15w5110: Laplacians and Heat Kernels: Theory and Applications
Mar 22 - Mar 27, 2015

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–10:20	Grebenkov, Maltsev
10:20–10:40	Coffee Break, TCPL
10:40–12:00	Jakobson, van den Berg
12:00–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–14:20	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:20–15:00	Saito
15:00–15:30	Coffee Break, TCPL
15:30–17:30	Kao, Levy, Bronstein
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–10:20	Berkolaiko, Beliaev
10:20–10:40	Coffee Break, TCPL
10:40–12:00	Laugesen, Siudeja
12:00–13:30	Lunch
13:40–15:00	Antunes, Shivakumar
15:00–15:30	Coffee Break, TCPL
15:30–17:30	Lai, Cloninger, Meyer
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–10:20	Burdzy, Mayboroda
10:20–10:40	Coffee Break, TCPL
10:40–12:00	Amitai, Maggioni
12:00–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:20	Henrot, Mahadevan
10:20–10:40	Coffee Break, TCPL
10:40–12:00	Benguria, Hermi
12:00–13:30	Lunch
13:40–15:00	David, Filoche
15:00–15:30	Coffee Break, TCPL
15:30–17:30	Harrell, Beg, Song
17:30–19:30	Dinner
20:00–??	Informal Discussions as a Whole Group

Friday

7:00–9:00	Breakfast
9:00–10:20	Jones, Jerison
10:20–10:40	Coffee Break, TCPL
10:40–11:30	Discussion/Conclusion
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

15w5110: Laplacians and Heat Kernels: Theory and Applications
Mar 22 - Mar 27, 2015

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Assaf Amitai** (Inst. Medical Eng. & Sci., MIT)
Title: *The mean encounter time between two polymer sites: a Brownian search process in high dimensional manifolds*
Abstract: The encounter between two sites on chromosomes in the cell nucleus can trigger gene regulation, exchange of genetic material and repair of DNA breaks. By forming a DNA loop, a protein located on the DNA can trigger the expression of a gene located far along the chain. We computed asymptotically the mean time encounter time (MFET) between two polymer sites, using the classical Rouse polymer model, in which the polymer is described as a collection of bead monomers connected by harmonic springs. When two monomers come closer than a distance epsilon, the search process ends. The novel asymptotic relies on the expansion of the spectrum of the Fokker-Planck operator as a function of the small parameter epsilon. The key of the asymptotic is the explicit computation of the Riemannian volume for Chavel-Feldman formula, which gives the shift in the spectrum of the Laplacian operator when Dirichlet boundary conditions are imposed on the boundary of tubular neighborhood of a constraint manifold (removal of a manifold with a small volume). We have shown that the encounter time is Poissonian and obtained an asymptotic expression of the MFET as function of the polymer length N and ϵ. The MFET is inversely proportional to the first eigenvalue of the FP-operator but the expansion is not uniform in ϵ and N. Finally, we show that the asymptotic estimation of the eigenvalue can be obtained by a generalized boundary layer analysis in high dimension. [Joint work with David Holcman.]
References:
[1] A. Amitai, I. Kupka, and D. Holcman, Computation of the Mean First-Encounter Time between the Ends of a Polymer Chain, *Phys. Rev. Lett.* **109**, 108302 (2012).
[2] A. Amitai, I. Kupka, and D. Holcman, Kinetics and encounter rates of diffusing polymers in confined microdomains, *J. Stat. Phys.* **153**, 1107–1131 (2013).
[3] A. Amitai and D. Holcman, Diffusing polymers in confined microdomains and estimation of chromosome territory sizes from chromosome capture data, *Phys. Rev. Lett.* **110**, 248105 (2013).
[4] A. Amitai, I. Kupka, and D. Holcman, Analysis of the mean first looping time of a rod-polymer, *Multiscale Model. Simul.* **10**, 612 (2012).

