

BIRS workshop 15w5067: Advances in Numerical Optimal Transportation

Feb 15 - Feb 20, 2015

February 10, 2015

1 Schedule

The (xx) number corresponds to the abstracts in section 2.

- Monday

7:00-9:00 Breakfast
9:00-10:00 Bruno Levy (23)
10:00-10:30 Coffee Break
10:30-11:00 Martial Agueh (10)
11:00-11:30 Guillaume Carlier (11)
11:30-13:30 Lunch
13:30-14:30 Free Time
14:00-14:30 Bertrand Maury (2)
14:30-15:00 Qinglan Xia (16)
15:00-15:30 Coffee Break
15:30-16:00 Louis-Philippe Saumier (7)
16:00-16:30 Nizar Touzi (21)
16:30-17:30 Free Time
17:30-19:30 Dinner

- Tuesday

7:00-9:00 Breakfast
9:00-10:00 Jean-Marie Mirebeau (20)
10:00-10:30 Coffee Break
10:30-11:00 Marie-Thérèse Wolfram (14)
11:00-11:30 Horst Osberger (8)
11:30-13:30 Lunch
13:30-14:30 Free Time
14:30-15:00 Filippo Santambrogio (15)
15:00-15:30 Coffee Break

15:30-16:00 Chris Budd (4)

16:00-16:30 Hongkai Zhao (12)

16:30-17:30 Free Time

17:30-19:30 Dinner

- Wednesday

7:00-9:00 Breakfast
9:00-10:00 Marco Cuturi (9)
10:00-10:30 Coffee Break
10:30-11:00 Francesco Patacchini (26)
11:00-11:30 Ruan Yuanlong (25)
11:30-13:30 Lunch

- Thursday

7:00-9:00 Breakfast
9:00-10:00 Cristian Gutierrez (5)
10:00-10:30 Coffee Break
10:30-11:00 J.H.M. Tenthijeboonkkamp (3)
11:00-11:30 Boris Thibert (19)
11:30-13:30 Lunch
13:30-14:30 Free Time
14:30-15:00 Gabriel Peyré (13)
15:00-15:30 Coffee Break
15:30-16:00 Justin Solomon (18)
16:00-16:30 Bjorn Engquist (22)
16:30-17:30 Free Time
17:30-19:30 Dinner

- Friday

7:00-9:00 Breakfast
9:00-10:00 Brendan Pass (1)
10:00-10:30 Coffee Break
10:30-11:00 Quentin Mérigot (21)
11:00-11:30 Mike Cullen (6)
11:30-13:30 Lunch

2 Abstracts

1. Brendan Pass (U. Alberta)

Title: Multi-marginal optimal transport and application

Abstract: I will give a general overview of the multi-marginal optimal transport problem and outline several applications. I will try to provide some insight into how and why the uniqueness and structure of solutions differ, depending on the cost function, a phenomenon largely absent in the classical, two marginal problem.

2. Bertrand Maury (U. Paris-Sud)]

Title: Pressureless Euler equation with a maximal density constraint

Abstract: An optimal transportation framework has been recently proposed to account for congestion in macroscopic crowd motion models. We shall present recent attempts to extend this approach to second order in time models, namely macroscopic granular flows with non elastic collisions.

3. J.H.M. Tenthijeboonkkamp (TU Eindhoven)]

Title: A least-squares method for optimal transport with applications to optics

Abstract: In this talk I will outline a numerical method to solve the optimal transport problem with quadratic cost function, and the equivalent Monge-Ampère BVP with transport boundary condition. The method computes a map m that minimizes a suitable functional containing the 2-norm of mT^*P , where P is a symmetric positive definite matrix, and a boundary integral taking into account the transport boundary condition. Both parts of the functional are successively minimized in an iterative fashion. Upon convergence we solve $u = \min$ a least-squares sense, where u is the solution of the related Monge-Ampère BVP. As an application I will show how to compute a freeform lens surface given by $z = u(x, y)$ that converts a parallel beam of light into a desired target distribution.

4. C. Budd (U. Bath)

Title: Monge Ampere based methods for Mesh Generation

Abstract: When numerically solving a PDE in three dimensions, it is often necessary to generate a mesh on which to discretise the solution. Often this can be expensive to do. However, by using idea from optimal transport it is possible both to construct a mesh quickly and cheaply, and also to prove that it has the necessary regularity properties to allow an accurate approximation of the solution of the PDE. In this talk I will describe these methods, show how to prove regularity, and then apply the methods to problems in meteorology.

