Kostka–Foulkes polynomials at $q = -1$

Gabriel Frieden

LaCIM, Université du Québec à Montréal

Joint work with
Michael Chmutov, Dongkwan Kim, Joel Lewis, and Elena Yudovina

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The $q = -1$ phenomenon

Suppose we have

- a finite set X of combinatorial objects
- a combinatorial involution $\sigma : X \to X$
- a "natural" q-enumerator of X, that is, a polynomial $f(q)$ with non-negative integer coefficients such that $f(1) = |X|$

Following Stembridge, we say that $(X, \sigma, f(q))$ exhibits the $q = -1$ phenomenon if the number of fixed points of σ is equal to $f(-1)$.

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Of course, such a polynomial always exists. Stembridge was interested in the case where $f(q)$ has a simple closed formula, so that the fixed points can then be enumerated easily.

Example 1

- $X = \{$partitions contained in the $k \times (n-k)$ rectangle$\}$
- $\sigma =$ complement (and rotate 180 degrees)
- $f(q) =$ the q-binomial coefficient $\left[{n \atop k} \right]$, where

$$[a] = \frac{1-q^a}{1-q}, \quad [a]! = [a]\cdots[2][1], \quad \left[{n \atop k} \right] = \frac{[n]!}{[k]![n-k]!}$$

This triple exhibits the $q = -1$ phenomenon.

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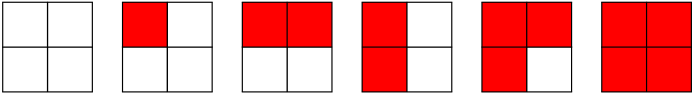
Example: $n = 4, k = 2$



$$f(q) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} = 1 + q + 2q^2 + q^3 + q^4$$

and the number of self-complementary partitions is $f(-1) = 2$.

Example 2

- $X = \{$partitions contained in the staircase $(n-1, n-2, \ldots, 1)\}$
- $\sigma =$ transpose
- $f(q) =$ the q-Catalan polynomial $\dfrac{1}{[n+1]} \cdot \begin{bmatrix} 2n \\ n \end{bmatrix}$

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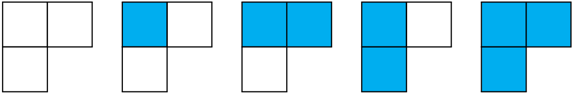
Example: $n = 3$



$$f(q) = \frac{1}{[4]} \cdot \begin{bmatrix} 6 \\ 3 \end{bmatrix} = 1 + q^2 + q^3 + q^4 + q^6$$

and the number of self-conjugate partitions is $f(-1) = 3$.

Evacuation (the Schützenberger involution)

Let $\mathrm{SSYT}(\lambda, \leq n)$ denote the set of semistandard tableaux of shape λ, with entries at most n. Evacuation is an involution

$$e_n : \mathrm{SSYT}(\lambda, \leq n) \to \mathrm{SSYT}(\lambda, \leq n).$$

Example:

$$e_4\left(\begin{ytableau} 1 & 1 & 1 & 1 & 2 & 2 & 3 \\ 2 & 3 & 4 \end{ytableau}\right) = \begin{ytableau} \, & \, & \, & \, & \, & \, & \, \\ \, & \, & \, \end{ytableau}$$

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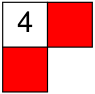

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Observe that evacuation reverses the content. That is, if μ is a composition with n (possibly zero) parts, then

$$e_n : \mathrm{SSYT}(\lambda, \mu) \to \mathrm{SSYT}(\lambda, w_0(\mu))$$

where w_0 is the longest element of $\mathfrak{S}_n$.

Self-evacuating tableaux

- $X = \mathrm{SSYT}(\lambda, \leq n)$
- $\sigma = e_n$
- $f(q) = q^{b(\lambda)} s_\lambda(1, q, q^2, \ldots, q^{n-1})$

$$\text{where} \quad b(\lambda) = \sum (i-1)\lambda_i$$

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Theorem (J. Stembridge, '96)

The number of self-evacuating semistandard tableaux in $\mathrm{SSYT}(\lambda, \leq n)$ is equal to $f(-1)$.

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Our goal: a weight space refinement of this result

The standard case

Evacuation acts on standard tableaux. For the q-enumerator, take the q-analogue of the hook-length formula:

$$f^\lambda(q) = [n]! \prod_{(i,j)\in\lambda} \frac{1}{[h_{i,j}]}$$

where $n = |\lambda|$, and $h_{i,j}$ is the hook-length of the box in position (i, j) of the diagram of λ.

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Theorem

The number of self-evacuating standard tableaux of shape λ is equal to $f^{\lambda}(-1)$.

