

Mathematicians and School Mathematics Education: A Pan-American Workshop

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Background to the Workshop

In January 2012, Patricio Felmer organized a conference in Chile entitled “Mathematicians and School Mathematics Education,” bringing together many of the mathematicians involved in the projects described above. The conference had a number of follow-on projects:

- Chile and Mexico collaborated to produce interactive resources that support the books for teacher preparation developed in Chile.
- Mexico developed the online repository ReLaMat.
- Angel Piñeda and José Antonio announced a study on transition from undergraduate to graduate school in Latin America as a result of the Chile meeting.

The participants in the Chile conference agreed that further pan-American collaboration and exchange is desirable, hence this BIRS workshop.

Before describing some of the issues discussed at the workshop, we give a brief overview of work being done in the various participating countries.

In Chile, mathematicians have led the way in writing standards for the preparation of secondary teachers, writing textbooks for teacher preparation courses, and providing professional development for teachers to meet the new standards.

In the United States, mathematicians played a significant role in the writing of the Common Core State Standards for Mathematics, adopted by over 40 states. A mathematician was one of the lead writers and chaired the Work Team, and the presidents of all mathematical societies have endorsed the standards.

In Brazil mathematicians have organized to write vignettes—accounts of modern mathematics for upper secondary teachers—for the Klein Project (see blog.kleinproject.org). The Klein Project (kleinproject.org) is an initiative of the International Mathematical Union and the International Commission on Mathematics Instruction, celebrating the 100th anniversary of the publication of Felix Klein’s Elementary Mathematics from an Advanced Standpoint. It promises to be a strong vehicle for bringing the expertise of

mathematicians to the school mathematics education enterprise, in a way that gives a voice to both mathematicians and teachers, and promotes productive dialogue. Mathematicians in Brazil have also started to collaborate on constructing virtual repositories of technology based teaching materials. (See, for example, <http://portaldoprofessor.mec.gov.br/links.html>, <http://m3.ime.unicamp.br> and <http://uff.br/cdme>.)

Argentina, Brazil, and Colombia organize Mathematics Olympiads with the active participation of mathematicians; in Brazil the Olympiad reaches 19 million students through an innovative inclusive model.

In Mexico the Mexican Academy of Sciences developed a professional development program for primary and secondary teachers—Science in Your School—under the direction of a mathematician from the National Autonomous University of Mexico, with both an on-site and on on-line component.

In Brazil, mathematicians have played a key role in replacing a system of sporadic professional development for high school teachers with a more academic degree for including both research and a deeper understanding of the content, relating abstract concepts and advanced topics to the content of the school curriculum. In addition, the Brazilian Mathematical Society initiated in 2011 a purely content-based degree for high school teachers.

In the United States there has been a proliferation of content-based masters' degrees for elementary, middle, and high school teachers. Many of these were developed through federally and state funded Math Science Partnerships, with the active participation of mathematicians. These degrees combine pedagogy courses with mathematics courses, relating the mathematics to what teachers will be teaching.

Canada has a long tradition of mathematicians' involvement in school mathematics education. The Canadian Mathematics Education Study Group was founded by mathematicians and maths educators and plays a major role for mathematics education in Canada and beyond. Mathematicians at the University of Laval started a free mathematics magazine, *Accromath*, aimed at high school teachers, and a mathematics show for high school students. Mathematicians at York University developed a course to teach further mathematics to current high school teachers.

Highlights of the Presentations

Videos of the presentations and supporting materials such as slides can be seen at the BIRS website. There were panels on the state of mathematics education in the region. There were panels about mathematicians' work in teacher education, both pre-service and in-service, in developing online resources, in curriculum, and in education policy. There were panels about international collaboration and collaboration between mathematicians and education researchers.

The plenary addresses spanned the region and the concerns of the conference:

- Hyman Bass started off the conference with an analysis of the different roles that mathematicians can play in mathematics education.
- Gabriela Gomez Pasquali gave a moving presentation entitled “Paraguay Solves: Aiming High from the Bottom of the Well,” about OMAPA, a project that supports both teachers and students in a country with one of the weakest education systems in the region. The project has had remarkable success engaging students in mathematics olympiads and teachers in workshops.
- Jim Lewis talked about how to build local partnerships between mathematicians, educators, and teachers to forge effective continuing education programs for mathematics teachers.
- Carlos Bosch, Universidad Nacional Autónoma de México, gave a presentation on an in-service teacher training course, the last of a series of efforts over the last decade to reach teachers in both urban and distant rural areas of Mexico.
- Patricio Felmer talked about the work of mathematicians to improve school mathematics in Chile, particularly in developing curricula for teacher preparation courses.

Common Concerns Discussed at the Meeting

The last day of the workshop was devoted to group discussions of issues nominated by the participants. Here is a summary of some of those discussions.

Content-based Masters Degrees for Teachers

This discussion centered around the growth of content-based masters degrees for teachers, based in departments of mathematics, aimed at increasing teachers' understanding of the mathematics they teach and the mathematics it leads to. Many of those participating in the discussion have been involved in such programs and they wanted to share ideas.

James Madden opened the discussion with a comparison with the medical profession. In 1900 the American Medical Association recognized the lack of professionalism in medical practice, and commissioned the Flexner report. Flexner made a comparison of medical training programs and brought them to public attention, but the resulting changes in the profession were led by the physicians themselves. The question was posed: are we in a similar situation with regard to the training of teachers? Do we have model programs?

