

Talks and Abstracts

BIRS Workshop on Geometric Scattering Theory and Applications

2-7 November 2014

October 31, 2014

1. ALBIN, Pierre, and SHER, David

Speaker: Pierre Albin

Title: Fibered cusp degeneration and a Cheeger-Muller theorem

Abstract: I will report on joint work with Frederic Rochon and David Sher on finding a topological expression for the analytic torsion of a manifold with fibered cusp ends. Examples of these manifolds include any locally symmetric space of rank one. We establish our theorem by controlling the behavior of analytic torsion as a space degenerates to form hyperbolic cusp ends.

Speaker: David Sher

Title: Analytic torsion on manifolds with cusps

Abstract: This talk is a continuation of the talk of P. Albin and based on joint work with him and Frederic Rochon. We specialize the class of spaces to manifolds with cusp-like ends, but we allow more general representations of the fundamental group. I will describe the Cheeger-Muller theorem and degeneration in this case.

2. ALEXAKIS, Spyros

Title: Global uniqueness theorems and decay properties for linear and nonlinear waves

3. BASKIN, Dean

Title: Asymptotics of scalar waves on long-range asymptotically Minkowski spaces

Abstract: In this talk I will describe a full asymptotic expansion for solutions of the wave equation on Lorentzian spacetimes endowed with an end structure modeled on long-range perturbations of the radial compactification of Minkowski space. Solutions of the wave equation on such spaces have two main asymptotic regimes: along the light cones and interior to the light cone. The rates of decay seen in the expansion (a classical object) can be expressed in terms of a purely quantum object, namely, the resonances of a related elliptic problem on an asymptotically hyperbolic manifold. This is joint work Andras Vasy and Jared Wunsch.

4. BORTHWICK, David

Title: Symmetry factorization of Selberg zeta functions and distributions of resonances (joint work with Tobias Weich)

Abstract: We discuss a factorization of the Selberg Zeta function that applies to hyperbolic manifolds generated by Schottky groups and possessing discrete symmetries. The factorization results in dramatically improved convergence rates for calculations of resonances. We apply these methods to shed new light on the phenomenon of resonance chains in hyperbolic surfaces, and on the recent essential spectral gap conjecture of Jakobson and Naud.

5. CHRISTIANSEN, Tanya

Title: Resonances in even-dimensional Euclidean scattering

Abstract: In odd-dimensional Euclidean scattering, the resonances lie on the complex plane, while in even dimensions they lie on the logarithmic cover of the complex plane. In even-dimensional Euclidean scattering there are some surprises for those who are more familiar with the odd-dimensional case. For example, in even dimensions for Dirichlet or Neumann obstacle scattering there are no resonances with argument $\pi/2+k\pi$ —that is, no “pure imaginary” resonances. Moreover, for Dirichlet or Neumann obstacle scattering or for scattering by a fixed-sign potential one can show there are many resonances in even dimensions. In fact, for these cases the m th resonance counting function ($m \in \mathbb{Z}$, $m \neq 0$) has maximal order of growth.

Some of this talk is based on joint work with Peter Hislop.

6. DATCHEV, Kiril

Semiclassical resolvent bounds in general trapping situations

We give a new, simplified proof of Burq’s and Cardoso and Vodev’s resolvent estimates for scattering on manifolds with boundary. The proof also allows lower regularity on the coefficients of the scattering problem.

7. DEGERATU, Anda

Title: QAC spaces

Abstract: In this talk I will introduce the class of quasi-asymptotically conical (QAC) manifolds, a less rigid Riemannian formulation of the QALE geometries introduced by Joyce in his study of resolutions of singularities of Calabi-Yau orbifolds. Our set-up is in the category of real stratified spaces and Riemannian geometry. Given a QAC manifold, we identify the appropriate weighted Sobolev spaces, for which we prove the finite dimensionality of the null space for generalized Laplacian as well as their Fredholmness.

The methods we use are based on techniques developed in geometric analysis by Grigor’yan and Saloff-Coste, as well as Colding and Minicozzi, and

Peter Li. We show that our geometries satisfy the volume doubling property and the Poincaré inequality, and we use these properties to analyze the heat kernel behaviour of a generalized Laplacian and to establish Li-Yau type estimates for it. With these estimates we construct a parametrix for our operator and establish our Fredholm results.

This work is joint with Rafe Mazzeo.