Speaker: **Pedro Antunes** (Math., Lusophone Univ. of Humanities and Technologies, Lisbon)
Title: *Numerical calculation of localized eigenfunctions of the Laplacian*
Abstract: It is well known that for some planar domains, the Laplacian eigenfunctions are localized in a small region of the domain and decay rapidly outside this region. We address a shape optimization problem of minimizing or maximizing the L^2 norm of the eigenfunctions in some sub-domains. This problem is solved by a numerical method involving the Method of Fundamental Solutions and shape derivatives. Several numerical simulations illustrate the good performance of the method.

Speaker: **Dmitry Beliaev** (Math., Oxford Univ.)
Title: *Random plane wave*
Abstract: Random plane wave is a universal model for high energy Laplace eigenfunctions in domains with chaotic dynamics. Bogomolny and Schmit conjectured that the morphology of nodal domains could be well described by the critical percolation on $\mathbb{Z}^2$. We will discuss this conjecture and possible ways to relate percolation and random plane wave.

Speaker: **Mirza Faisal Beg** (Eng. Sci., Simon Fraser Univ.)
Title: *Applications of Laplacian eigenfunctions and heat kernels in computational anatomy*
Abstract: In this talk, I will review the applications of Laplacian Eigenfunctions and Heat Kernels in problems related to shape analysis in the field of computational anatomy. In particular, examples from applications in brain mapping will be discussed.

Speaker: **Rafael Benguria** (Instituto de Física, Pontificia Universidad Católica de Chile)
Title: *The Brezis–Nirenberg problem on $\mathbb{S}^n$ in spaces of fractional dimension*
Abstract: We consider the nonlinear eigenvalue problem,

$$-\Delta_{\mathbb{S}^n} u = \lambda u + |u|^{4/(n-2)} u,$$

with $u \in H_0^1(\Omega)$, where Ω is a geodesic ball in $\mathbb{S}^n$ contained in a hemisphere. In dimension 3, Bandle and Benguria proved that this problem has a unique positive solution if and only if

$$\frac{\pi^2 - 4\theta_1^2}{4\theta_1^2} < \lambda < \frac{\pi^2 - \theta_1^2}{\theta_1^2}$$

where θ_1 is the geodesic radius of the ball. For positive radial solutions of this problem one is led to an ODE that still makes sense when n is a real number rather than a natural number. In this talk we will consider precisely that problem with $3 < n < 4$. Our main result is that in this case one has a positive solution if and only if λ is such that

$$\frac{1}{4}[(2\ell_2+1)^2 - (n-1)^2] < \lambda < \frac{1}{4}[(2\ell_1+1)^2 - (n-1)^2]$$

where ℓ_1 (respectively ℓ_2) is the first positive value of ℓ for which the associated Legendre function $\mathrm{P}_\ell^{(2-n)/2}(\cos\theta_1)$ (respectively $\mathrm{P}_\ell^{(n-2)/2}(\cos\theta_1)$) vanishes. [Joint work with Soledad Benguria (Math., Univ. Wisconsin, Madison); The work of RB has been supported by Fondecyt (Chiile) Project # 112–0836 and by the Nucleo Milenio en "Física Matemática", RC–12–0002 (ICM, Chile)]

Speaker: **Gregory Berkolaiko** (Math., Texas A&M Univ.)
Title: *Symmetry and Dirac points in graphene spectrum*
Abstract: Many exciting physical properties of graphene can be traced to the presence of conical singularities ("Dirac points") in its dispersion relation. Initial calculations (e.g., Wallace (1947)) were done within the tight-binding approximation (essentially, a discrete Laplace operator on two vertices). More recently, Kuchment and Post (2007) showed the presence of Dirac points in quantum graphs arranged to resemble graphene (honeycomb) structure. Grushin (2009) considered the Laplacian in $\mathbb{R}^2$ with a weak potential having symmetries of the honeycomb lattice; Fefferman and Weinstein (2012) proved the presence of Dirac points for a general potential with required symmetry. We will present a general yet very simple proof that works in all the above models: $\mathbb{R}^2$ Laplacian, discrete Laplacian and quantum graph Laplacian.

