

14w5133: Specialization of Linear Series for Algebraic and Tropical Curves

March 30 - April 4, 2014

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

17:30–19:30 Buffet Dinner, Sally Borden Building

20:00 Informal gathering in 2nd floor lounge, Corbett Hall

Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45 Breakfast

8:45–9:00 Introduction and Welcome by BIRS Station Manager, TCPL

9:00–10:00 Farbod Shokrieh: *Riemann-Roch theory for graphs and tropical curves.*

10:00–10:30 Coffee Break, TCPL

10:30–11:30 Melody Chan: *Specialization of linear series from algebraic to tropical curves.*

11:30–13:00 Lunch

13:00–14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

14:00–14:10 Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).

14:10–15:10 Dave Jensen: *Tropical Brill-Noether Theory and the Gieseker-Petri Theorem.*

15:15–15:30 Coffee Break, TCPL.

15:30–16:30 Dustin Cartwright: *Lifting divisors and matroid realizability.*

17:30–19:30 Dinner

19:30–20:30 Brian Osserman: *Some problems relating to linear series on curves.*

Tuesday

7:00–9:00 Breakfast

9:00–10:00 Renzo Cavalieri: *Piecewise linear geometry behind wall crossing phenomena in Hurwitz theory.*

10:00–10:30 Coffee Break, TCPL

10:30–11:30 Jesse Kass: *Two ways to degenerate the Jacobian are the same.*

11:30–13:30 Lunch

14:00–15:00 Eduardo Esteves: *Limit linear series: Perspectives from a work in progress.*

15:00–15:30 Coffee Break, TCPL.

15:30–16:30 Omid Amini: *Limits of linear series and distribution of Weierstrass points on curves over non-Archimedean fields*

17:30–19:30 Dinner

Wednesday

- 7:00–9:00** Breakfast
9:00–10:00 Janne Kool: *A combinatorial Li-Yau inequality and rational points on curves.*
10:00–10:30 Coffee Break, TCPL
10:30–11:30 Filip Cools: *Linear pencils encoded in the Newton polygon.*
11:30–13:30 Lunch
Free Afternoon - Organized hike
17:30–19:30 Dinner
19:30–20:30 Dan Abramovich: *Tropicalizing moduli space(s).*

Thursday

- 7:00–9:00** Breakfast
9:00–10:00 Yoav Len: *Algebraic rank and linear series on semistable curves.*
10:00–10:30 Coffee Break, TCPL
10:30–11:30 Shu Kawaguchi: *Rank of divisors on curves and graphs under specialization: the hyper-elliptic case and the genus 3 case.*
11:30–13:30 Lunch
14:00–15:00 Marc Coppens: *Clifford's theorem for metric graphs.*
15:00–15:30 Coffee Break, TCPL.
15:30–16:30 David Zureick-Brown: *Rational points on curves and tropical geometry.*
17:30–19:30 Dinner

Friday

- 7:00–9:00** Breakfast
9:00–10:00 Informal Discussions
10:00–10:30 Coffee Break, TCPL
10:30–11:30 Informal Discussions
11:30–13:30 Lunch
Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

14w5133: Specialization of Linear Series for Algebraic and Tropical Curves

March 30 - April 4, 2014

ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Dan Abramovich** (Brown University)

Title: *Tropicalizing moduli space(s)*

Speaker: **Omid Amini** (CNRS-Ecole Normale Supérieure)

Title: *Limits of linear series and distribution of Weierstrass points on curves over non-Archimedean fields.*

Abstract: The first part of this talk presents a joint work with Matt Baker where we propose a framework for the study of linear series on Berkovich curves, which in particular allows to generalize Eisenbud-Harris theory of limit linear series, restricted to curves of compact type, to any semistable curve.

In the second part of this talk, I will describe a refinement of the above framework and present some applications. In particular, I will show the non-Archimedean analogue of Mumford-Neeman theorem on distribution of Weierstrass points on smooth proper curves.

Speaker: **Dustin Cartwright** (Yale University)

Title: *Lifting divisors and matroid realizability.*

Abstract: The rank of a divisor on a curve can be bounded by the rank of a the specialization of the divisor on the dual graph of a degeneration. Conversely, given a divisor on a graph, we will say that it has a lifting if there is a curve and a divisor of the same with the given dual graph and specialization. I will present a series of examples of rank 2 divisors on graphs for which the existence of a lift depends in a strong way on the field of definition. The examples work by relating the lifting problem to the realizability of matroids.

