Women in Control: New Trends in Infinite Dimensions

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1 Overview

Control theory as a discipline has received considerable attention over the years. There are important applications to such diverse fields as engineering and medical sciences and challenging mathematical questions have made it relevant from both the abstract and applications point of view. The first developments in mathematical theory were for ordinary differential equation models. The need for more accurate descriptions and the constitutive relations in physical models such as structures and fluids give rise to partial differential equations (PDE's). The analysis of these models is strongly tied to several areas of mathematics, both pure and applied. Besides PDE's, geometric analysis, functional analysis, harmonic analysis, differential geometry, combinatorics and graph theory, optimization, numerical analysis and computational mathematics are all relevant. The last 20 years or so have witnessed great interaction and synergism where long-standing questions have found unexpected answers but these results have opened up new avenues of research. Because the state of systems with PDE models is a time-varying function that evolves on an infinite-dimensional state space, such systems are often referred to as infinite-dimensional systems.

With the impetus of technological problems demonstrating a need for models based on a continuum rather than a discrete description of phenomena, control theory has evolved into infinite-dimensions. This means the use of models described as the objects in infinite-dimensional Hilbert/Banach spaces with concrete representations stemming from PDE's. It was the goal of this workshop to review the major trends in this area and to introduce a wealth of problems and techniques which could provide a springboard to further research. This conference was a perfect opportunity to put together people working in the field of infinite-dimensional systems from diverse backgrounds and with different expertise.

Control theory in the last 20 years or so has undergone tremendous evolution. From vibrant developments in linear finite-dimensional control, with almost complete understanding of fundamental concepts such as stabilization, controllability, optimal control in this context. A wealth of applications pushed the borders into PDE's modeling by asking similar questions for infinite-dimensional systems. The unifying language then became that of semigroups which provide a common description of dynamics in both finite and infinite-dimensions. Not surprisingly, the answers are not the same, and in depth study of analysis, geometry, topology has paved the way to new control theoretical results within the realm of infinite-dimensions. These results were often formulated first in an abstract form in the language of functional analysis. However new conditions which transpired had to be verified by concrete representations described by PDE's or other infinite-dimensional functional equations. Thus the theory progresses in parallel-at an abstract level and also on a more specific PDE level, where verification of the sought-after properties became an art in itself. In infinite-dimensional systems research the focus is on development of an abstract and axiomatized framework. The talks presented at the workshop represented both trends: PDE's and infinite-dimensional system theory.The interaction between these two approaches and the resulting benefits as well as for collaborations were heavily emphasized. Applications to biological, fluid dynamics and mechanical models give a very tangible payoff in the applied world with solutions obtained via numerical methods; extension of algorithms to this class is non-trivial. Plenary talks were meant to represent diverse areas of infinite-dimensional control theory where new developments have emerged and are projected to flourish into the future. It was hoped that these talks would be accessible to non-specialists and providing a basis for the material in a number of shorter contributed talks.

Control theory, like other STEM disciplines, has a marked gender gap. Gender studies confirm that having scientists and engineers with diverse backgrounds, interests, and cultures assures better scientific and technological results. Improving female representation in the control field is not just an equity issue, but also an opportunity to improve the quality of work in the branch by having an impact on highly dedicated female researchers with varied expertise ranging from very applied work being done in engineering, to very theoretical mathematical work. All branches of control are inter-disciplinary. Inter-disciplinary work can add to the difficulties faced by minority researchers since it is easy to criticize contributions and grant proposals as not being "mainstream". Another difficulty, faced by female researchers in all STEM fields, is in finding collaborations since men tend to collaborate with other men.

The meeting had several inter-connected objectives: to provide support for young women interested in this field, introduce young female graduate students to potential future advisors and collaborators. We hope the workshop helped retention and will increase the participation of women in research activities in control of PDE's in the long-term. Another objective was to deepen worldwide networks of collaboration by inviting women from Latin America.