Domino tableaux

Definition

Say that a standard tableau T of shape $\lambda \vdash n$ is a domino tableau if for each pair

$$(n-1, n), (n-3, n-2), \ldots, \begin{cases} (1,2) & \text{if } n \text{ even} \\ (2,3) & \text{if } n \text{ odd} \end{cases},$$

the entries in the pair are adjacent in T.

Examples:

1	2	5	6
3	7	8	
4			

1	2	3
4	5	8
6	7	9

Theorem (J. Stembridge, ’96)

The number of self-evacuating standard tableaux of shape λ is equal to the number of domino tableaux of shape λ.

The standard case

Theorem

The number of self-evacuating standard tableaux (or domino tableaux) of shape $\lambda \vdash n$ is equal to $f^\lambda(-1)$, where $f^\lambda(q) = [n]!/\prod[h_{i,j}]$.

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Sketch of proof, following Stanley '09

- One has $f^\lambda(q) = q^{b(\lambda)} \sum_{T \in \mathrm{SYT}(\lambda)} q^{\mathrm{comaj}(T)}$,

  where $\mathrm{comaj}(T) = \sum_{i \text{ a descent of } T} (n-i)$

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The number of self-evacuating standard tableaux (or domino tableaux) of shape $\lambda \vdash n$ is equal to $f^{\lambda}(-1)$, where $f^{\lambda}(q) = [n]!/\prod[h_{i,j}]$.

Sketch of proof, following Stanley '09

- One has $f^{\lambda}(q) = q^{b(\lambda)} \sum_{T \in \mathrm{SYT}(\lambda)} q^{\mathrm{comaj}(T)}$,

  where $\mathrm{comaj}(T) = \sum_{i \text{ a descent of } T} (n-i)$
- Define an involution $\tau : \mathrm{SYT}(\lambda) \to \mathrm{SYT}(\lambda)$ as follows: if some pair of entries $(n-(2i+1)), (n-2i))$ is not adjacent in T, swap the entries in the largest such pair; if there is no such pair, then τ fixes T
- If T is not domino, then τ changes the parity of comaj; if T is domino, T is fixed by τ, and $\mathrm{comaj}(T) \equiv b(\lambda) \bmod 2$

Aside: cyclic sieving

Suppose we have

- a finite set X of combinatorial objects
- a combinatorial map $\sigma : X \to X$ of order n
- a "natural" q-enumerator $f(q)$ of X

Let ζ be a primitive nth root of unity. The triple $(X, \sigma, f(q))$ exhibits the cyclic sieving phenomenon if for each k, the number of fixed points of σ^k is equal to $f(\zeta^k)$.

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Theorem (B. Rhoades, '10)

*Let λ be a **rectangular** partition, and let pr be jeu-de-taquin promotion. The triples*

$$(\mathrm{SSYT}(\lambda, n), \mathrm{pr}, q^{b(\lambda)} s_\lambda(1, q, q^2, \dots, q^{n-1}))$$

and

$$(\mathrm{SYT}(\lambda), \mathrm{pr}, f^\lambda(q))$$

exhibit the cyclic sieving phenomenon.

Beyond the standard case

Question: When $\mu \neq (1^n)$, what is the natural q-analogue of the Kostka number $K_{\lambda\mu} := |\mathrm{SSYT}(\lambda, \mu)|$?

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Answer: the (cocharge) Kostka–Foulkes polynomials $\widetilde{K}_{\lambda\mu}(q)$, which are given combinatorially by

$$\widetilde{K}_{\lambda\mu}(q) = \sum_{T \in \mathrm{SSYT}(\lambda,\mu)} q^{c(T)},$$

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When T is standard, one has $\mathrm{coch}(T) = \mathrm{comaj}(T)$, so cocharge can be viewed as a semistandard generalization of comaj. This means that

$$f^\lambda(q) = q^{b(\lambda)} \sum_{T \in \mathrm{SYT}(\lambda)} q^{\mathrm{comaj}(T)} = q^{b(\lambda)} \widetilde{K}_{\lambda,(1^{|\lambda|})}(q).$$

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More generally, when μ is **palindromic** (i.e., $w_0(\mu) = \mu$), evacuation acts on $\mathrm{SSYT}(\lambda, \mu)$, and the number of self-evacuating tableaux is equal to

$$(-1)^{b(\lambda)} \widetilde{K}_{\lambda\mu}(-1).$$

This can be seen by combining several results of Stembridge and Lascoux–Leclerc–Thibon.

The general case

- $X = \mathrm{SSYT}(\lambda, \mu)$
- $f(q) = q^{b(\lambda)} \widetilde{K}_{\lambda\mu}(q)$
- $\sigma = \underline{\qquad}$?

What is $f(-1)$ counting? Evacuation reverses content, so does not act on X in general. We would like to have another involution which reverses content and commutes with evacuation....

