Every genus 1 algebraically slice knot is 1-solvable.

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and Jung Hwan Park (Rice University)

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Outline

1. Setting: Concordance and the solvable filtration
2. The solvable filtration and surgery curves
3. A modification lemma and counterexamples to a conjecture of Kauffman.
4. An example of Litherland of a slice whitehead double.
5. Every genus 1 algebraically slice knot is 1-solvable.
6. String link infection and higher genus results

Concordance and the solvable filtration

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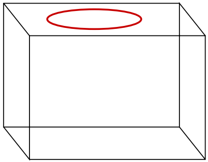

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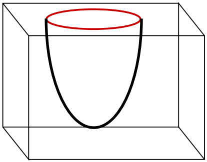

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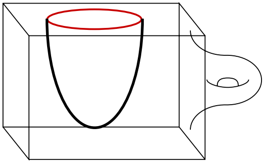

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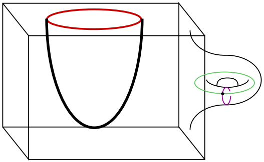

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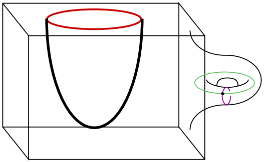

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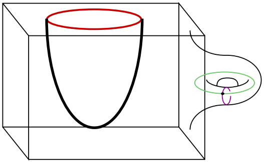

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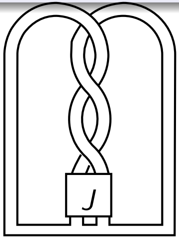

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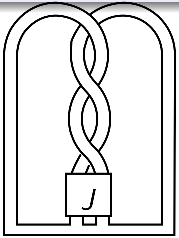

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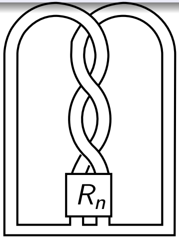

R_{n+1}

Iterating this (and similar) constructions gives a $\mathbb{Z}^\infty$ in $\mathcal{F}_n/\mathcal{F}_{n.5}$. Since R_1 is automatically 1-solvable you can drop the 0-solvability assumption from the CHL examples.

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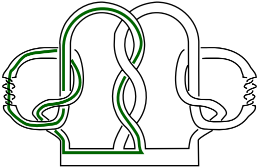

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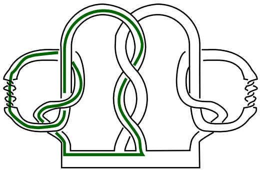

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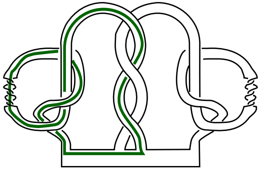

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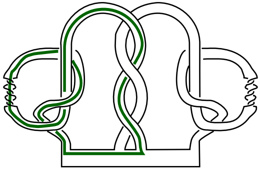

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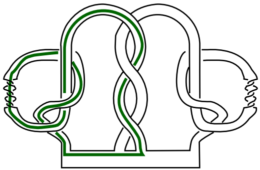

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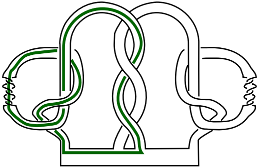

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This conjecture is false (Cochran-D.) I will recall the counterexample, since it uses a technique which we generalize.

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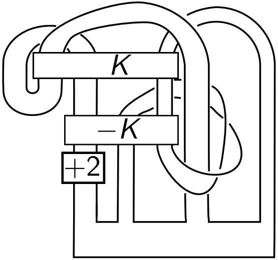

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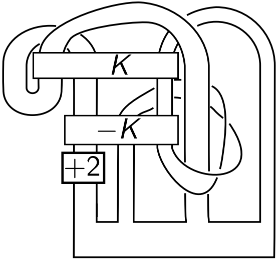

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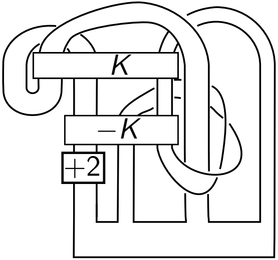
  The technique we use is infection.



