Beyond Lehmer's problem for α with $\mathbb{Q}(\alpha)$ Galois

Amoroso and David [1] proved that the answer to Lehmer's question is 'yes' in the case of algebraic numbers α of degree d with $\mathbb{Q}(\alpha)$ a Galois extension of $\mathbb{Q}$. That is, there is then a constant $C > 0$ such that the Mahler measure $M(\alpha)$ satisfies $M(\alpha) \geq C$ when $M(\alpha) > 1$. But does something stronger hold in this case? Might there even be a constant $c > 0$ such that $M(\alpha) \geq c^d$ when $M(\alpha) > 1$?

Now such a $\mathbb{Q}(\alpha)$ is either totally real or totally nonreal. In the totally real case, we have, by a result of Schinzel [2], that the stronger result does hold, with constant $c = \left(\frac{1+\sqrt{5}}{2}\right)^{1/2} = 1.272\ldots$. Here the fact that $\mathbb{Q}(\alpha)$ is Galois is not used. So we can confine our attention to the totally nonreal case – i.e., the problem is already half solved!

Let $n \geq 2$ and $\beta_1, \ldots, \beta_n$ be the roots of $z^n - z - 1$, known to be irreducible for all n, and to have Galois group the full symmetric group S_n. Put

$$\alpha = \beta_1^1 \beta_2^2 \ldots \beta_{n-1}^{n-1}.$$

Then the Galois closure of $\mathbb{Q}(\beta_1)$ is $\mathbb{Q}(\alpha)$ of degree $d = n!$ over $\mathbb{Q}$. The following table shows the value of $M(\alpha)^{1/d}$ for $n = 2, \ldots, 9$, computed with Maple.

n	$d = n!$	$M(\alpha)^{1/d}$
2	2	1.2720196495
3	6	1.1509639252
4	24	1.2428334720
5	120	1.2292495215
6	720	1.2846087150
7	5040	1.2833028970
8	40320	1.3243452986
9	362880	1.3307248410

Does anyone know of any smaller values of $M(\alpha)^{1/d} > 1$ for α of degree d with $\mathbb{Q}(\alpha)$ Galois?

References.

[1] Amoroso, Francesco ; David, Sinnou. Le problème de Lehmer en dimension supérieure. J. Reine Angew. Math. 513 (1999), 145–179.

[2] Schinzel, A. On the product of the conjugates outside the unit circle of an algebraic number. Acta Arith. 24 (1973), 385–399. Addendum: Acta Arith. 26 (1974/75), no. 3, 329–331.