Predictability and entropy for actions of amenable and non-amenable groups

BIRS workshop 17w5068 - Mean Dimension and Sofic Entropy Meet Dynamical Systems, Geometric Analysis and Information Theory

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Pinsker's formula

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For a countable amenable group G with an algebraic past Φ:

$$h_\mu(G, \alpha \vee \beta \mid \mathcal{F}) = h_\mu(G, \beta \mid \mathcal{F}) + H_\mu(\alpha \mid \beta_G \vee \alpha_\Phi \vee \mathcal{F}).$$

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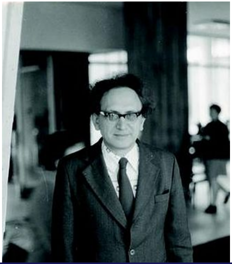

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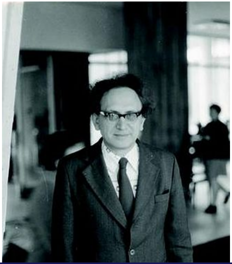

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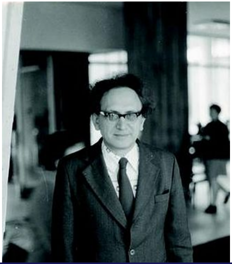

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- We say that the action $G \curvearrowright (X, \mathcal{B}, \mu)$ is S-predictable if every $E \in \mathcal{B}$ is S-predictable. Similarly, define S-predictable functions and S-predictable partitions.

Predictability implies zero entropy - amenable + measure preserving case

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- So entropy does not determine "the arrow of time"...

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  - Predictability not preserved by time reversal and by taking cartesian products.

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- Remark: The assumption that S generates G as a group is not necessary, because having a zero entropy subaction implies zero entropy (for whatever reasonable def of entropy you use).

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- Remark: For non free actions on orderable groups, choose "proper" notions of entropy and predictability.

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Thanks!