Algebraic and Model Theoretical Methods in Constraint Satisfaction
Nov 24–28, 2014

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00-10:00	Tutorial 1. M.Pinsker, Constraint satisfaction problems on infinite domains
10:00-10.30	Coffee Break
10:30-11.25	Tutorial 2. Peter Cameron, Synchronization, graph endomorphisms, and some remarkable graphs
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:00–14:55	Tutorial 1. M.Pinsker, Constraint satisfaction problems on infinite domains
15:00-15.30	Coffee Break
15:30-16.25	Tutorial 3. Ross Willard, Universal algebra and CSP
16:30-17.25	Tutorial 3. Ross Willard, Universal algebra and CSP (if desired)
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:55	Tutorial 1. M.Pinsker, Constraint satisfaction problems on infinite domains
10:00–10:20	Coffee Break
10:20–11:15	Tutorial 2. Peter Cameron, Synchronization, graph endomorphisms, and some remarkable graphs
11:20-11.45	Michal Wrona, Algebraic Algorithms for the Inference Problem in Propositional Circumscription
12:00–13:30	Lunch
13:30–14:25	Vera Koponen, Simple homogeneous structures
14:30–15:00	Coffee Break
15:00–15:55	Andrei Krokhin, On constant-factor approximable finite-valued CSPs
16:00–16:25	Johan Thapper, Affine Consistency and the Complexity of Semilinear Constraints
16:30–16:55	Andras Pongracz, Continuity of homomorphisms to the clone of projections
17:00–17:25	Hubie Chen, What's up with the left-hand side?
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–9:55	Tutorial 2. P.Cameron, Synchronization, graph endomorphisms, and some remarkable graphs
10:00–10:20	Coffee Break
10:20–10:45	Barnaby Martin, Distance Constraint Satisfaction Problems
10:50–11:15	Zdenek Dvorak, Towards dichotomy for planar boolean CSP
11:20–11:45	Alexandr Kazda, Linear Datalog and k-permutability = symmetric Datalog
12:00–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner
19:00–20:30	Problem session

Thursday

7:00–9:00	Breakfast
9:00–9:55	Greg Cherlin, Classifying homogeneous structures
10:00–10:20	Coffee Break
10:20–10:45	Martin Dyer, Counting Matrix Partitions
10:50–11:15	Libor Barto, The basic CSP reductions revisited
11:20–11:45	Patrice Ossona de Mendez, Restricted Dualities and First-Order Definable Colorings
11:50–13:30	Lunch
13:30–14:25	Ralph McKenzie, Absorption and directed Jonsson terms
14:30–15:00	Coffee Break
15:30–15:55	Jan Hubicka, Ramsey classes with algebraical closure and forbidden homomorphisms
16:00–16:55	Stanislav Zivny, Necessary Conditions for Tractability of Valued CSPs
17:00–17:25	TBD
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–9:55	Manuel Bodirsky, Challenges in Infinite-Domain Constraint Satisfaction
10:00–10:30	Coffee Break
10:30–10:55	Victor Dalmau, Descriptive Complexity of approximate counting CSPs
11:00–11:25	Matthew Moore, Optimal strong Maltcev conditions for congruence meet-semidistributivity
11:30–13:30	Lunch

Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Libor Barto** (Charles University)
Title: *The basic CSP reductions revisited*
Abstract: The classic reductions between fixed template CSPs over a finite domain are based on three constructions on relational structures: forming images under unary endomorphisms, adding singletons to cores, and pp-interpretation. On the algebraic side, these reductions are usually interpreted in the following (or some equivalent) way: the complexity (computational or descriptive) of CSP(A) depends only on the idempotent Maltsev conditions satisfied by the core of A. We offer a sharper interpretation: the complexity of CSP(A) depends only on the linear Maltsev conditions satisfied by A. By a "linear condition" we mean a condition where all terms involved are of height precisely 1. The corresponding observation for infinite templates will be also discussed.