Tool: Infection and the modification lemma

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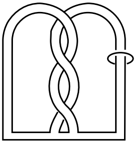

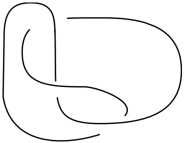

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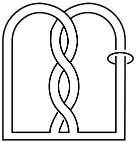

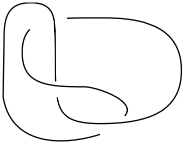

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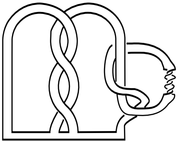

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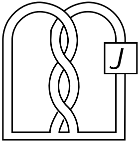

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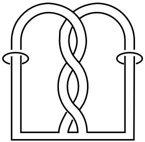
This operation can be done iteratively: $F_{\eta_1,\eta_2}(J_1, J_2)$.



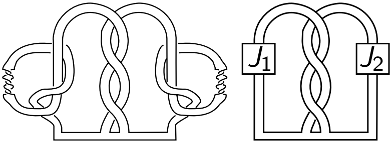
Tool: Infection and the modification lemma

We make use of a construction of knots called **infection**. Start with a knot K in S^3 and an unknotted curve η in the complement of K and an infecting knot J.

Cut out a neighborhood of η and glue back in the complement of a neighborhood of J (meridian-to-longitude, longitude-to-meridian.)

The resulting manifold is still S^3. $K_\eta(J)$ is the resulting knot.

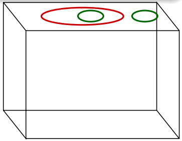
This operation can be done iteratively: $F_{\eta_1,\eta_2}(J_1, J_2)$.



Tool: The modification lemma

Theorem (Cochran-D.)

Let η_1 and η_2 be unknotted, unlinked curves in the complement of the knot R.



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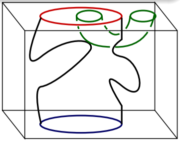

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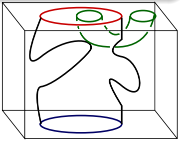

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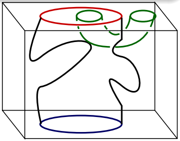

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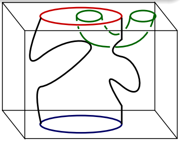

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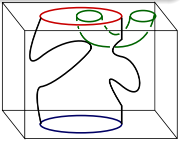

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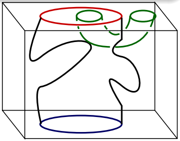

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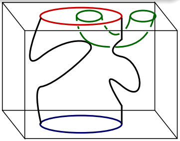

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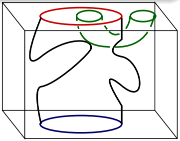

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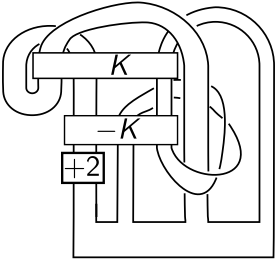
- (Park) There is a similar theorem for surgery.

Modifying surgery curves: The Kauffman counterexample

(Cochran-D.) There exists a slice knot with a genus 1 Seifert surface on which no surgery curve is even 0-solvable.



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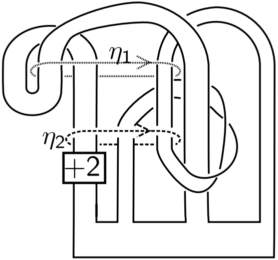

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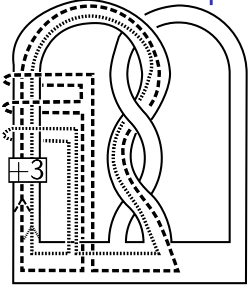

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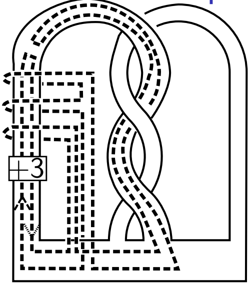