2 State of the Field

Developments in PDE optimal control theory started with considerations of parabolic partial differential equations - see for instance, the celebrated work of J.L. Lions [2]) . Later developments brought up front hyperbolic systems -first in one space dimension, starting with the pioneering papers of D. Russell [4] and W. Littman [3]. Already in one space dimension, it became clear that the nature of control mechanisms and capabilities is vastly different from that in finitedimensional spaces. This has provided an drive in abstract infinite-dimensional theory to seek counter-examples and also positive results within more restrictive frameworks. One can get a basic understanding of the area by realizing that properties such as stabilization are relatively simple for parabolic systems (this is essentially finite-dimensional theory due to spectral analysis), while (exact) controllability becomes almost impossible. In hyperbolic dynamics, controllability and more generally inverse problem are practical and solvable. On the other hand, stabilization is much more difficult for hyperbolic systems and requires special feedback that is intrinsically infinite-dimensional. While controllability is an *open loop* control problem, stabilization is realized by a *closed loop or feedback* control acting on an infinite time horizon $T = \infty$. Optimal control could be either open loop or closed loop and either finite or infinite horizon. As we shall see later, there is strong synergism between these concepts - solution of one problem feeds into another problem.

Consider an abstract model that provides a unified treatment for most of infinite-dimensional control problems. Let $T > 0$ be finite or infinite and let H and U be two separable Hilbert spaces with scalar product and associated norm respectively denoted by $(\cdot,\cdot)_H$, $(\cdot,\cdot)_U$, $\|\cdot\|_H$ and $\|\cdot\|_U$. Let us consider the abstract control system:

$$\begin{cases} y(t) = \ Ay(t) + Bu(t) \quad \text{in } \ (0,T)\,, \\ y(0) = y_0 \in H. \end{cases} \tag{1}$$

In this system $y_0 \in H$ is the initial datum and $u \in L^2(0,T;U)$ is the control, is exerted on the system through the operator B. Assume that A and B are unbounded operators respectively defined on $D(A) \subset H$ and $D(B) \subset U$. Let us also assume that system (1) is well-posed ; that is, for any $(y_0,u) \in H \times L^2(0,T;U)$ there exists a unique weak solution $y \in C^0([0,T];H)$ to problem (1) which depends continuously on the data. Denote by $y(t;y_0,u) \in H$ the solution to system (1) at time $t \in [0,T]$ corresponding to $(y_0,u) \in H \times L^2(0,T;U)$. Controllability is an open loop control problem where one seeks a control $u \in L_2(0,T;U)$

in order to "hit" a given target. Hitting may be *exact* or *approximate*. These concepts are quantitized by the following definitions.

1. System (1) is exactly controllable at time T if, for all $(y_0, y_1) \in H \times H$, there exists $u \in L^2(0,T;U)$ such that the solution y of (1) satisfies
$$y(T; y_0, u) = y_1.$$
2. System (1) is controllable to trajectories at time T if, for every $(y_0, \widehat{y}_0) \in H \times H$ and $\widehat{u} \in L^2(0,T;U)$, there exists $u \in L^2(0,T;U)$ such that the corresponding weak solution to (1) satisfies
$$y(T; y_0, u) = y(T; \widehat{y}_0, \widehat{u}).$$
3. System (1) is null controllable or exactly controllable to zero at time T if, for every $y_0 \in H$ there exists $u \in L^2(0,T;U)$ such that the corresponding weak solution to (1) satisfies
$$y(T; y_0, u) = 0.$$
4. System (1) is approximately controllable at time T if, for every $(y_0, y_1) \in H \times H$, and every $\varepsilon > 0$, there exists $u \in L^2(0,T;U)$ such that the corresponding weak solution to (1) satisfies
$$\|y(T; y_0, u) - y_1\|_H < \varepsilon.$$

Stabilization can be formulated as seeking a feedback control $u = F(y)$ where F is an operator from $U \to H$ [bounded or unbounded, linear or nonlinear] with the property that when inserted into the dynamics, it leads to stable dynamics. Stability may be asymptotic or uniform. In a simplest case when F is bounded, asymptotic stability amounts to the property that the semigroup $e^{(A+BF)}$ is asymptotically stable, that is,

$$e^{(A+BF)t}y_0 \to 0, \text{ as } t \to \infty \forall y_0 \in H \tag{2}$$

Uniform stability requires that stability property be uniform with respect to the underlined topology. This is to say

$$||e^{(A+BF)t}||_{t\to\infty} \to 0 \tag{3}$$

Several developments and new trends in various areas were discussed during the workshop.

