Symmetries of Surfaces, Maps and Dessins

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1 Introduction

The main objective of this workshop was to bring together a number of leading and emerging researchers in the area of study of actions of discrete groups on Riemann and Klein surfaces and algebraic curves, and related topics such as symmetric embeddings of graphs and dessins d'enfants on surfaces. It followed up on a meeting held at the CIEM in Spain in 2010.

The last two decades have seen a burgeoning of activity in these fields, with various strands coming together, exploiting the linkages established by Belyi and Grothendieck, and some increasingly useful techniques from combinatorial and computational group theory. In particular, computational experiments and searches have produced a wealth of examples (either for small genera or infinite families of a particular type), and these serve as a useful test-bed for conjectures and potential new approaches.

2 Workshop format

The workshop began with introductory lectures, one on each of the eight themes, presented by experts in the area. These addressed recent developments and described important open problems, and possible approaches to answering them. Then the participants formed groups to work separately on the first four themes from late morning on the Tuesday to lunchtime on the Wednesday, and rearranged into new groups to work on the other four themes from the Wednesday afternoon to the Friday morning.

Also during the week an opportunity was given for open problems to be presented, and ten short talks were presented by workshop participants on topics related to the workshop themes, on the Monday, Tuesday and Wednesday afternoons. A conference excursion was made to Lake Louise on the Thursday afternoon, and enjoyed by all who took part in that.

3 Themes

Introductory talks on the main themes of the workshop were given on the first two days, as follows:

- Theme 1: *Regular and edge-transitive maps*,
  introduced by Marston Conder and Tom Tucker

- Theme 2: *Dessins d'enfants, Belyi's theorem and Galois action*,
  introduced by Gabino González-Diez and Ernesto Girondo
- Theme 3: *Defining equations for Riemann surfaces*,
  introduced by Allen Broughton and Aaron Wootton
- Theme 4: *Polytopes, hypertopes, maps and maniplexes*,
  introduced by Dimitri Leemans
- Theme 5: *Cayley maps and skew morphisms*,
  introduced by Robert Jajcay
- Theme 6: *n-gonal automorphisms of Riemann surfaces*,
  introduced by Mariela Carvacho
- Theme 7: *External symmetries of regular maps*,
  introduced by Jozef Širáň
- Theme 8: *Pseudo-real surfaces, and large automorphism groups of surfaces*,
  introduced by Javier Cirre and Grzegorz Gromadzki.

4 Working groups

Below we report on the activities of the eight working groups that were formed during the workshop. The first four convened early in the week, with two of them merging together, and the last four convened in the second half of the workshop.

4.1 Regular and Edge-Transitive Maps

[Workshop leaders: Marston Conder, Jozef Širáň, and Tom Tucker]

By a 'map' we mean a 2-cell embedding of a connected graph or multigraph on a closed surface (which may be orientable or non-orientable). Such a map M is *edge-transitive* if its automorphism group $\mathrm{Aut}(M)$ acts transitively on edges. Graver and Watkins [5] introduced a classification of such maps into 14 types, according to the local effects of certain generators of the automorphism group. Then Širáň, Tucker and Watkins [10] showed how to construct orientable maps of each type from certain presentations satisfying an 'index two subgroup and a 'forbidden automorphism' condition, and used this construction to give finite orientable maps of each type with $\mathrm{Aut}(M) \cong S_n$ for certain n.

Recently Jones [7] extended this work to surfaces with boundary, and among other things showed that all but a small number of simple groups occur as the automorphism group of various edge-transitive maps of all 14 types (with all of these maps being chiral or non-orientable). Gareth Jones was a co-organiser of this workshop, but sadly was unable to attend, and so we made available to participants a copy of a talk that he gave on edge-transitive maps at a BIRS meeting in Oaxaca in August 2017.

There have been only a few papers on edge-transitive maps published since 2001. Many questions have been suggested by some of the above people and others for further study: classifying all such maps for a given group or family of groups, or on a given surface, or for a given underlying graph or family of graphs; determining the relative frequency of each type; and seeking generalisations, say to polytopes.