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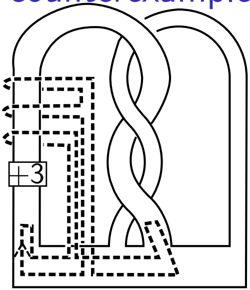

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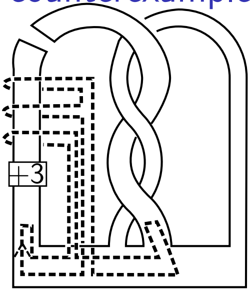

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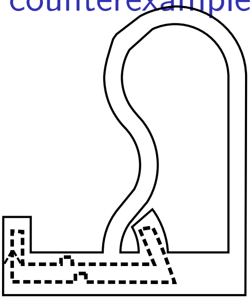

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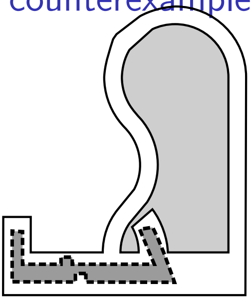

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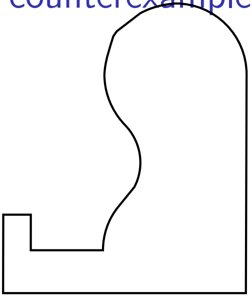

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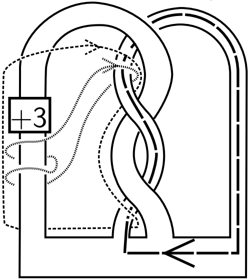

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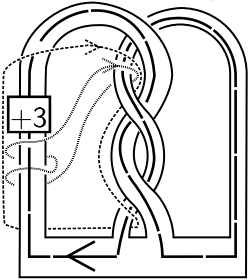

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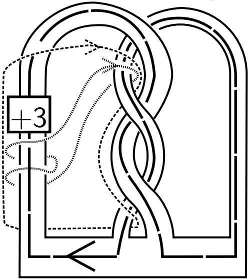

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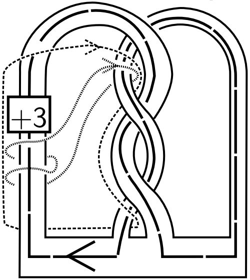

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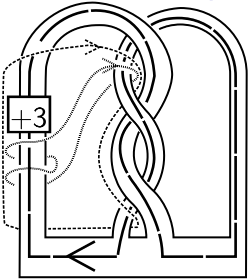

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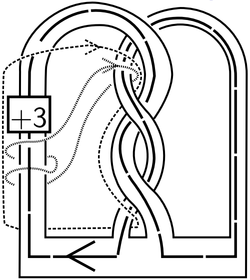

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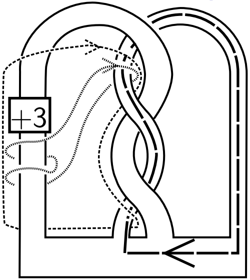

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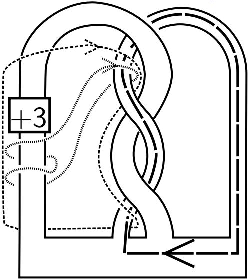

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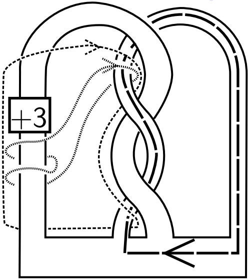

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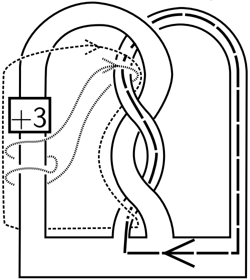

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As long as $\mathrm{Arf}(K) \neq 0$, neither d_1 nor d_2 is even 0-solvable.

We have a counterexample to Kauffman's slice conjecture.

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Theorem

Let η_1 and η_2 be framed curves in the complement of the knot R. Suppose that in the complement of a concordance from R to S η_1 and η_2 cobound a framed annulus. Then for any knot J $R_{\eta_1,\eta_2}(J,-J)$ is concordant to S (in a homology coordism)

The proof is the exact same, only now we don't even try to prove that the new 4-manifold is $S^3 \times [0,1]$.