Symmetric group action on tableaux

Lascoux and Schützenberger defined an action of $\mathfrak{S}_n$ on $\mathrm{SSYT}(\lambda, \leq n)$, where the generators s_i act on a tableau T as follows:

- Write the sequence of i's and $i+1$'s in T in reading order.
- Recursively cross out pairs of the form $(i+1)\ i$ with no uncrossed letters in between, until the remaining letters form a subsequence of the form $i^a\ (i+1)^b$.
- Replace this subsequence with $i^b\ (i+1)^a$, and let $s_i(T)$ be the corresponding tableau.

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Example:

$$s_i\left(\begin{array}{|c|c|c|c|c|c|c|}\hline 1 & 1 & 1 & 2 & 2 & 3 & 3 \\ \hline 2 & 3 & 3 & 3 \\ \cline{1-4}\end{array}\right) = \begin{array}{|c|c|c|c|c|c|c|}\hline 1 & 1 & 1 & 2 & 2 & 2 & 3 \\ \hline 2 & 2 & 3 & 3 \\ \cline{1-4}\end{array}$$

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Theorem (Chmutov–F–Kim–Lewis–Yudovina, '18)

If μ is a partition with n parts, then the number of elements of $\mathrm{SSYT}(\lambda, \mu)$ fixed by the involution e_n^* is equal to $(-1)^{b(\lambda)}\widetilde{K}_{\lambda\mu}(-1)$.

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Our proof uses Kirillov and Reshetikhin's bijection between semistandard tableaux and rigged configurations, and several difficult results of Kirillov–Schilling–Shimozono about the interaction of this bijection with evacuation, the symmetric group action, and cocharge.

Context

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The affine matrix-ball construction (AMBC) of Chmutov–Pylyavskyy–Yudovina sends affine permutations to pairs of tabloids of the same shape (plus some additional data), where a tabloid is a filling of a Young diagram with the numbers $1, \ldots, n$, each used once, so that rows increase.

There is a natural analogue of the reverse complement map for affine permutations. This map corresponds to a shape-preserving involution on tabloids, which we call affine evacuation.

Fixed points of affine evacuation

From computational evidence, we conjectured a recursive formula for the number of fixed points $t(\mu)$ of this map:

$$t(\mu) = \sum_{\substack{i \geq 2, \\ m_i \text{ is odd}}} t\left(\mu\downarrow^{(i)}_{(i-2)}\right) + \sum_{i=1}^{k} 2\left\lfloor \frac{m_i}{2} \right\rfloor \cdot t\left(\mu\downarrow^{(i,i)}_{(i-1,i-1)}\right).$$

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Dongkwan had come across exactly the same recursion! He proved that this recursion was satisfied by the Euler characteristics of certain Springer fibers in types B,C,D, and he proved that these Euler characteristics are given by

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The Green's polynomial $\mathcal{Q}^{\mu}(q)$ is (up to tensoring with the sign character) the graded character of the symmetric group action on the cohomology of the type A Springer fiber asscoiated to the partition μ. $\mathcal{Q}^{\mu}_{\sigma}(q)$ denotes its value on the permutation σ. (For comparison, the Euler characteristic of the type A Springer fiber is $\mathcal{Q}^{\mu}_{id}(1)$.)

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Combinatorially, the Green's polynomials are given by

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We showed that if $T \leftrightarrow (P, Q)$, then $e(T) \leftrightarrow (e(P), e^{*}(Q))$. By the Murnaghan–Nakayama rule, the number of self-evacuating (or domino) tableaux of shape λ is $(-1)^{b(\lambda)} \cdot \chi^{\lambda}_{w_0}$. This suggested that the number of fixed points of e^{*} on $\mathrm{SSYT}(\lambda, \mu)$ should be equal to $(-1)^{b(\lambda)} \cdot \widetilde{K}_{\lambda,\mu}(-1)$.

Fixed points of affine evacuation

Thus, our result about Kostka–Foulkes polynomials at $q = -1$ was the key step in proving the following result:

Theorem (Chmutov–F–Kim–Lewis–Yudovina, '18)

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$$f(q) = \sum_{T \in \mathcal{T}(\mu)} q^{\mathrm{comaj}(T) + c(Q(T))},$$

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Can this $q = -1$ phenomenon be proved by means of a sign-reversing involution?

More questions

- Is there a natural subset of “domino tabloids” which are in bijection with the fixed points of affine evacuation?
- What do our results say about Schur P- and Q-functions?
- Let $K_{\lambda,\mu}(q,t)$ be the Macdonald–Kostka polynomial. We have focused on $K_{\lambda,\mu}(q,0)$. Does $K_{\lambda,\mu}(q,1)$ exhibit a $q=-1$ phenomenon?
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Thanks for listening!