Speaker: **Renzo Cavalieri** (Colorado State University)

Title: *Piecewise linear geometry behind wall crossing phenomena in Hurwitz theory.*

Abstract: It has been observed a while ago that double Hurwitz numbers exhibit piecewise polynomial behavior with inductive wall crossing formulae. After an overview of the problem, I want to discuss some of the geometry that explains such phenomena- specifically deriving a tautological intersection theoretic formula for the dhn's (joint with Marcus) and discussing, how tropical geometry witnesses the chamber structure of the boundary behavior in the appropriate moduli spaces of maps.

Speaker: **Melody Chan** (Harvard University)

Title: *Specialization of linear series from algebraic to tropical curves.*

Abstract: I will give a survey talk introducing specialization of divisors on curves to graphs and metrized complexes, due originally to Baker and then refined by Amini-Caporaso and Amini-Baker. I'll emphasize concrete examples, including Weierstrass points and applications to Brill Noether theory.

Speaker: **Filip Cools** (Katholieke Universiteit Leuven)

Title: *Linear pencils encoded in the Newton polygon.*

Abstract: Let C be an algebraic curve defined by a sufficiently generic bivariate Laurent polynomial with given Newton polygon Δ . It is classical that the geometric genus of C equals the number of lattice points in the interior of Δ . In joint work with Wouter Castryck (KU Leuven), we give similar combinatorial interpretations for the gonality, the Clifford index and the Clifford dimension, by removing a technical assumption from a recent result of Kawaguchi. More generally, the method shows that apart from certain

well-understood exceptions, every base-point free pencil whose degree equals or slightly exceeds the gonality is ‘combinatorial’, in the sense that it corresponds to projecting C along a lattice direction. We then give an interpretation for the scollar invariants associated to a combinatorial pencil, and show how one can tell whether the pencil is complete or not. We also provide an upper bound on the minimal degree of a plane model.

Speaker: **Marc Coppens** (Katholieke Universiteit Leuven)

Title: *Clifford’s theorem for metric graphs.*

Abstract: Let Γ be a metric graph of genus g . Assume there exists a natural number $2 \leq r \leq g - 2$ such that Γ has a linear system g_{2r}^r . Then Γ has a linear system g_2^1 . For algebraic curves this is part of the well-known Clifford’s Theorem. In the talk we give a proof of this statement for the case of metric graphs.

Speaker: **Eduardo Esteves** (IMPA, Brazil)

Title: *Limit linear series: Perspectives from a work in progress.*

Abstract: The theory of limit linear series was devised by Eisenbud and Harris in the early 80’s to systematize certain arguments using degenerations of linear series to singular curves. In view of its many applications, it was unfortunate that the theory worked only for curves of compact type, and thus Eisenbud and Harris posed the problem of extending it to any stable curve. In the past 30 years, there have been three significant breakthroughs in the theory. First, in the early 90’s Caporaso constructed a relative compactification \overline{P}_g^d over \overline{M}_g of the relative Jacobian, the first step over which to construct a variety of limit linear series, according to Eisenbud and Harris Program. Then, in the late 90’s Mainò constructed a moduli space E_g of enriched curves over \overline{M}_g , capturing the essence of the inseparability of the relative Jacobian. Finally, in the mid 2000’s Osserman showed how one could use all the data coming from degenerations of linear series to construct meaningful varieties $G_d^r(X)$ of limit linear series for two-component curves X of compact type.

At each of these I thought: Now we’ve nailed it! It didn’t turn out to be the case, for several reasons: The compactification of the relative variety of linear series cannot simply be a relative Grassmannian over \overline{P}_g^d ; Mainò’s E_g is only quasi projective over \overline{M}_g . The obvious generalization of Osserman’s approach to any stable curve breaks up in too many pieces . . .

I will present an approach to put together all the above breakthroughs, with the goal of getting a projective variety over \overline{P}_g^d whose fibers parameterize Mainò-style enriched structures and their degenerations, whose general points parameterize Osserman-style limit linear series, and over which a compactification of the relative variety of limit linear series is fundamentally a relative Grassmannian. Relations to the tropical world remain to be braved. However, fundamental tools are tilings of toric varieties and Artin stacks, as well as combinatorics aplenty. To be emphasized: Work in progress!

Speaker: **Dave Jensen** (Yale University)

Title: *Tropical Brill-Noether Theory and the Gieseker-Petri Theorem.*

Abstract: Many problems in algebraic geometry concern the behavior of linear series on general curves. The recently developed theory of specialization of linear series from algebraic to tropical curves provides us with a new set of tools for approaching such problems. We will show how such tools can be used to provide a new proof of the Giesker-Petri theorem, which says that the varieties parameterizing linear series on a general curve are smooth.