Speaker: **Manuel Bodirsky** (Dresden Technische Universität)
Title: *Challenges in Infinite-Domain Constraint Satisfaction*
Abstract: In infinite-domain constraint satisfaction, I see two major challenges: one is the generalization of the successful universal-algebraic approach to large classes of infinite structures, typically subclasses of the class of omega-categorical structures. The other major challenge is the treatment of those infinite domains that are most relevant in computer science, as the integers, rationals, or reals. For which constraint languages over these domains is the CSP in P? I believe that several fundamental classes here do not have complexity dichotomy, but unfortunately few non-dichotomy results are known. In this talk I want to present some of the central open problems for these two challenges, and partial results (unless discussed previously in the workshop).

Speaker: **Peter Cameron** (Queen Mary University of London)
Title: *Synchronization, graph endomorphisms, and some remarkable graphs*
Abstract: A deterministic finite automaton is said to be synchronizing if there is a sequence of transitions which brings it to the same state from any starting point. This concept has been much studied, largely because of the Černý conjecture, asserting that if an n-state automaton is synchronized, then there is a synchronizing word of length at most $(n-1)^2$.

Synchronization is a property of the monoid generated by the transitions of the automaton, and can be stated in purely graph-theoretic terms: a monoid fails to be synchronizing if and only if it is contained in the endomorphism monoid of a non-trivial graph with complete core.

Examples attaining the conjectured Černý bound motivate considering the monoid generated by a permutation group and a single non-permutation. The property that a permutation group synchronizes every non-permutation lies strictly between primitivity and 2-homogeneity. The minimum-rank endomorphisms of a vertex-transitive graph are uniform (all kernel classes have the same size), but very recently some remarkable graphs admitting non-uniform endomorphisms have been discovered.

Speaker: **Hubie Chen** (Universidad del País Vasco and Ikerbasque)
Title: *What's up with the left-hand side?*
Abstract: We study the CSP restricted by the left-hand side, formulated as the relational homomorphism problem over a set of structures A, wherein each instance must be a pair of structures such that the first structure is an element of A. We present a comprehensive complexity classification of these problems, which strongly links graph-theoretic properties of A to the complexity of the corresponding homomorphism

problem. In particular, we define a binary relation on graph classes and completely describe the resulting hierarchy given by this relation. This binary relation is defined in terms of a notion which we call graph deconstruction and which is a variant of the well-known notion of tree decomposition. We then use this graph hierarchy to infer a complexity hierarchy of homomorphism problems which is comprehensive up to a computationally very weak notion of reduction, namely, a parameterized form of quantifier-free reductions. We obtain a significantly refined complexity classification of left-hand side restricted homomorphism problems, as well as a unifying, modular, and conceptually clean treatment of existing complexity classifications, such as the celebrated classifications by Grohe-Schwentick-Segoufin (STOC 2001) and Grohe (FOCS 2003, JACM 2007).

This talk is based on joint work with Moritz Müller that appeared in PODS '13 and CSL-LICS '14.

Speaker: **Gregory Cherlin** (Rutgers)
Title: *Classifying homogeneous structures.*
Abstract: We discuss various cases in which the homogeneous structures of a given type can be classified, either definitely or conjecturally, and the particular role of a Ramsey-theoretic method due to Lachlan in that enterprise.

Speaker: **Victor Dalmau** (University Pompeu Fabra)
Title: *Descriptive Complexity of approximate counting CSPs*
Abstract: Motivated by Fagin's characterization of NP, Saluja et al. have introduced a logic based framework for expressing counting problems. In this setting, a counting problem (seen as a mapping C from structures to non-negative integers) is 'defined' by a first-order sentence phi if for every instance A of the problem, the number of possible satisfying assignments of the variables of ϕ in A is equal to $C(A)$. The logic $RHPI_1$ has been introduced by Dyer et al. in their study of the counting complexity class #BIS. The interest in the class #BIS stems from the fact that, it is quite plausible that the problems in #BIS are not #P-hard, nor they admit a fully polynomial randomized approximation scheme. In this talk we shall present some results concerning the definability of counting constraint satisfaction problems #CSP(H) in the monotone fragment of $RHPI_1$.

Joint work with A. Bulatov and M. Thurley.