In the talk we will discuss the statement of the problem, give a short proof of the presence of the Dirac points using quotients of the operator by the (co)representations of the symmetry group, and a proof of stability of Dirac points which uses the Berry phase, illustrated by animations. [The talk is based on joint work with A. Comech, arXiv:1412.8096 .]

Speaker: **Michael Bronstein** (Informatics, Univ. Lugano)
Title: *Old new methods for manifold correspondence*
Abstract: In recent years, geometric data is gaining increasing interest both in the academia and industry. In computer graphics and vision, this interest is owed to the rapid development of 3D acquisition and printing technologies, as well as the explosive growth of publicly-available 3D shape repositories. In machine learning, there is a gradual understanding that geometric structure plays an important role in high-dimensional complicated datasets. In this talk, I will use the problem of manifold correspondence

(a fundamental and notoriously hard problem with a wide range of applications in geometric processing, graphics, vision, and learning) as a showcase for classical methods from the domain of signal processing (such as sparse coding, joint diagonalization, and matrix completion) applied to geometric problems. I will show applications to 3D shape correspondence, multi-view clustering, and image labeling.

Speaker: **Krzysztof Burdzy** (Math., Univ. Washington)
Title: *On obliquely reflected Brownian motion*
Abstract: I will present forthcoming results on obliquely reflected Brownian motion in fractal domains. Time permitting, I will also discuss discrete approximations of reflected Brownian motion in fractal domains. [Joint work with Zhenqing Chen, Donald Marshall and Kavita Ramanan.]

Speaker: **Alexander Cloninger** (Appl. Math., Yale Univ.)
Title: *Bigeometric organization of deep nets and function driven embeddings*
Abstract: In this talk, we explore ways to augment diffusion kernels on data sets using knowledge of function values on a subset of the data. The function is used to discover features in the data that are locally invariant, as well as features that are locally insignificant, and to build a diffusion kernel on the data that diffuses quickly along these local irrelevant features. The new metric and diffusion kernel, generated via iterations of stacked neural nets, create an embedding which captures the geometry of the data while yielding a low Lipschitz constant with respect to the functions of interest. Along with this, the new kernel does not depend explicitly on the function values, which makes it trivially extendable to new points. We examine the use of these kernels on several medical examples.

Speaker: **Guy David** (Math., Univ. Paris-Sud)
Title: *A free boundary problem connected to the localization of eigenfunctions*
Abstract: I will try to present a free boundary problem, which is close to the celebrated Alt, Caffarelli, and Friedman problem, but with N phases. In its simplest form, we are given a domain O and some data (such as a potential V on the domain), and an integer N, and we minimize a functional whose argument is a collection of N disjoint subsets E_i of O, and corresponding functions u_i such that $u_i = 0$ out of O_i, and whose main parts are the energy $\sum_i \int_{E_i} |\nabla u_i|^2$ and a simple function of the volumes of the E_i. I'll try to explain why we (with D. Jerison, M. Filoche, and S. Mayboroda) wanted to use such functionals to decompose O into subdomains where eigenfunctions for the operator $-\Delta + V$ may be localized, and some of the regularity properties of the free boundaries ∂E_i for minimizers.

Speaker: **Marcel Filoche** (Laboratory of Condensed Matter Physics, CNRS - Ecole Polytechnique)
Title: *Localization of Laplacian and bi-Laplacian waves and vibrations*
Abstract: Localization of Laplacian or bi-Laplacian waves plays a major role in the behavior of numerous physical systems. It can be the result of a complex medium, of the geometry of the vibrating structure, or even due to the presence of disorder. It has been observed in acoustics, optics, and is responsible for some important quantum properties. In mechanics, a marked localization of steady vibrations can be achieved in rectangular clamped plates by blocking only one interior point [1]. In this talk, we will show that this phenomenon can be explained within a general theory of localization applying to all vibratory systems whose wave equation derive from an energy form [2]. The main tool of this theory, the localization landscape, controls the amplitude of the stationary vibrations, and predicts the spatial regions where vibrations will be localized as well as the frequencies above which a delocalization transition occurs. We will present comparisons between numerical simulations and experimental measurements of wave localization in thin clamped plates, proving that it possible to directly observe the localization landscape in real devices. We will provide an example where the landscape can be used as a "design" tool by defining the positions of blocked points inside the plate in order to obtain specific spatial vibrating patterns. Finally we will briefly sketch the current developments of the theory as well as some of its applications. [Joint work with D. Arnold, G. David, D. Jerison, S. Mayboroda, P. Sebbah, M. Atlan.]
References:

[1] M. Filoche and S. Mayboroda, "Strong localization induced by one clamped point in thin plate vibrations," *Phys. Rev. Lett.* **103**, 254301 (2009).
[2] M. Filoche and S. Mayboroda, "Universal mechanism for Anderson and weak localization," *Proc. Natl Acad. Sci.* **109** (37) 14761–14766 (2012).

Speaker: **Denis Grebenkov** (Laboratory of Condensed Matter Physics, CNRS - Ecole Polytechnique)
Title: *Geometrical structure of Laplacian eigenfunctions*
Abstract: This talk is dedicated to low-frequency eigenfunctions of the Laplace operator in bounded Euclidean domains. In spite of a common picture of eigenfunctions as oscillating waves, the geometrical structure of Laplacian eigenfunctions is much richer and more complicated [1]. We consider a bounded domain of arbitrary shape with elongated "branches" of variable cross-sectional profiles. When an eigenvalue is smaller than a prescribed threshold (which is determined by the shape of the branch), the corresponding eigenfunction is proved to have an upper bound decaying exponentially fast along each branch [2,3]. This behavior is demonstrated for Dirichlet and Robin boundary conditions on the branch boundary. We also discuss how the exponential decay leads to localization or trapping of eigenmodes in finite quantum waveguides. In particular, we obtain a sufficient condition which determines the minimal length of branches for getting a trapped eigenmode. Varying the branch lengths may switch certain eigenmodes from non-trapped to trapped or, equivalently, the waveguide state from conducting to insulating [4].
References:
[1] D. S. Grebenkov and B.-T. Nguyen, Geometrical structure of Laplacian eigenfunctions, *SIAM Reviews* **55**, 601–667 (2013).
[2] B.-T. Nguyen, D. S. Grebenkov, and A. L. Delitsyn, On the exponential decay of Laplacian eigenfunctions in planar domains with branches, *Contemp. Math.* **630**, 337–348 (2014).
[3] A. L. Delitsyn, B.-T. Nguyen, and D. S. Grebenkov, Exponential decay of Laplacian eigenfunctions in domains with branches of variable cross-sectional profiles, *Eur. Phys. J. B* **85**, 371 (2012).
[4] A. L. Delitsyn, B.-T. Nguyen, and D. S. Grebenkov, Trapped modes in finite quantum waveguides, *Eur. Phys. J. B* **85**, 176 (2012).

Speaker: **Evans Harrell** (Math, Georgia Tech)
Title: *Optimal estimates of sums of eigenvalues and heat traces*
Abstract: I'll present two rather distinct results whose common theme is to say something optimal about heat traces. One of those is the analysis of an optimal placement problem for an obstacle in a domain, so as to maximize or minimize the heat trace. The other is a set of sharp semiclassical inequalities for sums of eigenvalues. (Sums of eigenvalues are closely related to traces of heat kernels, whether through the Laplace transform or through Karamata's theorem and its variants.) These are based on a new variational principle that incorporates averaging, which implies sharp estimates for Laplacians of various kinds, including those on graphs and on quantum graphs. [Parts of this work are joint with John Dever, Ahmad El Soufi, Said Ilias, and Joachim Stubbe.]

Speaker: **Antoine Henrot** (Institut Élie Cartan - Université de Lorraine)
Title: *Maximization of the first eigenvalue with an obstacle*
Abstract: In this talk, we are interested in maximizing the first eigenvalue of the Laplacian with Dirichlet boundary conditions by placing some obstacle K inside a fixed domain Ω. We will first recall some known results for that problem, mainly when the shape of the obstacle K is already given (then the question is to find its location). Here the shape of K is free and, for reasons which will be clear, we have to assume it connected and with a fixed perimeter. We will prove existence of a maximizer and some qualitative properties. We will also consider more specifically the case where Ω is a ball, for which we prove that the obstacle should be a concentric ball. We finish by giving a quantitative isoperimetric inequality in that context. [This is joint work with Davide Zucco (SISSA, Trieste).]

