Massively Parallel Communication and Geometric Graph Problems

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Joint work with **Alexandr Andoni** (Microsoft),
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Modern Systems for Big Data

- Modern data sets can be large

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- Examples:
  - MapReduce (Google) [Dean, Ghemawat 2004]
  - Pregel (Google) [Malewicz et al. 2009]
  - Hadoop (Yahoo!, Facebook, IBM)
  - Dryad (Microsoft) [Isard et al. 2007]

Modeling Attempts

- Multiple attempts at modeling (MapReduce as main focus):
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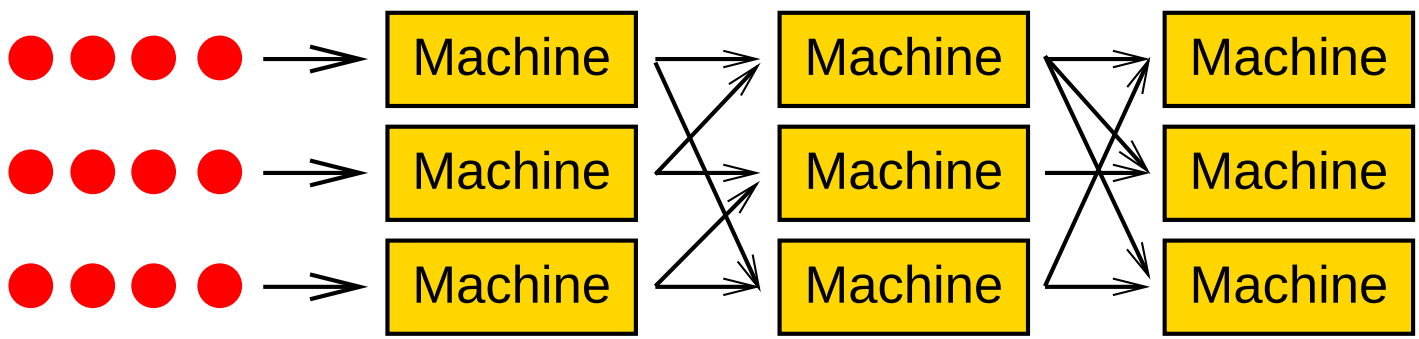
  Massively Parallel Communication (MPC)
- Can be seen as a specific case of BSP
  (Bulk Synchronous Parallel) [Valiant 1990]
  - Multiple parameters
  - Never thoroughly explored

Our Model

- n items on input
- m machines
- s space per machine, $s = O(n/m)$



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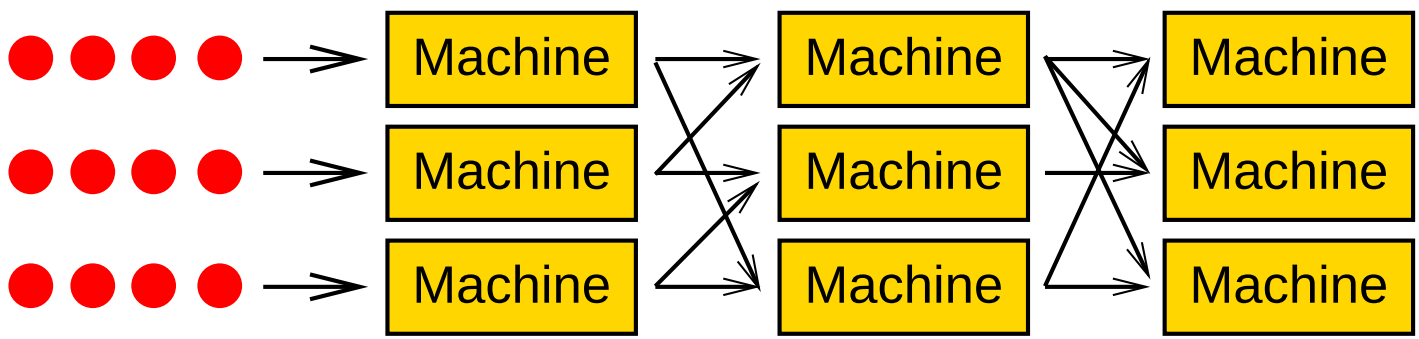

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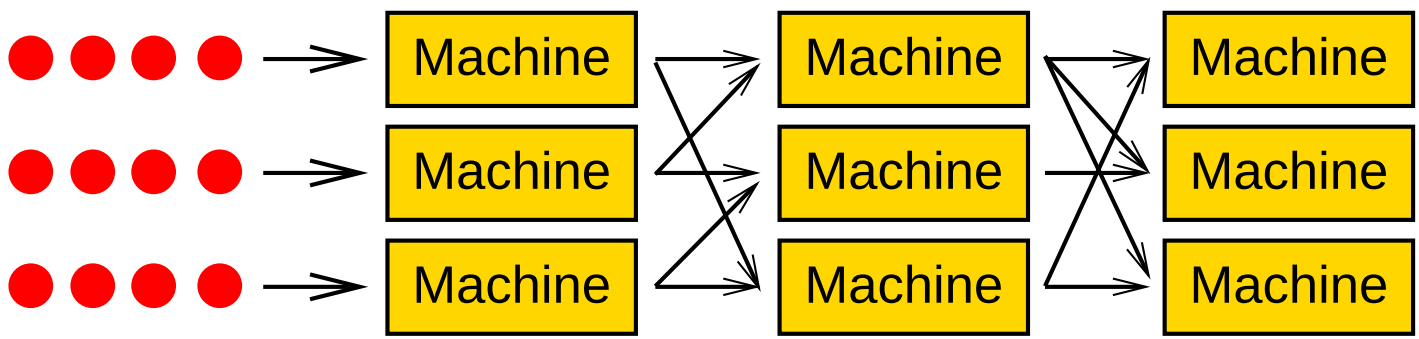

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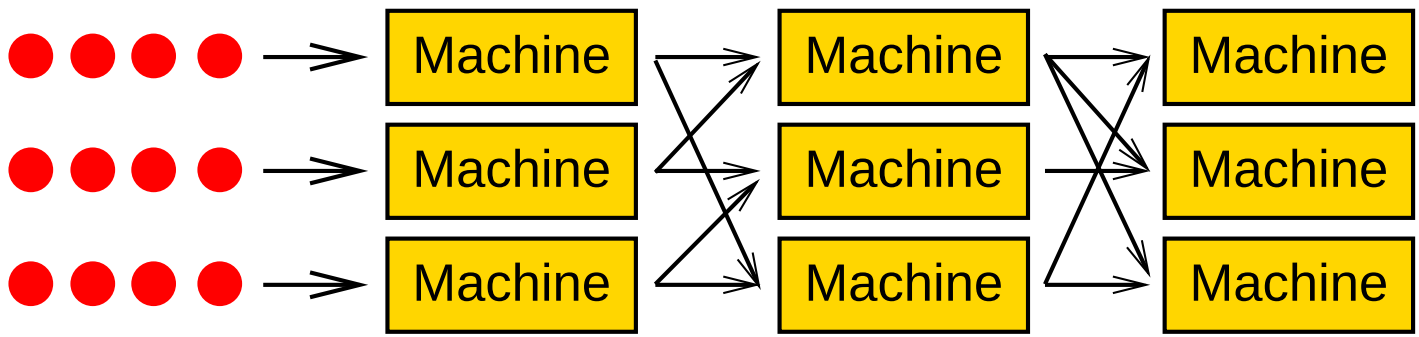

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  Example: if $s = \omega(m)$, then $\alpha \geq 1/2$

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- Main focus: communication (i.e., moving data around)
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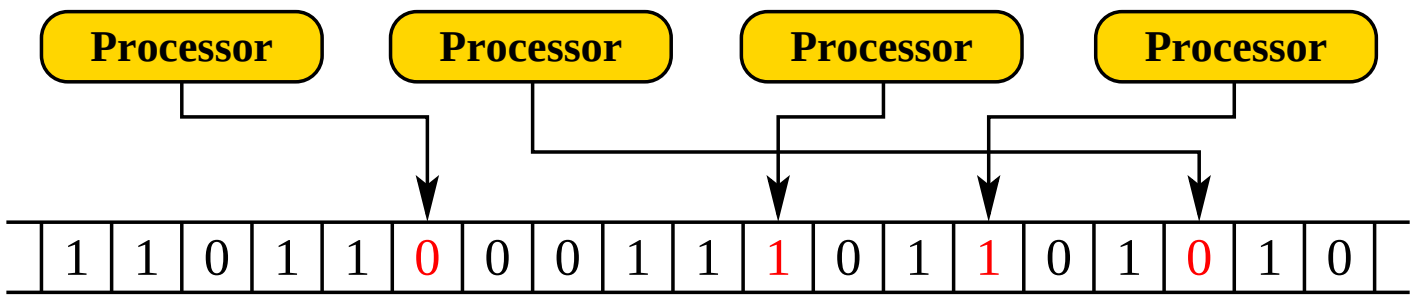
- But running time of single machine important as well
- Relatively easy to implement in the MapReduce model

Comparison to PRAM

- PRAM: classic attempt at modeling parallelism
  - m processors
  - processors access common memory



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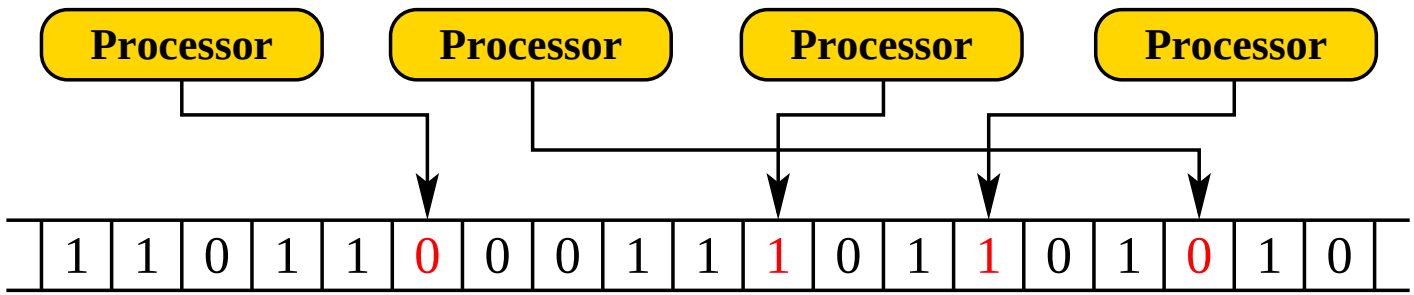