Speaker: **Zdenek Dvorak** (Charles University)
Title: *Towards dichotomy for planar boolean CSP*
Abstract: We give some partial results on the complexity of planar boolean CSP and conjecture its dichotomy characterization.

Speaker: **Martin Dyer** (University of Leeds)
Title: *Counting Matrix Partitions*
Abstract: Matrix partitions are a generalisation of graph homomorphisms. They were introduced into the context of structural graph theory, and are equivalent to homomorphisms of trigraphs, as introduced by Chudnovsky. The decision version of this problem was investigated by Feder, Hell, Klein and Motwani (2003).

We will discuss recent work on the counting version of this problem.

(Joint work with Leslie Goldberg and David Richerby)

Speaker: **Jan Hubicka** (University of Calgary)
Title: *Ramsey classes with algebraical closure and forbidden homomorphisms*
Abstract: Class K of finite structures is Ramsey if for every choice of A and B in K there exists C in K such that for every coloring of its substructures isomorphic to A with 2 colors there exists an isomorphic copy of B in C where all copies of A are monochromatic. It is a classical result of Nešetřil and Rödl that the class of all finite linearly ordered graphs is Ramsey. We discuss a recent extension of these techniques to classes with non-trivial algebraic closures and with forbidden homomorphic images. This framework

simplify many of existing proofs of Ramsey property (such as for acyclic graphs, partial orders or metric spaces) and introduce new (such as expansions for Sherlin-Shelah-Shi classes).

This is joint result with Jaroslav Nešetřil.

Speaker: **Alexandr Kazda** (Vanderbilt University)
Title: *Linear Datalog and k-permutability = symmetric Datalog*
Abstract: Datalog is a Prolog-like language that allows one to talk about relational structures. Given a relational structure $\mathbf{A}$, there is a Datalog program that decides CSP($\mathbf{A}$) if and only if CSP($\mathbf{A}$) can be solved by local consistency methods. Such structures are well understood. By restricting the kinds of rules a Datalog program can use, one obtains first linear and then symmetric Datalog languages. Characterizing the CSPs solvable by these fragments of Datalog is an open problem.

We show that if $\mathbf{A}$ is a finite relational structure whose algebra of polymorphisms is k-permutable for some k and CSP($\mathbf{A}$) can be solved using linear Datalog, then CSP($\mathbf{A}$) can be solved by symmetric Datalog (which is weaker). This supports the conjecture that CSP($\mathbf{A}$) is solvable by symmetric Datalog iff the algebra of polymorphisms of $\mathbf{A}$ is semidistributive and k-permutable for some k.

Speaker: **Vera Kopponen** (Uppsala University)
Title: *Simple homogeneous structures*
Abstract: Homogeneous structures play a role in infinite domain CSP. In this talk we consider (infinite countable relational) homogeneous structures which are simple in the model theoretic sense, i.e. they admit a well behaved independence relation on subsets. Those simple homogeneous structures which are stable have been classified, but not much is known in general about nonstable simple homogeneous structures. I will present some recent results about binary simple homogeneous structures, where 'binary' means that there is no relation symbol with arity higher than 2. Every such structure, say M, is supersimple with finite SU-rank and all definable sets of SU-rank 1 have trivial pregeometry. If M is, in addition, 1-based, then every definable substructure of SU-rank 1 (in the extension of M with imaginaries) is a reduct of a binary random structure. If M is, in addition, primitive, then M can be interpreted in a binary random structure.

Speaker: **Andrei Krokhin** (Durham University)
Title: *On constant-factor approximable finite-valued CSPs*
Abstract: We study the approximability of minimisation problems related to CSP, namely the (finite-)valued constraint satisfaction problems (VCSPs) and their special case, the minimum constraint satisfaction problems (Min CSPs), all with a fixed finite constraint language Gamma. In this talk, we will focus on characterising such problems that admit a constant-factor approximation algorithm.

A recent result of Ene et al. says that, under a mild technical condition, the basic LP relaxation is optimal for constant-factor approximation for VCSPs unless the Unique Games Conjecture fails. We show that our characterisation problem for VCSPs reduces to the one for Min CSPs, and then use the algebraic approach to the CSP to characterise constraint languages such that the basic LP has a finite integrality gap for the corresponding Min CSP. We also show how this result can in principle be used to round solutions of the basic LP relaxation, and how, for several examples that cover all previously known cases, this leads to efficient constant-factor approximation algorithms. Finally, we improve the above mentioned UG-hardness of constant-factor approximation to NP-hardness for a class of Min CSPs.