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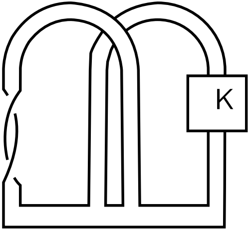

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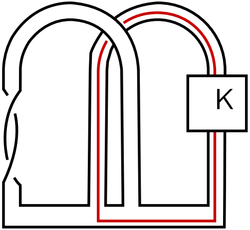

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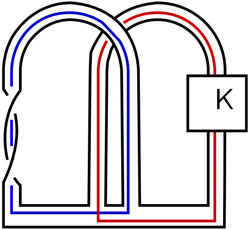

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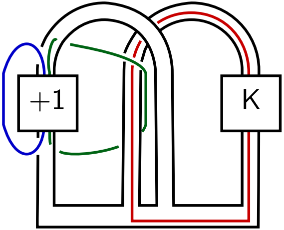

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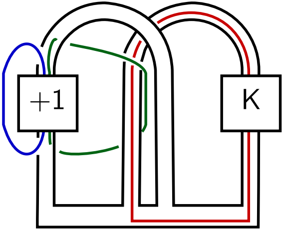

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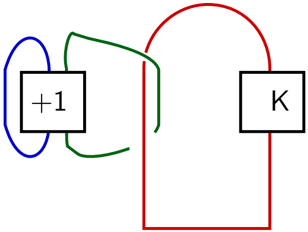

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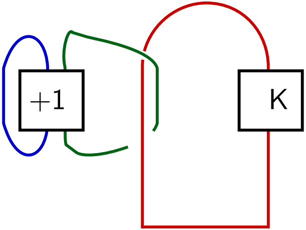

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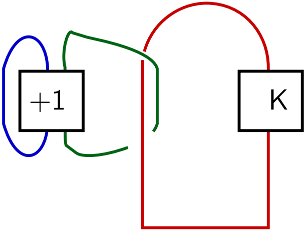

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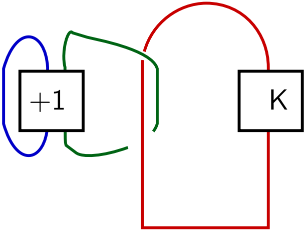

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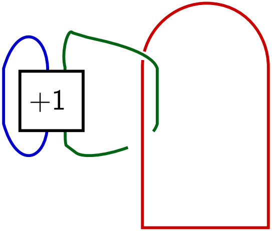

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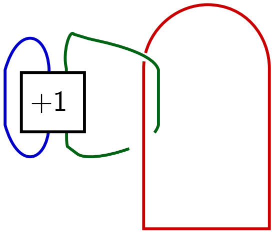

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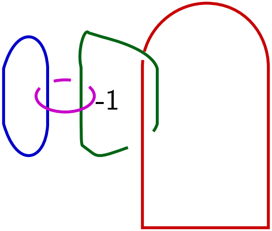

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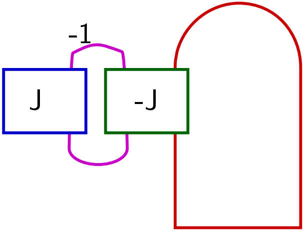

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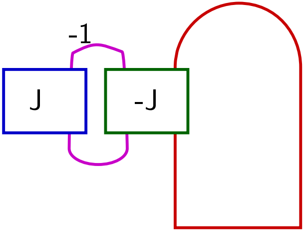


Corollary (Litherland, 1979)

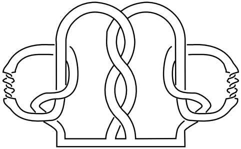
The Whitehead double of (the concordance inverse of) this knot is slice in a homology ball.

Remark: This knot has exactly the algebraic concordance class of J.

Application: Modifying surgery curves.

Theorem (D.-Martin-Otto-Park)

If a knot K is 0.5-solvable, and K bounds a genus 1 Seifert surface, then K is 1-solvable.