Speaker: **Lotfi Hermi** (Math., Univ. Arizona)
Title: *Isoperimetric upper bound for the fundamental tone of the membrane problem for a class of wedge-*

like domains
Abstract: Inspired by an old result of Pólya and Szegő, we introduce new geometric factors which lend themselves to the Payne interpretation in Weinstein fractional space to prove new isoperimetric inequalities which complement those of Payne-Weinberger and Saint-Venant, and offer to a new upper bound for the fundamental mode of vibration of a wedge-like membrane and a new lower bound for its "relative torsional rigidity". We show how to use this procedure to improve the bounds for certain triangles. [Joint work with A. Hasnaoui.]

Speaker: **Dmitry Jakobson** (Math., McGill Univ.)
Title: *Probability measures on manifolds of Riemannian metrics*
Abstract: We construct Gaussian measures on the manifold of Riemannian metrics with the fixed volume form. We show that diameter, Laplace eigenvalue and volume entropy functionals are all integrable with respect to our measures. We also compute the characteristic function for the L^2 (also called Ebin) distance from a random metric to the reference metric. [This is joint work with Y. Canzani, B. Clarke, N. Kamran, L. Silberman and J. Taylor.]

Speaker: **David Jerison** (Math., MIT)
Title: *Singularities of the wave trace at cluster points of the length spectrum*
Abstract: The wave trace is the spectral invariant function $h(t) = \sum \cos(\lambda_j t)$, in which the sum is over all eigenvalues $-\lambda_j^2$ of the Laplacian. It is called the wave trace because it is the trace of the solution operator for the wave equation. It is well known that $h(t)$ is singular at times t that are equal to the length of geodesics. The singularities are well understood if the length is an isolated point in the time line. This talk considers cluster points of the lengths. The simplest example, is the case of a circular billiard table. The regular inscribed polygons are periodic geodesics and the limit of their lengths is the perimeter. [Joint work with Colin de Verdiere and Victor Guillemin.]

Speaker: **Peter Jones** (Math., Yale Univ.)
Title: *Eigenfunctions and heat kernels: An overview*
Abstract: I will start by discussing the role of heat kernels and eigenfunctions on manifolds. The goal here is to review how to get local or global charts (using eigenfunctions) on compact manifolds. The second topic will be a discussion of the Gaussian Free Field (or Massless Free Field) in both astrophysics and 2D statistical physics. Here one obtains the GFF by taking each eigenfunction, multiplying it by an (independent) Gaussian random variable, and then summing over all eigenfunctions. (This sum is with probability = 0 anywhere convergent, but still just barely misses converging. It is however a nice distribution with probability = 1.) Exponentiation of the GFF leads to interesting classes of Borel measures that are closely related to SLE processes.

Speaker **Chiu-Yen Kao** (Math., Claermont McKenna College)
Title: *Shape optimization for eigenvalue problem involving biharmonic operators*
Abstract: The displacement of an isotropic plate vibration satisfies a fourth order partial differential equation with a biharmonic operator. In this talk, we will discuss several recent results for shape optimization of the corresponding eigenvalue problem. The aim is to find the minimum or maximum of a particular eigenvalue. We use efficient rearrangement algorithms to achieve the optimal configurations and numerous results will be shown to demonstrate the efficiency and robustness of the algorithms.

