Moves on k-graphs preserving Morita equivalence

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Theorem (Eilers-Restorff-Ruiz-Sørensen [ERRS16])

Let E, F be directed graphs with finitely many vertices. $C^*(E)$ and $C^*(F)$ are stably equivalent if and only if one can convert E into F by a finite sequence of the moves

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Our work constitutes a first step in developing such classification results for higher-rank graphs.

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For directed graphs, reduction and delay are (intuitively but not exactly) inverses.

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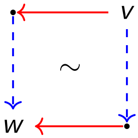

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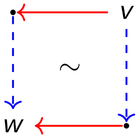

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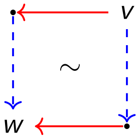

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Higher-rank graphs (k-graphs) were introduced by Kumjian & Pask in 2000 to give examples of combinatorial, computable C^*-algebras, more general than graph C^*-algebras.

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(CK4) For any vertex v and any color i, $p_v = \sum_{e:d(e)=i, r(e)=v} s_e s_e^*$.

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Theorem (Kumjian-Pask; "Gauge-invariant uniqueness theorem")

Let Λ be a k-graph. There is a continuous action α of $\mathbb{T}^k$ on $C^*(\Lambda)$, satisfying

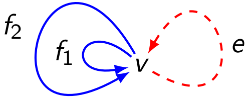
$$\alpha_z(s_e) = z_i s_e$$

if e is an edge of color i. If $\pi(p_v) \neq 0$ for all v, and there is also an action β of $\mathbb{T}^k$ on $C^*(\{Q_v, T_e\})$ such that

$$\pi \circ \alpha = \beta \circ \pi$$

then π is an isomorphism.

Example and notation



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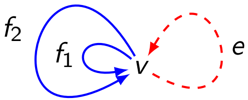

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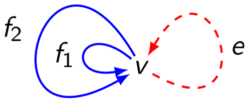

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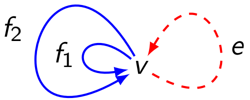

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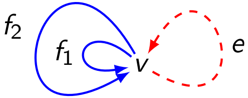

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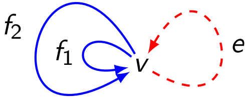

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Note that Λ^0 is the vertices of Λ.

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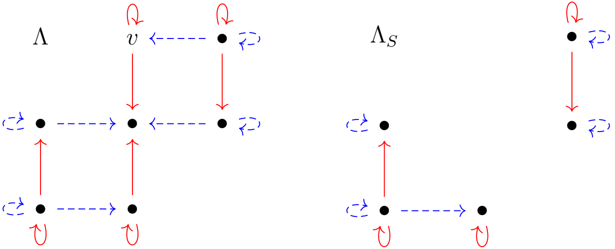
Proposition (Eckhardt-Fieldhouse-Gent-G-Gonzales-Pask)

If v is a sink in Λ^0, then deleting v, all vertices w such that $w \geq v$, and all incident edges results in a k-graph Λ_S such that $C^*(\Lambda_S) \sim_{ME} C^*(\Lambda)$.

Example of sink deletion



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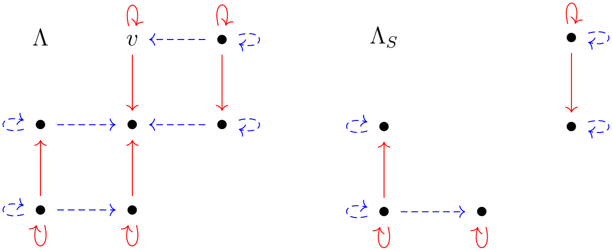

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In this case, $C^*(\Lambda) \sim_{ME} C(\mathbb{T}^2) \oplus C(\mathbb{T}^2)$, which we see if we delete the three remaining sinks.

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We define Λ_I by $\Lambda_I^0 = \Lambda^0 \backslash \{v\} \cup \{v_1, v_2\}$; importing from Λ all edges not incident on v; edges in $\mathcal{E}_i$ have range v_i; and making two copies e_1, e_2, of all edges e with source v.