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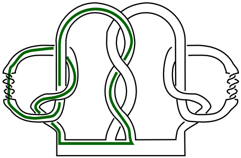

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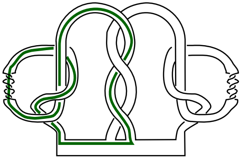

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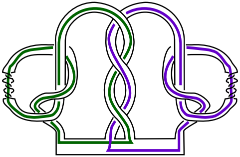

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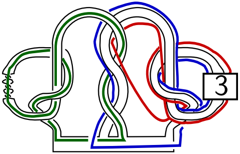

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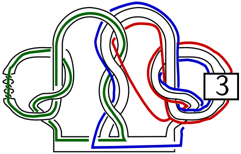

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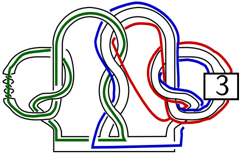

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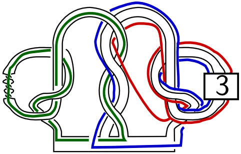

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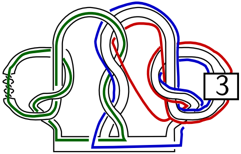

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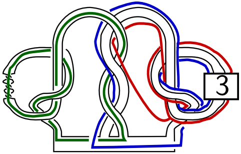

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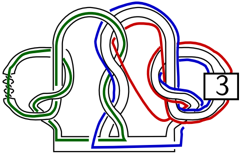

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$K_{\delta^+,\delta^-}(T,-T)$ has a 0-solvable surgery curve and so is 1-solvable. Since K is concordant to $K_{\delta^+,\delta^-}(T,-T)$, K is also 1-solvable

What if K has genus ≥ 2?

A genus 2 version of the theorem

Theorem

Let K be a genus g algebraically slice knot with surgery curves J, If J is a boundary link (or even just has $\overline{\mu}_{iijj}(J)$ even and $\overline{\mu}_{ijk}(J) = 0$) then K is 1*-solvable.*

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(Martin) J is 0 solvable if and only if for all $1 \leq i < j < k \leq g$ $\text{Arf}(J_i) = 0$, $\overline{\mu}_{iijj}(J)$ is even and $\overline{\mu}_{ijk}(J) = 0$

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What if we cannot find a derivative which is a boundary link? How can we modify the Sato-Levine and triply linking invariants of a surgery curve?

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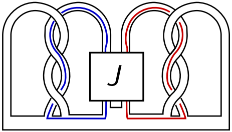

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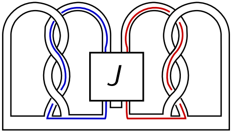
Let K be a genus 2 algebraically slice knot with surgery curves $J = J_1, J_2$, duals δ_1, δ_2 and 4×4 Seifert matrix (over this basis) $\begin{bmatrix} 0 & A \\ B & C \end{bmatrix}$. If either $\det(A) + \det(B)$ is odd or $\overline{\mu}_{1122}(J)$ is even then K is 1-solvable.



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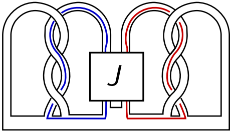

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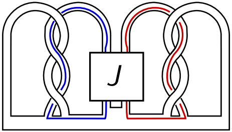

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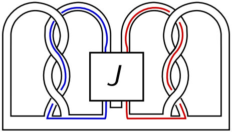
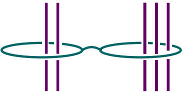
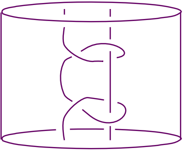
In the case that $\overline{\mu}_{1122}(J)$ is odd we need a string link version of the Modification lemma.



A string link modification lemma



Let α be wedge of circles embedded the complement of a knot (or link) R. Let T be a pure string link (with zero linking number).



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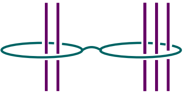

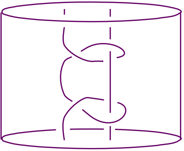

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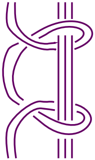

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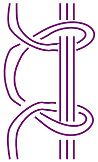

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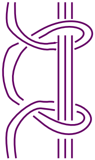

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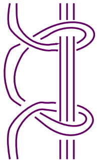

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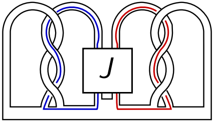
Let K be a genus 2 algebraically slice knot with surgery curves $J = J_1, J_2$.