Speaker: **Rongjie Lai** (Math., Rensselaer Polytechnic Inst.)
Title: *Geometric understanding of point clouds using Laplace-Beltrami operator*
Abstract: Point clouds are the simplest and most basic forms for data representation in 3D and higher. Examples include those used in 3D modeling, image science, the Internet and many others. Analyzing and inferring the underlying structure from the point clouds are crucial in many applications. However, these raw data or primitive representations in $\mathbb{R}^n$ are lack of global parameterization and far from intrinsic, which

obstruct further intrinsic and global analysis for given data. On the other hand, point clouds from practical problems are usually with certain coherent structures which allow us to model them as points sampled from lower dimensional Riemannian manifolds in a high dimensional space. In this talk, I will discuss our work on solving PDEs on point clouds. It can be applied to manifolds with arbitrary dimensions and co-dimensions. This approach enables us to propose geometric methods for analyzing and understanding point clouds through solutions of PDEs on point clouds. As applications, I will demonstrate our methods to intrinsic geometric quantities extraction, comparison and registration for point clouds based on eigensystem of the Laplace-Beltrami operator.

Speaker: **Richard Laugesen** (Math, Univ. Illinois, Urbana-Champaign)
Title: *Steklov spectral inequalities through quasiconformal mapping*
Abstract: Eigenvalues of the Steklov or Dirichlet-to-Neumann operator represent frequencies of vibration of a free membrane whose mass is concentrated at the boundary. They arise also in sloshing problems.

We show the disk maximizes various functionals of the Steklov eigenvalues, under normalization of the perimeter and a kind of boundary moment. The results cover the first eigenvalue, spectral zeta function and trace of the heat kernel. Interestingly, the method employs quasiconformal mapping to estimate the distortion of the energy functional (Dirichlet integral). [Joint with Alexandre Girouard and Bartłomiej Siudeja.]

Speaker: **Bruno Levy** (CS, INRIA, Nancy)
Title *Computing diffusion distance Voronoi diagrams*
Abstract: In this presentation, I explain how to compute Voronoi diagrams on surfaces and volumes with some non-Euclidean distances, such as the diffusion distance [Coifman and Lafon, Rustamov]. The diffusion distance is defined by an embedding of the initial surface using coordinates that correspond to the eigenfunctions of a diffusion operator. Typically between 100 to 1000 eigenfunctions are necessary to obtain a faithful approximation of the diffusion distance. The algorithm needs to compute the intersection between a Voronoi diagram in this high-dimension space and the initial surface mapped by the embedding. To avoid the d space and time complexity factor of classical algorithms that compute a Voronoi diagram, I propose an alternative algorithm that can directly compute the intersection between the Voronoi cells and the surface, based on a simple characterization of the contributing bisectors that allows to compute them from a KdTree. In a more general setting, I propose an alternative of the algorithm, that solely depends on a kernel and does not need to use the high-dimensional space. This comes at the expense of replacing the KdTree used in the previous algorithm with a metric-space data structure.

Speaker: **Mauro Maggioni** (Math., Duke Univ.)
Title: *Geometric methods for the approximation of high-dimensional dynamical systems*
Abstract: We discuss a geometry-based statistical learning framework for performing model reduction and modeling of stochastic high-dimensional dynamical systems. We consider two complementary settings. In the first one, we are given long trajectories of a system, e.g., from molecular dynamics, and we discuss new techniques for estimating, in a robust fashion, an effective number of degrees of freedom of the system, which may vary in the state space of the system, and a local scale where the dynamics is well-approximated by a reduced dynamics with a small number of degrees of freedom. We then use these ideas to produce an approximation to the generator of the system and obtain, via eigenfunctions, reaction coordinates for the system that capture the large time behavior of the dynamics. We present various examples from molecular dynamics illustrating these ideas. In the second setting we only have access to a (large number of expensive) simulators that can return short simulations of high-dimensional stochastic system, and introduce a novel statistical learning framework for learning automatically a family of local approximations to the system, that can be (automatically) pieced together to form a fast global reduced model for the system, called ATLAS. ATLAS is guaranteed to be accurate (in the sense of producing stochastic paths whose distribution is close to that of paths generated by the original system) not only at small time scales, but also at large time scales, under suitable assumptions on the dynamics. We discuss applications to homogenization of

rough diffusions in low and high dimensions, as well as relatively simple systems with separations of time scales, and deterministic chaotic systems in high-dimensions, that are well-approximated by stochastic differential equations.

