MATH 720
Unary Algebras: Dualizability and Quasi-Equational Theory
BIRS Women in Mathematics Workshop

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MATH 720 Unary Algebras

Today's Lecture

- Definitions of unary algebras;
- Examples of unary algebra;
- Theorem we will be working towards.

Definition

Definition:

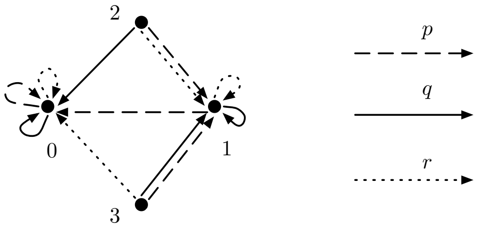
A **unary algebra** consists of a set with a finite collection of operations which each have exactly one parameter.

Example:

$\langle Z; s \rangle$ where $s(x) = x + 1$.

Representing unary algebras

We can represent unary algebras with directed graphs or with tables of functions.



$\mathbf{A_4}$	p	q	r
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	0

Finite unary algebras: $\mathbf{E}_3$

$\mathbf{E}_3 = \langle \{0, 1, 2\}; f, g \rangle$

	f	g	f^2	gf^2	g^2
0	1	0			
1	2	0			
2	2	1			

Equations

An **equation** is a universally quantified formula that consists of

- a well-formed expression in the language of the algebra,
- an equal sign,
- another well-formed expression.

$\mathbf{A}_4 = \langle \{0, 1, 2, 3\}; p, q, r \rangle$

When is $p(x) = q(x)$?

What else is true when $p(x) = q(x)$?

A quasi-equation holds!

Course Goal

Theorem: If **M** is a $\{0,1\}$-valued unary algebra with 0 then one of the following holds:

1. the $\leq$ relation on $\{0,1\}$ can be positive primitively defined;
2. the graph of addition modulo 2 on $\{0,1\}$ can be positive primitively defined; or
3. the rows of **M** form an order ideal.

In the first two cases there is no finite basis for the quasi-equations, and in the last case there is a finite basis for the quasi-equations.

Influences

No finite basis

I. P. Bestsennyi

Duality

Ross Willard

Small algebras

Jane Pitkethly

Escalators

Erin Beveridge

Quasi-equations

David Casperson