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The pairing condition ensures that we can define the factorization in Λ_I by importing the factorization in Λ.

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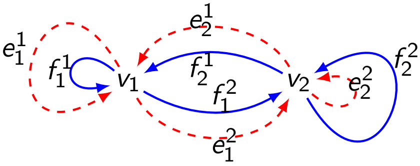
If $e^i f^j = f^j e^i$, then Λ cannot be insplit. If $e^i f^j = f^i e^j$, then take $\mathcal{E}_1 = \{e^1, f^1\}, \mathcal{E}_2 = \{e^2, f^2\}$. Then in Λ_I, we have $e^i_j f^j_k \sim_I f^i_j e^j_k$.



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Then show we get an onto map $\psi : C^*(\Lambda) \to C^*(\Lambda_I)$. Use the gauge-invariant uniqueness theorem to prove that $C^*(\Lambda_I) \cong C^*(\Lambda)$.

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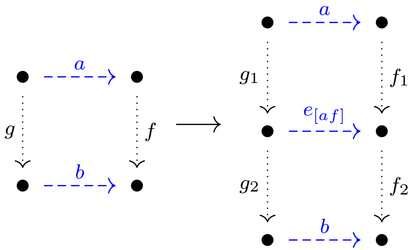

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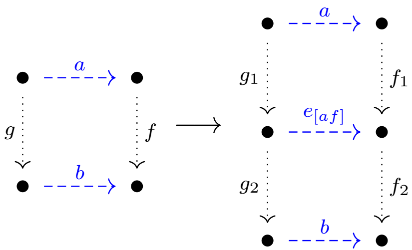

Many commuting squares α in Λ have now become rectangles;

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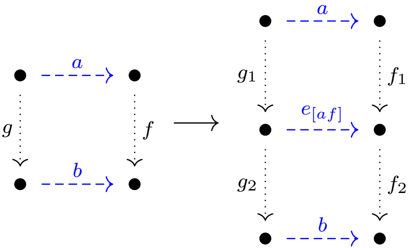

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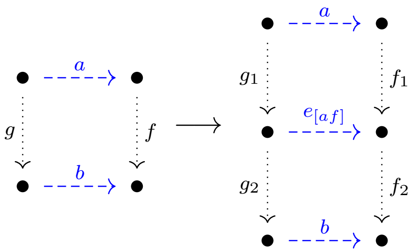

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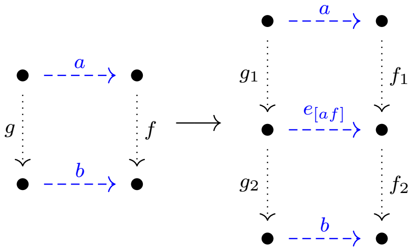

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The factorization in Λ_D essentially comes from the factorization in Λ, but there are lots of cases to check.

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Proof.

If $C^*(\Lambda_D) = C^*(\{p_v, s_e\})$, define

$$q_v = p_v \ \forall \ v \in \Lambda^0; \qquad t_e = \begin{cases} s_e, & e \notin \mathcal{E}^1 \\ s_{e_2} s_{e_1}, & e \in \mathcal{E}^1 \end{cases}.$$

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Then $\{q_v, t_e\}$ is a Cuntz-Krieger Λ-family in $C^*(\Lambda_D)$; the gauge-invariant uniqueness theorem tells us $C^*(\Lambda) \cong C^*(\{q_v, t_e\})$. If $p = \sum_{v \in \Lambda^0} p_v$, then $C^*(\{q_v, t_e\}) \cong pC^*(\Lambda_D)p$ is a full corner of $C^*(\Lambda_D)$. □

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Pick an edge $f \in s^{-1}(v)$; define a Cuntz-Krieger Λ_R-family in $C^*(\Lambda) = C^*\{p_v, s_e\})$ by

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Again, check that $C^*(\{q_v, t_e\}) \cong C^*(\Lambda_R)$, using gauge-invariant uniqueness theorem, and that it's a full corner in $C^*(\Lambda)$. □

The end

Thanks for listening!

This research was supported by the National Science Foundation.

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