⧫ Talk Titles and Abstracts

Optimization of Lyapunov exponents of matrix cocycles

Jairo Bochi (Pontificia Universidad Católica de Chile)

I will discuss the problem of optimizing (i.e., maximizing or minimizing) the upper Lyapunov exponent of a matrix cocycle. The main result to be presented, joint with Michal Rams (Warsaw), says that if a 2x2 one-step cocycle has certain hyperbolicity properties (namely, there exist strictly invariant cones whose images do not overlap) then the Lyapunov-optimizing measures have zero entropy. The proof has two steps: first, a generalization of the Barabanov norm (similar to Mañé lemma) and second, a study of geometrical constraints between the invariant directions.

Asymptotics for random intermittent maps

Chris Bose (University of Victoria)

This talk reports on joint work with Wael Bahsoun and Yuejiao Duan, University of Loughborough, UK.

Expanding interval maps with a neutral fixed point are some of the simplest examples of nonuniformly hyperbolic systems. They are frequently studied for their potential to give interesting statistical behaviour such as sub-exponential decay of correlation, intermittency or so-called anomalous diffusion (different terms that amount to essentially the same thing: slow relaxation to equilibrium).

A random map (skew product with a Bernoulli shift) constructed from a family of such nonuniformly hyperbolic maps undoubtedly inherits some of these intermittency features, but exactly how they combine may not be immediately obvious. We will show, for example, that the rate of correlation decay is completely determined by the 'least nonuniformly hyperbolic' map in the family, no matter how infrequently the map is chosen in the randomization. Once you guess the result, the reason behind this principle is actually rather transparent and amounts to an elementary calculation about large deviations in Bernoulli trials.

Invariant measures stochastic Navier-Stokes equations in unbounded domains via bw-Feller property

Zdzislaw Brzezniak (York University)

In this talk I will describe a general result on the existence of invariant measure for Markov processes having the bw-Feller property and will show how this can be applied to stochastic Navier-Stokes equations in unbounded domains. This talk is based on joint works with M Ondrejat and Ela Motyl. The results presented are in some sense generalisations of related results for stochastic nonlinear beam and wave equations (where a Pritchard-Zabczyk trick playes an essential rôle) obtained in a joint woth with M. Ondrejat and J. Seidler.

Attractors for nonautonomous random dynamical systems with an application to stochastic resonance

Anna Cherubini (University of Salento)

We consider random dynamical systems with nonautonomous deterministic forcing and provide existence results for nonautonomous random attractors. In particular, we prove the existence of an attracting random periodic orbit for a class of one-dimensional random dynamical systems with a time-periodic forcing, generalising results obtained by Hans Crauel and Franco Flandoli. As an application, we study a standard model for the stochastic resonance in this framework, given by the one-dimensional 'overdamped' approximation of the stochastic Duffing oscillator. (Joint work with J.S.W. Lamb, M. Rasmussen and Y. Sato.)

Oseledets splittings for semi-invertible linear cocycles: Existence, stability, and applications.

Gary Froyland (University of New South Wales)

I will report on a program of work to establish existence and stability results for linear cocycles in the semi-invertible situation - where the driving mechanism is invertible, but the linear actions may be non-injective - and to create numerical methods to apply to real-world models and data. The "existence of Oseledets splitting" results provide a stronger multiplicative ergodic theorem than the "classical" theorems, which only guarantee the existence of measurable Oseledets filtrations. The stability results concern continuity properties of the Lyapunov exponents and their corresponding splitting elements when the linear actions are subjected to a variety of perturbations. The applied motivations for this work are the detection and tracking of so-called coherent structures in time-dependent dynamical systems, and I will also report on the application of these constructions to fluid flow in the ocean and atmosphere. This is joint work with Cecilia González Tokman, Christian Horenkamp, Simon Lloyd, Adam Monahan, Anthony Quas, Vincent Rossi, Naratip Santitissadeekorn, Alex Sen Gupta, and Erik van Sebille.