2.1 Controllability and Stabilization of Parabolic Systems

The theory of infinite-dimensional systems often bifurcates into parabolic or hyperbolic frameworks. This is due to axiomatized assumptions imposed where parabolicity exhibits very special properties such as smoothing and infinite speed of propagation while hyperbolicity is marked by finite speed of propagation and intrinsic lack of smoothing by the dynamics. These properties have a fundamental effect on abstract control theory. Treatments of both type of dynamics often require different methods. These differences were also emphasized at the workshop where parabolicity [or more generally analyticity of the underlying semigroups] was exhibited by systems of heat transfer, Navier Stokes equations, viscoelastic systems, equations of thermoelasticity. In the case of parabolic dynamics expected properties are stabilization and null-controllability [or controllability to trajectories]. Talks by Assia Benabdallah, Catherine Lebiedzik, Suzanne Lenhart, Jing Zhang provide good representation of the area. However, the results presented by Assia Benabdallah showed that parabolic coupled equations can present hyperbolic behavior related to controllability; more specifically, minimal time of controllability and regional dependence of the control region.

2.2 Controllability and Stabilization of Hyperbolic Systems

Controllability and stabilization of hyperbolic systems is different than for parabolic systems. Here the situation is opposite: exact controllability is a natural property to expect due to time reversibility and propagation property. which does not change the regularity. However, it takes time for the signal to travel. Thus finite speed of propagation occurs and a typical results on localized/boundary controllability takes place after some time. Stabilization, instead, is more demanding due to the fact that instability in hyperbolic dynamics is typically infinite-dimensional. Typically the unstable part of the spectrum lies on the imaginary axis. In parabolic dynamics the unstable subspace is finite-dimensional. The workshop had many talks devoted to this issue. Valeria Cavalcanti presented some results on stabilization of wave equation via the transmission of viscoelasticity. Daniela Sforza considered systems of equation with memory terms. Paola Loreti presented an approach where controllability of a large class of hyperbolic systems can be resolved by methods of Fourier's Analysis and exponential functions.

2.3 Optimal control

In optimal control the goal is to find a control to minimize a given performance index, $J(u, y_0)$, subject to dynamics (1). Thus one seeks an optimal control $u^0 \in L_2(0, T; U)$ such that

$$J(u^0, y(u^0)) = \inf_{u \in L_2(0,T;U)} J(u, y(u)). \tag{4}$$

Optimal control has strong synergy with controllability and stabilization. For instance, consider $T = \infty$. Seeking an optimal control requires that the functional cost be finite for some control. This is guaranteed by the so called *finite cost condition*, and this may be deuced from controllability. Once the optimal control is found, for linear systems it can be synthesized in a feedback form. There exists an operator P so that the optimal control

$$u^0(t) = -B^* P y(t).$$

In addition, subject to coercivity of the functional cost, the corresponding optimal system is exponentially stable.

Classical optimal control involves minimization of a chosen functional (representing the cost and the target) subject to dynamics depending on control function and possibly constraints on both state and control. This constitutes the core of calculus of variations and optimal control theory. This class of control problems has been studied first within the context of ODE's, then evolved in the direction of PDE's, mostly abstract systems with bounded control operators. Unbounded controls such as point or boundary control have been considered more recently starting with parabolic systems, due to their nice regularity properties. Again, motivated by important applications in mechanics and medicine optimal control problems, both boundary and point controls have been considered, and some theory has been obtained for hyperbolic dynamics. We have new phenomena coming into picture, such as finite speed of propagation or unexpected hidden regularity. These properties have a heavy influence on the overall theory. New trends that are emerging in the field include optimal and constrained control of nonlinear systems and optimal control of hyperbolic- like models with boundary controls.

There were several talks on this topic, For instance, Suzanne Lenhart and Wendi Ding on optimal control of parabolic PDE's with application to biology and Francesca Bucci on optimal control including optimal feedback control of the third order system with boundary control. This latter reduces to a second-order system with a finite speed of propagation. All these talks, in addition to presenting mathematical contribution to the field, were strongly routed in life science applications. Lenhart's and Ding in biology while Bucci's presentation dealt with a problem of controlling radiation and high frequency ultrasounds. Constanza Sanchez presented optimal control problem applicable to a Schrödinger equation, an equatio thatn in some sense combines parabolic and hyperbolic effects - it displays infinite speed of propagation but lack of smoothing on bounded domains.

2.4 Abstract Infinite-Dimensional Systems

Infinite-dimensional system theory is an area where the goal is to axiomatize certain properties in control theory within the context of infinite-dimensional theory with standard matrices replaced by operators. The aim is to verify which properties from finite-dimensional control theory persists and which are no longer true and thus need to be replaced by an appropriate framework. The importance of the field lies in its generality. Of course, such abstract theory must be supported by examples that provide a verification and justification for the content. Thus, not surprisingly, there is a strong synergism between infinite-dimensional system theory and PDE control. This workshop was a good example of existing collaborations and the need for future research exchanges. Talks by Birgit Jacob and Jacquelien Scherpen provided overviews of some recent developments in these areas.