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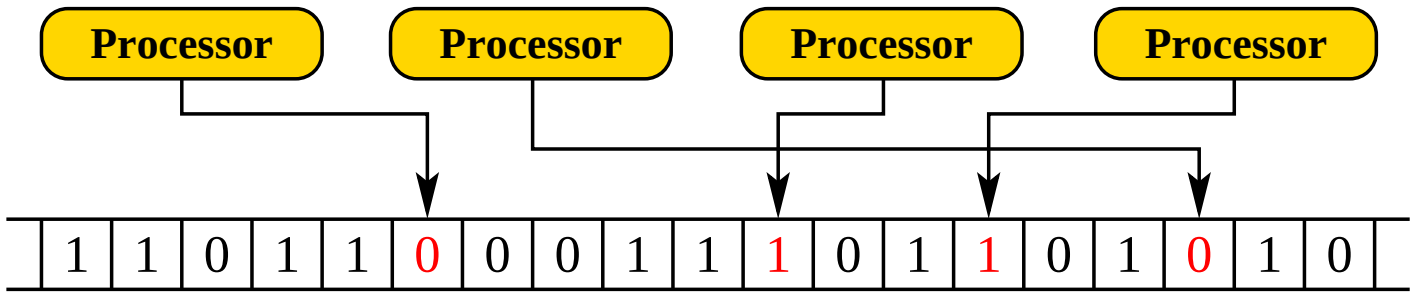

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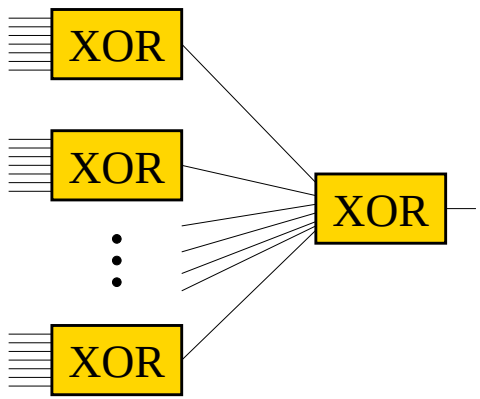

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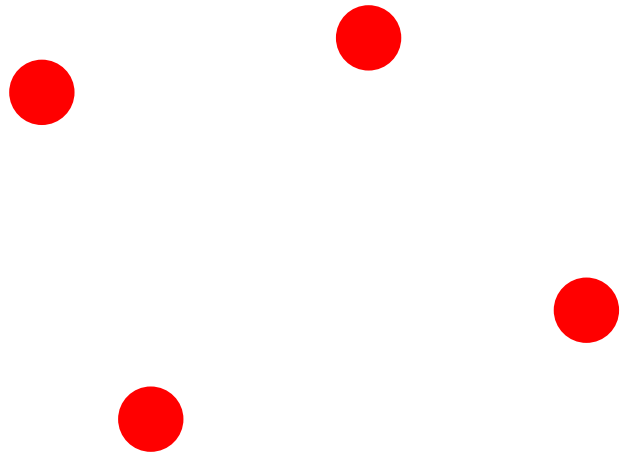
- Our goal: constant number of communication rounds

This Talk: Geometric Graph Problems

- Input: set of points in low dimensional metric space



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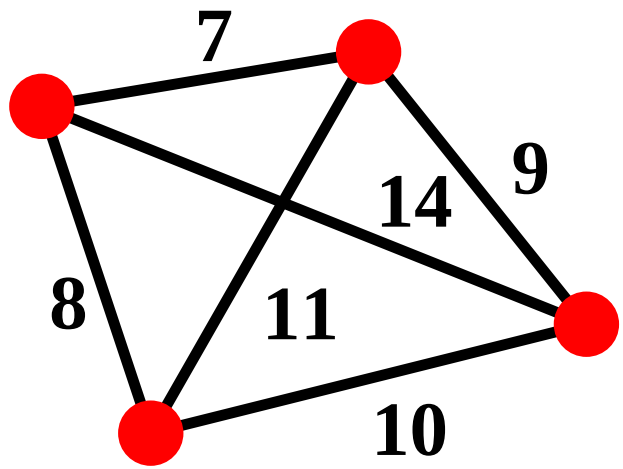

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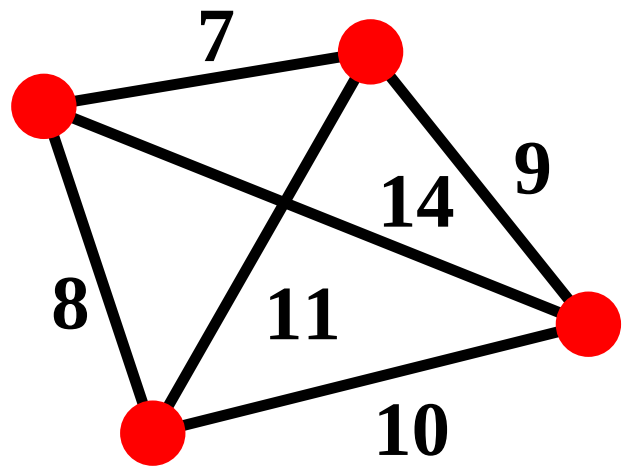

- Points induce a weighted graph
- Graph problems to consider:
  - Minimum Spanning Tree
  - Earth Mover Distance
  - Transportation Problem
  - Travelling Salesman Problem
  - ...

Minimum Spanning Tree

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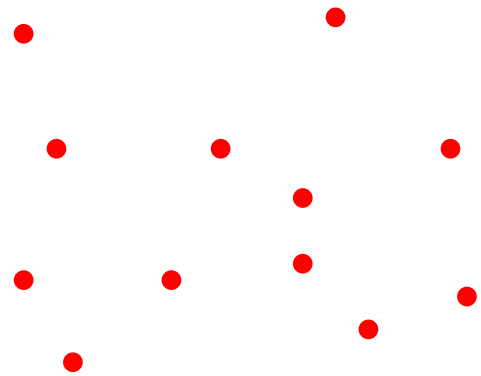
Input: n points in $\mathbb{R}^2$



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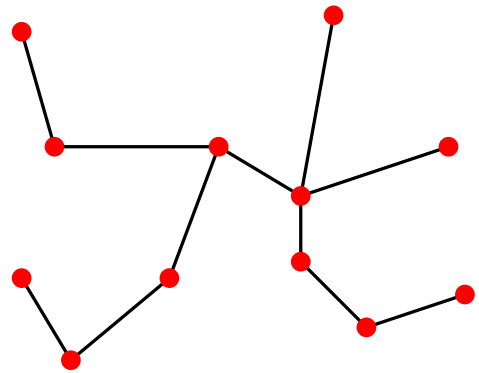

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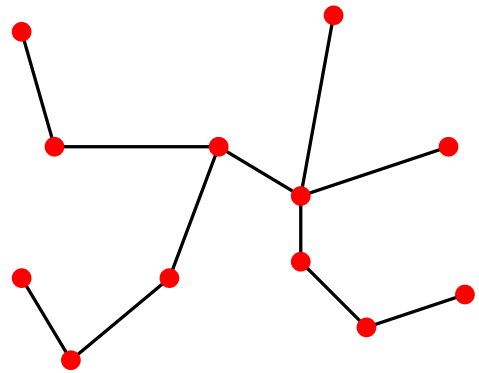

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Applications:

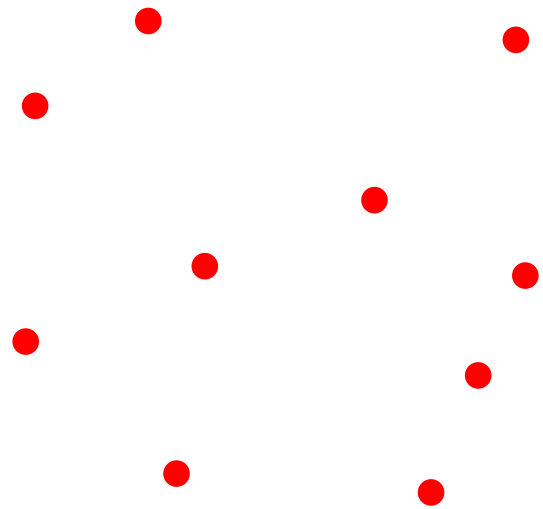
- Related to hierarchical agglomerative clustering with single linkage
- Classic practical clustering algorithms
  [Zahn 1971] [Kleinberg, Tardos]

  Remove heaviest $k-1$ edges to create k clusters

Random Gridding

We reuse the Arora-Mitchell approach:

Apply a randomly shifted grid



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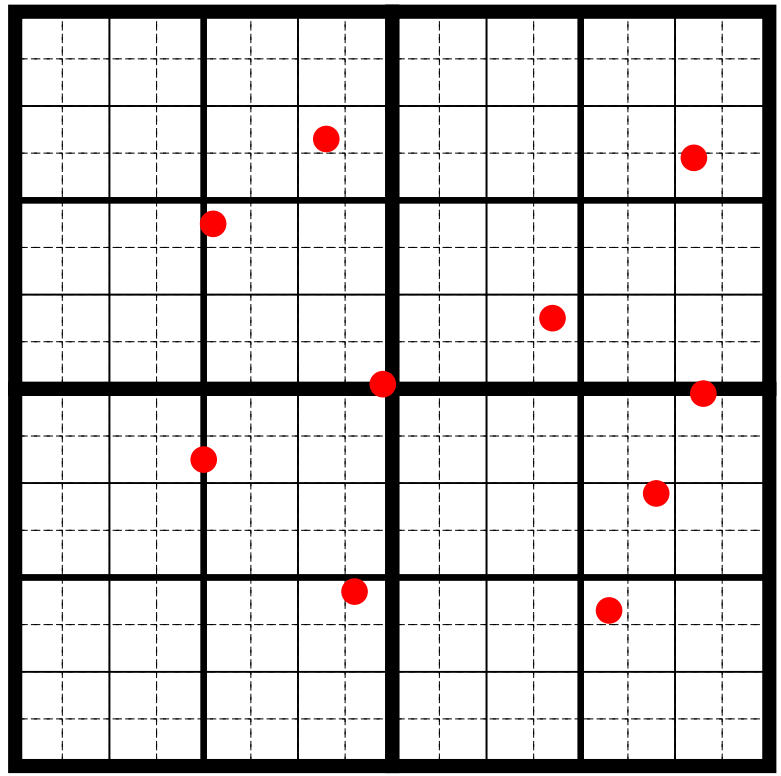

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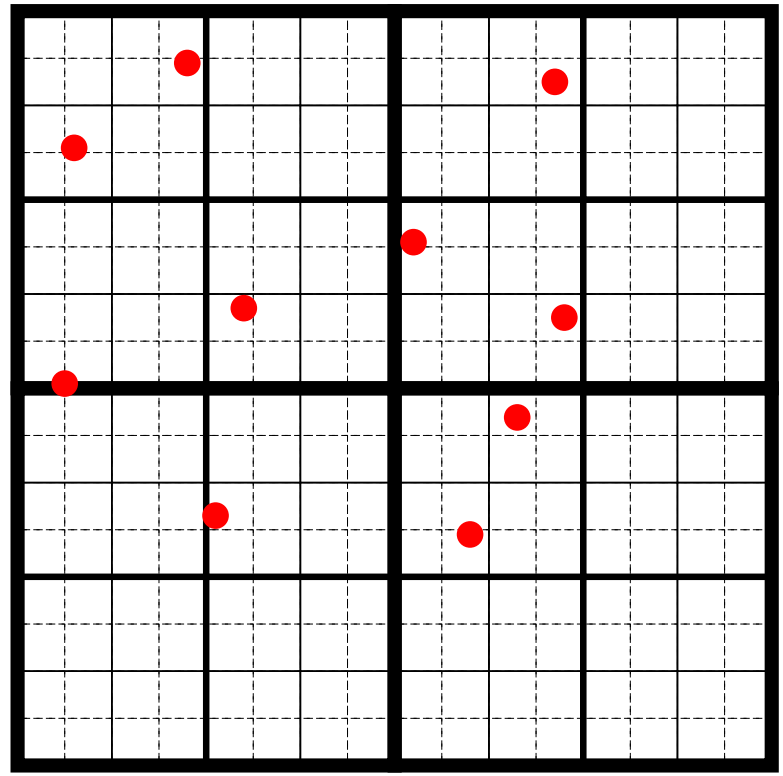

