Pseudothermalization of the QGP

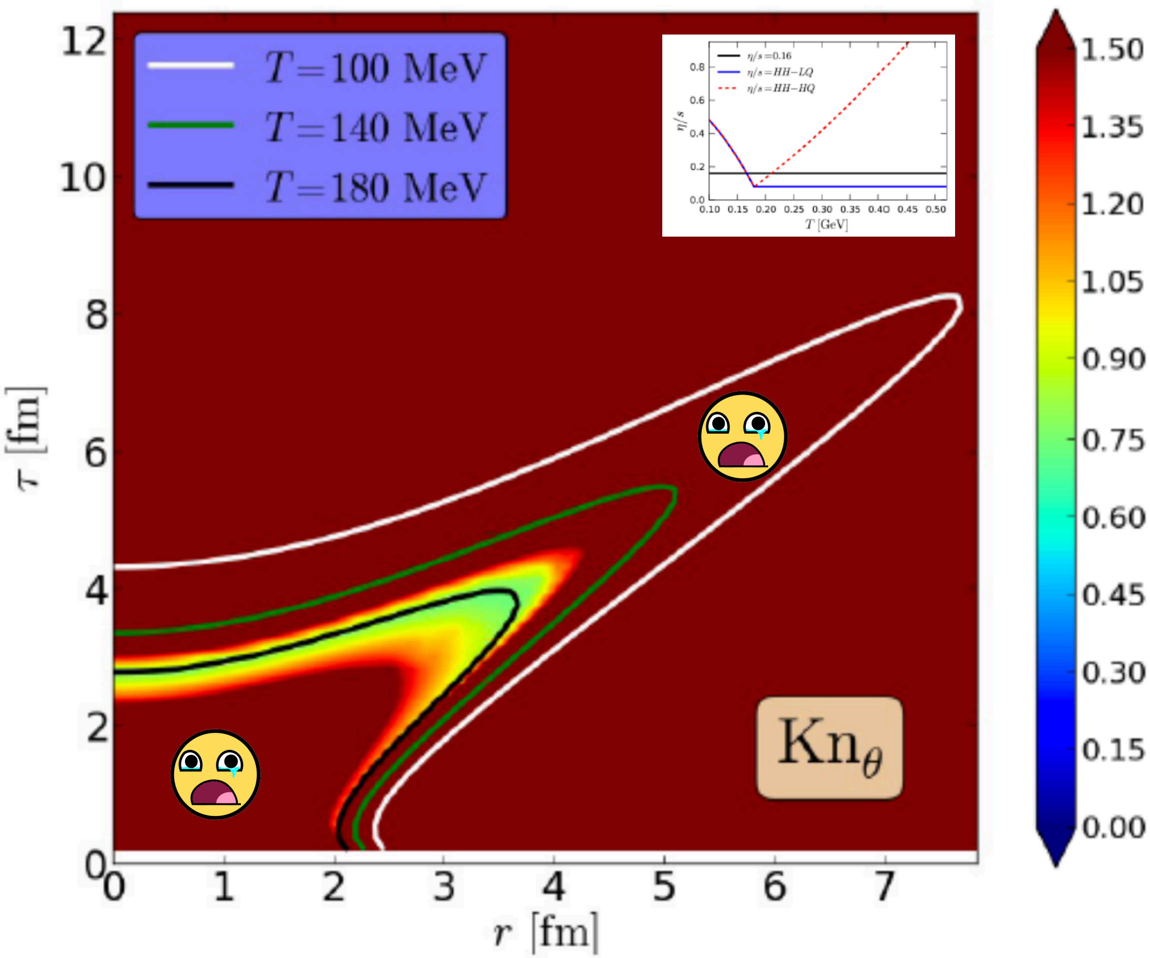
Michael Strickland
Kent State University

Primary refs:
D. Almaalol, A. Kurkela, and MS, forthcoming
MS, JHEP2018, 128

Banff International Research Station
Theoretical Foundations of Relativistic Hydrodynamics
Nov 25, 2019

p-A @ 2.76 TeV - Don't be happy, worry!

Figure (sans emoticons): H. Niemi and G. Denicol, 1404.7327



- Large gradients (Knudsen #) induce non-equilibrium deviations (measured by inverse Reynolds #)
- Evolution equations truncated at fixed order in these quantities → potential inaccuracy
- System has short lifetime → distribution function still far from equilibrium at freeze out

Practical goals

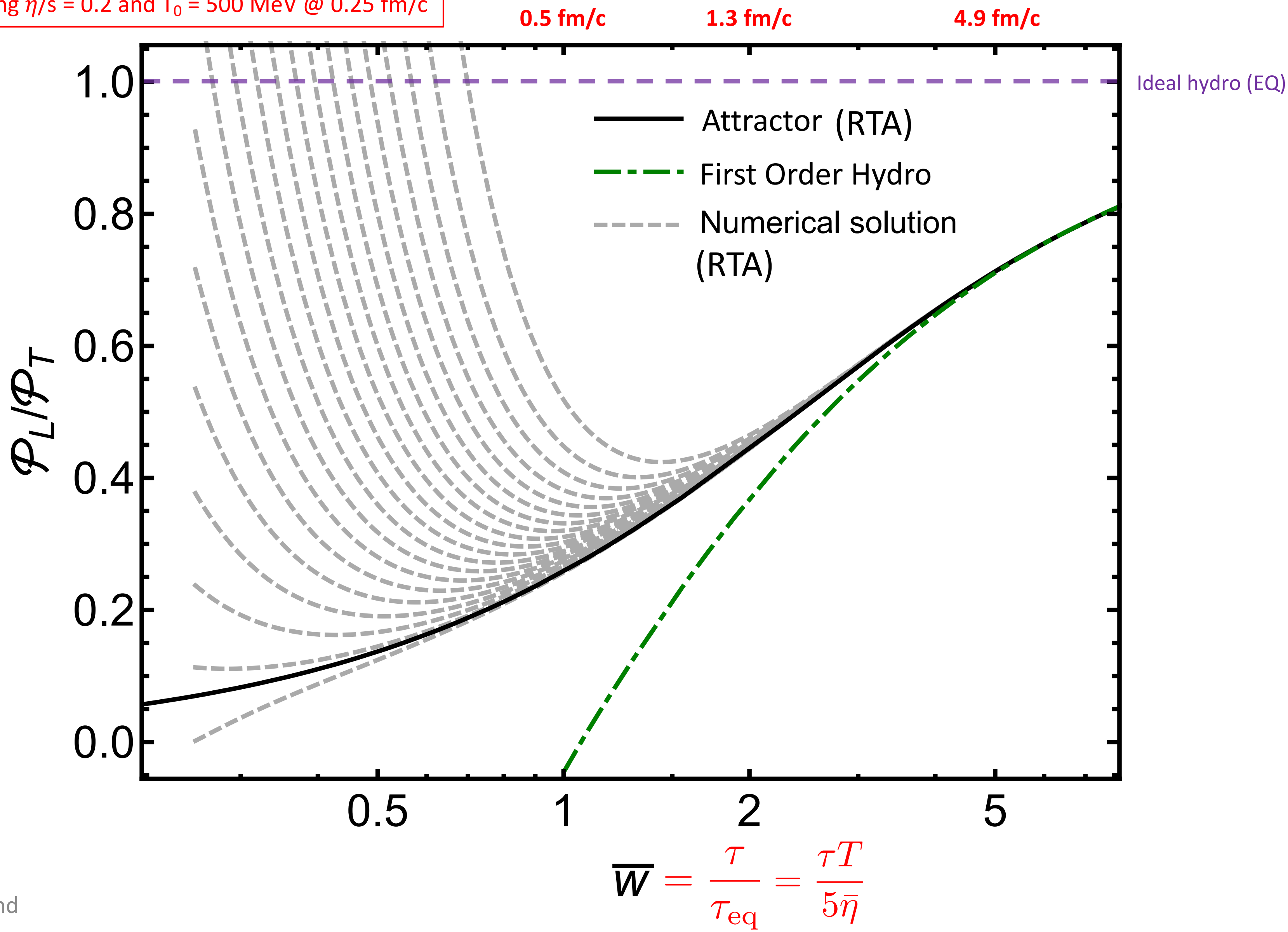
Improved hydrodynamic treatments in far from equilibrium systems:

- Can we construct hydrodynamic frameworks that more accurately describe EKT QCD thermalization and apply them to phenomenology? ** more computationally efficient than doing 3+1d kinetic theory simulations and can be extended across changes in fundamental DOFs
- Is there an attractor for the one-particle distribution function using EKT → improved description at freezeout?
- Today, I will present initial progress towards these goals