5. Cristian Gutierrez (Temple University)

Title: Refraction problems in geometric optics

Abstract: Given two homogeneous and isotropic materials I and II with refractive indices n_I and n_{II} , the problems described in this talk concern with the design of interface surfaces between the two materials, so that radiation emanating from one source lying in medium I and having a given intensity depending of the direction, is refracted by the surface into a prescribed destination, such as a set of directions or a target lying in medium II, and with a given intensity, in general a Radon measure. This leads to the design of lenses refracting radiation in a prescribed manner. We prove existence of these surfaces using an optimization process when the destination is a finite set, and then by a limiting process in the general case. The surface solutions satisfy pdes of Monge-Ampere type. Time permitting, I will discuss some recent regularity results.

6. M. Cullen (Met. Office)

Title: Computations of singular solutions of an optimal transport problem.

Abstract: The semi-geostrophic Eady problem has been demonstrated to produce finite-time singularities. The singularities then go through a quasi-periodic lifecycle which is

a simple model of the lifecycles of weather systems. Successful computations of these lifecycles have been carried out by Cullen and Purser using a geometric method which is specific to this problem and does not work well when generalised to other problems. Therefore there is a strong need to obtain these results using more conventional numerical methods. I will show how this can be done by computing solutions of the Euler problem of which the semi-geostrophic solution is the asymptotic limit as the Rossby number tends to zero. Since the semi-geostrophic solution is independent of Rossby number in this case, it can be computed as the limit of the Euler solutions. The results illustrate the expected rate of convergence to the singularity. However, the subsequent evolution is not sufficiently energetic, because the numerical methods are too dissipative. I show diagnostics illustrating this, and discuss how it may be possible to do better.

7. Louis-Philippe Saumier Demers (U. Victoria)

Title: Optimal Transport for Particle Image Velocimetry

Abstract: Particle Image Velocimetry (PIV) is a popular technique used to measure the velocity field of experimental flows. The idea is to seed small particles in a fluid and to illuminate them with a pulsing laser. The distribution of light scattered by the particles is recorded at every pulse and a discrete sequence of images is created. An approximate velocity field for the fluid is then obtained by analyzing the images, traditionally with algorithms based on cross-correlation of parts of these images. Recently, a new method based on Optimal Transport (OT) has been developed for PIV. In this method, two successive images are taken as the initial and final densities in the L2 OT problem. I will present during this talk a model used to analyze and predict the behaviour of the OT map and corresponding field for these types of images. I will also briefly present numerical strategies and associated results for real and synthetic data. This is joint work with M.Agueh and B.Khouider.

8. Horst Osberger (TU Muenchen)

Title: FULLY-DISCRETE VARIATIONAL LAGRANGIAN SCHEMES FOR NONLINEAR DRIFT-DIFFUSION EQUATIONS

Abstract: It is now well-known, that several diffusion equations of second and fourth order are gradient flows of an entropy in the L^2 -Wasserstein metric.

On basis of the equations' underlying variational structure, we define fully-discrete numerical schemes in one and higher space dimension, which inherits several important structural properties of the continuous flow — like conservation of mass, nonnegativity of solutions and dissipation of the entropy. We perform our spatial discretization by restricting the gradient flow to a suitable finite-dimensional submanifold inside the space of probability densities. The definition of this submanifold is purely Lagrangian — any density is given as the push-forward of a fixed reference density with respect to a Lagrangian map, chosen from a finite-dimensional subset of (optimal) transportation maps. The evolution of the discrete solution corresponds to a temporal movement of packages of mass. We present numerical results, and prove convergence of the scheme and long-time-asymptotics at least in one spatial dimension.

9. Marco Cuturi (Kyoto U.)

Title: Optimal Transport with Entropy Regularization: Pros and Cons

Abstract: The idea of regularizing optimal transport problems with an entropic penalty has been known for decades now (e.g. Wilson 1969). It was mostly introduced in the

relevant literature as a modeling tool (e.g. the "gravity model") to enforce desirable properties for optimal couplings, such as balanced flows between equally expensive routes and overall smoothness. Only recently was it shown that such a regularization can provide an extremely efficient computational framework to approximate optimal transport using the toolbox of (strict) convex optimization. I will provide in this short talk an overview of this regularization. I will insist in particular on the numerical advantages and disadvantages compared to using well established LP formulations, and also provide examples where this regularization is not only beneficial computationally but also yields a better performance in applications (e.g. inference, clustering).