This is joint work with Victor Dalmau (UPF Barcelona) and Rajsekar Manokaran (KTH Stockholm).

Speaker: **Barnaby Martin** (Middlesex University)
Title: *Distance Constraint Satisfaction Problems*
Abstract: We give a complexity-based classification for Distance CSPs which are CSPs whose template is fo-definable in (Z;succ). This completes a five-year project and is the first time a mostly full classification of this form has been given for a template that is not omega-categorical. (joint with M.Bodirsky and A.Mottet)

Speaker: **Ralph McKenzie** (Vanderbilt University)
Title: *Absorption and directed Jonsson terms*
Abstract: We prove that every congruence distributive variety has directed Jónsson terms, and every congruence modular variety has directed Gumm terms. The directed terms we construct have the property that they respect every absorption witnessed by the original Jónsson or Gumm terms. This result is directly equivalent to a strong absorption theorem (formulated differently for CD and CM varieties.) Our results were already known for locally finite varieties, but it is a surprise that they hold for unrestricted varieties. The absorption theorems have interesting applications in a broader domain.

The absorption theorem in question was proved by Libor Barto for finite algebras with Jonsson terms, as a main step toward his proof that a finitely related finite algebra with Jonsson terms has a near unanimity term operation. Marcin Kozik used this to prove that a finite algebra has Jonsson terms iff it has directed Jonsson terms: that is, $J_1(x, y, z), \ldots, J_n(x, y, z)$ satisfying equations $J_i(x, y, x) = x, J_1(x, x, y) = x, \ldots, J_n(x, y, y) = y$, and $J_i(x, x, y) = J_{i+1}(x, y, y)$ for $1 \leq i < n$. We have extended both results to arbitrary (not locally finite) algebras, and shown that they are fundamentally the same result.

Speaker: **Matthew Moore** (Vanderbilt University)
Title: *Optimal strong Maltcev conditions for congruence meet-semidistributivity*
Abstract: Locally finite, congruence meet-semidistributive varieties have been characterized by numerous Maltcev conditions and, recently, by two strong Maltcev conditions. We provide three new strong Maltcev characterizations and a new Maltcev characterization each of which improves the known ones in some way.

Speaker: **Patrice Ossona de Mendez** (Ecole des Hautes Etudes en Sciences Sociales)
Title: *Restricted Dualities and First-Order Definable Colorings*
Abstract: We address the problem of characterizing H-coloring problems that are first-order definable on a fixed class of relational structures. In this context, we give also several characterizations of a homomorphism dualities arising in a class of structure.

Speaker: **Michael Pinsker** (Technische Universitat Wien)
Title: *Constraint satisfaction problems on infinite domains*
Abstract: TBD

Speaker: **Andras Pongracz** (École Polytechnique)
Title: *Continuity of homomorphisms to the clone of projections*
Abstract: Automatic continuity of homomorphisms between closed subgroups of the full symmetric group of countably infinite degree has been studied intensively. We initiated the investigation of the analogous problem for clones: Given a homomorphism between two closed function clones acting on a countably infinite set, is it continuous with respect to the topology of pointwise convergence? The question is particularly interesting if the domain of the homomorphism is the polymorphism clone of an omega-categorical structure F, and the image is the clone of all projections. The existence of a continuous clone homomorphism from Pol(F) to the clone of all projections implies that CSP(F) is NP-hard. In the talk, positive and negative results will be discussed, as well.

Speaker: **Johan Thapper** (Université Paris-Est Marne-la-Vallée)
Title: *Affine Consistency and the Complexity of Semilinear Constraints*
Abstract: The linear programming feasibility problem can be expressed as an infinite-domain constraint satisfaction problem over the reals (or the rationals) in which the allowed constraint relations are (1) the unary constant 1-relation, (2) the binary linear order relation, and (3) the ternary addition relation stating that $x + y = z$. It is then possible to look for tractable extensions of the linear programming feasibility problem by allowing additional relations from some larger family. In Bodirsky et al. [LMCS, 2012], this idea was explored for the addition of semilinear relations. It turns out that tractability is maintained under the addition of any finite number of such relations as long as they are "essentially convex".