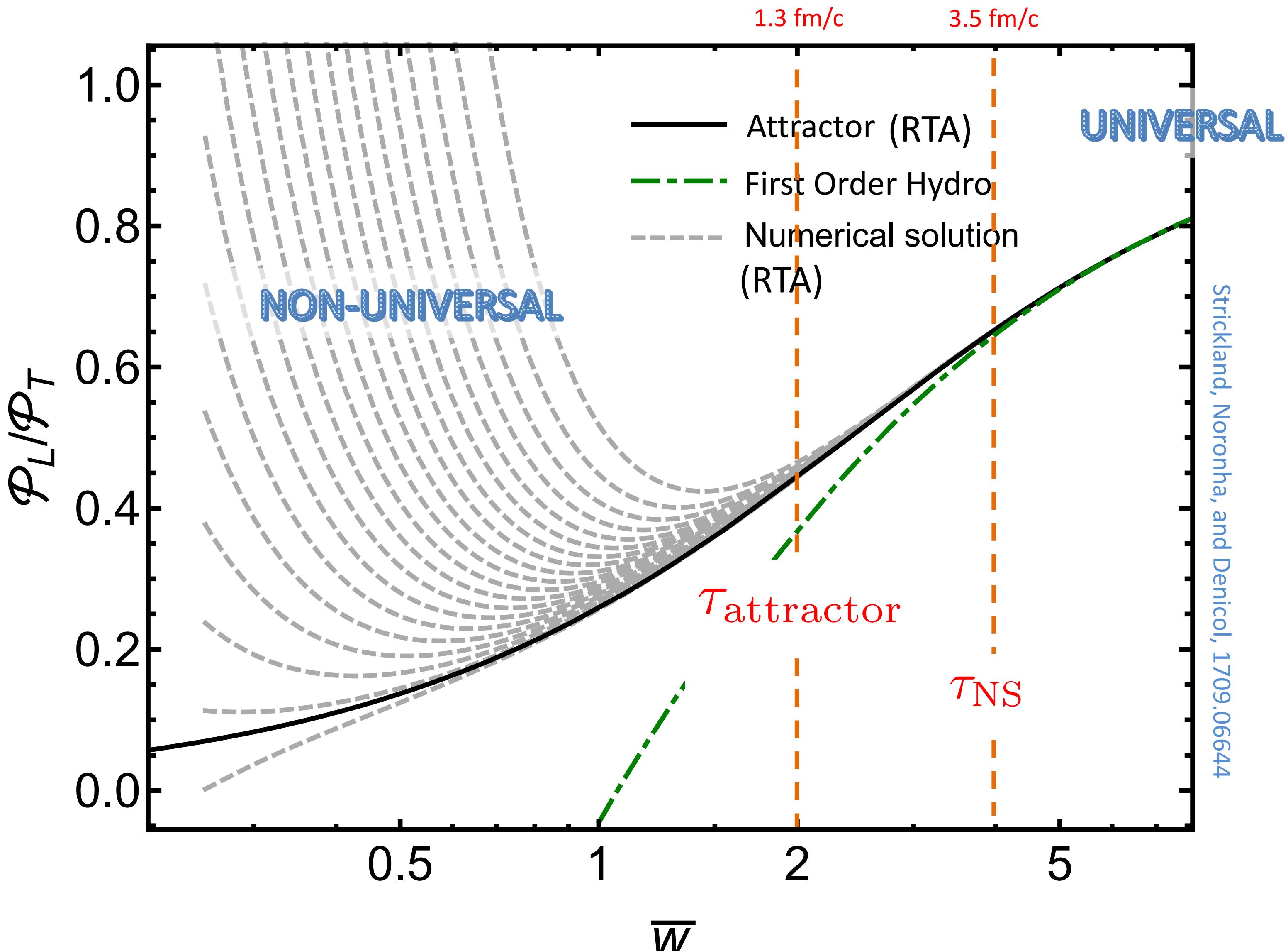
The
non-equilibrium
attractor

The attractor concept

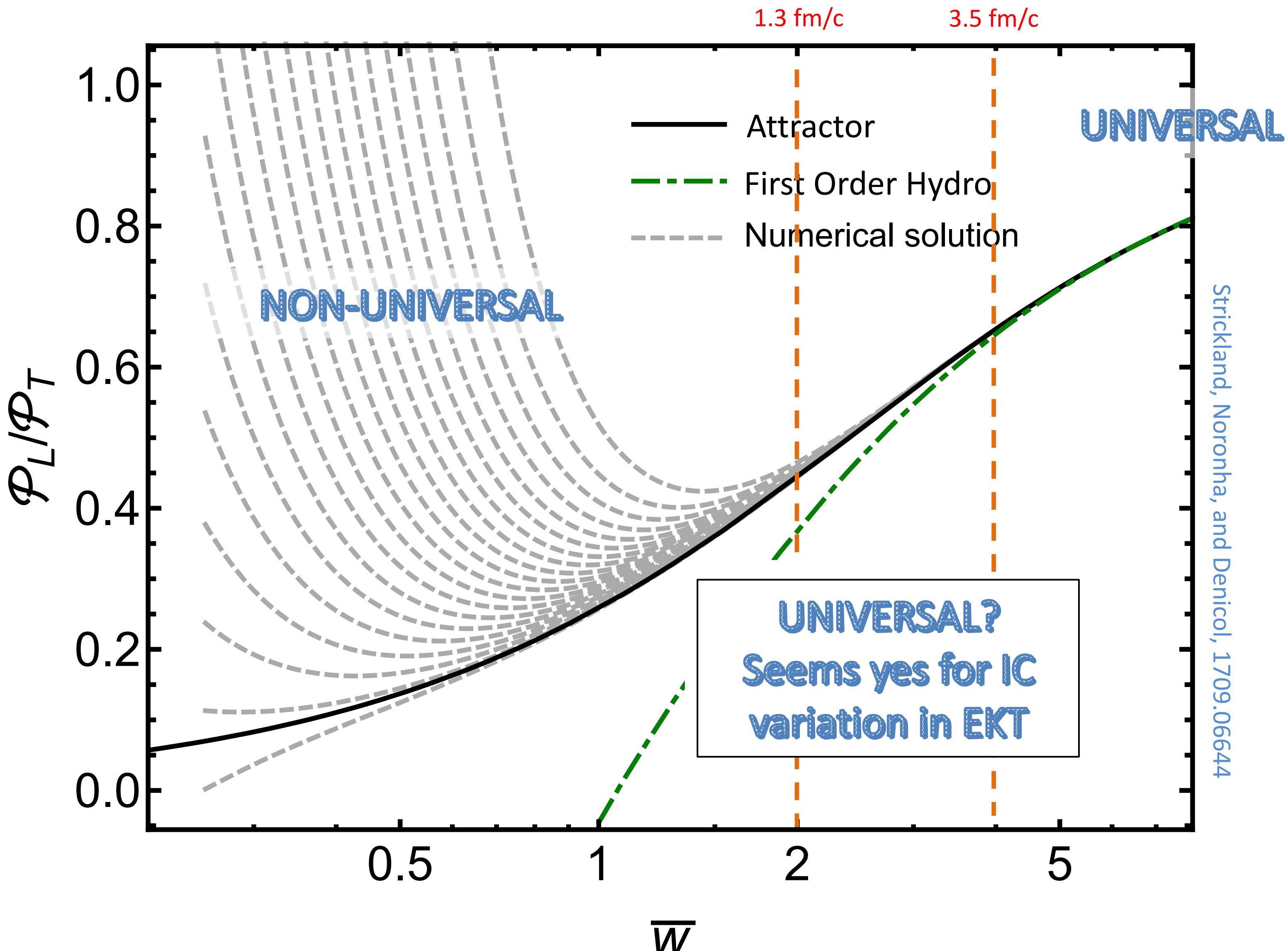
Assuming η/s = 0.2 and T_0 = 500 MeV @ 0.25 fm/c



The attractor concept



The attractor concept

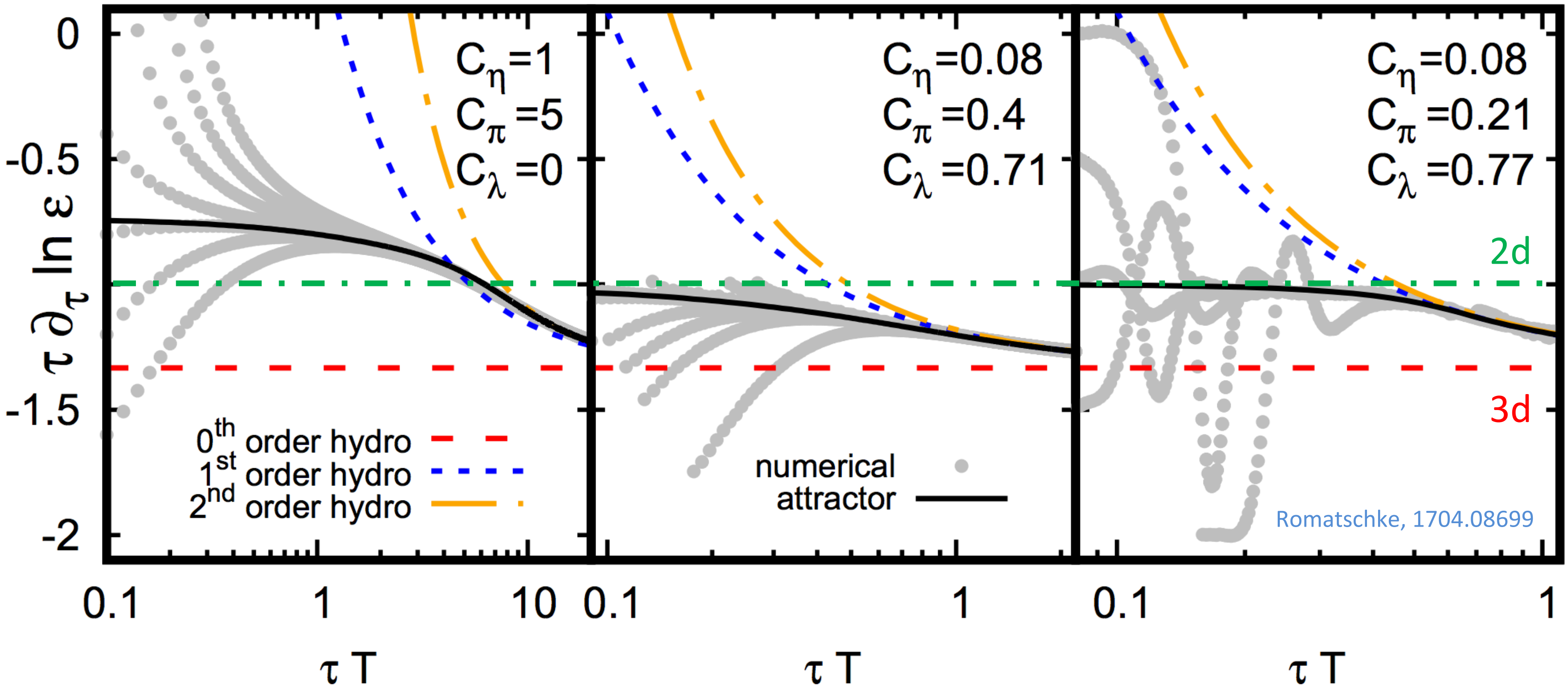


Attractor exists in many theories

rBRSSS Viscous Hydro
Baier, Romatschke, Son, Starinets, and Stephanov

Exact RTA Boltzmann EQ Sol
Florkowski, Ryblewski, and MS 1304.0665; 1305.7234

AdS/CFT

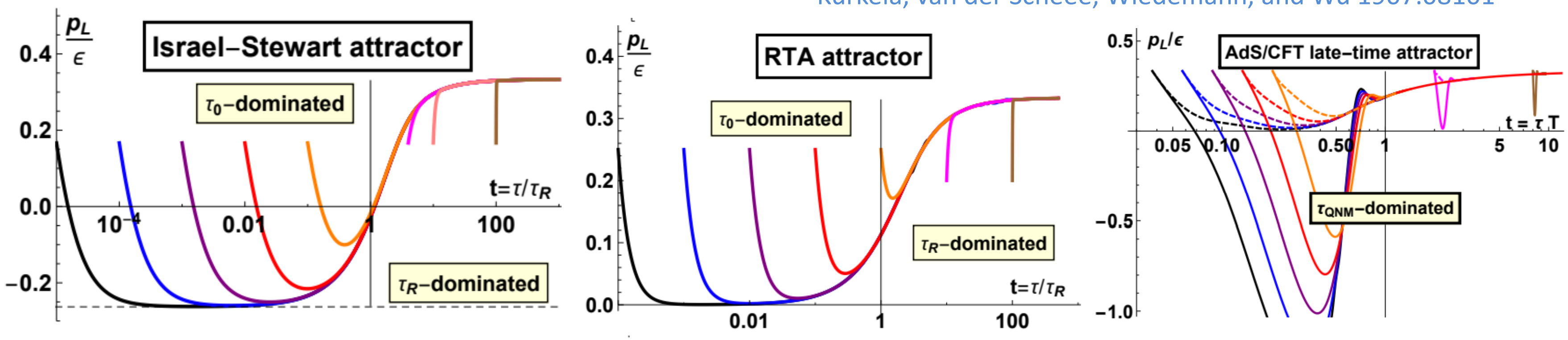


Quantity plotted is $-2\frac{\mathcal{P}_L+\mathcal{P}_T}{\mathcal{P}_L+2\mathcal{P}_T} \longrightarrow -4/3 \;\; \text{(ideal fluid)}$

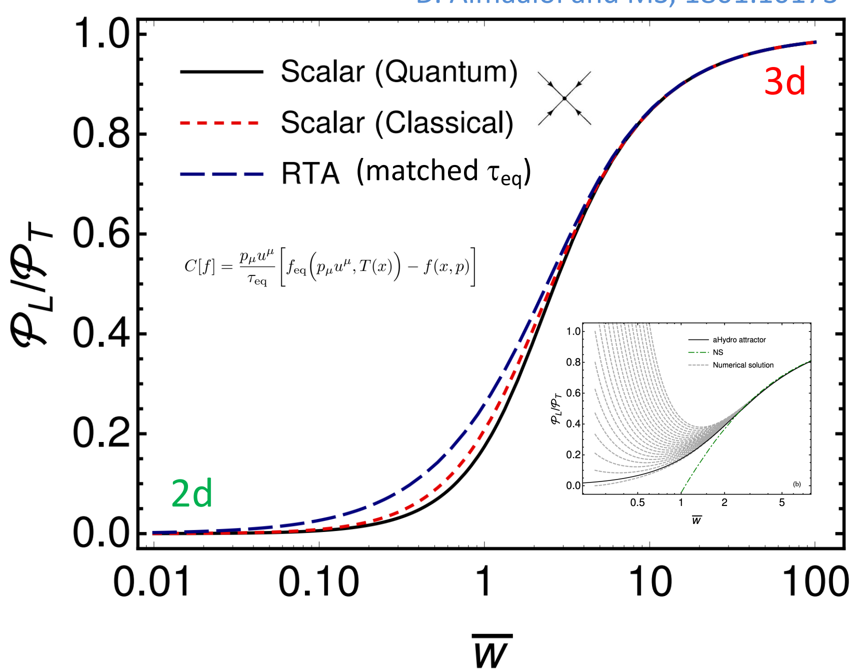
$\longrightarrow -1 \;\; \text{(2d fluid)}$