Others raised doubts about this analogy, pointing out that we have to consider the national or local context. Some populations of teachers are very mathematically literate, others not. In all cases we want the teachers to be supported by mathematicians in learning mathematics in a way that is conducive to good teaching.

Another point that came up was the relative dearth of such programs compared to the proliferation of masters degrees offered by colleges of education. The sentiment was expressed that we need more examples of content-rich programs established in mathematics departments before an exercise like the Flexner report.

Some programs are purely content based, with no education research component, such as a program for high school teachers introduced by the Brazilian Mathematical Society in 2011 that serves about 1,000 students per year, or the program at York University, whose intent is simply to teach mathematics to current high school teachers, covering a lot of traditional topics.

Other programs make an effort to integrate the content with mathematics education research. In addition to the Brazilian Mathematical Society program described above, another Brazilian program for high school teachers described by Yuriko Baldin integrates education research, content, and field experience. It relates abstract concepts and advanced topics to the content of the school curriculum. Students engage in research into their teaching, drawing on psychology and education research, and bring it to bear on classroom practice. The program also includes connections between high school and elementary school mathematics.

In Chile there had been a divergence into a content degree and a mathematics education degree around 1979. Now the latter program is being changed into a more content-based degree (for high school teachers).

The program at California State University Fullerton offers core classes in high school content areas and problem solving. Rather than offering separate courses in teaching methods, instructors in this program model the pedagogy of high school in the way that they teach the mathematics.

At Boston College the program goes beyond the masters degree through an induction phase, which systematically provides follow-up and support for the first few years of teaching, incorporating pedagogy, administrative matters, and content.

There are two audiences for a masters degree: experienced teachers of mathematics and finishing undergraduate majors looking to become secondary teachers. The University of Utah has a program for each, and has been thinking of merging them so that the experienced teacher pairs with the more mathematically prepared undergraduate. Louisiana State University has taken this approach, but has not yet found the best strategy for bringing about productive contact between the two groups. And, for the CSU Fullerton program mentioned earlier, the enrollment is a mix of high school teachers and community college students or mathematics majors at CSU.

Ed Barbeau offered the following concluding thoughts: a good teacher is a witness (in the evangelical sense) for what is good mathematics. To become such a witness they need to go through some of the struggles of doing mathematics. This can be done with good problems that don't necessarily belong in a topic area. A teacher should be able to orchestrate the use of tools for the students in the way a music teacher leads a band.

How can we support mathematicians in learning to teach mathematics a way that addresses the needs of teachers?

Bernard Hodgson opened with the following question: suppose you are teaching a real analysis course, and all the students are prospective teachers. What kind of analysis course would be good for them? He then suggested that the answer might well include topics not normally taught in such a course: for example, a deep knowledge of how trigonometric functions evolve from ratios of sides of a triangle to functions of a real variable. We need a literature of such examples.

It was pointed out that one way to generate examples is by asking teachers to describe areas where their students ask a question that they cannot give a good answer to, or if there is an area they do not feel quite certain about even though their students don't necessarily ask about it. Problems in courses for teachers should either be problems that help teachers reflect on topics in the curriculum, or problems that help teachers glue these topics together.

This led to a discussion of specific mathematical topics which are often not addressed in undergraduate classes but which secondary school teachers need to know: conic sections, complex numbers, place value, quadratic equations, modeling with calculus. There is also a need to consider the practices of mathematics: for example, what does it mean to have a good mathematical discussion? What does a good mathematical explanation look like?

Jim Lewis enunciated some general principles for courses for teachers. They should

- contain something that informs the curriculum they are teaching, and puts the ideas in that curriculum on a larger framework
- be informed by an awareness that teachers will be advising students about their mathematical future, so they need insights that would help with that
- give teachers a sense of some things that come after the high school mathematics they teach, so that they can judge what is fundamental from what is a side goal.

The session concluded with a discussion of ways of inducting mathematicians into the work of teacher preparation. These included

- asking your colleagues to come and participate in discussion groups with teachers in any venue where there are teachers coming together (courses, teacher circles, evening sessions, problem committees)
- co-presenting with a teacher at workshops, conferences
- co-teaching with someone who is more experienced
- inviting people to observe your class
- observing classes in the school, simply to understand the work of teaching; observing an entire course; engaging in team teaching with the teacher
- looking at videos of classroom practice is a way of bypassing preconceived ideological positions
- writing and critiquing mathematics problems and looking at student work.

Fostering International Collaboration

The workshop concluded with a discussion of ways of fostering international collaboration.

One goal of collaboration is to develop more systematic practice for the work of mathematicians in mathematics education, rather than isolated local efforts. Furthermore, local efforts can often benefit significantly from the moral support provided by international recognition.

There was some discussion of how countries with more developed programs can help countries with less developed programs. One model is the Visiting Lecture Program started in 2005 in Cambodia by the Centre International de Mathématiques Pures et Appliquées (CIMPA). There are not enough Cambodian mathematicians in the country to teach courses for masters' degree programs, so CIMPA arranged for foreign mathematicians volunteer to give these courses.

There is also a hybrid model for such programs, with some semesters running locally and some at a university in the supporting country. The supporting university doesn't need to start a new degree program but provides supporting faculty.

In the discussion of these models it was recognized that there is a tension between the need to develop local capacity versus the need to support a country with little or no local capacity. It was felt that such efforts to be led locally, with outside support.

A case in point is the work in Paraguay that was described at the workshop. Is their initiative something that people from other countries can contribute to, and if so, how?

In general it was agreed that mathematicians from a particular country or region should think about what the barriers are themselves first, before proposing a project. Furthermore, in order to attract funding, we need metrics for what we do.