Speaker: **Rajesh Mahadevan** (Math., Universidad de Concepcion - Chile)
Title: *Eigenvalue minimization for the clamped plate under compression*
Abstract: In this paper we ask the question, among domains of equal volume, for which domain the first eigenvalue of the clamped plate with compression is a minimum. Previously, the Rayleigh conjecture for the clamped plate has been proved by Nadirashvili and, Ashbaugh and Benguria showing that this is least when the domain is a ball. For the clamped plate problem under compression it is reasonable to conjecture that the first eigenvalue is a minimum when the domain is a ball, for at least small values of the compression parameter. We sketch a proof of this conjecture. [Joint work with Mark Ashbaugh and Rafael Benguria.]

Speaker: **Svitlana Mayboroda** (Math., Univ. Minnesota)
Title: *Uniform rectifiability and harmonic functions*
Abstract: In relatively friendly geometric settings, e.g., on Lipschitz domains, harmonic measure is absolutely continuous with respect to the Lebesgue measure. Informally, one can say that Brownian travelers "see" portions of the boundary proportionally to their size. Moreover, quantitative absolute continuity of elliptic measure is equivalent to solvability of the Dirichlet problem in L^p, square function estimates, Carleson measure estimates, and a certain approximation property, so called ϵ-approximability of solutions. The present talk addresses the question what are the key geometrical properties of the boundary responsible for such a behavior of harmonic functions.

Despite recent successes of harmonic analysis on general uniformly rectifiable sets, for a long time connectivity seemed to be a vital hypothesis from the PDE point of view. Indeed, in 1990 Bishop and Jones have produced a counter-example to show that the 1916 F. and M. Riesz Theorem does not hold in the absence of connectivity: harmonic measure maybe singular with respect to Hausdorff H^1 measure on the boundary of a planar domain. The main result of our work shows that, in spite of Bishop-Jones counterexample, some key properties of harmonic functions remain valid in general uniformly rectifiable domains. Most notably, Carleson measure estimates and the aforementioned ϵ-approximability hold in the absence of any connectivity hypotheses. [This is joint work with S. Hofmann and J.-M. Martell.]

Speaker: **François Meyer** (ECE/Appl. Math., Univ. Colorado, Boulder)
Title: *Decoding epileptogenesis in a reduced state space*
Abstract: In many areas of science the only method to study a complex system entails making indirect time-resolved measurements of the state of the system. In the absence of a detailed mathematical model that can be used to explain the measurements, we have to resort to machine learning methods to learn the association between the state of the system and the measurements. An example of such a problem involves the definition of a biomarker to monitor epileptogenesis following a traumatic brain injury.

In this talk I will describe the recent results of a multidisciplinary effort to actively and continuously decode the progressive changes in neural network organization leading to epilepsy. Using an animal model of acquired epilepsy, we chronically recorded hippocampal auditory evoked potentials during epileptogenesis. Our approach combines in a unique manner applied harmonic analysis, spectral graph theory, and state-space models to infer the hidden distinct stages of epileptogenesis. Using our decoding algorithm, we were able to show that archetypal changes in the waveform morphology have universal predictive value for the development of spontaneous recurrent seizures.

Speaker: **Anna Maltsev** (Math., Univ. Bristol)
Title: *On localization phenomena*
Abstract: The localization of eigenvectors plays an important role in several areas of mathematics, ranging from random matrix theory to the study of quantum graphs. I will discuss the localization phenomena in

various settings, with a few illustrative examples and some more general estimates on eigenfunction decay.

Speaker: **Naoki Saito** (Math., Univ. California, Davis)
Title: *Laplacian eigenfunctions that do not feel boundary*
Abstract: I will discuss Laplacian eigenfunctions defined on a Euclidean domain of general shape, which "do not feel the boundary." These Laplacian eigenfunctions satisfy the Helmholtz equation inside the domain, and can be extended smoothly and harmonically outside of the domain. Although these eigenfunctions do not satisfy the usual Dirichlet or Neumann boundary conditions, they can be computed via the eigenanalysis of the integral operator (with the potential kernel) commuting with the Laplace operator. In this talk, I will discuss their properties, the relationship with the Krein-von Neumann self-adjoint extension of unbounded symmetric operators, as well as with the Neumann-Laplacian eigenfunctions. [A part of this talk is joint work with Eugen Shvarts.]