Early time behavior sensitive to model

Kurkela, van der Scheee, Wiedemann, and Wu 1907.08101

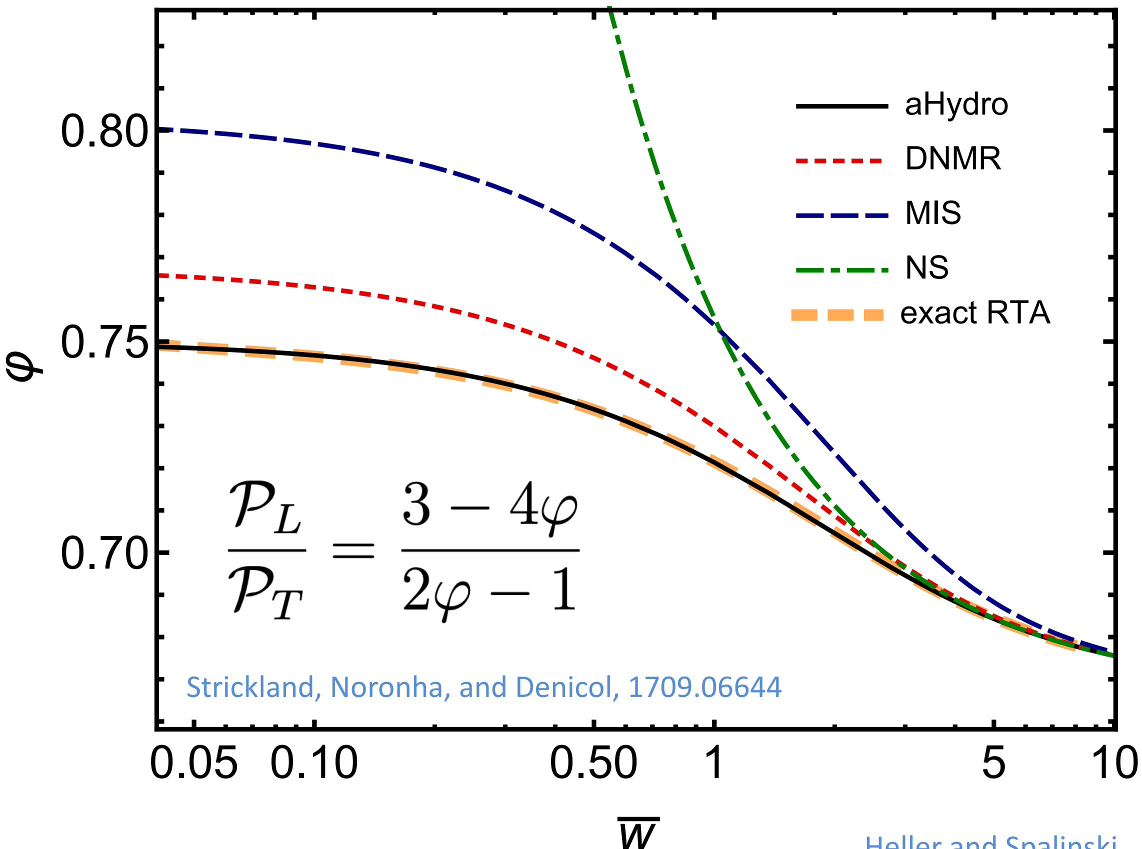


D. Almaalol and MS, 1801.10173



- Top three panels show IS, RA, and AdS/CFT evolution
- RTA has positive pressures, IS and AdS/CFT have negative P_L
- Early time AdS/CFT attractor sensitive to details of initial condition
- Left panel shows comparison of the attractor for RTA and a scalar QFT with/without quantum statistics

Can test hydro approximations using attractors



- Can compare exact RTA result to different hydro frameworks
- In each case one has to solve a 1d ODE subject to a self-consistent boundary condition
- **aHydro performs the best because it "resums" an infinite # of terms in Re^{-1}**

Heller and Spalinski

2nd order vHydro

$$\overline{w}\varphi\varphi' + 4\varphi^2 + \left[\overline{w} + \left(\beta_{\pi\pi} - \frac{20}{3}\right)\right]\varphi - \frac{4c_{\eta/\pi}}{9} - \frac{2}{3}(\beta_{\pi\pi} - 4) - \frac{2\overline{w}}{3} = 0$$

Strickland, Noronha, and Denicol

LO order aHydro

$$\overline{w}\varphi\frac{\partial\varphi}{\partial\overline{w}} = \left[\frac{1}{2}(1+\xi) - \frac{\overline{w}}{4}\mathcal{H}\right]\overline{\Pi}'$$

Beyond hydrodynamics?

Beyond hydrodynamics?

MS, JHEP2018, 128; 1809.01200

- Can the concept of a non-equilibrium attractor be extended beyond the 14 degrees of freedom described using the energy-momentum tensor, number density, and diffusion current?

- In kinetic theory we describe things in terms of a one-particle distribution function **f(x,p)** and the energy-momentum tensor is obtained from low-order moments:

$$T^{\mu\nu} = \int dP \, p^\mu p^\nu f(x,p) \qquad \int dP \equiv \int \frac{d^3p}{(2\pi)^2 E}$$

- What about more general moments of f? Particularly ones that are sensitive to higher momenta?

Beyond hydrodynamics?

MS, JHEP2018, 128; 1809.01200

- For a conformal system it suffices to consider

$$\mathcal{M}^{nm}[f] \equiv \int dP \, (p \cdot u)^n \, (p \cdot z)^{2m} \, f(x,p)$$

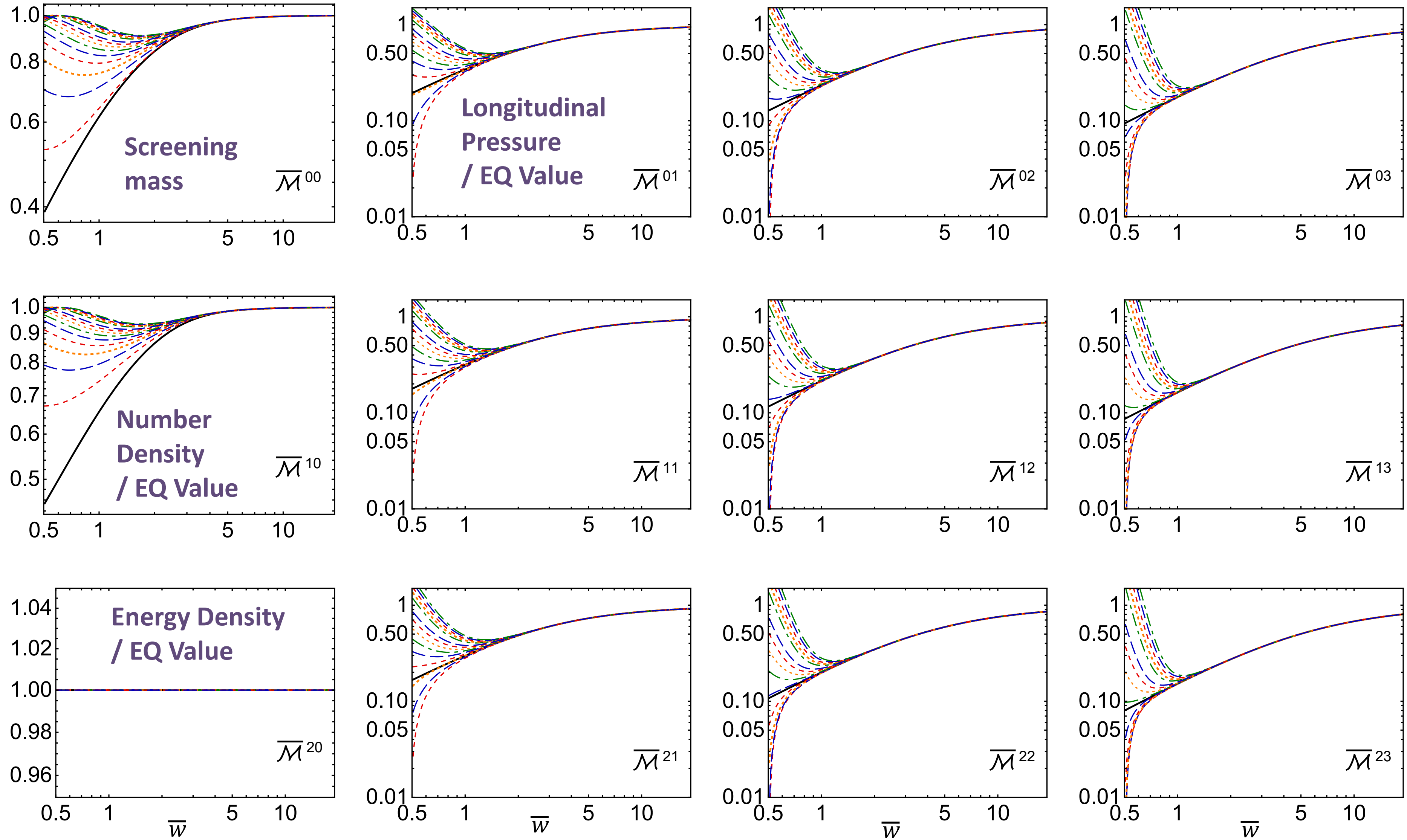
- This encompasses the moments necessary to construct the energy momentum tensor, e.g. below, and more

$$\varepsilon = \mathcal{M}^{20} = \int dP \, (p \cdot u)^2 \, f(\tau, w, p_T) = T^{00}_{\rm LRF}$$

$$P_L = \mathcal{M}^{01} = \int dP \, (p \cdot z)^2 \, f(\tau, w, p_T) = T^{zz}_{\rm LRF}$$

Behavior of higher-order moments

MS, JHEP2018, 128; 1809.01200

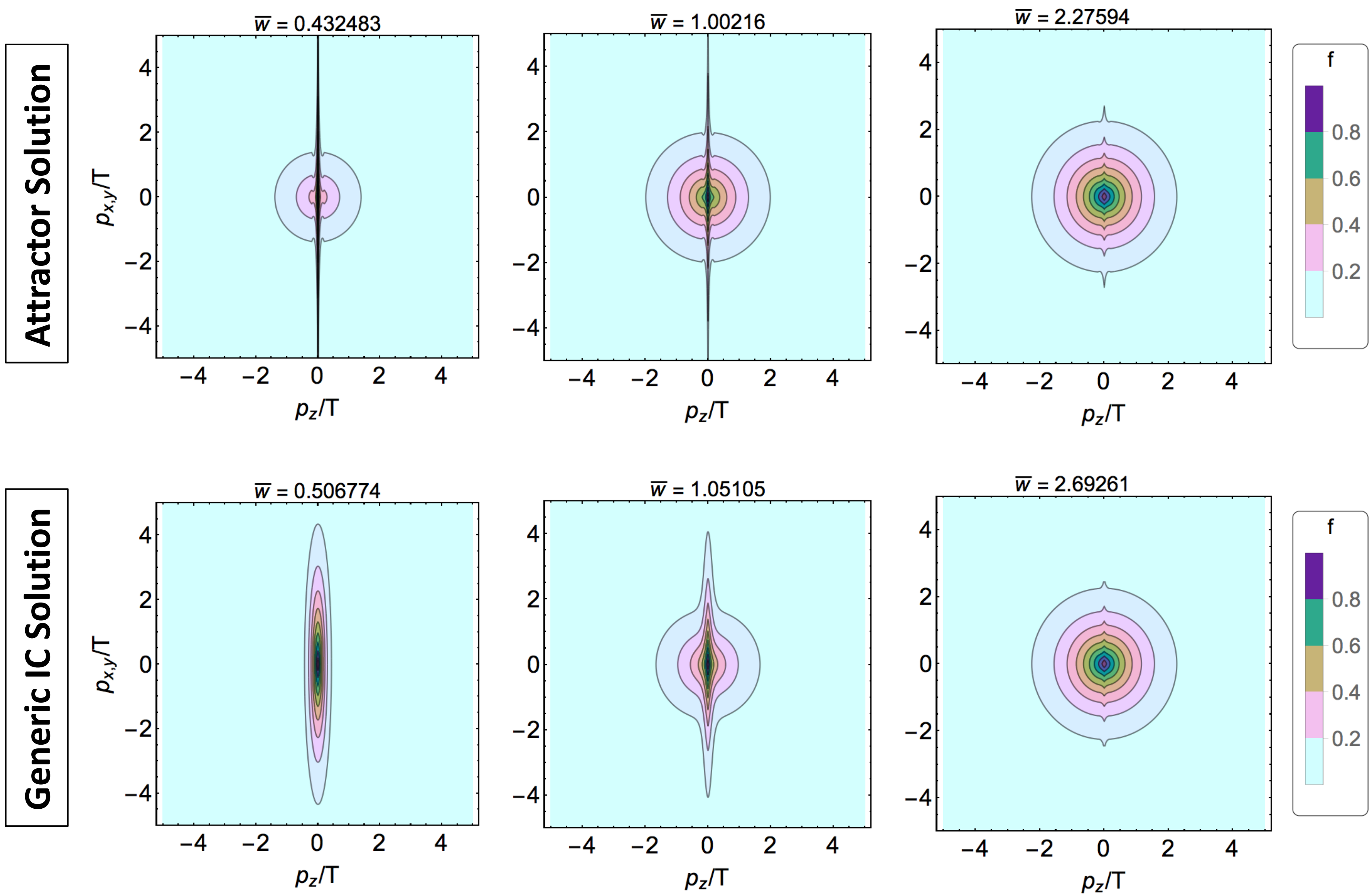


Black Line = Attractor Solution **Dashed colored lines = scan of initial conditions**