I will present joint work with Peter Jonsson in which we have considered what happens when one does not necessarily allow the full power of linear programming feasibility. More specifically, we look at constraint satisfaction problems allowing the ternary addition relation $x+y=z$ together with an arbitrary finite set of semilinear relations. For a large subclass of these problems, we identify the boundary between tractable and hard problems. To handle the new tractable problems, we introduce a notion of affine consistency and an accompanying algorithm which in favourable cases computes the affine hull of the set of satisfying assignments, and hence can be used to decide satisfiability.

Speaker: **Ross Willard** (University of Waterloo)
Title: *Universal algebra and CSP*
Abstract: TBD

Speaker: **Michal Wrona** (Linköping University)
Title: *Algebraic Algorithms for the Inference Problem in Propositional Circumscription*
Abstract: Circumscription, introduced by McCarthy [McC80], is perhaps the most impor- tant formalism in nonmonotonic reasoning. The inference problem for proposi- tional circumscription in multi-valued logics may be defined in constraint-based way as follows. Let $(D;\leq)$ be a partial order on domain D and Γ a constraint language over D. An instance of the general minimal constraint inference problem $(GMININF(\Gamma,(D;\leq)))$ is a set of constraints C over variables V and relations in Γ, a partition of V into three sets of variables: P that are subject to minimizing, Q that must maintain the fixed value and Z that can vary, and a constraint ψ over domain D. The question is whether every minimal solution to C is a solution to ψ.

The classification of the complexity of four variants of this problem where $D=\{0,1\}$ and $0<1$ (propositional circumscription in Boolean logic) has been completed in [DHN12]. Each version of GMININF exhibit a trichotomy among Π_2^P -complete, coNP-complete and problems solvable in polynomial time. In two versions: the most general one $(GMININF(\Gamma,(D;\leq)))$ and $VMININF(\Gamma,(D;\leq))$ where Q is always $\emptyset$, the complexity is fully captured by clones of polymorphisms of Γ. To the best of our knowledge, in the full generality, GMININF has been stud- ied so far only in [NJ04] which focuses on the complexity dichotomy between Π_2^P-complete problems and those contained in coNP for the three element do- main. The question of whether there are any interesting polynomial classes of languages is left open. We answer this question a?rmatively by providing three such classes of languages for GMININF and three for VMININF for arbitrarily large finite domains. These classes fully generalize two-element tractable classes from [DHN12] and are defined by closures under certain polymorphisms. There- fore the algorithms we provide for them are fully algebraic. We believe that it is the first but serious step towards obtaining trichotomies for GMININF and VMININF for restricted cases such as the three element domain or conservative languages.

References

[DHN12] Arnaud Durand, Miki Hermann, and Gustav Nordh. Trichotomies in the complexity of minimal inference. Theory Comput. Syst., 50(3):446–491, 2012.

[McC80] John McCarthy. Circumscription — A form of non-monotonic reasoning. Artif. Intel l., 13(1-2):27–39, 1980.

[NJ04] Gustav Nordh and Peter Jonsson. An algebraic approach to the complexity of propositional circumscription. In LICS, pages 367–376, 2004.

Speaker: **Stanislav Zivny** (University of Oxford)
Title: *Necessary Conditions for Tractability of Valued CSPs*
Abstract: The connection between constraint languages and clone theory has been a fruitful line of research on the complexity of constraint satisfaction problems. In a recent result, Cohen et al. [SICOMP'13] have characterised a Galois connection between valued constraint languages and so-called weighted clones. In this paper, we study the structure of weighted clones. We extend the results of Creed and Zivny

from [CP'11/SICOMP'13] and provide necessary conditions for tractability of weighted clones and thus valued constraint languages. We demonstrate that some of the necessary conditions are also sufficient for tractability, while others are provably not. (Joint work with Johan Thapper.)