The effect of noise on mixed-mode oscillations

Barbara Gentz (University of Bielefeld)

Many neuronal systems and models display so-called mixed-mode oscillations (MMOs) consisting of small-amplitude oscillations alternating with large-amplitude oscillations. Different mechanisms have been identified which may cause this type of behaviour. In this talk, we will focus on MMOs in a slow-fast dynamical system with one fast and two slow variables, containing a folded-node singularity. The main question we will address is whether and how noise may change the dynamics.

We will first outline a general approach to stochastic slow-fast systems which allows

(1) to construct small sets in which the sample paths are typically concentrated, and

(2) to give precise bounds on the exponentially small probability to observe atypical behaviour.

Applying this method to our model system shows the existence of a critical noise intensity beyond which the small-amplitude oscillations become hidden by noise. Furthermore, we will show that in the presence of noise sample paths are likely to jump away from so-called canard solutions earlier than the corresponding deterministic orbits. This early-jump mechanism can drastically change the mixed-mode patterns, even for rather small noise intensities.

The methods used to derive the results range from deterministic bifurcation theory and averaging to martingale techniques and estimates on Markov transition kernels.

Joint work with Nils Berglund (Université d'Orléans) and Christian Kuehn (TU Wien).

Synchronization by noise

Benjamin Gess (University of Chicago) and Michael Scheutzow (Technische Universitat Berlin)

We present recent results on synchronization by noise. That is, we introduce sufficient conditions under which weak random attractors for random dynamical systems consist of single random points. These conditions focus on SDE with additive noise, for which they are also essentially necessary. In addition, we provide sufficient conditions for the existence of a minimal weak point random attractor consisting of a single random point. The analysis passes through the statistical equilibrium, negativity of the top Lyapunov exponent and a local stable manifold result. As a model example, we prove synchronization by noise for an SDE with drift given by a (multidimensional) double-well potential and additive noise. This is joint work with Franco Flandoli.

Characterizing dynamics with covariant Lyapunov vectors

Francesco Ginelli (University of Aberdeen)

Recent years have witnessed a growing interest in covariant Lyapunov vectors (CLVs) as an important tool for characterizing chaotic dynamics in high dimensional system. CLVs define an intrinsic, non orthogonal basis at each point in phase space which is covariant with the dynamics and coincides with the so called Oseledets splitting for invertible systems. After a brief introduction, we discuss in details the dynamical algorithm we have introduced to efficiently compute CLV's and show that it generically converges exponentially in time. We also discuss its numerical performance and compare it with other algorithms presented in the literature. We finally illustrate selected applications; in particular, CLV's have been used to characterize the collective dynamics of globally coupled systems, to quantify the degree of hyperbolicity, and to evaluate the number of effective degrees of freedom in chaotic, spatially extended dissipative systems such as the Kuramoto-Sivashinsky equation.

On triangularizability in the Multiplicative Ergodic Theorem

Joseph Horan (Victoria)

TBA

On the dynamics of the Chafee-Infante equation with Lévy noise

Peter Imkeller (HU Berlin)

Dynamical systems of the reaction-diffusion type with small noise have been instrumental to explain basic features of the dynamics of paleo-climate data. For instance, a spectral analysis of Greenland ice time series performed at the end of the 1990s representing average temperatures during the last ice age suggest an $\alpha-$stable noise component with an $\alpha \sim 1{:}75$: We model the time series as a dynamical system perturbed by α-stable noise, and develop an efficient testing method for the best fitting α. The method is based on the observed p-variation of the residuals of the time series, and their asymptotic $\frac{\alpha}{p}$-stability established in local limit theorems. Generalizing the solution of this model selection problem, we are led to a class of reaction-diffusion equations with additive α-stable Lévy noise, a stochastic perturbation of the Chafee-Infante equation. We study exit and transition between meta-stable states of their solutions. Due to the heavy-tail nature of an α-stable noise component, the results differ strongly from the well known case of purely Gaussian perturbations. (Joint work with A. Debussche, J. Gairing, C. Hein, M. Högele, I. Pavlyukevich)

Entropy for control problems and random escape rates

Christoph Kawan (New York University)

This talk is about the control-theoretic problem to determine the smallest rate of information in a feedback loop above which a control system can solve a given control task. Such minimal data rates can be described by quantities that resemble topological entropy. For the control problem to render a given subset of the state space invariant, the associated entropy is related to escape rates of random dynamical systems which arise by putting shift-invariant measures on the space of admissible control functions. Here the multiplicative ergodic theorem comes into play, which allows to estimate the escape rates in terms of Lyapunov exponents.