The attractor for the distribution function itself

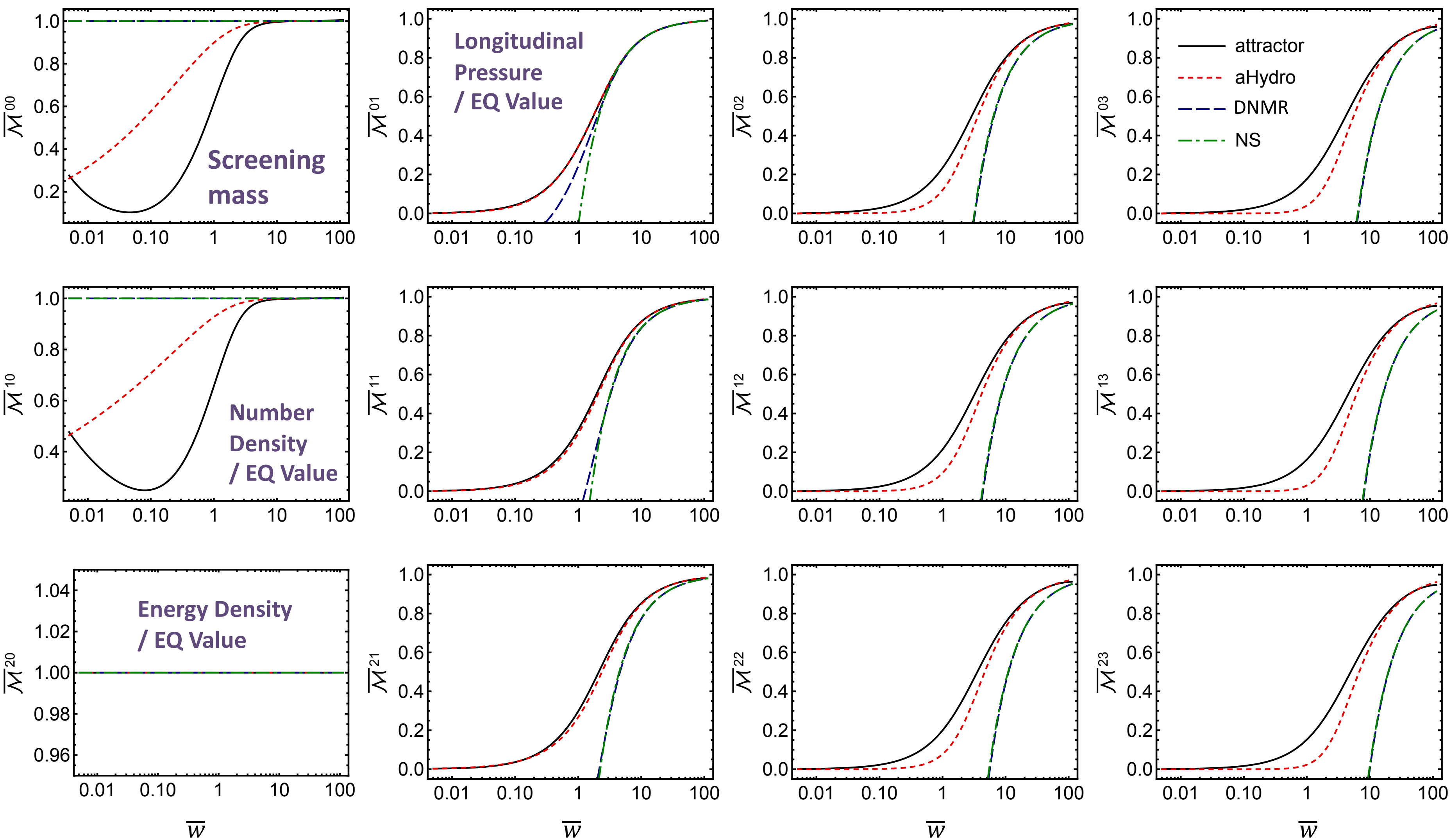
Attractor distribution function

MS, JHEP2018, 128; 1809.01200



Comparison of exact attractor for moments with different hydrodynamics approximations

Hydrodynamic comparisons

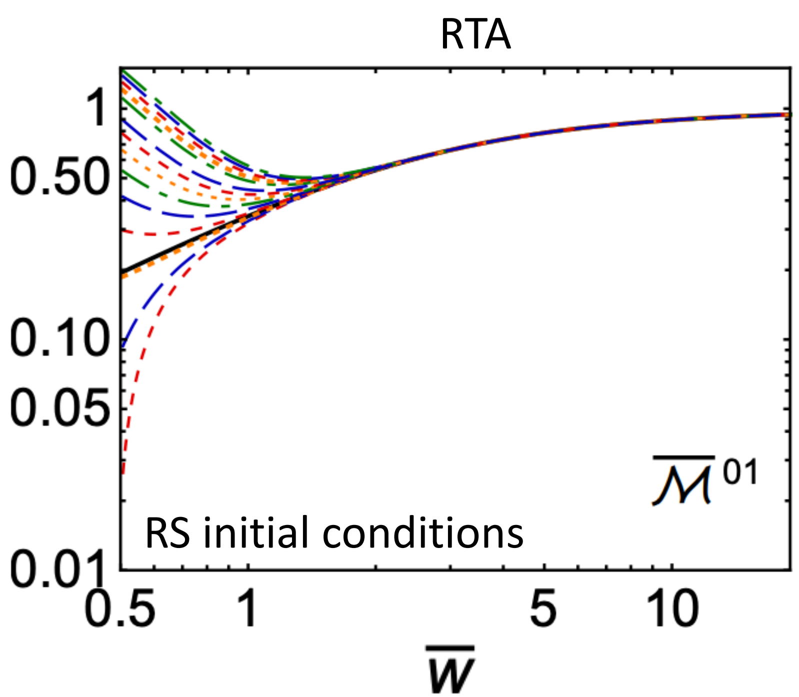


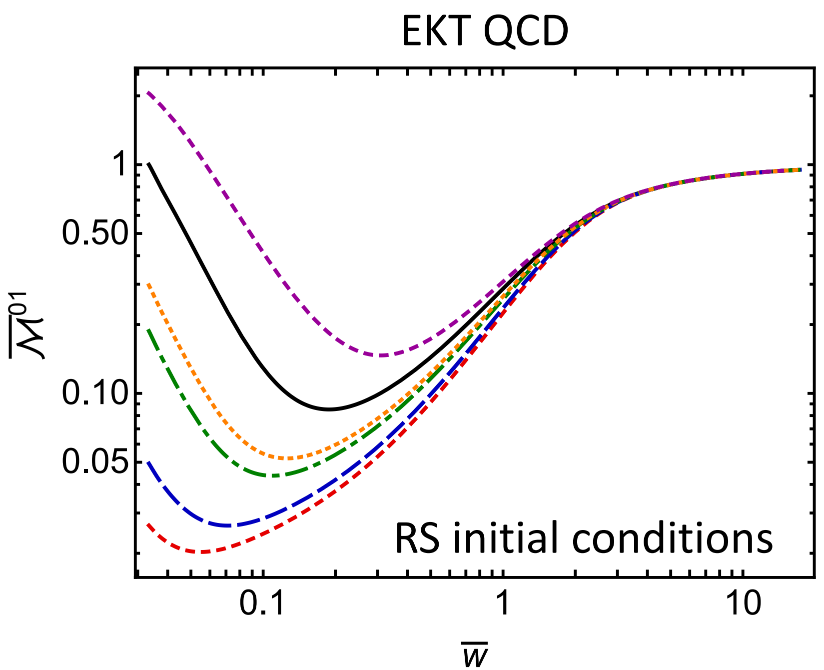
QCD?

Evidence for a QCD EKT attractor

D. Almaalol, A. Kurkela, and M. Strickland, forthcoming.

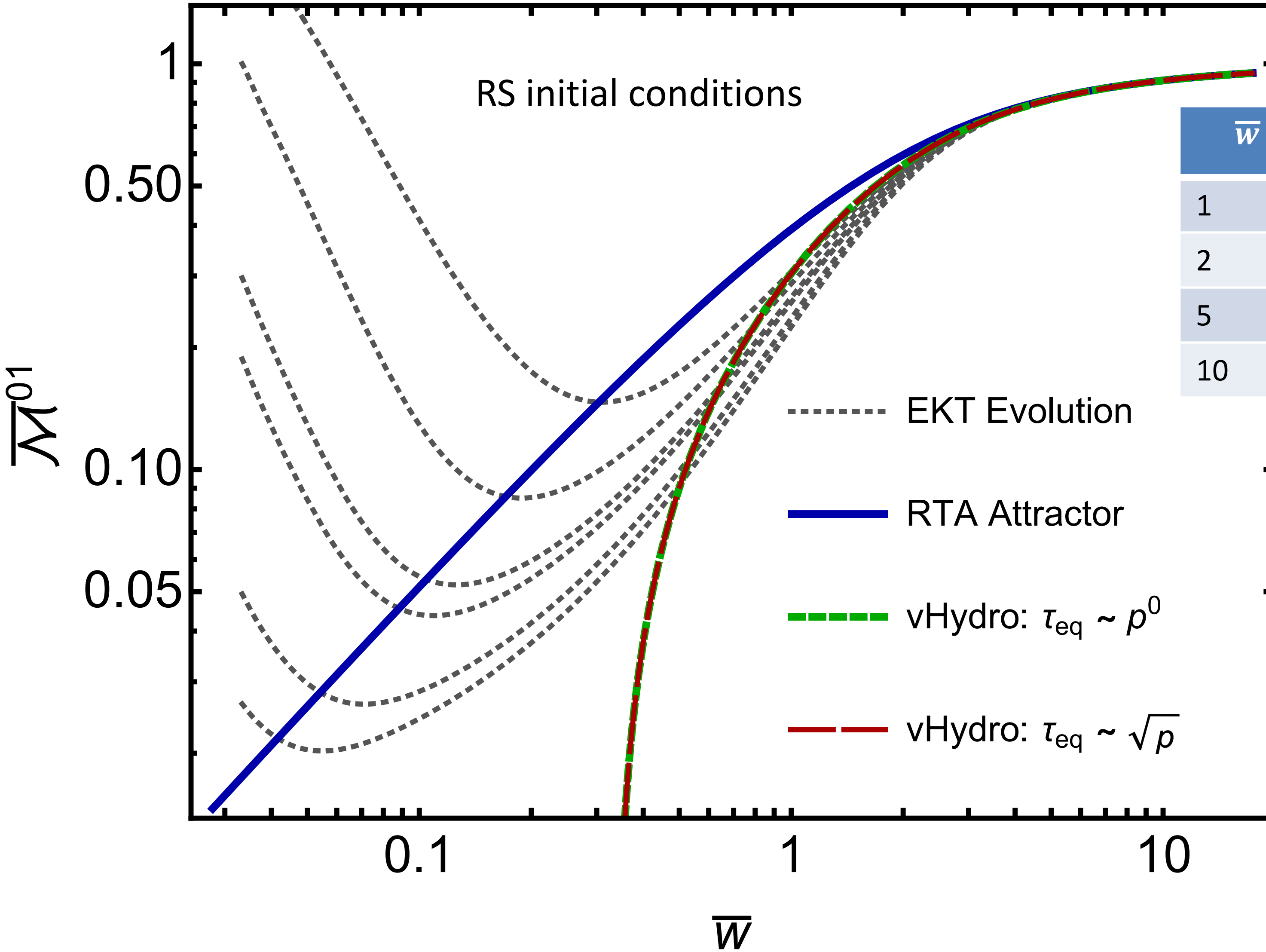
- Numerical implementation of pure glue AMY effective kinetic theory
- Includes elastic gluon scattering and gluon splitting with LPM and detailed balance.
- We use the “pure glue” EKT code of Kurkela and Zhu PRL 115, 182301 (2015)
- 250 x 50^2 grid in momentum space