Speaker: **P. N. Shivakumar** (Math., Univ. Manitoba)
Title: *Eigenvalues for infinite matrices, their computations and applications*
Abstract: In this talk we discuss properties eigenvalues which occur in infinite linear algebraic systems of the form $Ax = b$, where A is an infinite diagonally dominant matrix and A is considered as a bounded linear operator. Location of eigenvalues and lower and upper bounds are given. The theory is illustrated for the eigenvalues of the well known Mathieu equation. We also discuss the classical question "Can you hear the shape of a drum," the answer is known to be "yes" for certain convex planar regions with analytic boundaries and "no" for some polygons with reentrant corners. We develop a constructive analytic approach to indicate how a pre-knowledge of eigenvalues lead to the parameters of the boundary. In the case of a square we get an insight why analytic methods do not yield an answer, while in cases where the boundary is a circle or an ellipse or an annulus, the answer is "yes." In many elliptic partial differential equations, one gets infinite series whose zeros are the eigenvalues. Given the infinite series in the eigenvalue, we reformulate the problem as $Ax = b$, where A is the van der Waerden matrix, x is unit vector and the vector b is expressed in terms of the coefficients of the infinite series. This formulation leads to developing some bounds for the eigenvalues. Some applications to digital circuit dynamics, inequalities for the minimal eigenvalue of the Laplacian in an annulus are also given. [This is a joint work with Yang Zhang.]

Speaker: **Bartłomiej Siudeja** (Math., Univ. Oregon)
Title: *Nearly radial Neumann modes on highly symmetric domains*
Abstract: Schiffers conjecture states that if Neumann eigenfunction is constant on the boundary of a domain, then the domain is a disk, or the eigenfunction is constant. The disk is special, due to presence of radial modes. We will discuss existence of Neumann modes on regular polygons and boxes which are nearly radial (do not change sign on the boundary). We will build on the following recent result of Hoffmann-Ostenhof: an eigenfunction of a rectangle that is strictly positive on the boundary must be constant.

Speaker: **Yiqiao Song** (Schlumberger-Doll Research, Cambridge, MA)
Title: *Diffusion dynamics from multi-point correlation functions*
Diffusion dynamics is an important tool to study the microstructure of a wide range of porous materials, including geological formations and biological tissues. When the pore size and diffusion distance are close, the diffusion behavior can non-Gaussian. This non-Gaussian behavior is often characterized by the fourth order cumulant, Kurtosis. However, the apparent non-Gaussian behavior may also arise from pore size heterogeneities. These two scenarios cant be identified by only the two-point correlation function, such as PDF of displacement.

We have shown that the use of multi-point correlation functions allows the identification of different diffusion dynamics and developed the corresponding NMR experiments. For example, bulk diffusion and restricted diffusion can be identified by a correlation of displacement of different diffusion times. Kurtosis exhibits a unique 4-cycle sinusoidal modulation of the 4-point correlation function and easily distinguished from pore size distribution. We will discuss both theoretical understanding of the diffusion dynamics in term of these correlation functions and feasibility of experimental implementation and experimental results.

Speaker: **Michiel van den Berg** (Math., Univ. Bristol)
Title: *Heat flow in Riemannian manifolds*
Abstract: Let Ω be an open set in a complete, connected, non-compact Riemannian manifold with $\dim M = m$, and non-negative Ricci curvature. We study the heat flow from Ω into $M - \Omega$ if the initial temperature distribution is the characteristic function of Ω. For Ω of infinite measure we obtain a necessary and sufficient condition to have finite heat content for all $t > 0$ and obtain upper- and lower bounds for the heat content. Two-sided bounds are obtained for the heat loss of Ω in M if the measure of Ω is finite.