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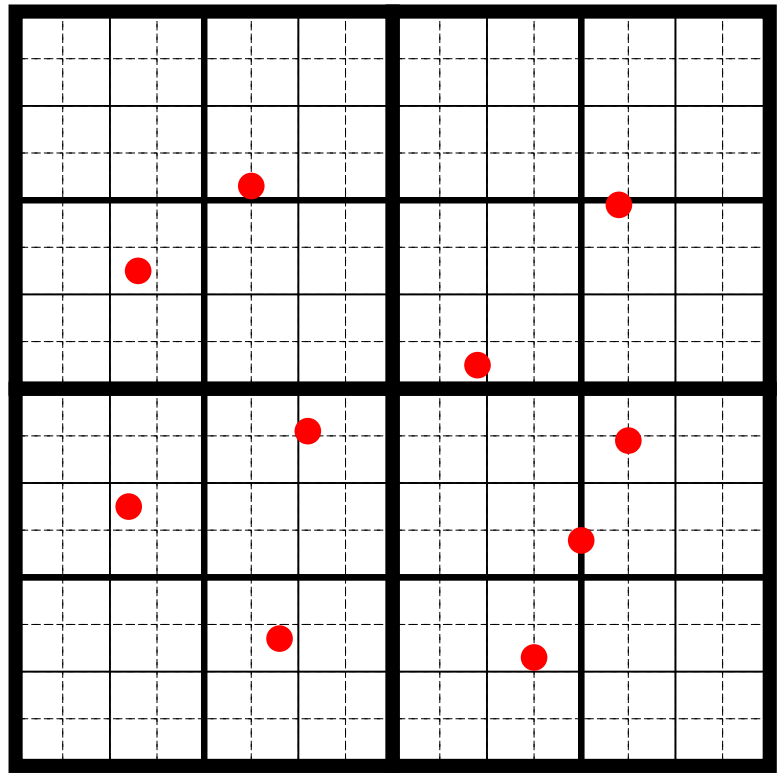

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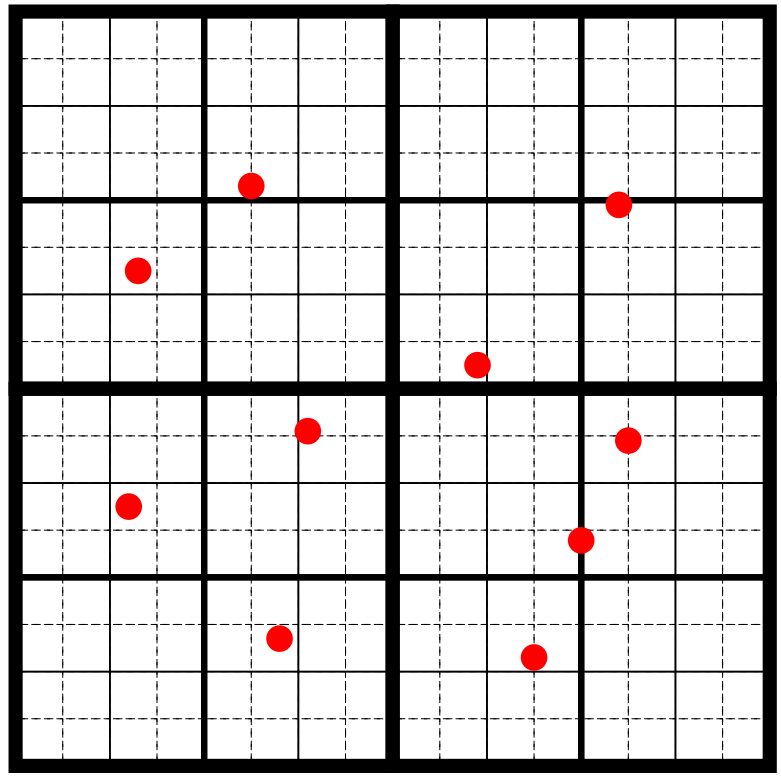

Key fact: cell of side Δ separates points x and y w.p. $O(1) \cdot \frac{\rho(x,y)}{\Delta}$

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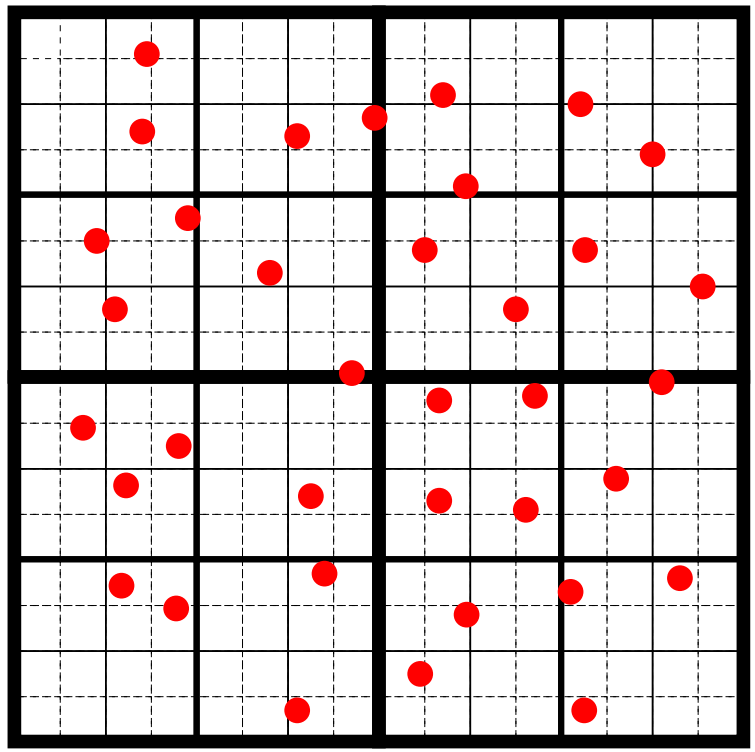

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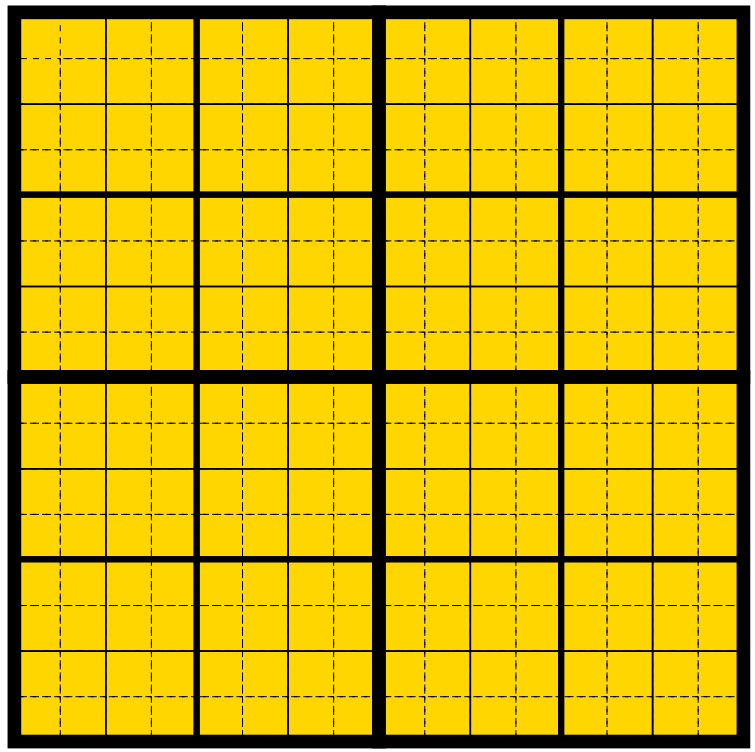

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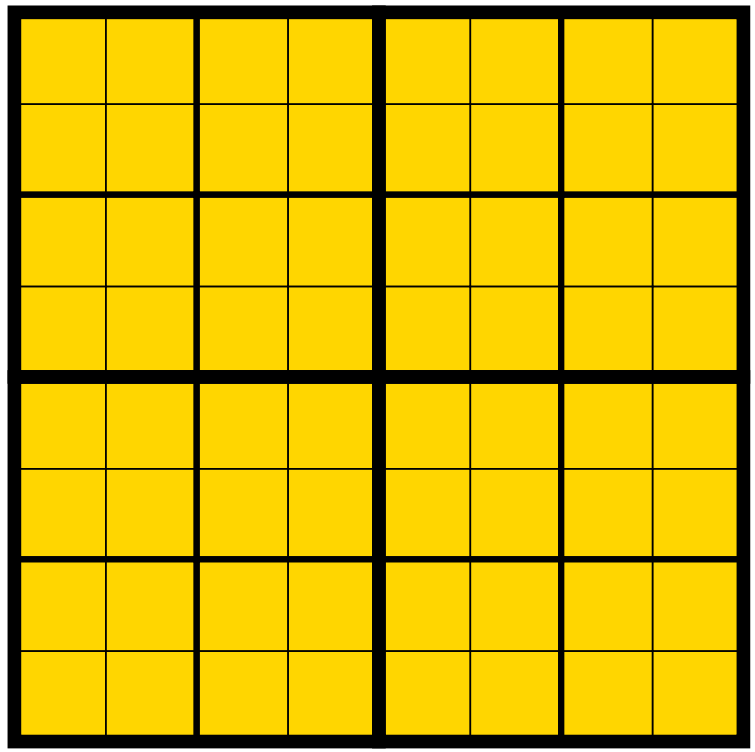