2.5 Control of Hybrid Systems and Systems with Transmission

An important area of current theoretical and practical interest is combining several different types of dynamics (as dictated by modeling -e.g. fluid–structure interaction) where control acquires a unique role in transferring desirable properties from one part of dynamics onto another. Propagation of stability, controllability and regularity [the latter in optimal control problems] through an interface becomes a centerpiece of the underlying analysis.

Canonical examples of such systems are fluid/ flow structure interactions, structural acoustic interactions and composite structures. Applications are abundant in various other areas of sciences such as medicine, biology, engineering, aeroelasticity etc. The main characteristic feature of such systems is the interaction of two different PDE dynamics via some interface (a manifold of co-dimension one) or via transmission conditions separating two different media. From the mathematical point of view control of such systems present formidable challenges. Any progress in this area must be preceded by good understanding of control theoretic aspects of a single PDE equation -both parabolic and hyperbolic.. Only then one can venture into trying to control the interaction by taking advantage of the desirable properties each PDE brings to the table. For instance, dissipativity and natural stabilizing mechanism in parabolic flows (Navier Stokes equation) enables controlling vibrations/oscillation in mechanical structures (bridges, vessels, arterias). In order to exhibit such phenomena one must assure appropriate geometric conditions. Several talks at the workshop [Valeria Cavalcanti, Lorena Bociu, Irina Ryzhkova, Weiwei Hu, Katie Szulc, Jing Zhang] were devoted to this topic which then was discussed in smaller groups of interest with plans for further collaboration. The topic, itself, is very rich and interdisciplinary. It involves modeling, PDE analysis, fluid mechanics and experiment/numerics. Speakers at the workshop were able to provide different perspectives and aspects of the problem, so that the cumulative experience is greater than the sum of ingredients. Formulation of relevant mathematical problem requires strong back up of modeling and of applied relevance. Thus it was natural to engage in collaborative work between USA, Brazil and France.

2.6 Control of models Arising in Biology and Medicine

This is a field where theoretical methods of optimal control find their way into truly applied problems. Due to the array of PDE models describing biological phenomena, a maximum principle in optimal control as well as feedback stabilization and synthesis provide for critical tools in improving therapies and treatments. Typical theoretical results are local (as relevant in applications), due to a nonlinear nature of the models. Suzanne Lenhart's talk on optimal control provided an overview of math biology applications, Francesca's Bucci lecture dealt with medical applications in inverse problems-ultrasound technology. Bianca Calsavara's presentation described local controllability results in genetic networks.

2.7 Nonlinear control

All these problems can be also be studied within the nonlinear framework, integral equations, delayed equations, Banach spaces etc. The applications give a vast scenario of models in the infinite-dimensional framework. Nonlinear models with nonlinear controls can be written

$$\dot{y} = A(y) + B(u)$$

where $A(y)$ and $B(u)$ are nonlinear operators may also be considered. With feedback control this becomes $\dot{y} = A(y) + BF(y)$ -where the ultimate equation may represent a well studied dynamical system. Clearly, many possible scenarios are under consideration, leading to an array of results with strong applied ramifications. Many talks at the workshop addressed these nonlinear aspects of control theory.

Every nonlinear problem needs its own precise statement and development. Some of the techniques presented in the workshop include fixed point arguments (from Browder to Kakutani fixed point theorems), Coron's "return method" [1] or the use of the Inverse Function Theorem.

3 Panel Discussions

We had several panel discussions that participants found useful. Our area is very inter-disciplinary which can be exciting but also an issue for women, particularly in hiring and promotion, granting agencies etc. The fact that research may not fit into a mainstream area can be used as an excuse for turning down a proposal or promotion. The efficacy of some kind of mentoring was clear. Of course this can be of great help for any young researcher but in particular for women since gender imbalance and society pressures and attitudes discourage women in a number of aspects of the academic life.

Open research problems

Identification of research problems that are interesting (either for applications or theory) but not already been studied by another group, is critical to success as an academic. This, and effective time management, are two skills that need to be developed by young academics in order to obtain tenure and promotion. New faculty need to distance themselves from their advisor to develop their own program and must write papers independently in order to obtain tenure. The following points were also made during the discussion.