At this workshop, the working group spent time reviewing the 14 types of edge-transitive maps, including important details about (a) the automorphism group $G = \mathrm{Aut}(M)$ with marked generators as a quotient of some universal group U for the appropriate type, (b) the image G^+ of the orientation-preserving subgroup U^+ of U and its meaning for orientability of the map, (c) the nature of forbidden automorphisms permuting the marked generators and their inverses, and (d) the roles of duality and Petrie duality.

The working group then focused on one of the open problems described in the opening presentation on the first day, namely *Classifying the edge-transitive maps for which the subgroup G^+ is abelian.* This problem turned out to be a great deal more complicated than the presenters had thought, but was highly illuminating. The discussion concentrated mostly on a small number of the 14 types, and eventually led to the discovery that there are no ET maps with abelian G^+ for six of the 14 types, and there do exist such maps for six of the other eight types. Two of the 14 types remained unresolved.

This problem provided an excellent introduction to the study of edge-transitive maps, and clarified a number of issues for the main presenters — indeed so much so that a publication on the findings is likely (once the investigations are completed).

4.2 Belyi Theory

[Workshop leaders: Allen Broughton, Gabino González-Diez and Aaron Wootton]

This theme was combined with 'Defining Equations for Surfaces' — see the next subsection.

4.3 Defining Equations for Surfaces

[Workshop leaders: Allen Broughton, Gabino González-Diez and Aaron Wootton]

(a) *Defining a surface.*

In both of these themes, each surface S of interest typically has either a suitably 'large' group of automorphisms G, with quotient surface S/G having genus 0, or a 'nicely structured' branched covering $\pi : S \to \mathbb{P}^1$ (n-gonal morphism). Such surfaces may be constructed in two different ways.

(b) *Defining equations.*

Let $f_1, \ldots, f_{N-1}$ be homogeneous polynomials in $X \in \mathbb{P}^N$ of varying degrees. In general, the algebraic variety

$$V(f_1, \ldots, f_{N-1}) = \left\{X \in \mathbb{P}^N : f_1(X) = \cdots = f_{N-1}(X) = 0\right\} \tag{1}$$

will be an irreducible, complex algebraic curve, and its normalisation will be a smooth surface. The desirable case is a plane curve where $N = 2$.

(c) *Group actions and monodromy groups.*

A conformal G-action on S is described by the set of points $\{Q_1, \ldots, Q_t\} \subset \mathbb{P}^1$ over which $\pi : S \to S/G = \mathbb{P}^1$ is ramified, and a t-tuple $(c_1, \ldots, c_t)$ of elements of G, called a *generating vector*, satisfying

$$G = \langle c_1, \ldots, c_t \rangle \text{ and } c_1 c_2 \cdots c_t = 1. \tag{2}$$

The t-tuple $(c_1, \ldots, c_t)$ is determined by the monodromy of $\pi : S \to S/G$.

More generally, given any branched cover $\pi : S \to \mathbb{P}^1$ of degree n, ramified over $\{Q_1, \ldots, Q_t\}$, a transitive monodromy group $M_\pi \leq \Sigma_n$ is determined by a monodromy system $(p_1, \ldots, p_t)$:

$$M_\pi = \langle p_1, \ldots, p_t \rangle \text{ with } p_1 p_2 \cdots p_t = 1. \tag{3}$$

The cycle structures of the permutations p_i are determined by the cycle structures of the exceptional fibres $\pi^{-1}(Q_i)$. Conversely, given points $\{Q_1, \ldots, Q_t\}$ and systems as in (2) or (3), a branched covering surface S may be defined. This is a typical starting point for defining S, and we want to determine a defining equation for S as in (1).

(d) *Quasi-platonic and Belyi surfaces.*

Currently there is intense interest in quasi-platonic surfaces, Belyi surfaces, and their dessins d'enfant. Quasi-platonic surfaces form the G-action case with $t = 3$, and Belyi surfaces include all monodromy groups with $t = 3$. Both types of surfaces have equations defined over number fields, and the absolute Galois group acts on both the surfaces and their dessins.

