2
	t^3	t	t^2+qt^3	$t+qt^2$	$1+qt$	qt^2	q
	t^3	qt^3	$t+t^2$	$t+qt^2$	$qt+qt^2$	1	q
	t^6	t^3	t^4+t^5	t^2+t^4	$t+t^2$	t^3	1

							
	1	q^3	$q+q^2$	q^2+q^4	q^4+q^5	q^3	q^6
	t	1	$qt+q^2t$	$q+q^2t$	$q+q^2$	q^3t	q^3
	t	q^2t	$1+qt$	$q+q^2t$	q^2+q^3t	q	q^3
	t^2	qt	$t+qt^2$	$1+q^2t^2$	$q+q^2t$	qt	q^2
	t^3	t	t^2+qt^3	$t+qt^2$	$1+qt$	qt^2	q
	t^3	qt^3	$t+t^2$	$t+qt^2$	$qt+qt^2$	1	q
	t^6	t^3	t^4+t^5	t^2+t^4	$t+t^2$	t^3	1

							
	1	q^3	$q+q^2$	q^2+q^4	q^4+q^5	q^3	q^6
	t	1	$qt+q^2t$	$q+q^2t$	$q+q^2$	q^3t	q^3
	t	q^2t	$1+qt$	$q+q^2t$	q^2+q^3t	q	q^3
	t^2	qt	$t+qt^2$	$1+q^2t^2$	$q+q^2t$	qt	q^2
	t^3	t	t^2+qt^3	$t+qt^2$	$1+qt$	qt^2	q
	t^3	qt^3	$t+t^2$	$t+qt^2$	$qt+qt^2$	1	q
	t^6	t^3	t^4+t^5	t^2+t^4	$t+t^2$	t^3	1

					
	1	$q+q^2+q^3$	q^2+q^4	$q^3+q^4+q^5$	q^6
	t	$1+qt+q^2t$	$q+q^2t$	$q+q^2+q^3t$	q^3
	t^2	$t+qt+qt^2$	$1+q^2t^2$	$q+qt+q^2t$	q^2
	t^3	$t+t^2+qt^3$	$t+qt^2$	$1+qt+qt^2$	q
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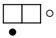
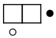
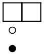
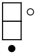
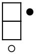
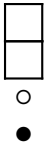
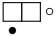
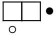
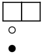
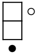
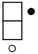
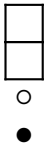
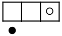
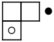
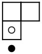
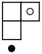
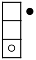
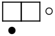
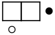
							
	1	q^2t	q	q^2	q	q^3t	q^3
	q^2t	1	q	q^2	q^3t	q	q^3
	qt	qt^2	1	q^2t	q	qt	q^2
	t	t^2	qt^2	1	qt	qt^2	q
	t	qt^3	t	qt	1	qt^2	q
	t^3q	t	t	qt	qt^2	1	q
	t^3	t^4	t^4	t	t^2	t^3	1

							
	1	q^2t	q	q^2	q	q^3t	q^3
	q^2t	1	q	q^2	q^3t	q	q^3
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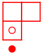
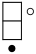
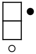
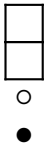
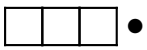
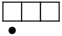
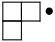
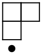
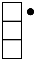
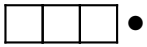
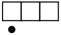
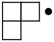
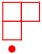
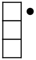
	t	t^2	qt^2	1	qt	qt^2	q
	t	qt^3	t	qt	1	qt^2	q
	t^3q	t	t	qt	qt^2	1	q
	t^3	t^4	t^4	t	t^2	t^3	1

							
	1	q^3	$q+q^2$	q^2+q^4	q^4+q^5	q^3	q^6
	t	1	$qt+q^2t$	$q+q^2t$	$q+q^2$	q^3t	q^3
	t	q^2t	$1+qt$	$q+q^2t$	q^2+q^3t	q	q^3
	t^2	qt	$t+qt^2$	$1+q^2t^2$	$q+q^2t$	qt	q^2
	t^3	t	t^2+qt^3	$t+qt^2$	$1+qt$	qt^2	q
	t^3	qt^3	$t+t^2$	$t+qt^2$	$qt+qt^2$	1	q
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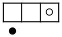
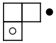
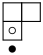
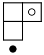
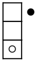
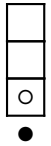
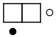
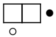
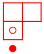
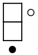
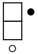
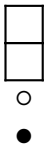
							