Speaker: **Jesse Kass** (University of South Carolina)

Title: *Two ways to degenerate the Jacobian are the same.*

Abstract: The Jacobian variety of a smooth projective curve can be described in two different ways: as the Picard variety or as the Albanese variety. Corresponding to these two different descriptions are two different degenerations of the Jacobian: the relative compactified Jacobian and the Néron model. I will explain how these two different degenerations are the same. If time permits, I will explain some connections with semi-factoriality and autoduality.

Speaker: **Shu Kawaguchi** (Kyoto University)

Title: *Rank of divisors on curves and graphs under specialization: the hyperelliptic case and the genus 3 case.*

Abstract: Let R be a complete discrete valuation ring with fractional field K and algebraically closed residue field. Let X be a regular, generically smooth, semi-stable R -curve, and let Γ be the metric graph associated to the vertex-weighted dual graph of the special fiber. The specialization map τ maps the divisor group $Div(X_{\bar{K}})$ to the divisor group $Div(\Gamma_{\mathbb{Q}})$. For a divisor D on $X_{\bar{K}}$, by Baker and Amini–Caporaso’s specialization lemma, the rank of $\tau(D)$ is larger or equal to the rank of D . In this talk, we study an equality condition. To be precise, we ask when any divisor of $Div(\Gamma_{\mathbb{Q}})$ lifts to a divisor on $X_{\bar{K}}$ with the same rank. We give a complete answer for the hyperelliptic case and the genus 3 case. We also explain our motivation, and relationship to the algebraic rank on a vertex-weighted graph introduced by Caporaso. This is joint work with Kazuhiko Yamaki.

Speaker: **Janne Kool** (Max Planck Institute for Mathematics, Bonn Germany)

Title: *A combinatorial Li-Yau inequality and rational points on curves*

Abstract: I will present a method to give lower bounds for the gonality of non-Archimedean curves in terms of the spectrum of several combinatorial Laplace operators on a graph corresponding to the curve. Moreover, I will present several applications for these bounds. The main part of this talk is based on joined work with Gunther Cornelissen and Fumiharu Kato.

Speaker: **Yoav Len** (Yale University)

Title: *Algebraic rank and linear series on semistable curves*

Abstract: The algebraic rank is a combinatorial invariant of divisors on graphs which arises in the study of linear series on nodal curves with a fixed dual graph.

The invariant is constant under linear equivalence of divisors, and shares similar properties with the standard combinatorial rank (in the sense of Baker and Norine). For instance, it satisfies a specialization lemma and the Riemann-Roch Theorem.

In my talk I will define this invariant and explain the motivation for studying it. I will show how it compares to other notions of rank, and report about the latest developments in the subject.

Speaker: **Brian Osserman** (University of California, Davis)

Title: *Some problems relating to linear series on curves.*

Abstract: The 1970’s and 1980’s saw great leaps forward in our understanding of linear series on curves, with corresponding applications to our understanding of Weierstrass points, and of moduli spaces of curves. We recall some of the main results of this period, and then outline some related problems which are still open.

Speaker: **Farbod Shokrieh** (Cornell University)

Title: *Riemann-Roch theory for graphs and tropical curves.*

Abstract: We start with a brief motivation for the Riemann-Roch theorem for graphs. We will then state the Riemann-Roch theorem for graphs and discuss the key proof ingredients. We discuss a generalization of the Riemann-Roch to tropical curves. We conclude with a comparison of the Riemann-Roch theory for graphs and its counterpart for tropical curves.

Speaker: **David Zureick-Brown** (Emory University)

Title: *Rational points on curves and tropical geometry.*

Abstract: Let X be a curve over \mathbb{Q} with genus $g \geq 2$, $p > 2r$ a prime, J the Jacobian of X , $r = \text{rank } J(\mathbb{Q})$, and \mathcal{X} a regular proper model of X at p . Suppose $r < g$. We prove that $\#X(\mathbb{Q}) \leq \#\mathcal{X}(\mathbb{F}_p) + 2r$, extending the refined version of the Chabauty-Coleman bound to the case of bad reduction.

In this talk I will review the setup of Chabauty-Coleman and explain a new technical insight from tropical geometry which generalizes the classical rank of a divisor on a curve to a notion better suited for singular curves and which satisfies Clifford’s theorem.