Evidence for a QCD EKT attractor

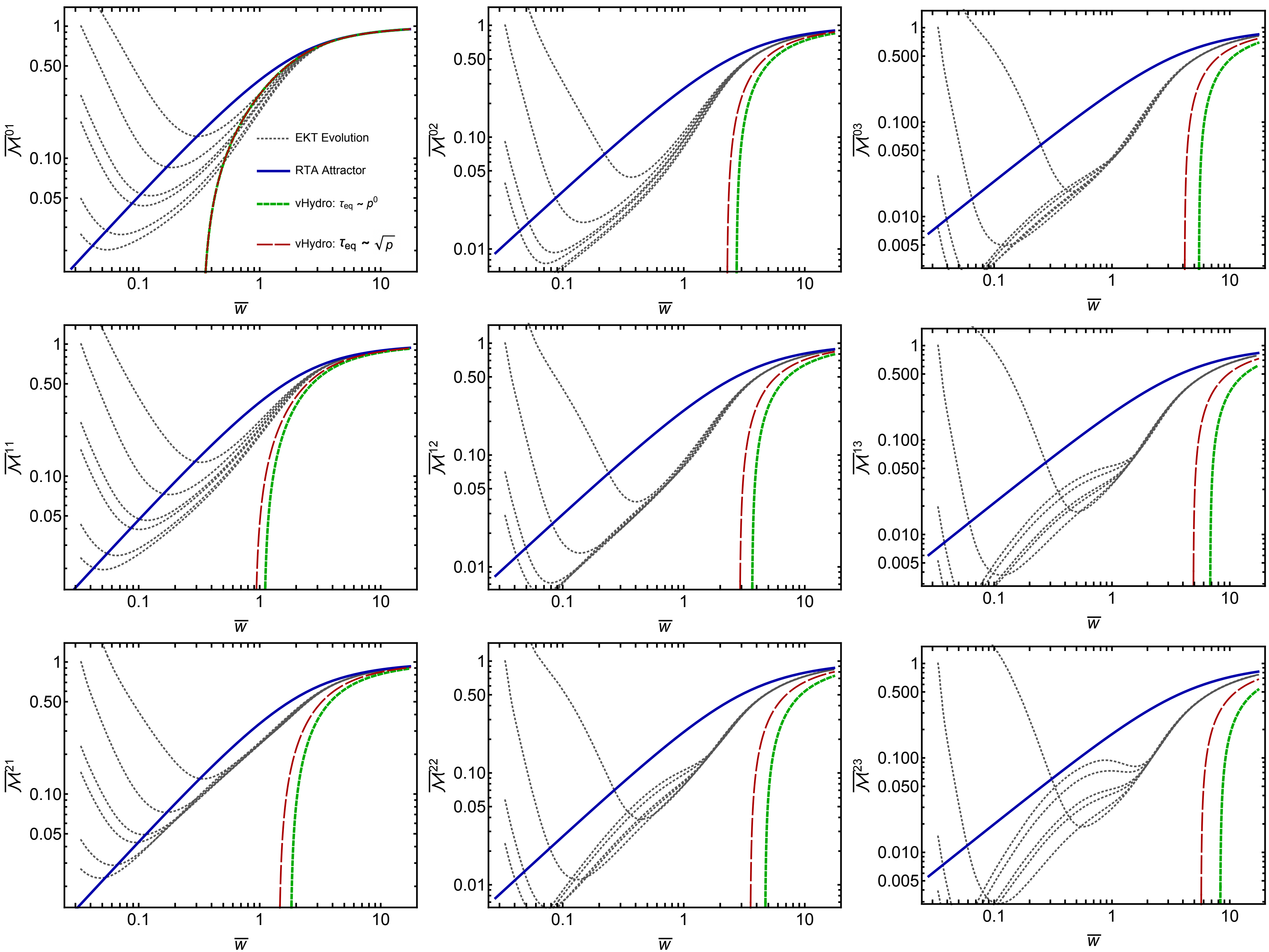
D. Almaalol, A. Kurkela, and M. Strickland, forthcoming.



$\overline{w}$	τ
1	0.5 fm/c
2	1.3 fm/c
5	4.9 fm/c
10	13.5 fm/c

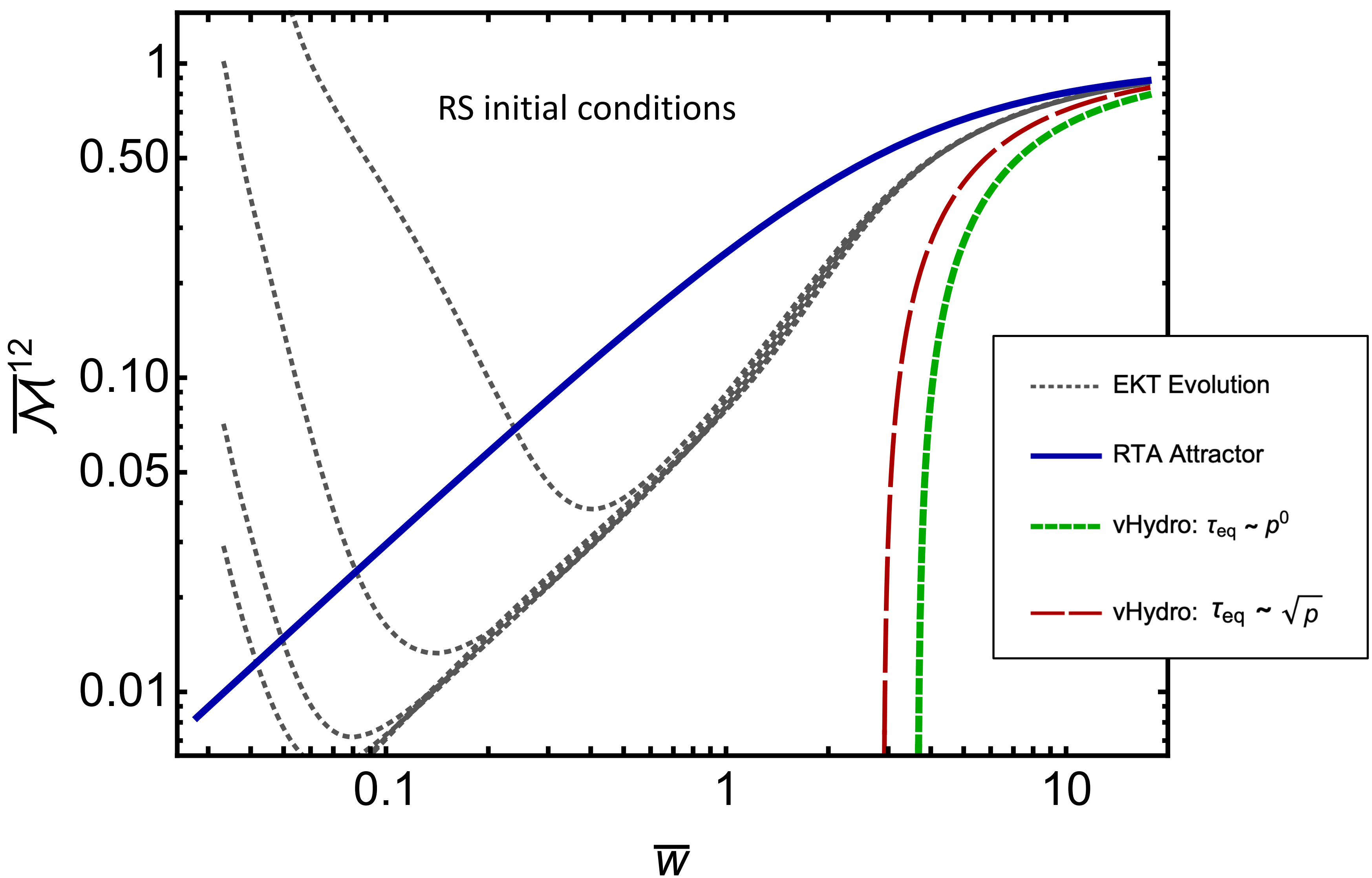
Evidence for a QCD EKT attractor

RS initial conditions



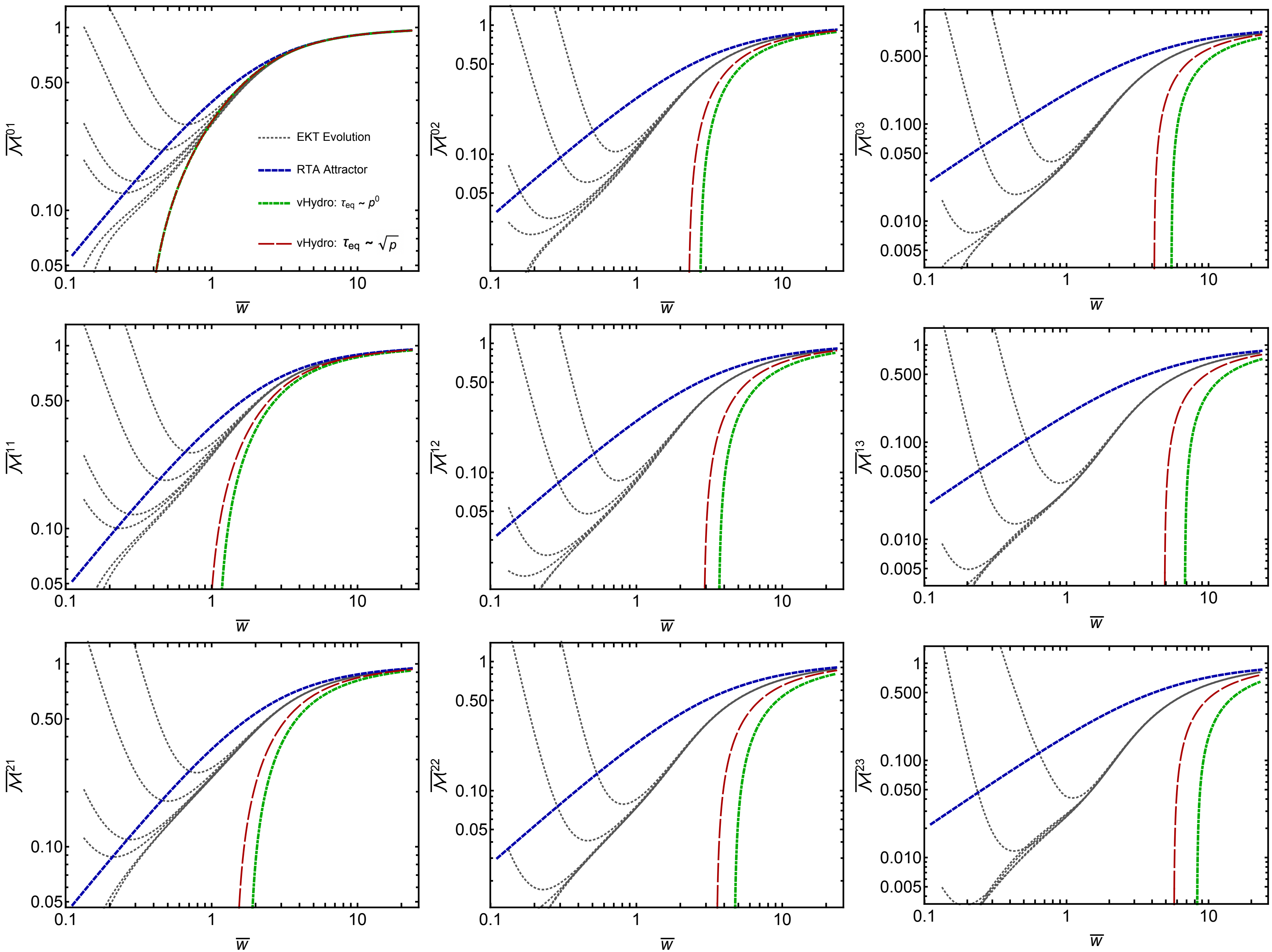
D. Almaalol, A. Kurkela, and M. Strickland, forthcoming.