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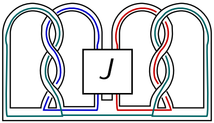

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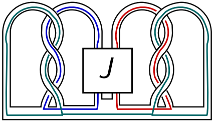

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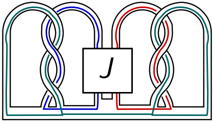

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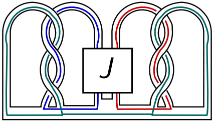

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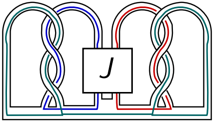

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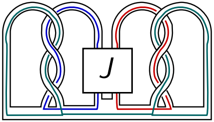

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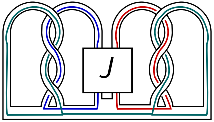

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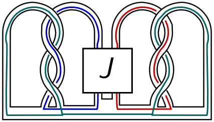

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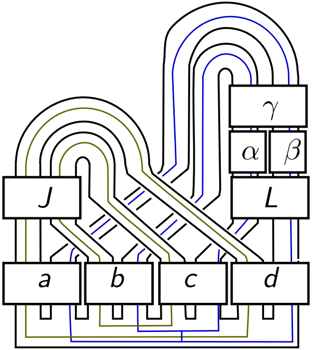
A genus 2 algebraically slice link which might not be 1-solvable.

Let J and L be (pure linking number zero) string links.
Here is an algebraically slice knot K with set of surgery curves J and Seifert matrix

$$\begin{bmatrix} 0 & 0 & a & b \\ 0 & 0 & c & d \\ a-1 & c & \beta & \gamma \\ b & d-1 & \gamma & \alpha \end{bmatrix}$$

If $\mu_{iijj}(J)$ is even then K is 1 solvable.



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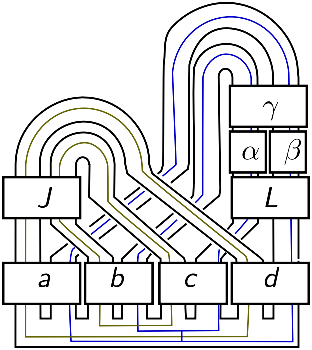

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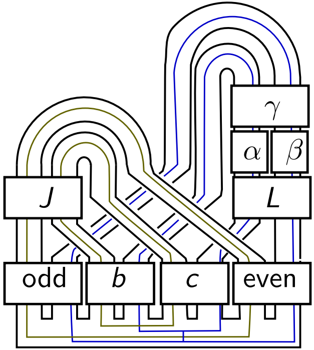

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If there is a genus 2 knot which is not 1-solvable then this is a candidate (J is the Whitehead link.)

A high genus example.

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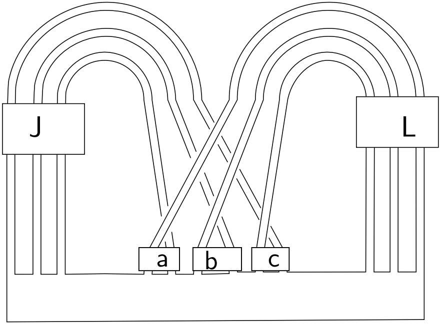
I will close with an example of a algebraically slice knot which **is** 1-solvable.

A surprising genus 3 algebraically slice knot.