(e) *Questions.*

At the workshop, the following questions were posed:

- Given a group action of G or monodromy group M_π with generating systems as in (2) and (3), is there an algorithm to find defining equations as in (1)? The cyclic case is easy and well known. In the important Belyi surface case, Monien [9] and Voight et al [12] have developed computationally intensive methods for determining equations.
- In the quasi-platonic case, can the G-action be used to speed up computation in the methods of Monien and Voight? Are there estimates for the running time of these algorithms?

- Can a set of defining equations as (1) be found so that the G-action is induced by a linear action of G on the ambient space $\mathbb{P}^N$? See [11].
- What are the defining equations of a curve S with cyclic group of automorphisms G such that S/G has genus 1? This generalises the cyclic n-gonal case.

(f) *Discussion, outcomes, and further work.*

The easy case of cyclic n-gonal surfaces was presented and discussed. The easy examples of explicit equations of higher genus Belyi surfaces have almost all been determined and exploited. Further work on Galois action on dessins needs a good library of examples.

Hartmut Monien and John Voight gave extended presentations of their work (as mentioned above) on constructing a defining equation for any monodromy triple given in (3). The monodromy triple is used to compute a fundamental region for a Fuchsian group Π such that $\mathbb{H}/\Pi \cong S$. One then works with modular forms for Π, using either numerical linear algebra or numerical PDEs.

The working group enthusiastically supported the future construction of a widely available data base of equations for those surfaces of low genus or those defined by monodromy systems (generating vectors) of interesting groups.

4.4 Maniplexes and Incidence Geometries

[Workshop leader: Dimitri Leemans]

This working group aimed to gain a better understanding of the link between maniplexes and incidence geometries, and to develop a kind of 'dictionary' that would permit people working in these fields to understand each other's research.

Steve Wilson introduced maniplexes as a means of constructing maps, either by drawing a structure on a surface, or by assembling polygons (just as a 4-cube can be constructed by gluing 3-cubes along faces).

An $(n+1)$-*maniplex* is a set Ω of *flags*, together with an ordered set $R = [r_0, \ldots, r_n]$ of sets of pairs of flags, where each r_i may be seen as an involution permuting the flags. Then for $0 \leq i < n$, an i-face is a connected component of $R_i = \cup_{j \neq i} r_j$, and the 0-faces are called the vertices, the 1-faces called the edges, and the n-faces called the facets. In particular, each facet is itself a maniplex.

The flag-graph of any polytope is a maniplex, but there are many other kinds of examples. A 1-maniplex is just a set Ω of cardinality 2 (and is the flag graph of the graph K_2, while a 2-maniplex is a polygon, and a 3-maniplex may be viewed as a transitive action of a non-degenerate string group.

The working group focused on understanding maniplexes as chamber systems of incidence geometries obtained from polytopes. The group also worked on ways of removing the 'string' condition from the definition of maniplexes, and developing a notion of hyperplexes which could generalise hypermaps. This work is still in progress, and is likely to enable people working on maniplexes to exploit research on incidence geometries, and vice-versa.

4.5 Cayley Maps and Skew Morphisms

[Workshop leader: Robert Jajcay]

A *skew-morphism* of a group A is a permutation φ of A preserving the identity and a function $\pi : A \to \mathbb{Z}_{|\varphi|}$, called the *power function* associated with φ, satisfying the property $\varphi(ab) = \varphi(a)\varphi^{\pi(a)}(b)$ for all $a, b \in A$. Skew-morphisms were originally introduced for the study of regular Cayley maps, but eventually became significant in the study of complementary cyclic group extensions.