Evidence for a QCD EKT attractor



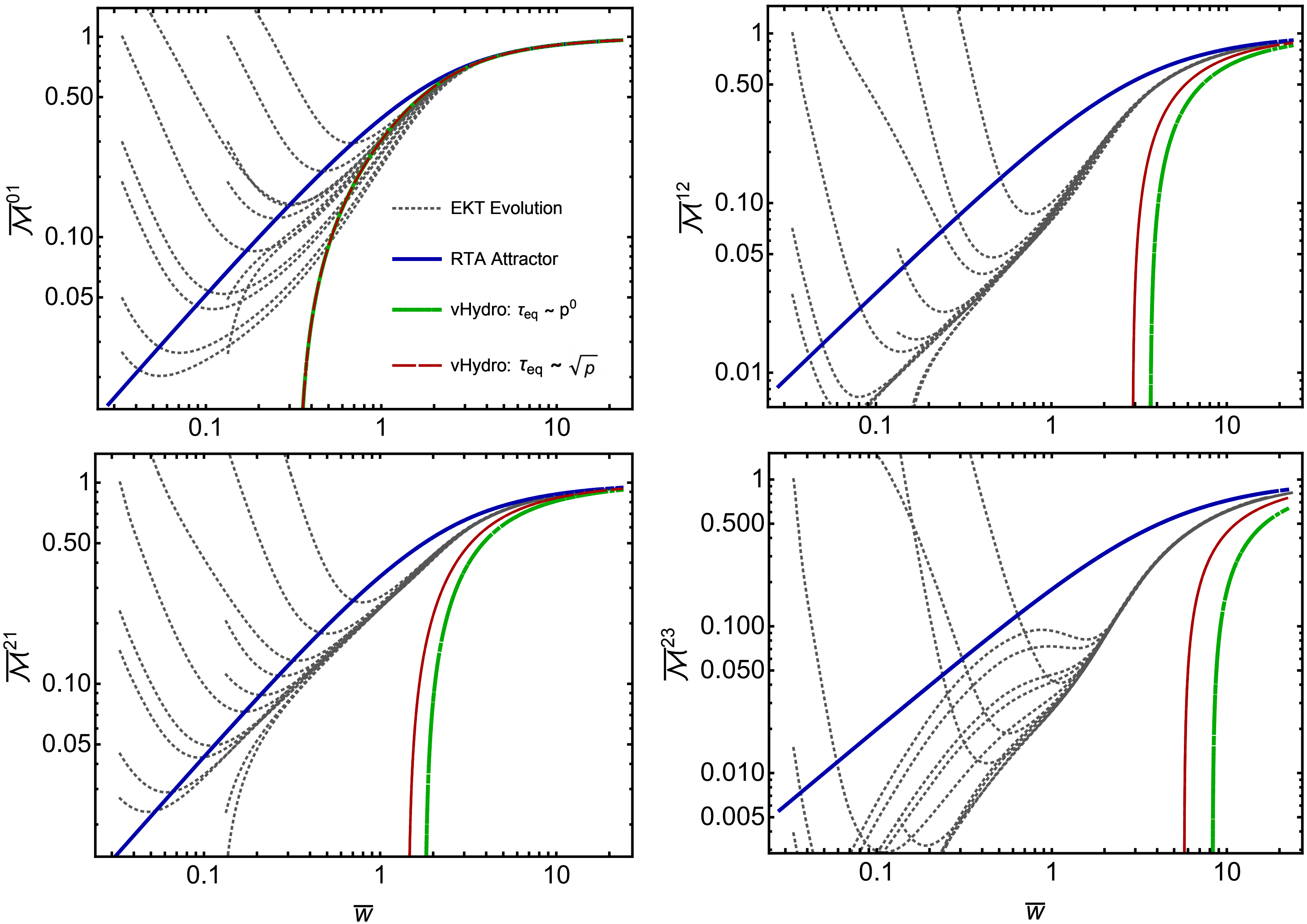
Evidence for a QCD EKT attractor

CGC (overoccupied) initial conditions



D. Almaalol, A. Kurkela, and M. Strickland, forthcoming.

Both types of initial conditions



D. Almaalol, A. Kurkela, and M. Strickland, forthcoming.

Conclusions

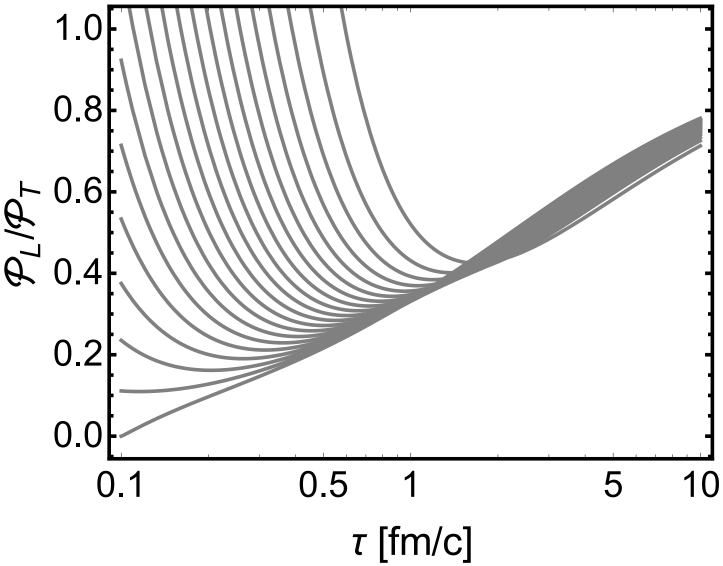
- Attractor for low-order moments well-approximated by hydro but <u>system is not in equilibrium</u> → hydrodynamization instead of thermalization
- RTA an EKT higher-order moments poorly described by standard viscous hydrodynamics
- There is, however, a fast convergence to a non-equilibrium attractor for higher moments → **<u>pseudo-thermalization</u> instead of hydrodynamization**

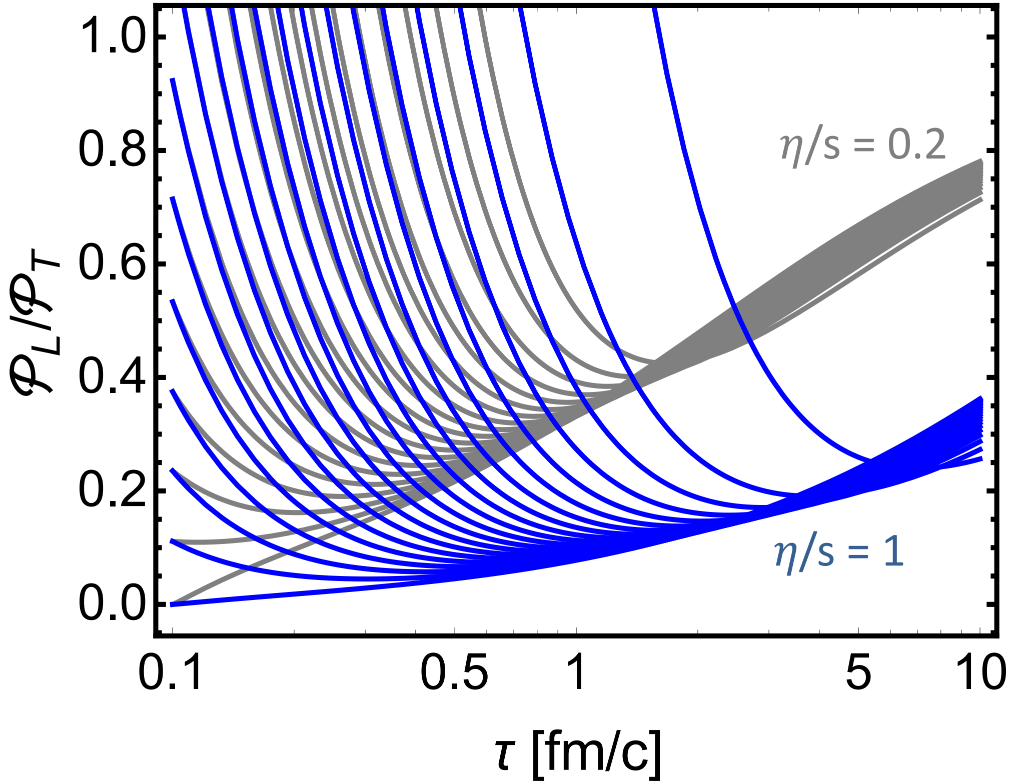
More conclusions

- Different models → different attractors and/or multiple basins of attractor
- Like RTA, EKT QCD has a "beyond hydrodynamics" attractor that seems to be independent of initial condition
- For EKT, we considered two types of initial conditions
- EKT early time attractor for low-order moments; higher-order moments show later "collapse"; all moments collapse @ $\overline{w}$ ~ 3
- Can we use properties of EKT attractor to improve hydro and freeze-out?

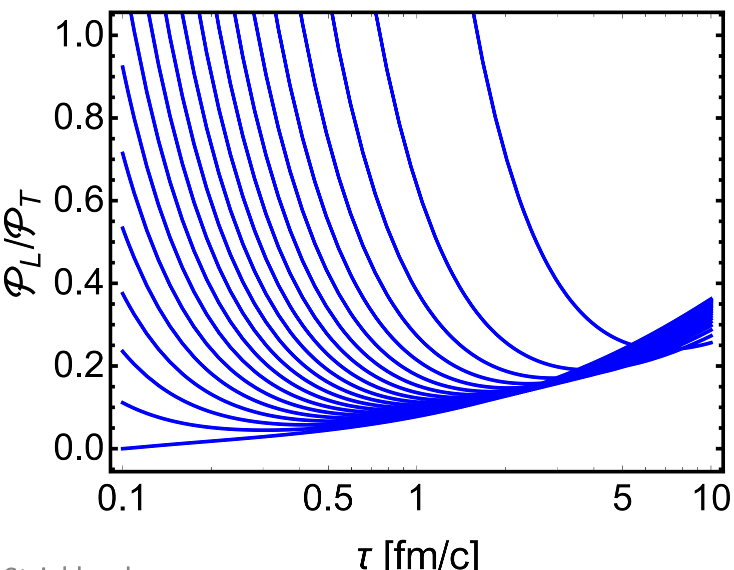
Backup Slides

Assuming η/s = 0.2 and T_0 = 500 MeV



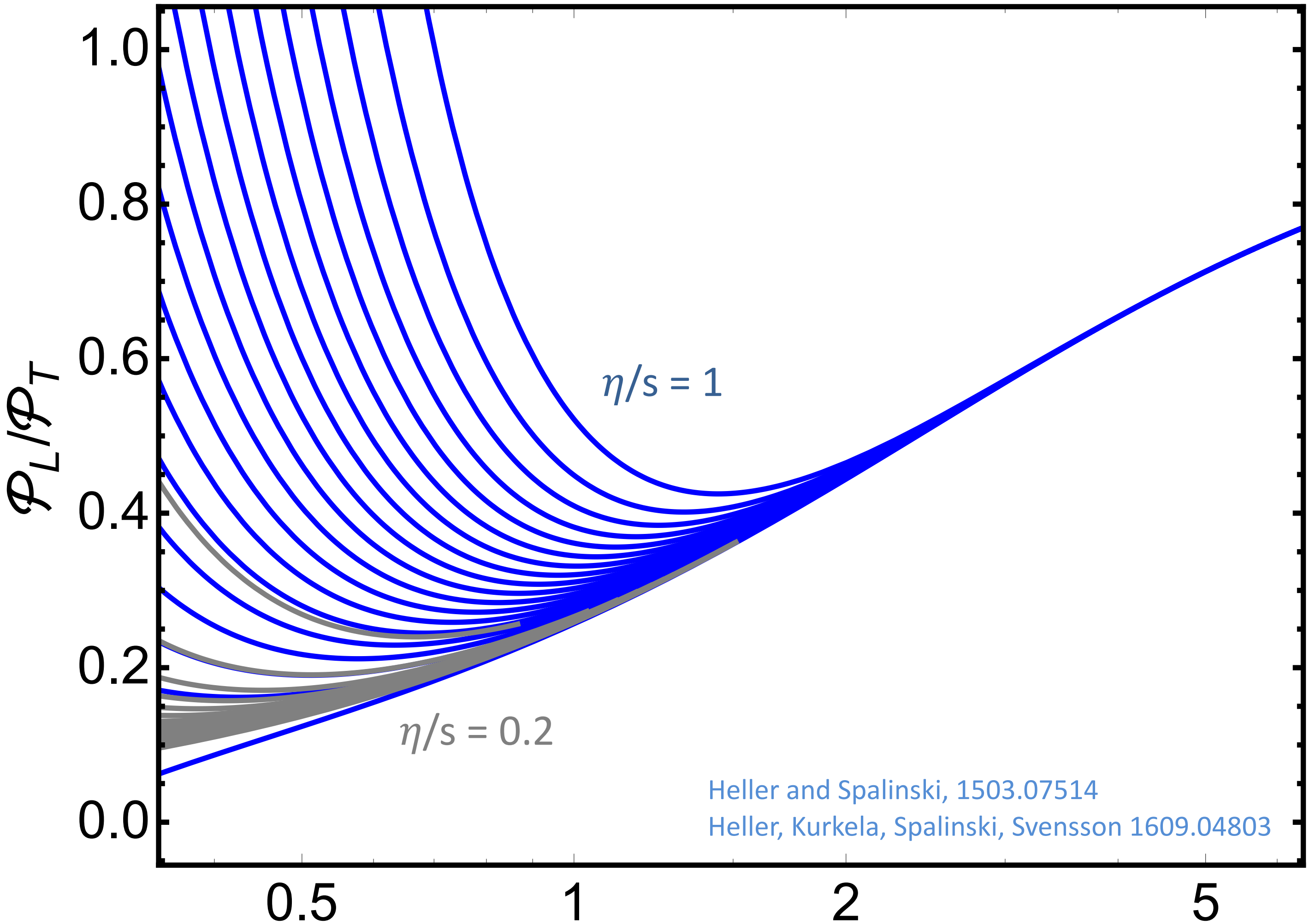


Assuming η/s = 1 and T_0 = 500 MeV



- Keep it simple: Bjorken 0+1d dynamics.
- Solve dynamical equations for different initial conditions and different values of the shear viscosity (gray vs blue)
- Hints of universal behavior at late times visible (similar levels of momentum anisotropy)