Entropy, Chaos and weak Horseshoe for Infinite Dimensional Random Dynamical Systems

Kening Lu (Brigham Young University)

In this talk, we give an answer to a problem on characterizing the chaotic behavior of orbits topologically or geometrically in the presence of only positive entropy for infinite dimensional dynamical systems. We show that if a random dynamical system has a compact random invariant set such as random attractor with positive topological entropy, then the system is chaotic and has a weak horseshoe. As a corollary, we have the same conclusion for a deterministic dynamical system with a compact invariant set of positive topological entropy. The chaotic behavior we have here is due to the positive entropy, not the randomness of the system. The is a joint work with Wen Huang.

Stochastic limits for deterministic fast-slow systems

Ian Melbourne (University of Warwick)

Consider a fast-slow system of ordinary differential equations of the form

$$\dot{x} = a(x, y) + \epsilon^{-1} b(x, y), \dot{y} = \epsilon^{-2} g(y),$$

where it is assumed that b averages to zero under the fast flow generated by g. Here $x \in \mathbb{R}^d$ and y lies in a compact manifold. We give conditions under which solutions to the slow equations converge to solutions of a d-dimensional stochastic differential equation as $\epsilon \to 0$. The limiting SDE is given explicitly.

Our theory applies when the fast flow is Anosov or Axiom A, as well as to a large class of nonuniformly hyperbolic fast flows (including the one defined by the well-known Lorenz equations), and our main results do not require any mixing assumptions on the fast flow.

This is joint work with David Kelly and combines methods from smooth ergodic theory with methods from rough path theory.

The transfer operator for the binary Euclidean algorithm

Ian Morris (Surrey)

Abstract: The binary Euclidean algorithm is a modification of the classical Euclidean algorithm which replaces division by an arbitrary integer with division by powers of two only. Statistical properties of the classical Euclidean algorithm – such as the average number of steps required to process a pair of integers both of which are less than N – can be studied via the thermodynamic formalism of the Gauss map acting on the unit interval. To investigate similar properties for the binary Euclidean algorithm one must instead study the thermodynamic formalism of an IID random dynamical system on the interval. I will describe a recent result on the transfer operator of the binary Euclidean algorithm which can be applied to resolve conjectures of R.P. Brent, B. Valleé and D.E. Knuth.

Necessary and Sufficient Conditions for Stable Synchronisation in Random Dynamical Systems

Julian Newman (Imperial College)

For a product of i.i.d. random maps or a memoryless stochastic flow on a compact space X, we find conditions under which the presence of locally asymptotically stable trajectories (e.g. as given by negative Lyapunov exponents) implies almost-sure mutual convergence of any given pair of trajectories (”synchronisation”). Namely, we find that synchronisation occurs and is stable if and only if the system exhibits the following properties: (i) there is a smallest deterministic invariant set $K \subset X$, (ii) any two points in K are capable of being moved closer together, and (iii) K admits asymptotically stable trajectories. Our first condition (for which unique ergodicity of the one-point transition probabilities is sufficient) replaces the intricate vector field conditions assumed in Baxendale's similar result of 1991, where (working on a compact manifold) sufficient conditions are given for synchronisation to occur in a SDE with negative Lyapunov exponents.

Annealed and quenched limit theorems for random expanding dynamical systems

Matthew Nicol (University of Houston)

We present results concerning annealed and quenched limit theorems for random expanding dynamical systems. In particular we will present results on annealed and quenched versions of a central limit theorem, a large deviation principle, a local limit theorem, and Erdös- Renyi type limit laws. We use functional analytic and martingale techniques to establish such limit laws. This is joint work with Romain Aimino (Aix Marseille Universite) and Sandro Vaienti (CPT Luminy).