An orientable map $\mathcal{M}$ is called *orientably-regular* if for every pair of arcs of $\mathcal{M}$ there exists an orientation-preserving automorphism of $\mathcal{M}$ that takes the first arc to the second. A *Cayley map* $CM(A, X, p)$ is an embedding of a connected Cayley graph $C(A, X)$, that has the same local orientation p at each vertex. All left multiplications within the Cayley group A induce automorphisms of the Cayley map, and many of the well-known families of orientably-regular maps turn out to be Cayley maps. A Cayley map $CM(A, X, p)$ is regular if and only if there exists a skew-morphism φ of A with the property that $\varphi(x) = p(x)$ for all $x \in X$. Thus regular Cayley maps on A correspond to orbits of skew-morphisms of A that generate A and are closed under inverses.

In the context of group extensions, if A is a group and φ is a skew-morphism of A, then the *skew-product* of A and $\langle\varphi\rangle$ is defined as $G = (A \times \langle\varphi\rangle, *)$ under the operation $(a, \varphi^i) * (b, \varphi^j) = (a\varphi^i(b), \varphi^{\sigma(i,j)})$, for a suitably defined function $\sigma(i, j)$. This product is a group, with a complementary factorisation $G = AY$. Conversely, if G is any finite group with a complementary factorisation $G = AY$ where Y is cyclic, then the quotient $\overline{G} = G/\mathrm{Core}_G(Y)$ is a skew product group associated with the skew morphism φ. Hence the classification of skew-morphisms of a finite group A allows for classification of all regular Cayley maps for A, as well as all complementary extensions of a group G in the form $G = AY$ with Y cyclic.

Much effort has been devoted recently to classifying skew-morphisms of various classes of groups. Classifications have been achieved for finite abelian groups of odd prime-power order, elementary abelian 2-groups, and groups whose order is a product of two primes. (Various product results have also been found.) It was announced at the workshop that the classification of skew-morphisms of finite dihedral groups has been completed. The classification for all abelian finite groups now seems within reach. On the other side of the spectrum, the classification of skew-morphisms of finite simple groups was also announced.

Also discussed at the workshop was a generalised definition of skew-morphisms, related to the concept of a generalised Cayley map (which is a map admitting a group of automorphisms that acts regularly on vertices). A suitable definition has been found, and basic properties have been determined. The concept appears to be related to the theory of 2-extensions of groups.

4.6 n-gonal Surfaces

[Workshop leaders: Mariela Carvacho and Aaron Wootton]

An n-gonal morphism of a Riemann surface S is map $\phi : S \to \mathbb{P}^1$ of degree n, and an n-gonal surface is one that exhibits such a ϕ. When ϕ is regular, we say that S is regular with group of deck transformations $\mathrm{deck}(\phi) = G$. Literature on n-gonal morphisms is scant, most being limited to regular morphisms with $G = \langle\sigma\rangle$ cyclic, and the so-called *cyclic n-gonal surfaces*) Most of the discussion at and after this workshop focused on generalising known theory of cyclic n-gonal surfaces to other regular n-gonal surfaces.

(a) *The cyclic n-gonal case.*

A complete picture is known for cyclic n-gonal surfaces and their automorphism groups when n is a prime number. This was made possible through a consequence of the Castelnuovo–Severi theorem, which states that if the genus g of a cyclic p-gonal surface S satisfies $g > (p-1)^2$, then $G = \langle\sigma\rangle$ is normal in the full automorphism group of S. When n is not prime, additional assumptions on the automorphism group, or on the branching data of the quotient map $\phi : S \to S/\langle\sigma\rangle = \mathbb{P}^1$ yield a similar inequality. In particular, a similar complete description appears to be tractable.

(b) *Strong branching.*

Much of the above theory depends on the concept of 'strong branching' and the normality of G. A covering $f : S_1 \to S_2$ of Riemann surfaces of degree n is said to be *strongly branched* if $g_1 > n^2 g_2 + (n-1)^2$, where g_i is the genus of S_i (for $i = 1, 2$). Accola proved that when f is a strongly branched regular map with $\mathrm{deck}(f) = G$ and G is simple, then $G \lhd \mathrm{Aut}(S_1)$.