Collapsing the data to the attractor



$$\overline{w} = \frac{\tau}{\tau_{\rm eq}(\tau)} = \frac{\tau T(\tau)}{5\bar{\eta}}$$

Bjorken Expansion: Exact RTA Solution

- Simple model: Boost-invariant, transversally homogeneous Boltzmann equation in relaxation time approximation (RTA).
- Many results in this model, so we can compare with the literature.

Boltzmann EQ: $p^\mu \partial_\mu f(x,p) = C[f(x,p)]$

RTA: $C[f] = \frac{p_\mu u^\mu}{\tau_{\rm eq}} \left[f_{\rm eq}\Big(p_\mu u^\mu, T(x)\Big) - f(x,p) \right]$

Massless Particles
W. Florkowski, R. Ryblewski, and MS, 1304.0665 and 1305.7234

Massive Particles
W. Florkowski, E. Maksymiuk, R. Ryblewski, and MS, 1402.7348

Solution:

$$f(\tau, w, p_T) = D(\tau,\tau_0) f_0(w,p_T) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} D(\tau,\tau')\, f_{\rm eq}(\tau', w, p_T)$$

Time-dependent relaxation time $\quad \tau_{\rm eq}(\tau) = \frac{5\bar\eta}{T(\tau)}$	Damping Function $\quad D(\tau_2,\tau_1) = \exp\left[- \int_{\tau_1}^{\tau_2} d\tau\, \tau_{\rm eq}^{-1}(\tau) \right]$

0+1d RTA Exact Solution

$$T^4(\tau) = D(\tau,\tau_0) T_0^4 \, \frac{\mathcal{H}\left(\frac{\alpha_0 \tau_0}{\tau}\right)}{\mathcal{H}(\alpha_0)} + \int_{\tau_0}^{\tau} \frac{d\tau'}{2\tau_{\rm eq}(\tau')} \, D(\tau,\tau') \, T^4(\tau') \mathcal{H}\left(\frac{\tau'}{\tau}\right)$$

Once this integral equation is solved (by numerical iteration), we can construct the full one-particle distribution function f(τ,p) and we can compute general moments:

$$f(\tau,w,p_T) = D(\tau,\tau_0) f_0(w,p_T) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} \, D(\tau,\tau') \, f_{\rm eq}(\tau',w,p_T)$$

- GPL'd CUDA code: personal.kent.edu/~mstrick6/code
- Computes all moments and the full distribution function
- CUDA enables computationon very fine grids (N_tau ~ 4000, N_pt, N_pz ~ 500, 500).

$$\mathcal{M}^{nm}(\tau) = \frac{\Gamma(n+2m+2)}{(2\pi)^2} \Bigg[D(\tau,\tau_0) 2^{(n+2m+2)/4} T_0^{n+2m+2} \frac{\mathcal{H}^{nm}\left(\frac{\alpha_0 \tau_0}{\tau}\right)}{[\mathcal{H}^{20}(\alpha_0)]^{(n+2m+2)/4}} + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} \, D(\tau,\tau') \, T^{n+2m+2}(\tau') \mathcal{H}^{nm}\left(\frac{\tau'}{\tau}\right) \Bigg],$$

$$\mathcal{H}^{nm}(y) = \tfrac{2y^{2m+1}}{2m+1} \, {}_2F_1\!\left(\tfrac{1}{2}+m, \tfrac{1-n}{2}; \tfrac{3}{2}+m; 1-y^2\right).$$

Time scales

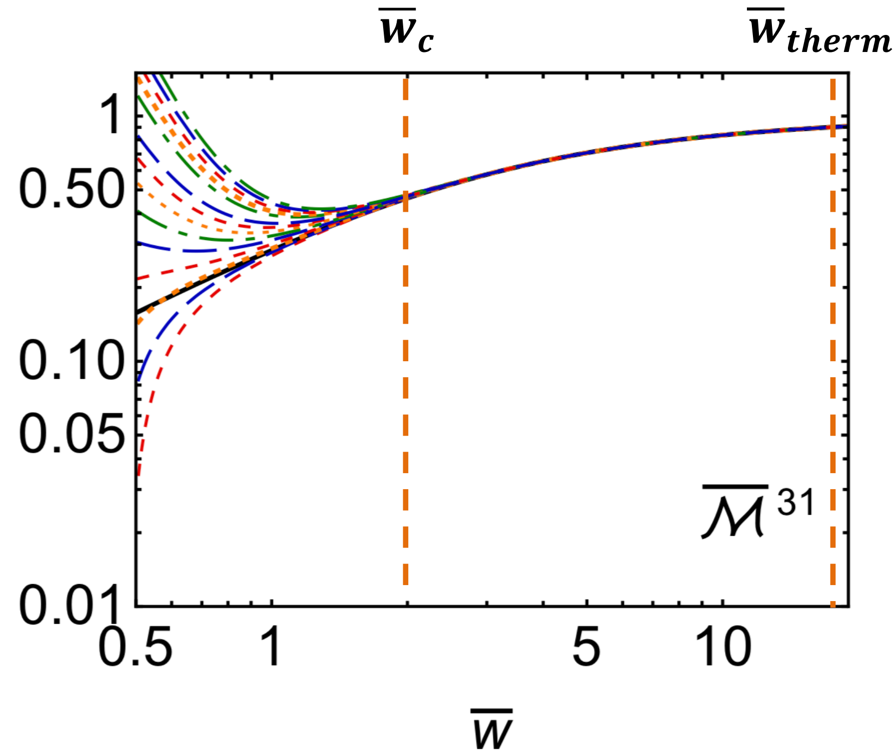
From each of these results, we can obtain

1. An estimate of the thermalization time by requiring that the scaled moment reaches 0.9.

   → **Thermalization time $\bar{w}_{therm}$**

2. An estimate of the time it takes each moment to approach its respective attractor solution to within a given tolerance.

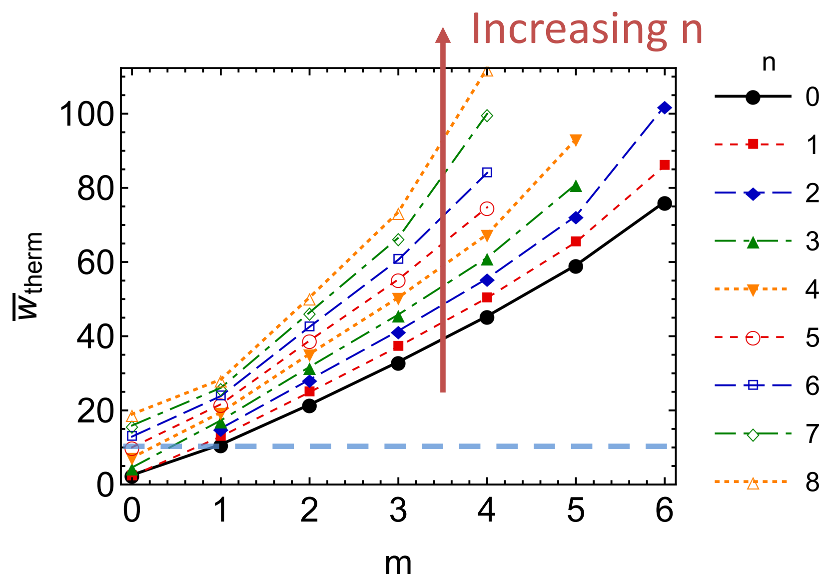
   → **Convergence (or pseudo-thermalization) time $\bar{w}_c$**