Time regularity of solutions to SPDEs

Szymon Peszat (Institute of Mathematics, Jagiellonian University and Institute of Mathematics, Polish Academy of Sciences)

Time regularity of solutions to SPDEs driven by Wiener process can be studied using either Kolmogorov criterion, Kotelenez theorem or Da Prato-Kwapień-Zabczyk factorization. It turns out that very often the solution is continuous is a given state space E even if the noise takes values in a bigger space $U \hookleftarrow E$.

If the noise is of jump type and does not take values in the space space then typically the solution is not càdlàg. In fact during the talk different concepts of càdlàg property will be discussed. A special emphasis will be put on infinite systems of linear equations driven by independent Lévy processes.

The talk will be based on the following papers:

- S. Peszat and J. Zabczyk, *Time regularity for stochastic Volterra equations by the dilation theorem*, J. Math. Anal. Appl. **409** (2014), 676–683.
- S. Peszat and J. Zabczyk, *Time regularity of solutions to linear equations with Lévy noise in infinite dimensions*, Stochastic Processes Appl. **123** (2013), 719–751.
- Z. Brzeźniak, B. Goldys, P. Imkeller, S. Peszat, E. Priola, and J. Zabczyk, *Time irregularity of generalized Ornstein–Uhlenbeck processes*, C. R. Math. Acad. Sci. Paris **348** (2010), 273–276.

Bifurcations of random dynamical systems

Martin Rasmussen (Imperial College)

Despite its importance for applications, relatively little progress has been made towards the development of a bifurcation theory for random dynamical systems. In this talk, I will demonstrate that adding noise to a deterministic mapping with a pitchfork bifurcation does not destroy the bifurcation, but leads to two different types of bifurcations. The first bifurcation is characterized by a breakdown of uniform attraction, while the second bifurcation can be described topologically. Both bifurcations do not correspond to a change of sign of the Lyapunov exponents, but I will explain that these bifurcations can be characterized by qualitative changes in the dichotomy spectrum and collisions of attractor-repeller pairs. This is joint work with M. Callaway, T.S. Doan, J.S.W Lamb (Imperial College) and C.S. Rodrigues (MPI Leipzig).

A renewal scheme for non uniformly hyperbolic flows

Dalia Terhesiu (University of Vienna)

In recent work, I. Melbourne and D. Terhesiu, 2014 obtain optimal results for the asymptotic of the correlation function associated with both finite and infinite measure preserving suspension semiflows over Gibbs Markov maps. The involved observables are supported on a thickened Poincare section. The involved renewal scheme relies on inducing to such a section. In more recent work with H. Bruin, we investigate a different renewal scheme for suspension flows over non uniformly hyperbolic maps: we induce to a well chosen region Y of the same dimension as the manifold (on which the flow is defined); we do not require that Y is of bounded length. By forcing expansion on the flow direction, we can ensure that the induced version of the flow is a hyperbolic map F. Although at first counter-intuitive, such a scheme ensures that the obtained hyperbolic map F satisfies good spectral properties. Combined with the type of renewal equation established in Melbourne and Terhesiu, 2014 and several abstract assumptions on the hyperbolic map F (and thus on the underlying map of the suspension flow), this scheme allows us to estimate the correlation function of observables supported on the whole region Y.

Almost sure invariance principle for sequential and non-stationary dynamical systems

Andrew Török (University of Houston)

We establish almost sure invariance principles, a strong form of approximation by Brownian motion, for non-stationary time-series arising as observations on dynamical systems. Our examples include observations on sequential expanding maps, perturbed dynamical systems, non-stationary sequences of functions on hyperbolic systems as well as applications to the shrinking target problem in expanding systems. (Authors: Nicolai Haydn (USC), Matthew Nicol (UH), Andrew Török (UH), Sandro Vaienti (Marseille))