(c) *Questions.*

At the workshop, the following questions were posed:

- What conditions can be imposed on G to ensure that $G \lhd \mathrm{Aut}(S)$, and in particular, what weakening of 'strongly branched' can be used?
- Given a regular n-gonal surface S, how can $\mathrm{Aut}(S)$ be computed for non-normal G?

(d) *Discussion, outcomes and further work.*

- It was proved that if the core of G in $\mathrm{Aut}(S)$ is trivial, then $G \lhd \mathrm{Aut}(S)$ when ϕ is strongly branched.
- It was agreed that a list should be built of examples of pairs $G < \mathrm{Aut}(S)$ where S/G has genus 0, to help provide a conjectural picture.
- Kay Magaard explained the work of Guralnick and Thompson on monodromy groups of rational maps. It was envisioned that this could be used as a tool in answering the second question in (c) above.

4.7 External Symmetries of Regular Maps

[Workshop leader: Jozef Širáň]

A map M on a surface is called *regular* if its automorphism group $\mathrm{Aut}(M)$ acts transitively on incident vertex-edge-face triples. Every such map has constant valency m and constant face-size ℓ, and this pair of numbers defines the *type* of M. Also every such M may be identified with a presentation for its automorphism group G in the form $G = \langle\, a, b, c \mid a^2 = b^2 = c^2 = (ab)^\ell = (bc)^m = (ca)^2 = \ldots = 1 \,\rangle$, as a quotient of the full $(\ell, m, 2)$-triangle group, and denoted by the symbol $M(G; a, b, c)$.

This working group explored duality, Petrie-duality and combinations of these, as well as *exponents* for self-invariance under the Coxeter 'hole' operators. Together, these concepts are sometimes colloquially called *external symmetries* of regular maps, and a regular map of type (ℓ, m) is said to be *kaleidoscopic* if it admits all exponents in the multiplicative group of units mod m.

A regular map $M(G; a, b, c)$ is self-dual if its group G admits an automorphism that swaps a with c while fixing b, and self-Petrie-dual if G admits an automorphism that swaps a with ac while fixing b and c, and kaleidoscopic if for every unit j mod m the group G has an automorphism that fixes a and c while taking b to $(bc)^j c$. Then further, M is said to have *trinity symmetry* (or to be completely self-dual, or 'self-everything') if it is both self-dual and self-Petrie-dual, and to be *super-symmetric* if it is kaleidoscopic and has trinity symmetry as well.

In his introductory lecture, Jozef Širáň cited four research-driving questions:

(a) Does there exist a completely self-dual regular map of valency n for every odd $n \geq 5$?

(b) Does there exist a kaleidoscopic completely self-dual regular map of valency n for every odd $n \geq 5$?

(c) What kind of structure has the external symmetry group of a kaleidoscopic completely self-dual regular map?

(d) Is it true that for every $m \geq 3$ and every subgroup U of $C_m^* \times C_2$, there exists a non-orientable regular map of valency m with exponent group U?

Affirmative answers to the first two questions for all even valencies are already known, and the analogue of (d) in the orientable case was shown to be true by Conder and Širáň in [3]. Also for valency 8 it was also shown by Conder, Kwon and Širáň [4] that the order of the external symmetry group can be larger than any pre-assigned positive integer.

The working group focused on Question (a). Because explicit generating triples of involutions can be found in the case where $G = PSL(2, q)$ or $G = PGL(2, q)$, it was suggested that suitable candidates for completely self-dual regular maps of arbitrary valency $m \geq 5$ should be sought in these families of groups. Marston Conder checked and confirmed the feasibility of such an approach for odd valencies 5 to 17 (with the help of the MAGMA computer system [2]), and he and Steve Wilson and Jozef Širáň made further suggestions about how this problem might be approached algebraically. The group also had a brief discussion about a possible approach to Question (b), using parallel products (which were developed several years ago by Steve Wilson and recently re-considered by Gareth Jones).

Some of the participants in this working group will take these investigations further.