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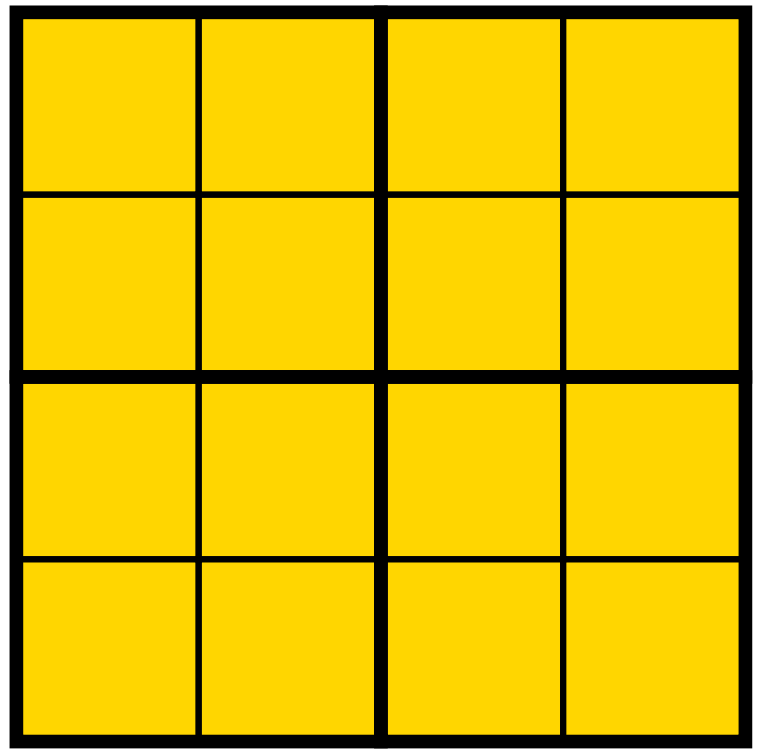

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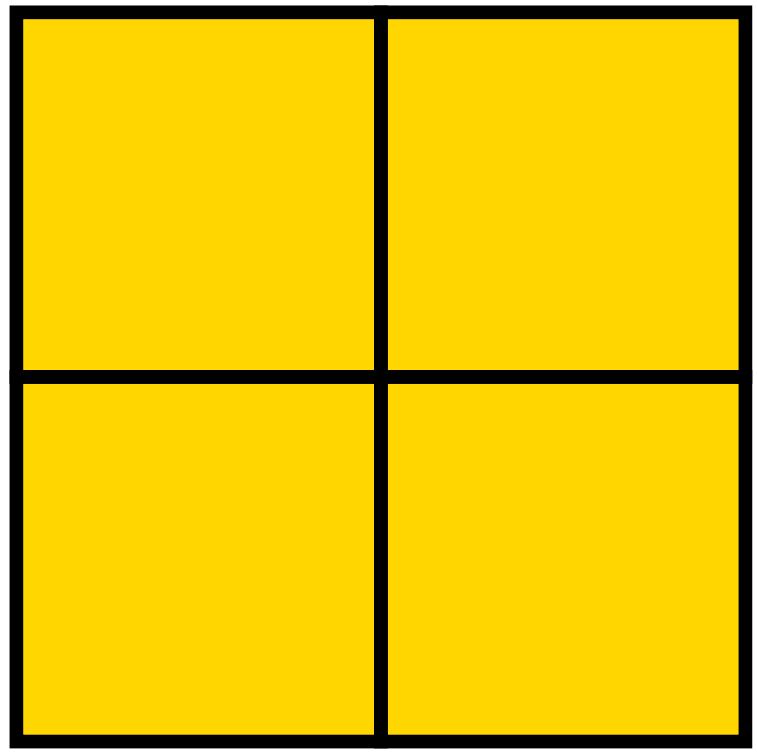

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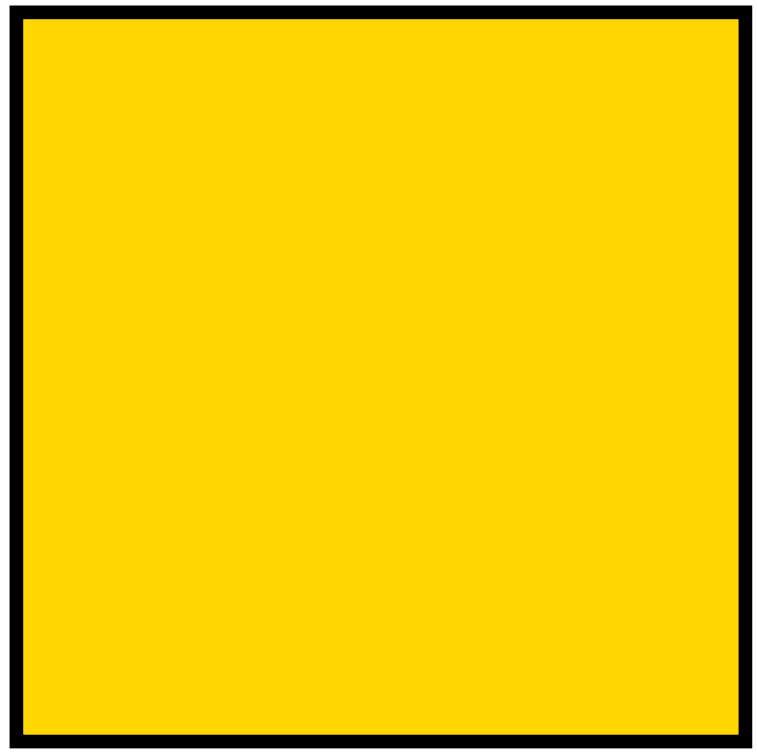

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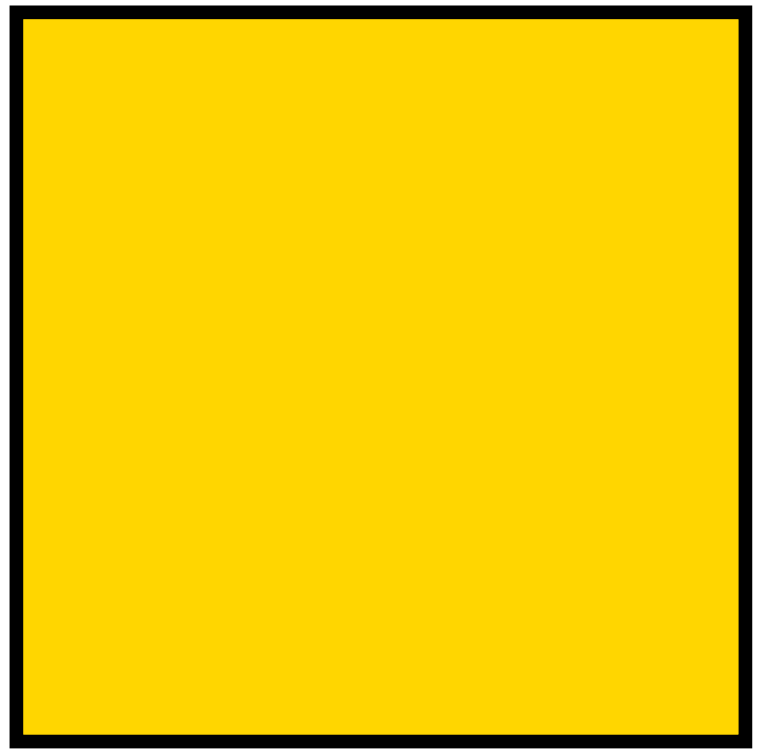

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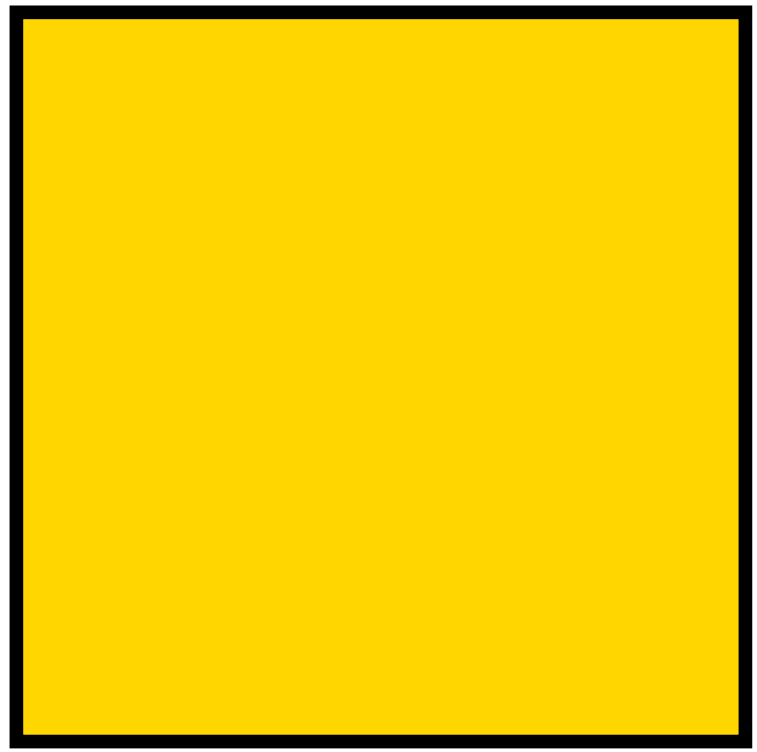

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- Can partially isolate what happens inside a cell

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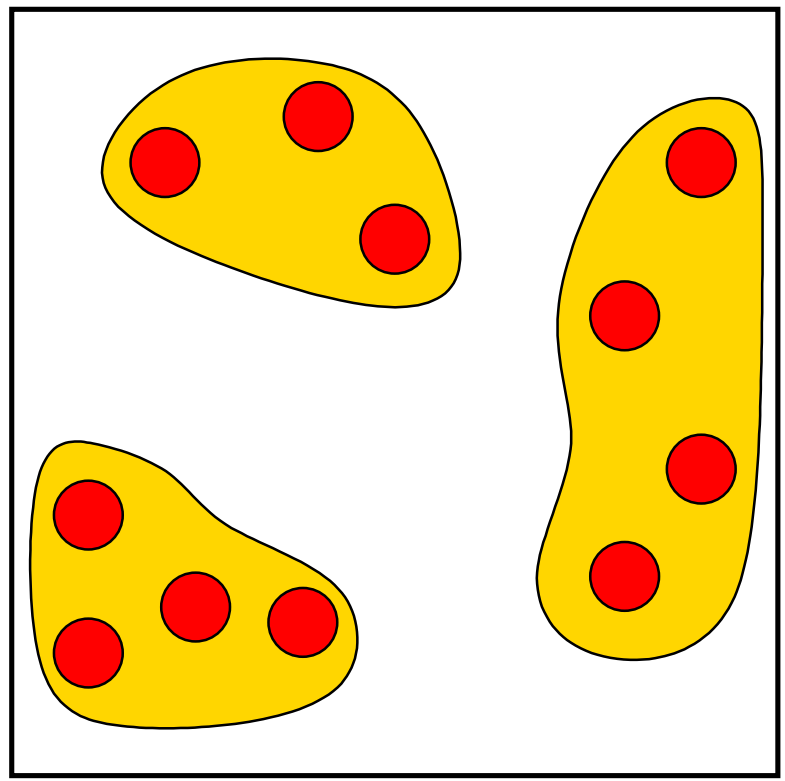