- Mathematical biology has a lot of job and grant opportunities and there are lots of interesting unsolved problems.
- For inter-disciplinary work, such as biology, working with an expert in the non-math area is very helpful.
- Slow/fast dynamics arise in many dynamics and methods are needed for them.
- Coupled PDE's, particularly hyperbolic/parabolic equations, have many interesting mathematical issues with consequences for applications (a number of talks at the workshop were concerned with such. systems)
- Systems with memory are receiving interest.
- Fractional Laplacians and other fractional derivatives have interesting theoretical issues and connections with applications, such as thermo-elastic materials, but some people won't touch them.
- Interdisciplinary work can be a good source of interesting problems, but there is a danger of falling out of the mainstream work in your department or the granting agencies
- Illustrations of differences with finite-dimensional systems useful.
- Keep in touch with numerical and experimental people. They can be a source of insight and new problems even if you are only interested in theory.
- Try to find problems that will develop collaborators in your department.
- Go to workshops in your field and related fields.
- Think of your strengths/uniqueness and play to those.
- Choose problems that excite you!

Grant Writing

Grants are important for academics in several respects. On a practical level, they provide funds for travel, collaboration and graduate students. They are also used by institutions as a criterioni for tenure and promotion, partly because they are regarded as an indication of the respect that a researcher's work holds and partly because grant overhead is important to university budgets.

It was clear in the discussion that several younger faculty had been discouraged by unsuccessful grant applications. However several respected and well-funded senior participants shared that often applications are turned down. The success rate for most programs is quite low, and applications for interesting research are turned down. Sometimes it is just bad luck. The important thing is use any reviews as useful information for revising and rewriting and to persist; either with a revised proposal to the same program or else to a different program. A number of other comments and advice were made:

- Look at all sources of funding, be creative in thinking of programs to which to submit applications.
- Think of who will be reading the application. Is it government personnel ? Experts in the field? This should affect how the research is presented.
- Useful to have someone read and advise on the proposal. Many universities have personnel to help.
- Start with applications for smaller grants.
- Proposal writing is time-consuming.
- Find out and address the priorities of the grant program.
- If you're trying to involve an industrial partner, "talk their language" and think about their priorities.
- Use common language for inter-disciplinary work.
- Collaborators can be helpful, particularly for inter-disciplinary work.
- Read reviews carefully and use to make changes for new proposals. Often you can make changes and resubmit to the same program. Or, the research may be more suitable for another program.
- Even unsuccessful proposals can help in focusing ideas and identification of open problems.
- Networking is important; go to conferences and meet people.

4 Highlights

As outlined above, there is a synergistic diagram

$$exact\ controllability \ \leftrightarrow\ solvability\ of\ optimal\ control \leftrightarrow exponential\ stabilizability$$

and also important applications of stabilizability and optimal control. There was a balance in the topics: not everything was theoretical nor everything was applied. A wide variety of different aspects of control theory of PDE's was covered. The workshop gave an opportunity to explore links and future collaborations. Although this was not planned, a number of talks featured work on different aspects of coupled systems. This was marked by a number of participants as a possibly fruitful area of research and also as a good topic for a conference invited session.

This was a very good meeting from the scientific point of view. All participants expressed enthusiasm for the high technical content and atmosphere of the workshop.The research presented was of the usual calibre of major international meetings. This provided encouragement to the younger women about the quality of women's contributions to our field.

The participants obtained interesting new research ideas and bibliography. Some of the participants have already invited to give seminars at other participants' universities and we all have new names of excellent researchers in the area of control of PDE's that can be invited to meetings, workshops and seminars. The experience was great in the sense that we put in touch different groups working on this interesting subject all around the world, with different techniques, backgrounds and area of application. Talks included very abstract setting to very applied one, from biology to engineering, from physics to abstract analysis.