10. Martial Agueh (U. Victoria)

Title: Kinetic models of granular and optimal transport I: general local existence,

Abstract: Granular materials consist of a large number of discrete solid particles (such as grains) which interact by nearly instantaneous collisions, much like in the classical model of a gas; but contrarily to ideal gas particle collisions, collisions of granular particles are inelastic, that is, characterized by a loss of kinetic energy. In this short talk, we will prove the local existence of weak solutions to a kinetic model of granular media. The existence proof relies on a splitting method (separating advection in position and interaction in velocity) where the spatially homogeneous equation is interpreted as the gradient flow of a convex interaction energy (in the velocity space) with respect to the Wasserstein distance.

11. Guillaume Carlier (U. Paris Dauphine)

Title: Kinetic models of granular and optimal transport II: one-dimensional models,

Abstract: The aim of this short talk is to show that for one-dimensional kinetic models of granular media, global results can be obtained for certain interaction kernels. For kernels whose second derivative is subquadratic near zero (which does not cover the quadratic case), we will prove a global entropy bound. In the quadratic kernel case, there is a first integral of motion which enables us to reformulate the model as a gradient flow.

12. Hongkai Zhao (UC Irvine)

Title: Multi-scale Non-Rigid Point Cloud Registration Using Robust Sliced-Wasserstein Distance via Laplace-Beltrami Eigenmap

Abstract: We propose computational models and algorithms for point cloud registration with non-rigid transformation. First, point clouds sampled from manifolds originally embedded in Euclidean space are transformed to new point clouds by Laplace-Beltrami (LB) eigenmap using the n leading eigenvalues and corresponding eigenfunctions of LB operator defined intrinsically on the manifolds. LB eigenmap is invariant under isometric transformation of the original manifolds. Then we design computational models and algorithms for registration of the transformed point clouds in distribution/probability form based on the optimal transport theory which provides both generality and flexibility. Our methods use robust sliced-Wasserstein distance and incorporate a rigid transformation to handle ambiguities introduced by the LB eigenmap. By going from smaller n , which provides a quick and robust registration (based on coarse scale features) as well as a good initial guess for finer scale registration, to a larger n , our method provides an efficient, robust and accurate approach for multi-scale non-rigid point cloud registration.

13. Gabriel Peyré (U. Paris-Dauphine)

Title: A Review of Wasserstein Barycenter Algorithms, with a New One

Abstract: The computation of Wasserstein barycenters is the cornerstone that allows to unleash the power of optimal transport (OT) methods to applications such as imaging sciences and machine learning. Indeed, it enables to extend traditional problems in these fields, such as texture synthesis, color normalization, clustering and principal component analysis from Euclidean settings to the OT's world. This is crucial to advance the front of research in these areas, because OT takes into account some underlying ground metric, which exploit correlations and structures between the considered variables (color and edges locations/directions for images, bag of features for machine learning). The computation of OT barycenters is however a challenging task when one considers large-scale setups where thousands of histograms of giga-samples need to be averaged. A recent breakthrough in this area is the idea of extending the entropic regularization of OT (which dates back to early works of Schrödinger, and was recently revitalized by Marco Cuturi in machine learning) to the setting of more advanced variational problems such as Wasserstein barycenters. Relying on this idea, I will show how several algorithms can be derived from either the primal or the dual formulation, and how this relates to classical convex optimization methods exploiting the geometry of the Kullback-Leibler divergence. I will show several promising results, and argue that this new computational paradigm has the potential to deeply impact the machine learning and imaging communities. This is a joint work with Marco Cuturi, Luca Nenna, Jean-David Benamou and Guillaume Carlier.

14. M.T. Wolfram (RICAM)

Title: Finite element methods for optimal transportation problems

Abstract: In this talk we discuss finite element methods for two optimal transportation problems, in particular a class of nonlinear convection-aggregation equations and the Monge-Ampère equation. In the first case we propose a numerical algorithm, in which the density is represented by an evolving diffeomorphism mapping the unknown density to the constant distribution of mass. The scheme is based on the variational formulation and automatically adapts the mesh to the shape of the mass distribution. In case of the Monge-Ampère equation we present a vanishing viscosity approach using C^0 -conforming finite elements. We discuss the appropriate construction of the corresponding Newton method and illustrate the properties of the scheme with various numerical simulations.