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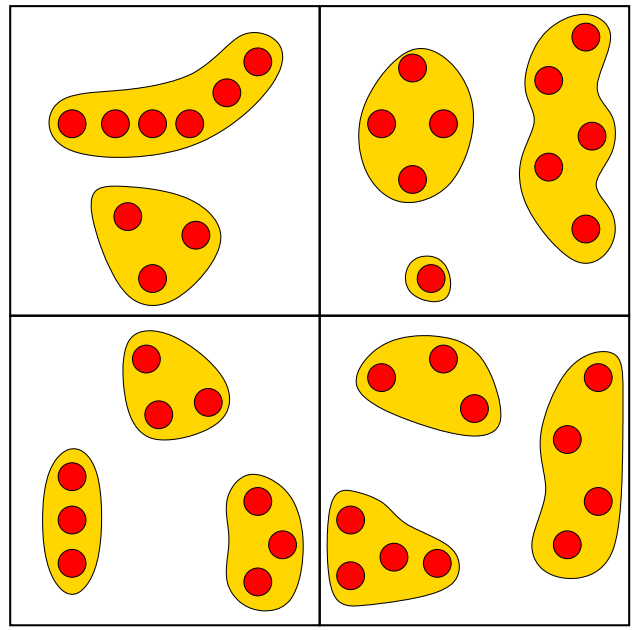

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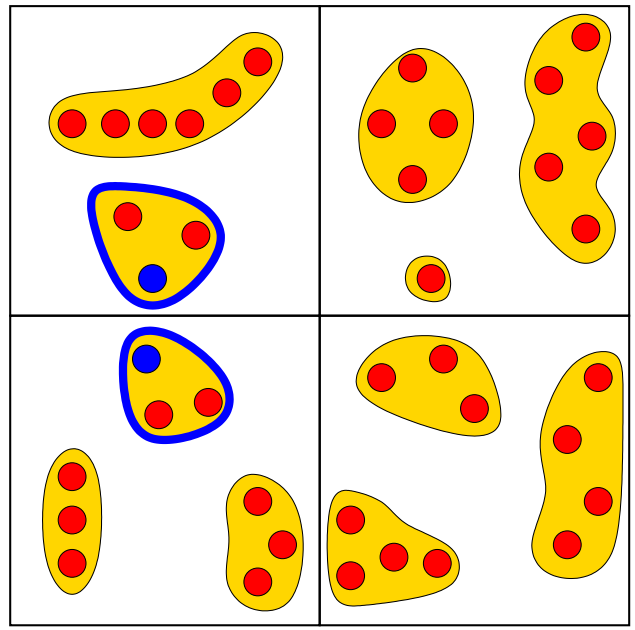

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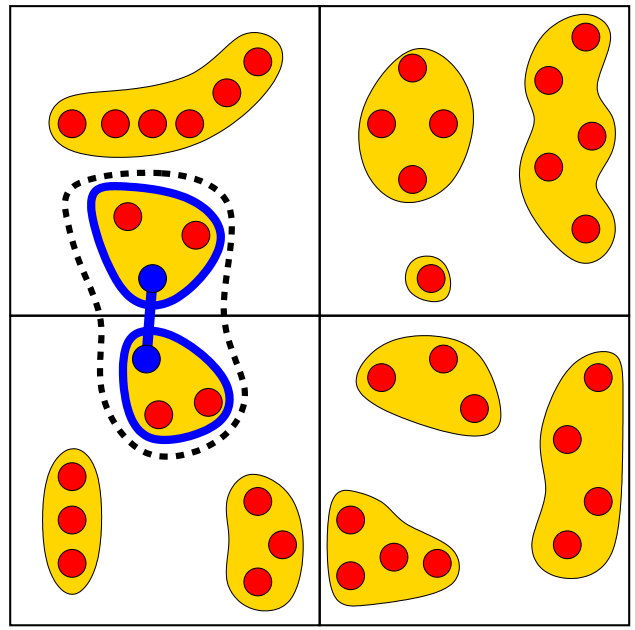

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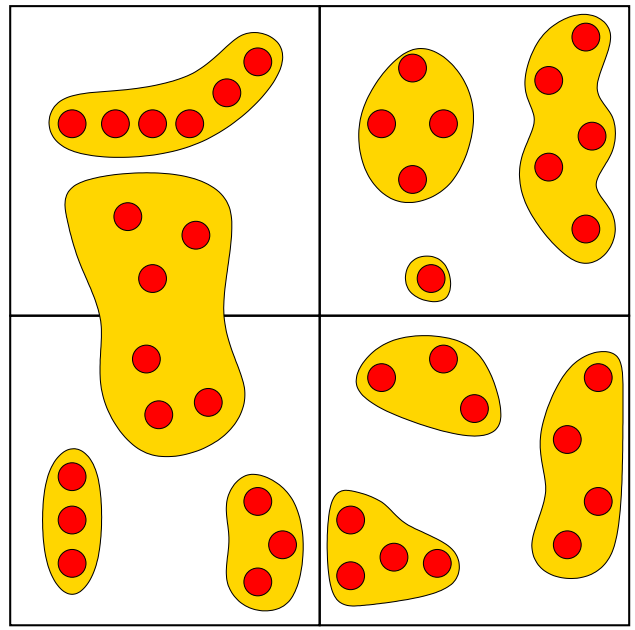

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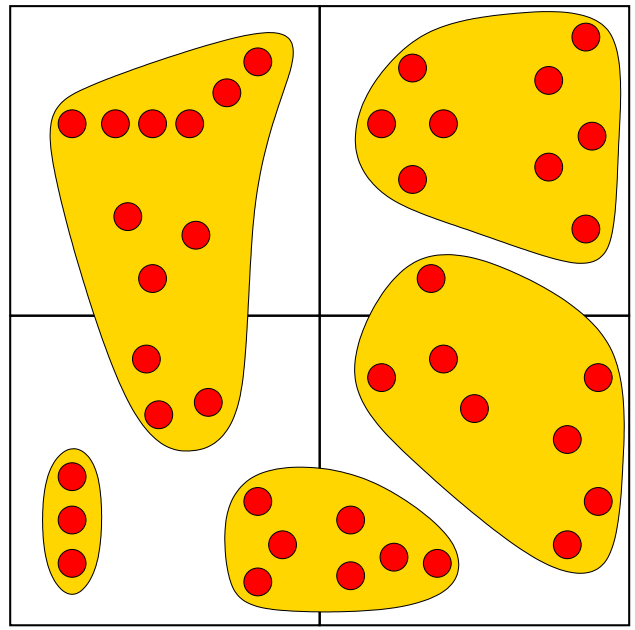

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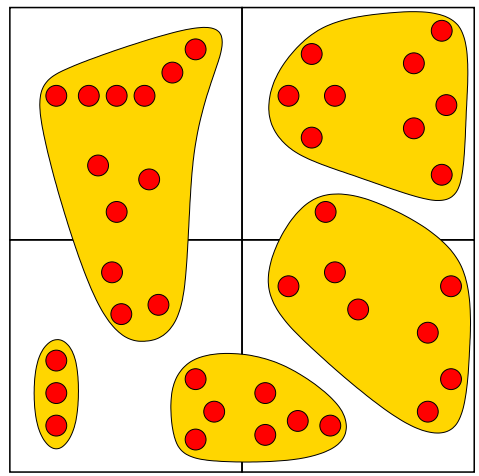

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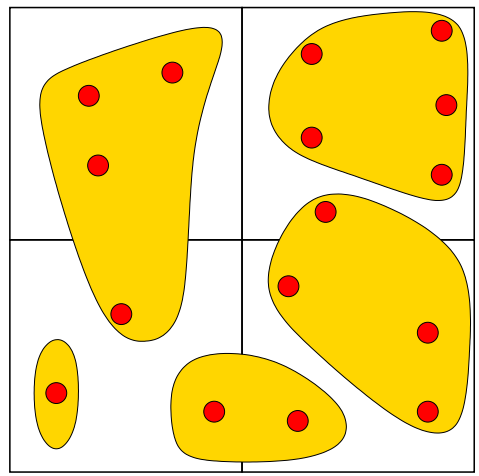

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- Connect points closer than $\frac{\epsilon \cdot \mathrm{diam}(S)}{100 \cdot n}$ arbitrarily
- Sub-solution for cell of side Δ:
  $\epsilon^2\Delta$-covering with induced components
- Combining sub-solutions:
  Truncated version of Kruskal's algorithm
  1. Find two closest clusters
  2. If their distance less than $\epsilon\Delta$, connect them and repeat
- Pass up $\epsilon^2\Delta$-covering with information about connected components
- Expected cost of solution: optimum $\cdot\ (1 + \epsilon \cdot$ #levels$)$

Select Implementation Details

- Merge $n^{\Omega(1)} \times n^{\Omega(1)}$ cells at once
- Sub-solutions for all subcells should fit on a single machine
- Use sorting [Goodrich, Sitchinava, Zhang 2011] for grouping points and subcells that are close
- Near-linear time:
  - Relax Kruskal's algorithm
  - Efficient nearest neighbor data structure [Krauthgamer, Lee 2004], [Cole, Gottlieb 2006]

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- We give conditional lower bound based on Sparse Connectivity

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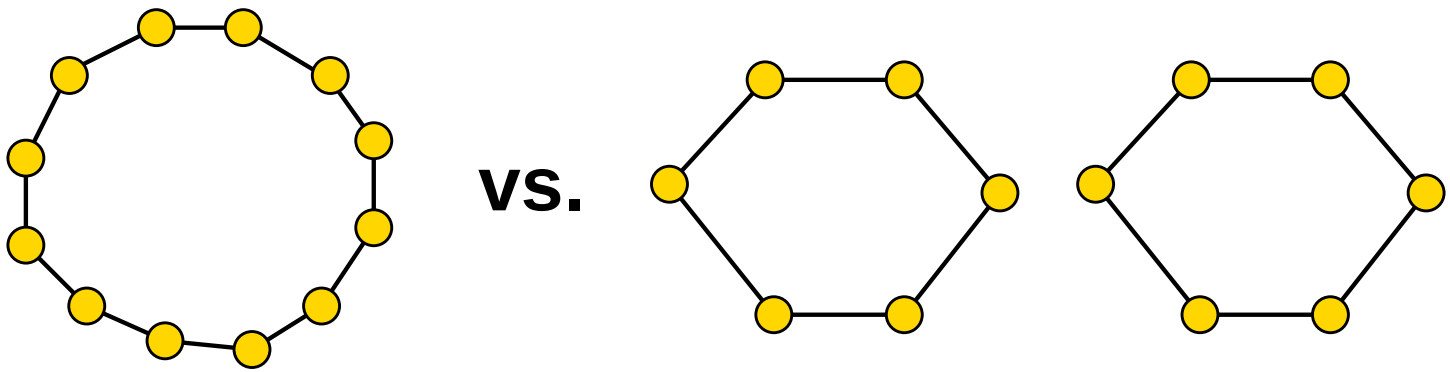