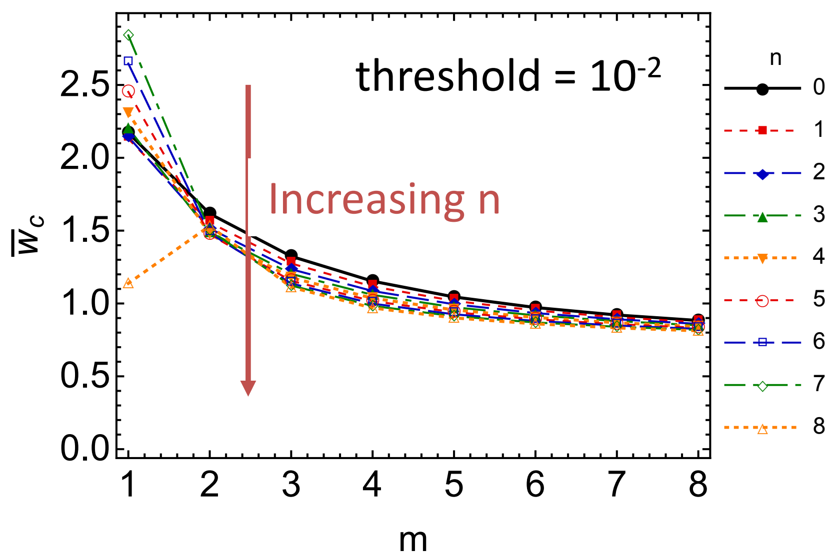


$\bar{w}$	τ
1	0.5 fm/c
2	1.3 fm/c
5	4.9 fm/c
10	13.5 fm/c

Assuming η/s = 0.2 and T = 500 MeV

Time scales



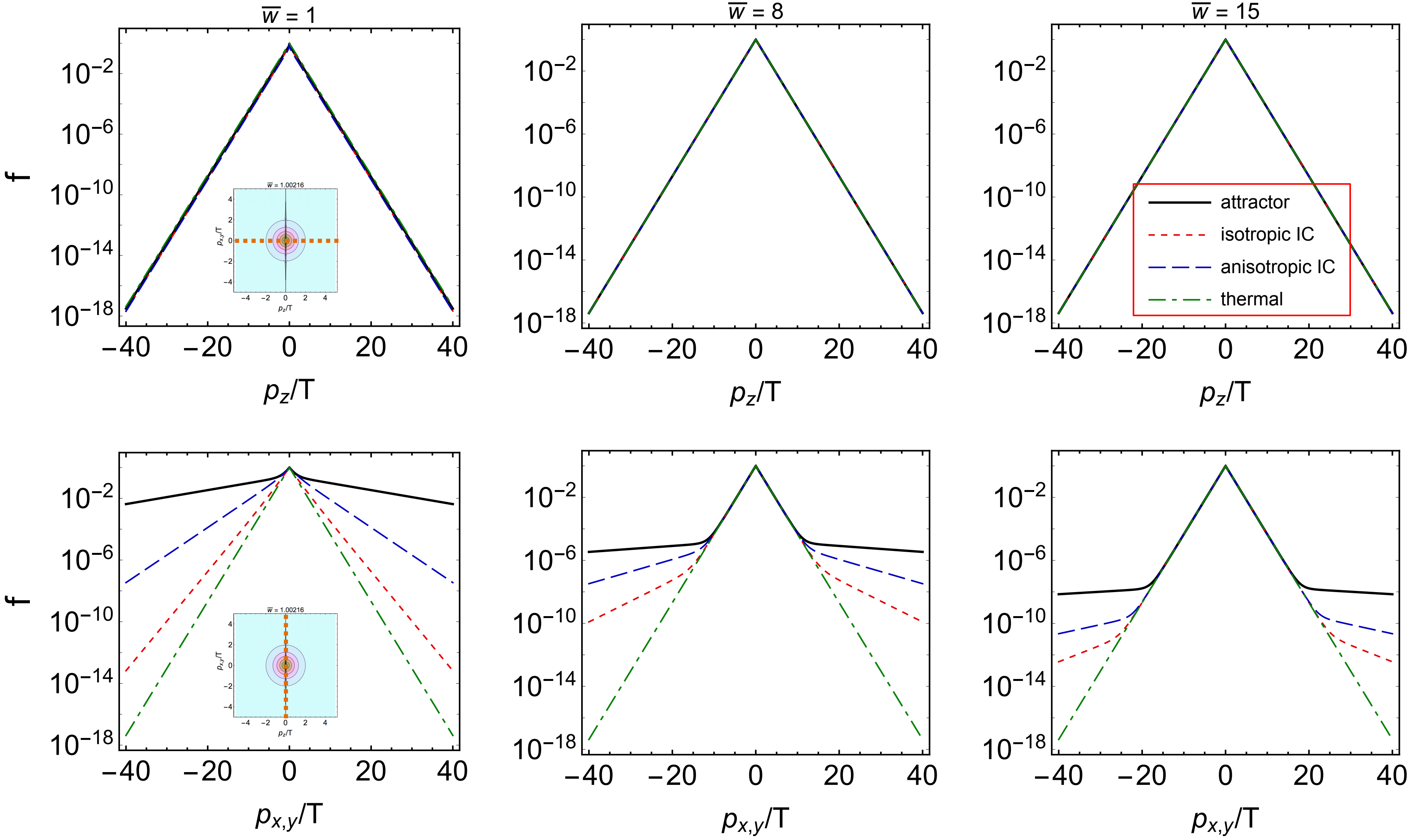


- As m or n increase the thermalization time increases
- For m>2 (n>1) convergence time decrease as n (m) increases
- **This demonstrates that at large n,m there is a parametric separation of scales**

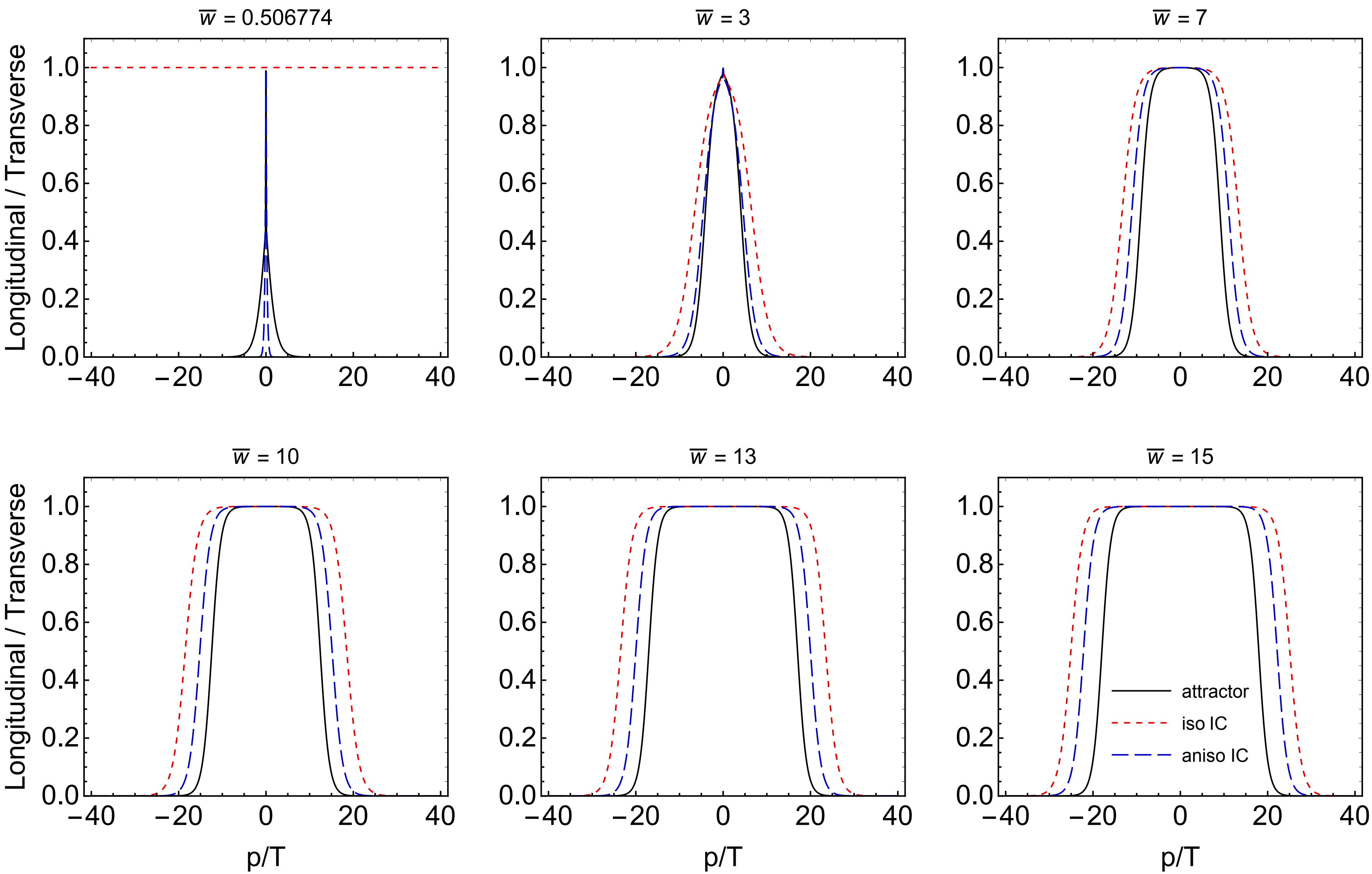
$\overline{w}$	τ
1	0.5 fm/c
2	1.3 fm/c
5	4.9 fm/c
10	13.5 fm/c

Assuming η/s = 0.2 and T = 500 MeV

Attractor distribution function



Isotropization front



How does one obtain the attractor?

- Let's look at hydrodynamics-like theories for simplicity (e.g. MIS, DNMR, aHydro, etc.)
- Start with the 0+1d energy conservation equation

$$\tau\dot{\epsilon} = -\frac{4}{3}\epsilon + \Pi \qquad \Pi = \Pi^{\varsigma}{}_{\varsigma}$$

- Change variables to

$$\boxed{w = \tau T} \qquad \boxed{\varphi(w) \equiv \tau \frac{\dot{w}}{w} = 1 + \frac{\tau}{4}\partial_\tau \log \epsilon}$$

$$\boxed{w\varphi \frac{\partial \varphi}{\partial w} = -\frac{8}{3} + \frac{20}{3}\varphi - 4\varphi^2 + \frac{\tau}{4}\frac{\dot{\Pi}}{\epsilon}}$$

How does one obtain the attractor?

- Need the evolution equation for the viscous correction.
- To linear order in the shear correction (e.g. MIS, DNMR) one has

$$\dot{\Pi} = \frac{4\eta}{3\tau\tau_\pi} - \beta_{\pi\pi}\frac{\Pi}{\tau} - \frac{\Pi}{\tau_\pi}$$

For DNMR in RTA $\beta_{\pi\pi} = \frac{38}{21}$

- Plugging this into the energy-momentum conservation equation gives

$$\boxed{\overline{w}\varphi\varphi' + 4\varphi^2 + \left[\overline{w} + \left(\beta_{\pi\pi} - \frac{20}{3}\right)\right]\varphi - \frac{4c_{\eta/\pi}}{9} - \frac{2}{3}(\beta_{\pi\pi} - 4) - \frac{2\overline{w}}{3} = 0}$$

$$\overline{w} \equiv \frac{w}{c_\pi} = \frac{\tau T}{5\overline{\eta}} \qquad c_{\eta/\pi} \equiv \frac{c_\eta}{c_\pi} = \frac{1}{5}$$

How does one solve for the attractor?

$$\boxed{\overline{w}\varphi\varphi' + 4\varphi^2 + \left[\overline{w} + \left(\beta_{\pi\pi} - \frac{20}{3}\right)\right]\varphi - \frac{4c_{\eta/\pi}}{9} - \frac{2}{3}(\beta_{\pi\pi} - 4) - \frac{2\overline{w}}{3} = 0}$$

$$\overline{w} \equiv \frac{w}{c_\pi} = \frac{\tau T}{5\overline{\eta}} \qquad c_{\eta/\pi} \equiv \frac{c_\eta}{c_\pi} = \frac{1}{5}$$

- First try to approximate using “slow-roll” approx ($\varphi' = 0$)
- From this, we can read off the boundary condition as $w \to 0$

$$\lim_{\overline{w}\to 0} \varphi(\overline{w}) = \frac{1}{24}\left(-3\beta_{\pi\pi} + \sqrt{64c_{\eta/\pi} + (3\beta_{\pi\pi} - 4)^2} + 20\right)$$

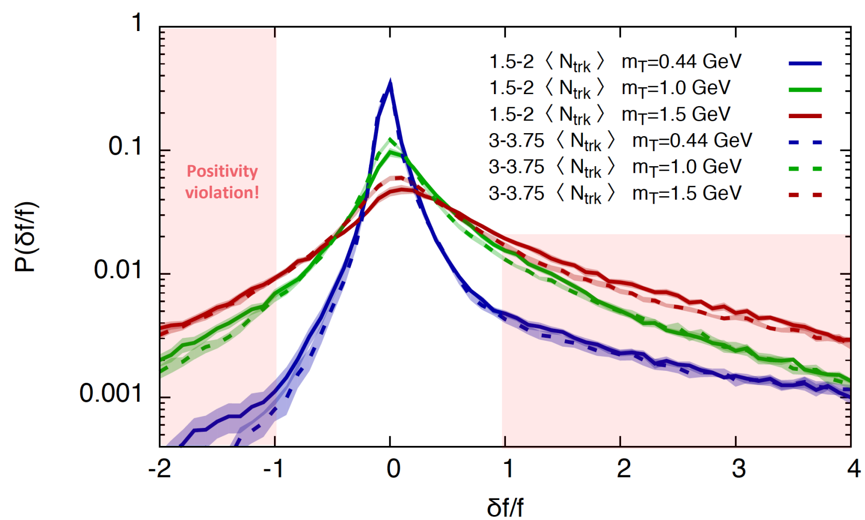
- Then numerically solve the ODE at the top of the slide

Are the viscous corrections under control?

p-Pb with IP-Glasma + MUSIC + URQMD

H. Mäntysaari, B. Schenke, C.Shen, P. Tribedy, PLB 772, 681 (2017)

> $v_n(p_T)$, however, studying the distribtution of corrections relative to the thermal distribution from all freeze-out surface cells, we find a significant share (10% for $p_T \sim 0.45\,\mathrm{GeV}$, 25% for $p_T \sim 1\,\mathrm{GeV}$, 45% for $p_T \sim 1.5\,\mathrm{GeV}$ for pions) of large (shear) corrections ($|\delta f|/f \gtrsim 100\%$). This demonstrates that our results are plagued by large viscous corrections, in particular for $p_T \gtrsim 1\,\mathrm{GeV}$. Nevertheless, p_T-integrated quantities are dominated by low p_T contributions and less sensitive to these problems.



- In small systems, no.
- **Short lifetime + large viscous corrections at freezeout → large δf corrections on freeze-out hypersurface**
- As a result, simulations suffer from negative effective pressures <u>and f</u> in a large hypervolume.
- Groups deal with this differently. SONIC, for example, uses an "exponentiation trick" introduced by Pratt and Torrieri in PRC 82, 044901 (2010) to prevent $f < 0$ on the switching surface (still large correction).