4.8 Pseudo-real Riemann Surfaces, and Large Group Actions on Surfaces

[Workshop leaders: Javier Cirre and Grzegorz Gromadzki]

A compact Riemann surface X is said to be *pseudo-real* if it admits anti-conformal automorphisms but no such automorphisms of order 2. Any such surface lies in the so-called *real moduli* of the moduli space of compact Riemann surfaces, but cannot be defined by real polynomials. There is no pseudo-real surface of genus 0 or 1, but it has been known since 2010 that there exist pseudo-real surfaces of genus g for every $g \geq 2$.

An important aspect of current research on pseudo-real surfaces has to do with the largest order $M(g)$ of the full automorphism group of a pseudo-real surface of genus $g \geq 2$. It is known that $M(g) \leq 12(g-1)$, and that this upper bound is attained for infinitely many values of g, but before this workshop, relatively little was known about lower bounds for $M(g)$. Accordingly, several questions regarding potential lower bounds for $M(g)$ were considered during the workshop. It was proved that $M(g) \geq 2g$ for all even g, and that $M(g) \geq 4(g-1)$ for all odd g. This left the question of sharpness of these lower bounds.

In some work that took place after the workshop, a proof of sharpness of the bound $M(g) \geq 4(g-1)$ for odd g was obtained for a large (and likely infinite) set S of odd values of g. Sadly a similar theorem for even genera seems well out of reach at this stage.

In the case of conformal actions on all Riemann surfaces, the values of the corresponding parameter $M(g)$ are known to lie between $8(g+1)$ and $84(g-1)$, by the celebrated theorems of Accola–Maclachlan [1, 8] and Hurwitz [6] respectively. The precise values of $M(g)$ are known for $2 \leq g \leq 300$ and for some particular series of values for g, but despite a lot of work on the topic, this function is still not well understood.

Accordingly, instead of looking at individual values of $M(g)$, it makes sense to study the asymptotic properties of the function M, via the set $\mathcal{A}^d$ of accumulation points of the set $\mathcal{A}$ of values of the ratio $M(g)/g$. It was known before the workshop that for the second derived set $\mathcal{A}^{(2)} = (\mathcal{A}^d)^d$, we have $12 \in \mathcal{A}^{(2)} \subseteq \{8, 12\}$, and during the workshop we succeeded in essentially reducing the problem of deciding whether or not $8 \in \mathcal{A}^{(2)}$ to deciding whether or not certain very particular Belyi actions exist.

Articles on both of the above topics are planned to be written.

5 Short talks

The following 20-minute talks on topics related to the workshop themes were given by workshop participants, on the Monday, Tuesday and Wednesday afternoons:

- Jen Paulhus: *A database of group actions*
- Alina Vdovina: *Buildings and generalisations of dessins*
- Dimitri Leemans: *Almost simple groups and polytopes*
- Alexander Zvonkin: *Diophantine invariants of dessins d'enfants*
- Becca Winarski: *Homomorphisms between mapping class groups of surfaces*
- Roman Nedela: *Skew morphisms of cyclic groups and complete regular dessins*
- Charles Camacho: *Counting quasiplatonic cyclic group actions of order* n
- Shaofei Du: *Nilpotent primer hypermaps with hypervertices of prime valency*
- Milagros Izquierdo: *Dessins d'enfants and a curve of Wiman*
- Dimitri Leemans: *Almost simple groups and polytopes*
- David Torres: *Teichmüller curves and Hilbert modular surfaces.*

6 Organisers

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- Gabino González-Diez (Universidad Autónoma de Madrid, Spain)
- Gareth Jones (University of Southampton, UK)
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— helpfully assisted by

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7 Participants

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- Alina Vdovina (Newcastle University, UK)
- John Voight (Dartmouth College, USA)
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- Steve Wilson (Northern Arizona University, USA)
- Becca Winarski (University of Wisconsin - Milwaukee, USA)
- Aaron Wootton (University of Portland, USA)
- Alexander Zvonkin (Université de Bordeaux, France).

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