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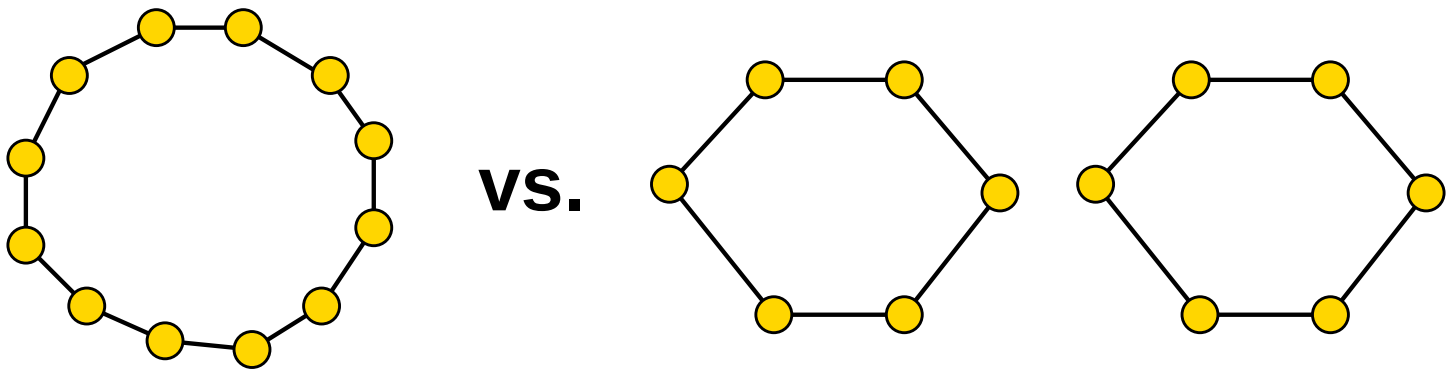

- Best known algorithm: $O(\log n)$ rounds

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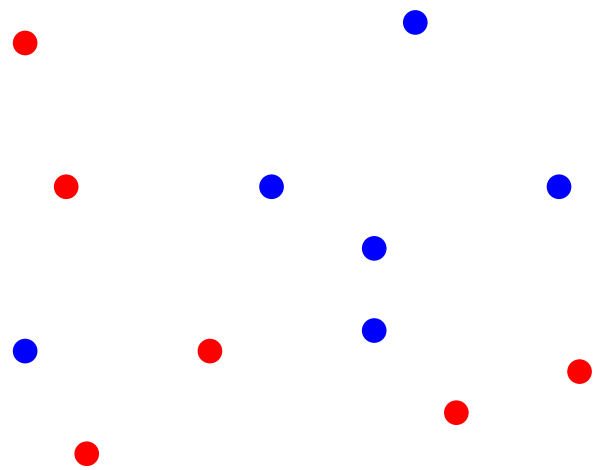
MST weight:

- Connected: $\leq 2(n-1)$
- Not connected: $\geq 2n$

The Earth Mover Distance

The Earth Mover Distance (EMD)

Input: n blue and n red points in $\mathbb{R}^2$



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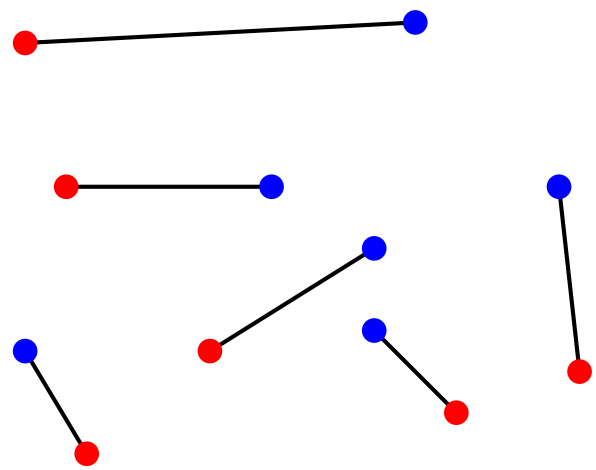

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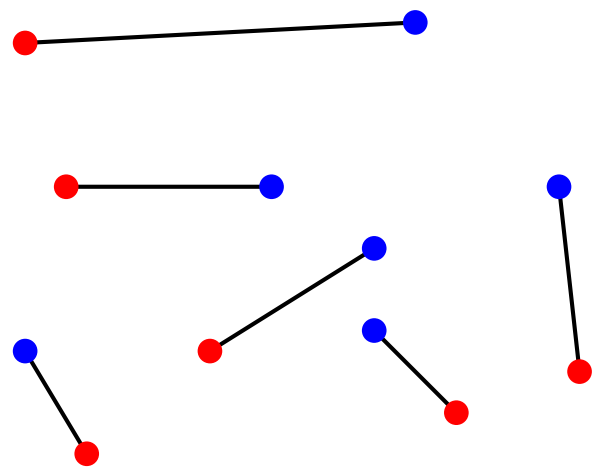

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Application:

- How similar are two images?
- Discover characteristic features
- See how much it costs to match them

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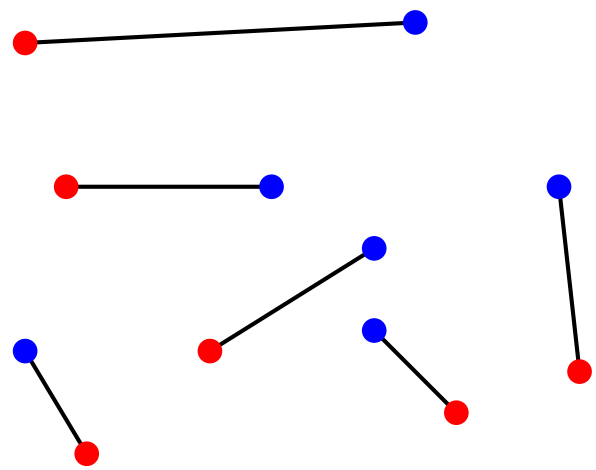

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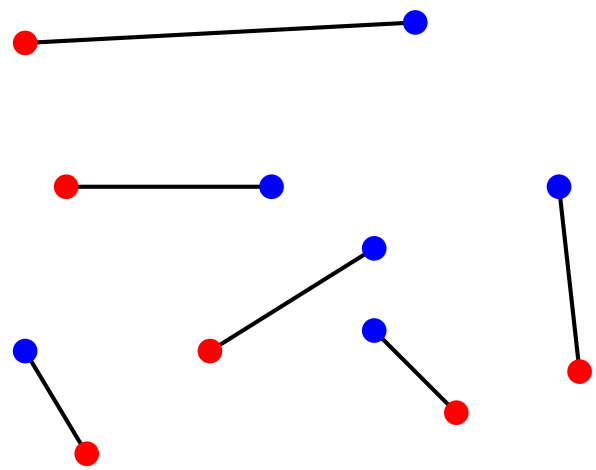

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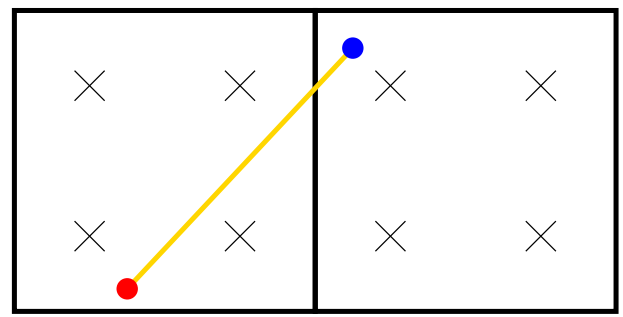

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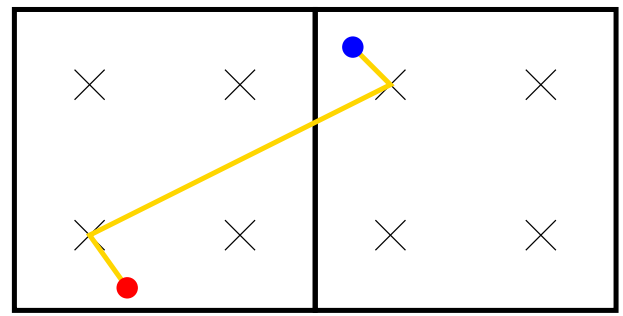

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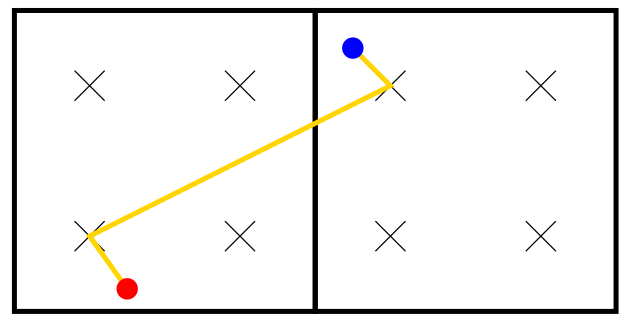
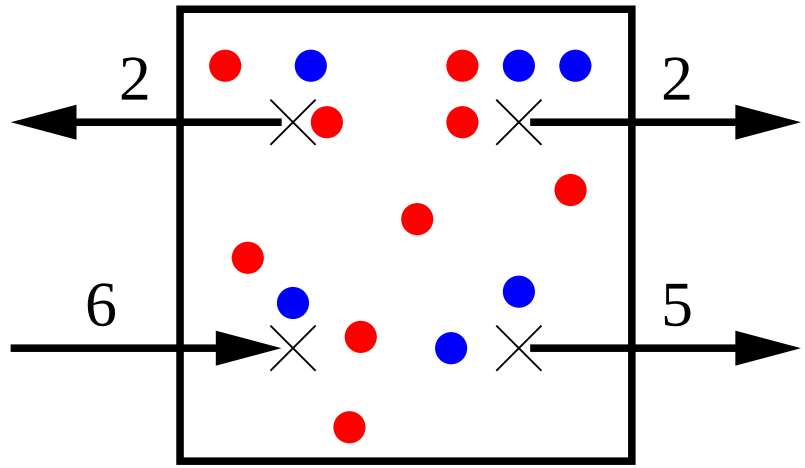


   - For random gridding and dense net small change in value

Cost Function for Subcell

- How much does it cost to send/receive given amounts of flow through net points?