	1	q^2t	q	q^2	q	q^3t	q^3
	q^2t	1	q	q^2	q^3t	q	q^3
	qt	qt^2	1	q^2t	q	qt	q^2
	t	t^2	qt^2	1	qt	qt^2	q
	t	qt^3	t	qt	1	qt^2	q
	t^3q	t	t	qt	qt^2	1	q
	t^3	t^4	t^4	t	t^2	t^3	1

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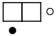
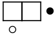
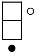
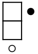
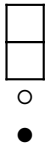
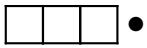
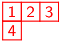
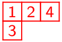
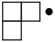
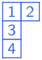
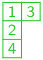
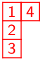
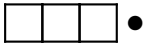
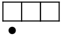
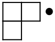
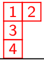
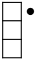
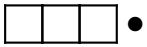
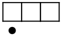
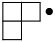
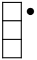
	1	q^2t	q	q^2	q	q^3t	q^3
	q^2t	1	q	q^2	q^3t	q	q^3
	qt	qt^2	1	q^2t	q	qt	q^2
1 2 3 4	t	t^2	qt^2	1	qt	qt^2	q
	t	qt^3	t	qt	1	qt^2	q
	t^3q	t	t	qt	qt^2	1	q
	t^3	t^4	t^4	t	t^2	t^3	1

			 	 	 		
	1	q^3	$q+q^2$	q^2+q^4	q^4+q^5	q^3	q^6
	t	1	$qt+q^2t$	$q+q^2t$	$q+q^2$	q^3t	q^3
	t	q^2t	$1+qt$	$q+q^2t$	q^2+q^3t	q	q^3
	t^2	qt	$t+qt^2$	$1+q^2t^2$	$q+q^2t$	qt	q^2
	t^3	t	t^2+qt^3	$t+qt^2$	$1+qt$	qt^2	q
	t^3	qt^3	$t+t^2$	$t+qt^2$	$qt+qt^2$	1	q
	t^6	t^3	t^4+t^5	t^2+t^4	$t+t^2$	t^3	1

	1 2 3 4	1 2 3 / 4	1 2 4 / 3 ; 1 3 4 / 2	1 2 / 3 4 ; 1 3 / 2 4	1 2 / 3 / 4 ; 1 3 / 2 / 4	1 4 / 2 / 3	1 / 2 / 3 / 4
	1	q^3	$q+q^2$	q^2+q^4	q^4+q^5	q^3	q^6
	t	1	$qt+q^2t$	$q+q^2t$	$q+q^2$	q^3t	q^3
	t	q^2t	$1+qt$	$q+q^2t$	q^2+q^3t	q	q^3
	t^2	qt	$t+qt^2$	$1+q^2t^2$	$q+q^2t$	qt	q^2
1 2 / 3 / 4	t^3	t	t^2+qt^3	$t+qt^2$	$1+qt$	qt^2	q
	t^3	qt^3	$t+t^2$	$t+qt^2$	$qt+qt^2$	1	q
	t^6	t^3	t^4+t^5	t^2+t^4	$t+t^2$	t^3	1

$$M_{\begin{smallmatrix}1&2\\3\\4\end{smallmatrix}} = t^3 s_{\begin{smallmatrix}1&2&3&4\end{smallmatrix}} + qt^3 s_{\begin{smallmatrix}1&3&4\\2\end{smallmatrix}} + t\, s_{\begin{smallmatrix}1&2&3\\4\end{smallmatrix}} + t^2 s_{\begin{smallmatrix}1&2&4\\3\end{smallmatrix}} + t\, s_{\begin{smallmatrix}1&2\\3&4\end{smallmatrix}}$$

$$+ qt^2 s_{\begin{smallmatrix}1&3\\2&4\end{smallmatrix}} + s_{\begin{smallmatrix}1&2\\3\\4\end{smallmatrix}} + qt\, s_{\begin{smallmatrix}1&3\\2\\4\end{smallmatrix}} + qt^2 s_{\begin{smallmatrix}1&4\\2\\3\end{smallmatrix}} + q\, s_{\begin{smallmatrix}1\\2\\3\\4\end{smallmatrix}}$$

No more freedom!!

$$M_{\young(12,3,4)} = t^3 s_{\young(1234)} + qt^3 s_{\young(134,2)} + t\, s_{\young(123,4)} + t^2 s_{\young(124,3)} + t\, s_{\young(12,34)}$$

$$+\, qt^2 s_{\young(13,24)} + s_{\young(12,3,4)} + qt\, s_{\young(13,2,4)} + qt^2 s_{\young(14,2,3)} + q\, s_{\young(1,2,3,4)}$$

No more freedom!!

Donald J. Trump
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THANK YOU!!

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