Here is an algebraically slice knot genus 3,



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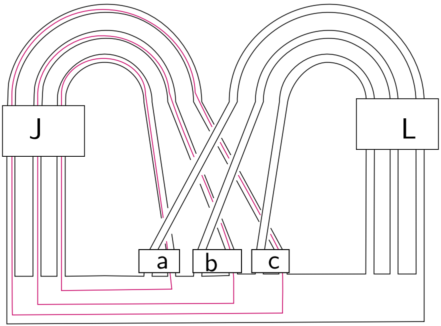

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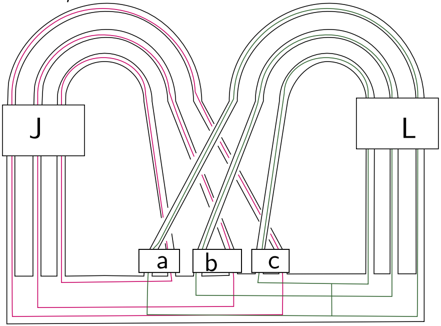

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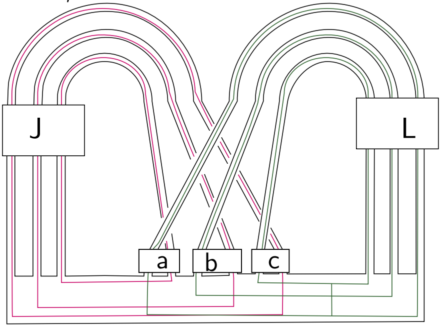

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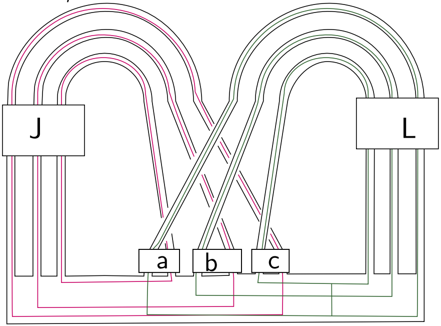

Infection along δ^+ and δ^- changes $\mu_{123}(J)$ by $q := ab+bc+ac-a-b-1$.

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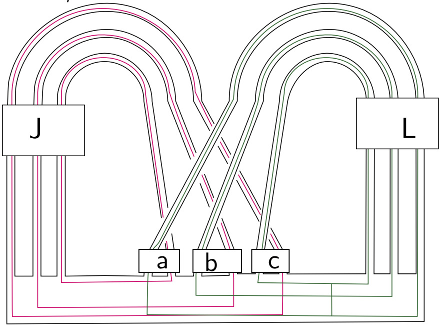

Infection along δ^+ and δ^- changes $\mu_{123}(J)$ by $q := ab+bc+ac-a-b-1$. As long as $\overline{\mu}_{123}(J)$ is a multiple of q this can be used to kill $\mu_{123}(J)$ Unfortunately, $\overline{\mu}_{iijj}$ has now changes in some mysterious way.

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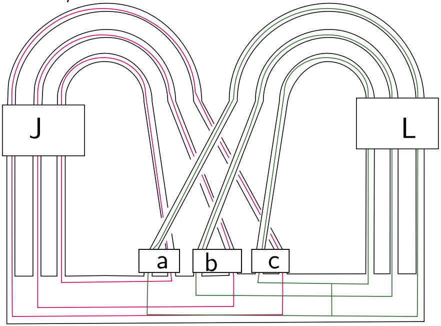

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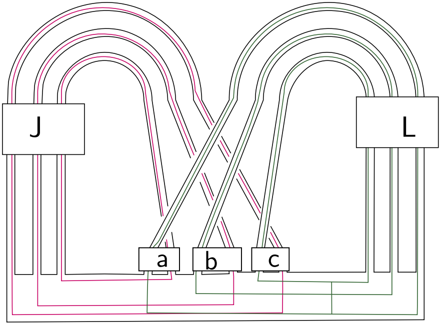
As long as a, b, and c are all even or are all odd we can undo $\overline{\mu}_{1122}(J)$ using $\delta_1 \wedge \delta_2$, $\overline{\mu}_{1133}(J)$ using $\delta_1 \wedge \delta_3$, and $\overline{\mu}_{2233}(J)$ using $\delta_2 \wedge \delta_3$

genus 3 example

Corollary

Let $q := ab + bc + ac - a - b - 1$. If $\overline{\mu}_{123}(J)$ is a multiple of q and a, b, and c are all even or are all odd then K is 1-solvable.



Thanks for your attention!