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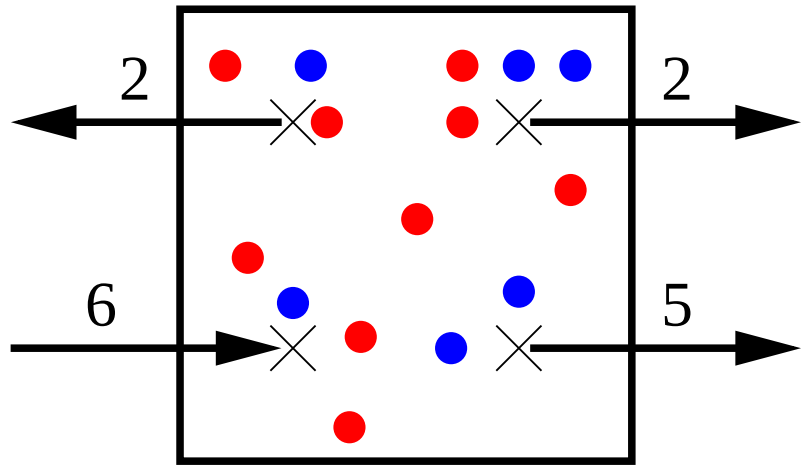

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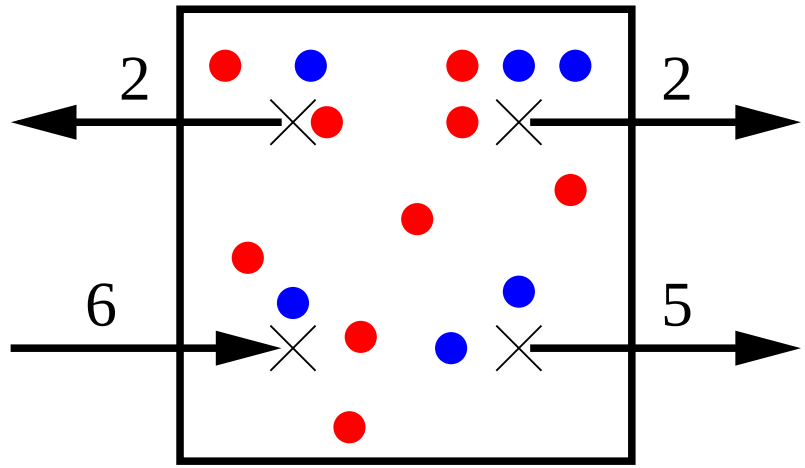

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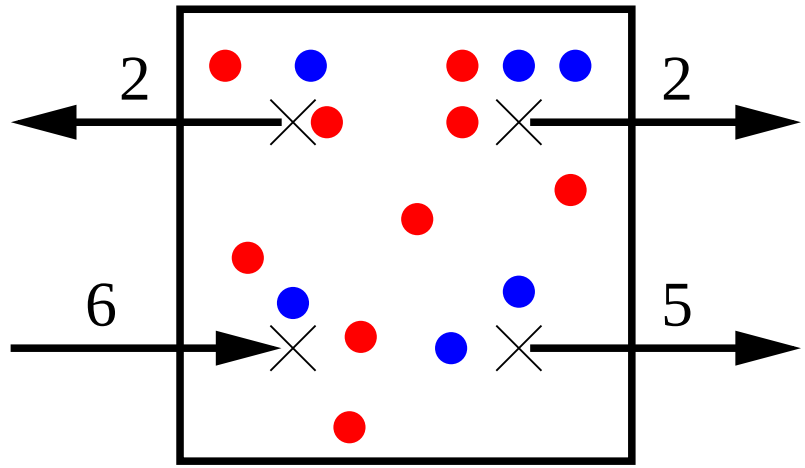

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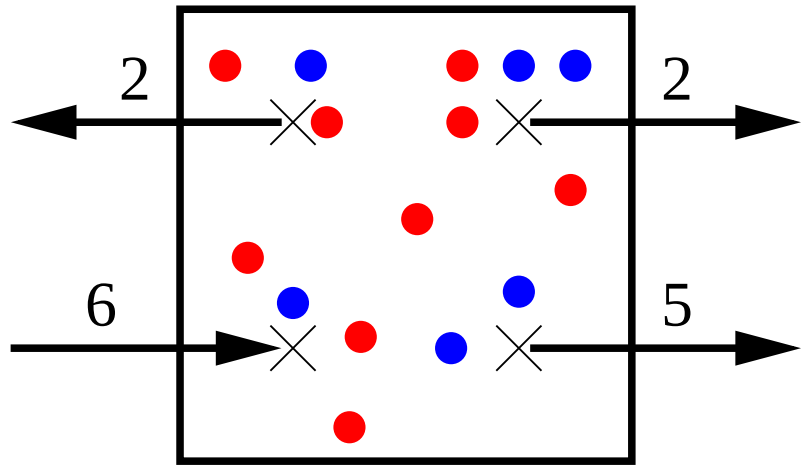

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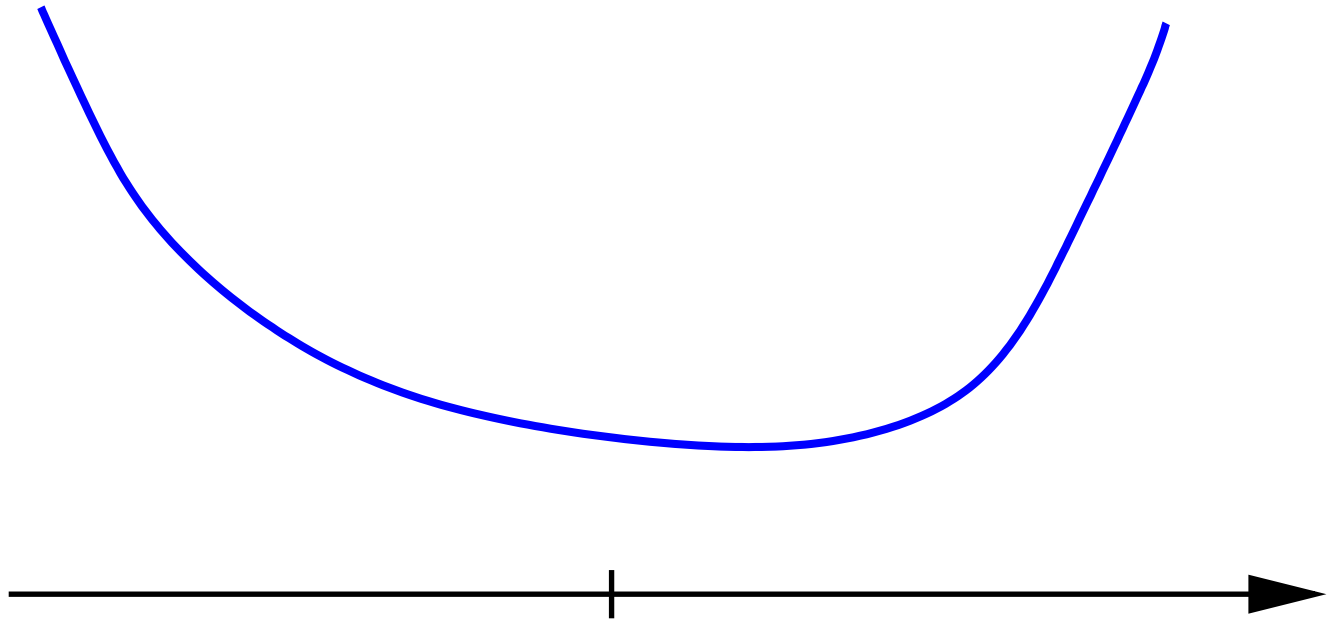
- Problem: potentially a lot of information

Compressed Form



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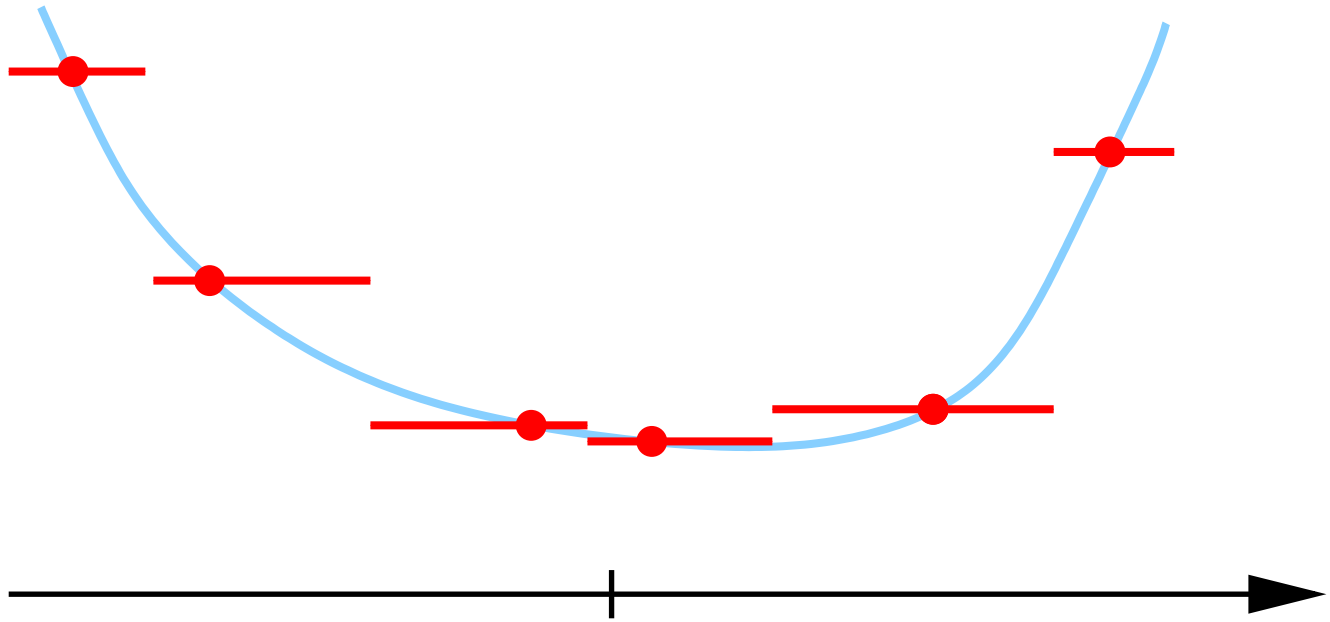

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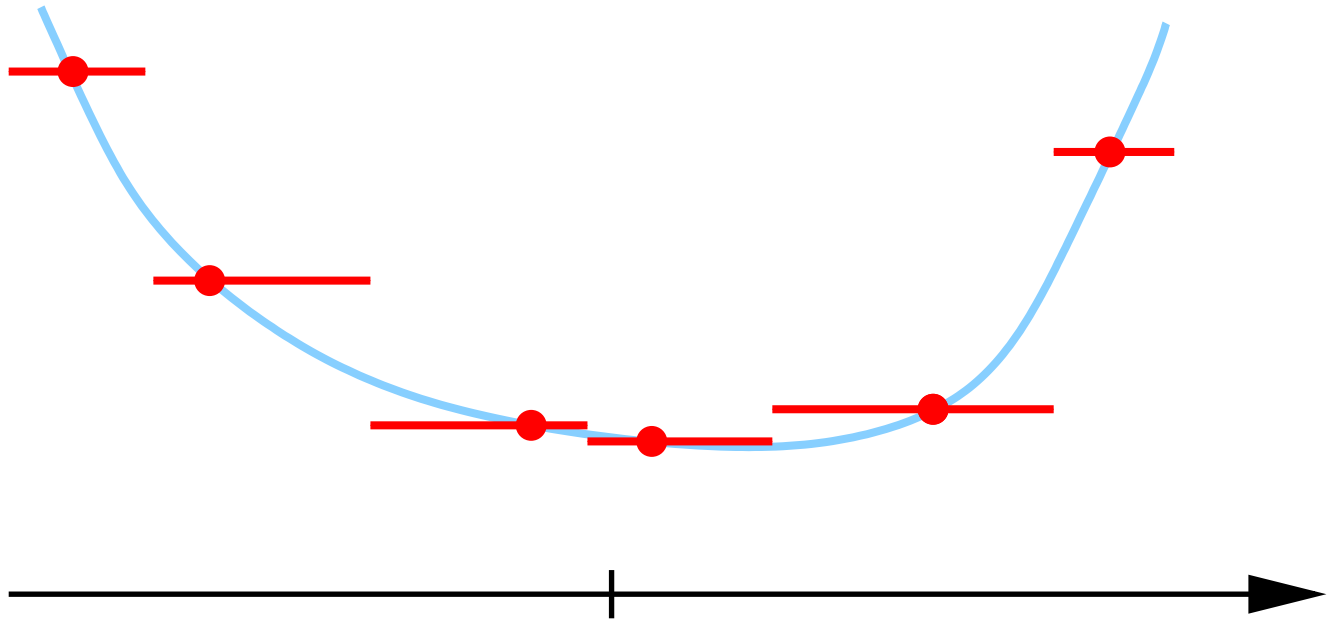