15. Filippo Santambrogio (U. Paris Sud)

Title: An L^∞ estimate for Keller-Segel with arbitrary diffusions

Abstract: I will present some new L^∞ estimates for the Keller-Segel model of chemotaxis, based on a JKO-like scheme, and on a fine analysis of the optimality conditions at every time step, combined with the use of the Monge-Ampère equation. These estimates work independently on the type of diffusion (linear or non-linear) and of the supercriticality or not of the mass, but, unfortunately (there is no free lunch!), explode in finite time. This allows anyway to prove existence of short-time L^∞ solutions, and to apply recent uniqueness results by Carrillo-Lisini-Mainini which require this kind of bounds. The result has been obtained in collaboration with J.-A. Carrillo.

16. Qinglan Xia (U.C. Davis)

Title: Numerical simulations of optimal transport paths

Abstract: In the ramified optimal transportation, an optimal transport path is a geodesic in the space of probability measures with respect to the metric d_α . In this talk, I will describe how to simulate an optimal transport path using some heuristics approaches.

If time permits, I will also describe some applications of this simulation to model blood vessel structures in human placentas.

17. Alfred Galichon (Science Po Paris)

Title: Equilibrium Transportation: Existence and Computation

Abstract: We study the problem of "Equilibrium Transportation", which includes as a particular case "Optimal Transportation". In Economics, this class of models is called matching with imperfectly transferable utility (ITU), including as special cases both the transferable utility (Monge-Kantorovich) and nontransferable utility (Gale-Shapley) models. Versions of this model capture a number of situations in economics: marriage, the labor market, industrial alliances, school choice, etc.

We provide a unified framework for equilibrium characterization and comparative statics in a version of the model where agents have a random utility component which introduces stochasticity in their choice problem. This framework allows the study and the computation of non-variational equilibrium transportation problems which are relevant in Economics, and cannot be tackled using the theory of optimal transportation.

18. Justin Solomon (Stanford U.)

Title: Transportation Techniques for Geometric Data Processing

Abstract: Countless problems in graphics, geometry processing, and learning can be posed as optimizations over triangulated surfaces, volumes, the image plane, and other geometric domains. Relaxations and alternative forms of these problems commonly can be posed using optimal transportation. In this talk, I will introduce machinery designed with these applications in mind, helping scale and apply transportation at-scale to challenging problems in image processing, registration, and shape analysis.

19. Boris Thibert (U. Joseph Fourier)

Title: Numerical resolution of the far field reflector problem

Abstract: The far-field reflector problem is a well-known inverse problem arising in geometric optics. It consists in creating a mirror that reflects a given point light source to a prescribed target light at infinity. This problem can be formulated as an optimal transport problem on the unit sphere. L. A. Caffarelli, S. Kochengin and V. Oliker proposed in 1999 an algorithm that involves the computation of the intersection of the convex hull of confocal paraboloids. We will show that the combinatorics of this intersection can be efficiently computed using tools from computational geometry, thus providing an efficient algorithm for the far field reflector problem. This work is in common with Pedro Machado et Quentin Mérigot.

20. Jean-Marie Mirebeau (U. Paris Dauphine)

Title:

Abstract:

21. Nizar Touzi (Ecole Polytechnique)

Title:

Abstract:

22. Bjorn Engquist (UT Austin)

Title: Optimal transport for seismology

Abstract: In seismic exploration a wave field is generated at the surface and reflections from the earth's interior are recorded. The purpose is to find properties such as wave velocity and location of reflecting sub layers. This is done in an inverse process where the measurements are compared to a computed wave field with unknown coefficients in the wave equation. We propose Wasserstein metric for this comparison. We will also remark on applications to registration in seismology.

23. Bruno Levy (INRIA)

Title:

Abstract:

24. Quentin Mérigot (U. Paris Dauphine)

Title:

Abstract:

25. Ruan Yuanlong (U. Mc Gill)

Title: Multiscale LP solvers for OT

Abstract:

26. Francesco Pataccini (Imperial College) *Title:* Gamma-convergence of the Discrete Internal Energy and Application to Gradient Flows

Abstract: We want to approximate diffusion equations with discrete numbers of particles. As the quadratic Wasserstein energy functional for these equations is not defined for point masses, we spread uniformly the mass of each particle in a specific ball around it, whose radius depends on the distance to the nearest neighbour. This "tessellation" gives rise to a discrete energy functional well defined on point masses, which we prove gamma-converges in the quadratic Wasserstein topology to its continuum version as the number of particles tends to infinity. For the specific case of the linear diffusion equation we use this result, in addition to Serfaty's conditions, to show the convergence of the resulting discrete gradient flow to the standard heat equation. By using a JKO scheme on the discrete gradient flow, we show numerical simulations for the heat and porous medium equations. This is a joint work with J. A. Carrillo, Y. Huang, P. Sternberg and G. Wolansky.