

A lower bound for the Stan-Zaharescu constant?

Let p be prime, $k \geq 2$ and a_1, \dots, a_k be elements of \mathbb{Q}_p . Then Stan and Zaharescu [1, Lemma 15] proved that there is a constant $\rho_{k,p} > 0$, depending only on p and k , such that

$$\max_{(i,j)} \min_{(l,s) \neq (i,j)} |(a_i - a_j) - (a_l - a_s)|_p \geq \rho_{k,p} \min_{i \neq j} |a_i - a_j|_p.$$

Their proof was by contradiction, and so gave no lower bound for $\rho_{k,p}$. It would be interesting to find such a bound, or indeed the optimal value of $\rho_{k,p}$. By taking $l = s$ we see that $\rho_{k,p} \leq 1$.

Reference.

[1] Stan, Florin and Zaharescu, Alexandru. Weil numbers in finite extensions of \mathbb{Q}^{ab} : the Loxton-Kedlaya phenomenon. *Trans. Amer. Math. Soc.* 367 (2015), no. 6, 4359–4376.