Characterization of the minimal series of Virasoro vertex operator algebras

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Introduction

1. We discuss two ways to characterize simple Virasoro vertex operator algebras (VOA) $L_{c_{p,q}}$ with the central charge $c_{p,q} = 1 - 6(p-q)^2/pq$ $(1 < p < q$ and $(p,q) = 1)$, 4 simple modules and satisfying modular linear differential equations (MLDE) of 4th order.
2. One is that the character of V with a central charge c has a form

$$\mathrm{ch}_V = q^{-c/24}\left(1+0\cdot q+q^2+O(q^3)\right) \Leftrightarrow \dim V_1 = 0,\ \dim V_2 = 1\,.$$

3. The other is that the 2nd coefficients of characters of simple V-modules except V are all 1.
4. The former shows that V is isomorphic to one of $L_{-46/3}$ $(c_{2,9} = -46/3)$, $L_{-3/5}$ $(c_{3,5} = -3/5)$ and the two VOAs which are extensions of $L_{-114/7}$ $(c_{3,14} = -114/7)$ and $L_{4/5}$ by $L_{-114/7,3}$ and $L_{4/5,3}$ $(c_{3,14} = 4/5)$, respectively.
5. The 2nd condition implies that V is isomorphic to one of $L_{-46/3}$, $L_{-3/5}$, or it is pseudo-isomorphic to and the lattice VOA V_L, $L = \mathbb{Z}\alpha$ with $\langle \alpha\,, \alpha \rangle = 6$, where $\omega = \frac{\alpha_{-1}^2}{12}\mathbf{1} + \frac{1}{3}\alpha_{-2}\mathbf{1}$.

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Minimal models–characterization

We study simple VOAs $V = \bigoplus_{n=0}^{\infty} V_n$ with central charges c satisfying

(A) The central charges and conformal weights are rational,
(B) Any basis of the space of characters of simple V-modules forms a fundamental system of a MLDE of 4th order,
(C) $\dim V_1 = 0$ and $\dim V_2 = 1$,
(D) $a_1^i = 1$ for all $i = 2, 3$ and 4, where $f_i = \sum_{n=0}^{\infty} a_n^i q^{\lambda_i - c/24 + n}$. where $\lambda_1 = 0$, i.e. f_1 is the character of V.

Remarks. (1) We always assume (A) and (B).
(2) The minimal models satisfy (A), (B), (C) and (D).
(3) We take combinations of (A), (B) and (C) or (A), (B) and (D).
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Generality for the classification

1. Let $f = e^{-c/24}\left(1 + 0 \cdot q + q^2 + mq^3 + \cdots\right)$ $(m \in \mathbb{Z})$ be a solution of a MLDE

$$D^4(f) - E_2 D^3(f) + \left(3E_2' + xE_4\right) D^2(f) - \left(E_2'' + \frac{x}{2}E_4' - yE_6\right) D(f) + zE_8\, f \,=\, 0\,, \quad (D = q\frac{d}{dq})\,.$$

2. First 3 coefficients give a system of simultaneous 3 linear equations in x, y and z.
3. $c \neq 0, -22/5$ and $7/578 \Longrightarrow$

$$x = -\frac{56c^3 + 993c^2 - 11660c - 1440}{96(578c - 7)},$$
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Solutions of the Diophantus equation

We have the Diophantus equation (as a necessary condition)

$$1050c^5 + (5m + 31020)c^4 + (275600 - 703m)c^3 + (32992m + 673104)c^2 + (504352 - 517172m)c + 3984m - 210432 = 0\,.$$

The Diophantus equation is completely solved as

m	c
1	$-46/3, -68/7, -3/5, 1/2$
2	$-114/7$, $4/5$
501971	36^*
3132760	$122/3^*$
37950512	$238/5^*$
42987520	48^*

Remark. (a) The all solutions with superscript $(*)$ were found by D. Zagier. There are more solutions with negative m.
(b) It was proved D. Zagier that they gives all rational c and integral m solutions. However, these do not give any VOA.

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Central charges $c = -46/3$ and $-3/5$

Theorem 1

Let V be a VOA satisfying (A), (B) and (C) with the central charge $-46/3$ or $-3/5$. Then V is isomorphic to either the minimal model $L_{-46/3}$ or $L_{-3/5}$, respectively.

List of $c = \{$~~$-46/3$~~, ~~$-68/7$~~, ~~$-22/5$~~, ~~$-3/5$~~, ~~$1/2$~~, $-114/7, 4/5\}$.

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Remark. The dimensions of the sets of characters of the minimal models $L_{-114/7}$ and $L_{4/5}$ are 13 and 10, respectively, since $c_{3,14} = -114/7$ and $c_{3,14} = 4/5$.

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Remark. The dimensions of the sets of characters of the minimal models $L_{-114/7}$ and $L_{4/5}$ are 13 and 10, respectively, since $c_{3,14} = -114/7$ and $c_{3,14} = 4/5$.

Theorem for $c = -144/7$ and $4/5$

List of $c = \{-46/3, -68/7, -22/5, -3/5, 1/2, -114/7, 4/5\}$.

Theorem 3

Let V be a vertex operator algebra satisfying (A), (B) and (C) with a central charge $c = -114/7$ or $4/5$. Then V is isomorphic to either $L_{-114/7} \oplus L(-114/7, 3)$ or $L_{4/5} \oplus L(4/5, 3)$.

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Central charges $c = -46/3$, $-3/5$ and -7

Theorem 4

Let V be a vertex operator algebra satisfying (A), (B) *and* (D). *Then the central charge of V is $-46/3$, $-3/5$ or -7.*

Theorem 5

Let V be a vertex operator algebra satisfying (A), (B) *and* (D) *with a central charge $-46/3$ or $-3/5$. Then V is isomorphic to either $L_{-46/3}$ or $L_{-3/5}$, respectively.*

List of c ~~$-46/3$~~, ~~$-3/5$~~, -7.

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Theorem 6

Let V be a vertex operator algebra satisfying (A), (B) *and* (D). *If the central charge of V is -7, then the space linearly generated by characters of simple V-modules coincides with that of simple* (*sifted*) *V_L-modules, where $L = \mathbb{Z}\alpha$ with $\langle \alpha, \alpha \rangle = 6$ and the Virasoro element is*

$$\omega = \frac{\alpha_{-1}^2}{12}\mathbf{1} + \frac{1}{3}\alpha_{-2}\mathbf{1}$$

$-46/3$, $-3/5$, -7

Theorem 6

Let V be a vertex operator algebra satisfying (A), (B) *and* (D)*. If the central charge of V is -7, then the space linearly generated by characters of simple V-modules coincides with that of simple* (*sifted*) *V_L-modules, where $L = \mathbb{Z}\alpha$ with $\langle \alpha, \alpha \rangle = 6$ and the Virasoro element is*

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