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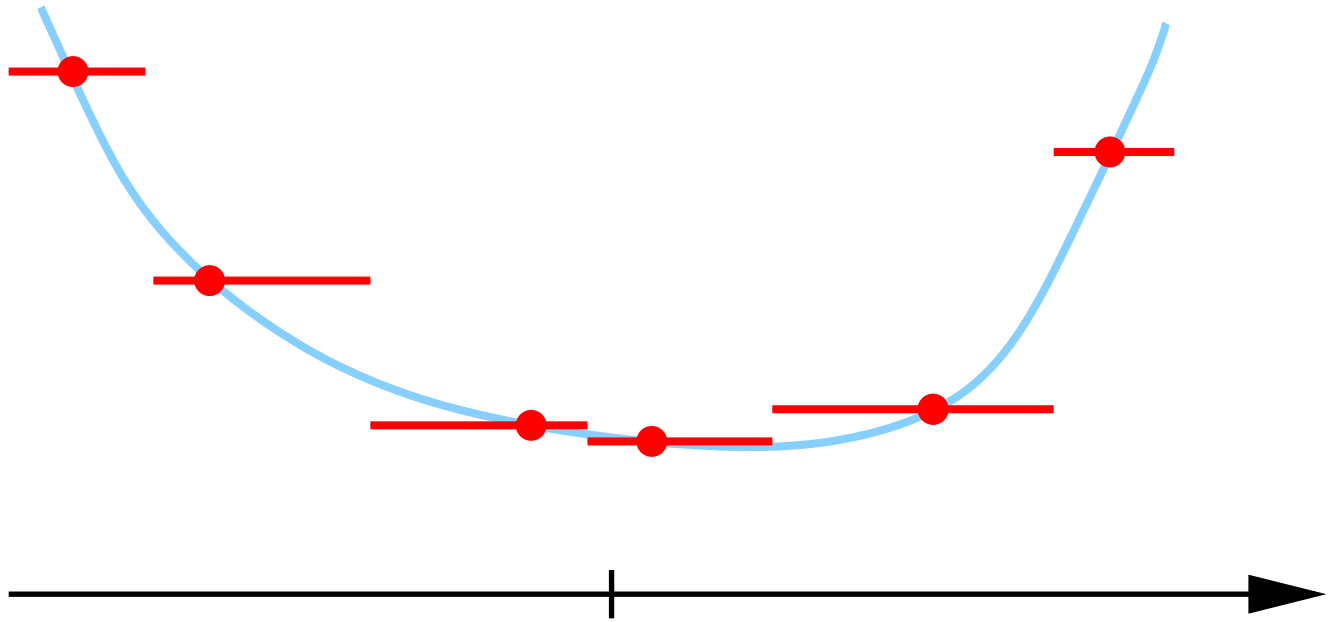

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Recursive Step

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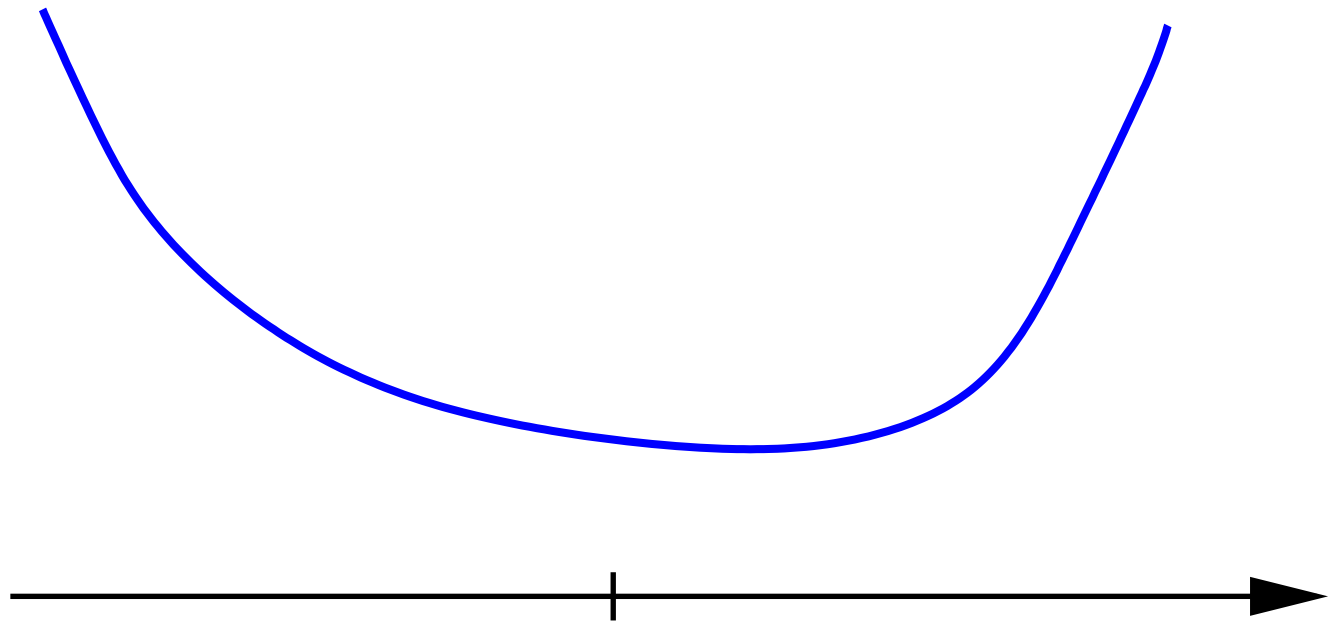

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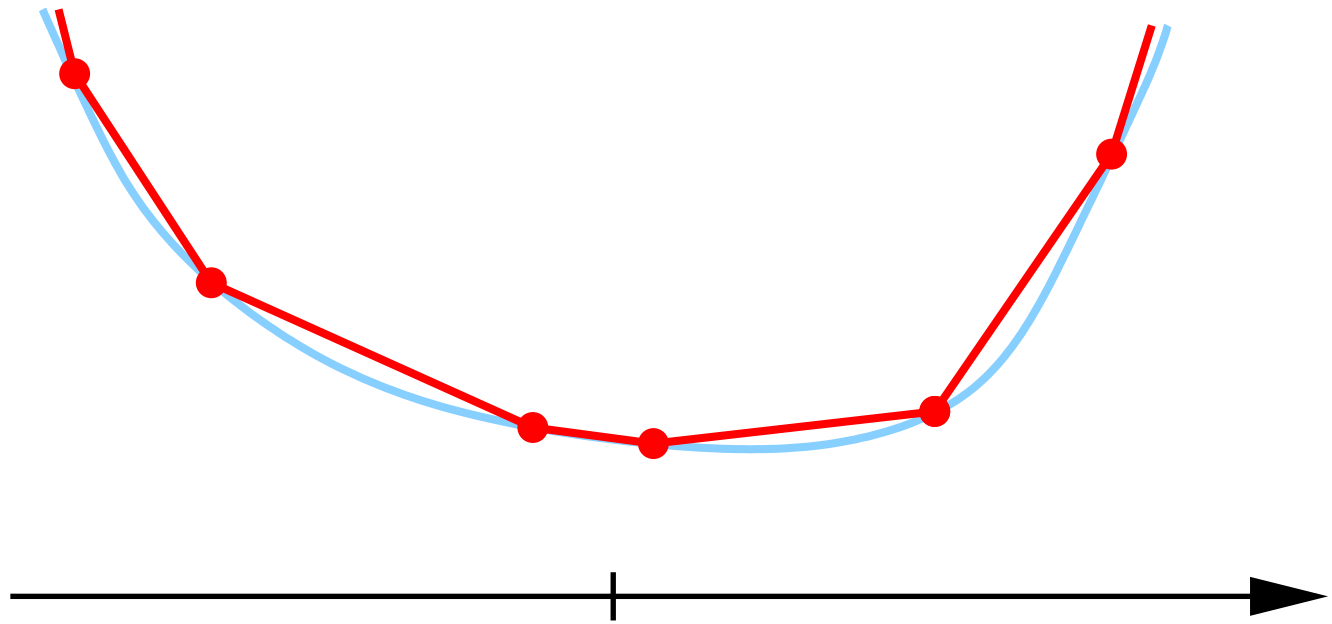

Recursive Step

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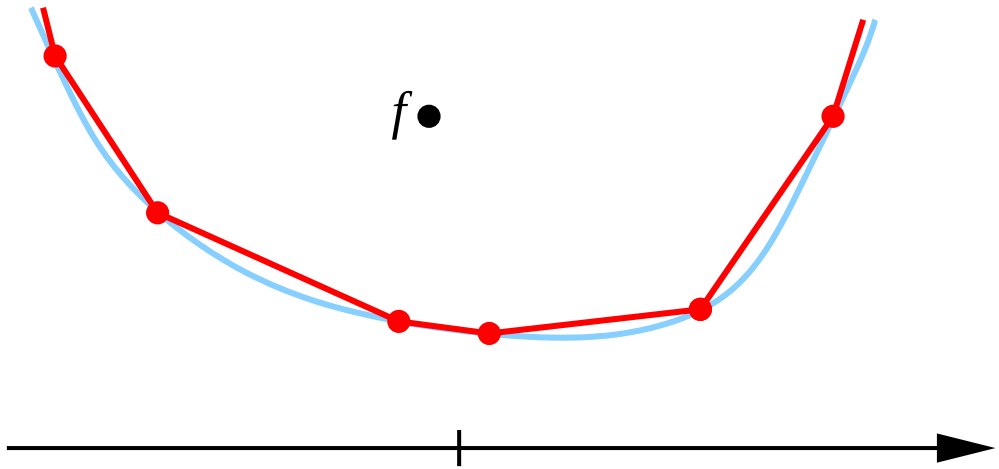
- Combining via a large linear system:
  - subfunctions $F_i(y_i)$'s can be rewritten as new variables f_i: $f_i \geq$ convex hull as function of y_i



  - To compute $F(x)$, solve

$$F(x) = \min(\sum f_i + Ax + \sum B_i y_i)$$

    with some linear constraints on x, y_i's and f_i's

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- Note:
  - Corollary: First near-linear time algorithm for the transportation problem
  - Open question in Sharathkumar & Agarwal (2012)

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- Superconstant lower bound for Sparse Connectivity?
- Can our techniques yield an efficient streaming algorithm for EMD?

Questions?