Typical comments were:

> *"I want to thank you for inviting me to the workshop women in control, it's really the best conference I have attended till now. Thank you."*

> *"First of all, thank you again for organizing the workshop! It was a great learning experience, and I found it much more beneficial than the usual conferences I attend. The schedule was great, and allowed participants to interact, collaborate, discuss potential common projects, and give/receive advice regarding professional development. I found all the small group research talks and panels very helpful. Also, I really enjoyed the atmosphere of the workshop. I wish we had more workshops like this."*

> *"In general, I think that the Workshop "Women in Control: New Trends in Infinite Dimensions" was a really useful and reach experience in many senses. One of the interesting things I think I obtained from the Workshop is that I could make contact with people with a lot of knowledge in control theory and learn and see other applications and techniques of different areas of control problems. I keep in touch with them and I am opened to establish common work with some of them."*

5 Meeting Outcome

The high quality of the talks, despite the restriction to women working in a small field, was very encouraging to all participants. There was discussion at the workshop about organization of sessions at international conferences in 2018, such as MTNS and IFIP, that will involve participants as speakers.

The format of the meeting facilitated discussion in small groups around open problems which helped participants develop their networks and think about research problems. Networks in five sub-areas were create to establish new collaboration, to mentor young women with their research proposals, to share specific material and papers, propose new research, and so on. This list will also help in organization of sessions at conferences. The list of sub-areas, along with acronyms, is

- CIDS: Control of infinite Dimensional Systems
- OCO: Optimal control and Optimization
- CMBM:Control of mathematical-biological Models
- SLTB: Stabilization and long time behaviour
- NM: Numerical Methods

Table 1 lists the members of each group.

We have also created a closed group in Facebook and a "basecamp" webtool for our group to share files and emails. Only invited members can receive the updates and to see the files.

Some representatives from Asia were missing. Although possible participants in China were identified, lack of support for their travel expenses did not allow them to attend. Also the control of PDE's community in France is large but not well represented. We believe that after the present positive experience a future meeting will be even more successful, particularly with financial support. We would like to encourage more participation from Asia and France, as well as women working on numerical issues. The financial and administrative support of BIRS that enabled this meeting to happen was very much appreciated. A future meeting, maybe in China, is already being discussed.

As described above the topics and lectures presented are characterized by great synergy and gradual buildup of knowledge in the area which is progressive with a strong slant to modern applications and technologies. For example: the results presented in topics 1-4 display great synergy. Developments achieved at

Person	Country	Networkings
Apraiz, Jone	Spain	CIDS/OCO
Belkhatir, Zehor	Saudi Arabia	CMBM/NM
Benabdallah, Assia	France	CIDS/CMBM
Bociu, Lorena	U.S.A	OCO/CMBM
Bucci, Francesca	Italy	OCO
Calsavara, Bianca	Brazil	CIDS/CMBM/OCO
Cavalcanti, Valéria Neves Domingos	Brazil	SLTB
de Teresa, Luz	Mexico	CIDS/OCO
Ding, Wandi	U.S.A	CMBM
Doubova, Anna	Spain	CIDS/CMBM/NM
Fu, Xiaoyu	China	SLTB
Gomes, Susana N.	U.K.	SLTB/NM/OCO
Hu, Weiwei	USA	SLTB/NM/OCO
Jacob, Birgit	Germany	CIDS/SLTB
Jamieson, Jessie	USA	NM
Laasri, Hafida	Gernamy	CIDS/SLTB
Lasiecka, Irena	USA	SLTB/CIDS
Lebiedzik, Catherine	USA	SLTB
Lenhart, Suzanne	USA	CMBM
Loreti, Paola	Italy	CMBM
Morris, Kirsten	Canada	SLTB/OCO/NM
Rivas, Ivonne	Colombia	CIDS/SLTB
Ryzhkova-Gerasymova, Iryna	Ukraine	SLTB
Sánchez de la Vega, Constanza	Argentina	CIDS/OCO
Scherpen, Jacquelien	Netherlands	NM/SLTB
Sforza, Daniela	Italy	CMBM
Szulc, Katarzyna	Poland	NM
Tang, Shuxia	Canada	SLTB/OCO
Tegling, Emma	Sweden	OCO
Zhang, Jing	USA	SLTB/OCO

Table 1: Networkings

the abstract level, when combined with parabolic and hyperbolic discoveries culminate in the study of *systems* where various types of dynamics interact. This led to strong interaction among participants who find proper mentoring in the areas complementary [but important] to their expertise. Similarly topics 5-7 pave the way to a treatment of applied problems that can be resolved with a help of computational mathematics.

Both senior and junior researchers found the workshop useful for advancing themselves and the field. The senior ones by being exposed to disciplines a bit outside their expertise and the junior ones by being introduced gradually into the arcanes and developments in each area. On site contacts, formalized by creation of the Working Groups [see above] are expected to provide an effective mentoring forum with the goal of advancing the field and careers of women researchers.

References

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