8. DYATLOV, Semyon

Title: A microlocal toolbox for hyperbolic dynamics

Abstract: I will discuss recent applications of microlocal analysis to the study of hyperbolic flows, including geodesic flows on negatively curved manifolds. The key idea is to view the equation $(X + \lambda)u = f$, where X is the generator of the flow, as a scattering problem. The role of spatial infinity is taken by the infinity in the frequency space. We will concentrate on the case of noncompact manifolds, featuring a delicate interplay between shift to higher frequencies and escaping in the physical space.

I will show meromorphic continuation of the resolvent of X ; the poles, known as Pollicott-Ruelle resonances, describe exponential decay of correlations. As an application, I will give a proof of Smale's conjecture for noncompact manifolds: the Ruelle zeta function continues meromorphically to the entire complex plane (the compact case was recently settled by Giulietti-Liverani-Pollicott and a simple microlocal proof was given by Zworski and the speaker). Joint work with Colin Guillarmou.

9. EWARATHASAN, Suresh

Title: Perturbations of the Schrödinger equation on negatively curved surfaces

Abstract: In this talk, we will take small perturbations of the semiclassical Schrödinger equation on negatively curved surfaces and consider some of the corresponding long-time quantum evolutions. We will show that, under certain admissibility conditions on the perturbation, these solutions become equidistributed in the semiclassical limit for "typical" perturbations.

10. FORD, Austin

Title: The wave trace on manifolds with conic singularities

Abstract: We consider the trace of the (half-)wave group on a compact manifold with conic singularities. The trace of the wave group, which on the one hand equals $\sum e^{-it\lambda_j}$ where λ_j^2 are the eigenvalues of the Laplacian, is on the other hand a distribution in t which is singular at the lengths of closed geodesics. Those closed geodesics that interact with the cone points generically do so "diffractively", carrying singularities into regions of phase space inaccessible to ordinary geodesic flow. We describe a formula for the leading order singularity of the wave trace at the lengths of

closed diffractive geodesics, generalizing the formula due of Duistermaat and Guillemin in the smooth setting and that of Hillairet in the setting of flat surfaces with conic singularities. (This project is joint with Jared Wunsch.)

11. GALKOWSKI, Jeff

Title: Distribution of Resonances for Scattering by Thin Barriers

Abstract: We consider resonances for the operator $-\Delta + V \otimes \delta_{\partial\Omega}$ where $\Omega \subset R^d$ is a bounded domain. This operator is a model for quantum corrals as well as other systems with thin barriers. We give a bound on the size of the resonance free region for very general Ω and, in the case that $\partial\Omega$ is strictly convex, give a dynamical characterization of the resonance free region that is generically sharp. We describe how this characterization can be thought of as a Sabine Law in certain cases. Finally, we give some preliminary results in the case that the potential is strongly frequency dependent

12. GELL-REDMAN, Jesse

Title: The Feynman Propagator on Asymptotically Minkowski Spaces

Abstract: We show that the Feynman propagator for the wave equation on perturbations of Minkowski space is well-defined as an isomorphism between certain weighted b-Sobolev spaces, the target space (i.e. the domain of the d'Alembertian) having additional apriori regularity. We derive small data well-posedness for semilinear wave equations with certain nonlinearities, obtaining solutions in the image of the Feynman propagator. This builds on work of Hintz and Vasy, which obtained similar results for the forward propagator; in our setting the relevant structures are more microlocal, and in particular an analysis of the b-wavefront sets of products is required.

More on the Feynman propagator: various inverses of the wave operator are obtained by choosing directions along the Hamiltonian flow on the set of null bicharacteristics along which to propagate regularity. For perturbations of Minkowski space, the null bicharacteristics have two connected components, one flowing towards the future and one towards the past. The forward propagator is obtained by propagating regularity forward in time along both components, which means propagating *backward* along the flow on the component which flows to the past. The Feynman propagator simply propagates regularity in the direction of the flow on both components

Joint with Haber and Vasy.

13. GOVER, Rod

Title: Boundary operators on conformally compact manifolds, and a boundary Loewner-Nirenberg-Yamabe problem

Abstract: On conformally compact manifolds I will describe a natural boundary calculus for computing the asymptotics of a class of natural boundary problems. This is applied to the non-linear problem of finding, conformally, a conformally compact constant scalar curvature metric on the interior of a manifold with boundary. This problem was studied from a different point of view by Andersson, Chrusciel, Friedrich (ACF) in 1992. They identified a conformal submanifold invariant that obstructs smooth boundary asymptotics for the problem on 3-manifolds (and gave some information on the obstructions in other dimensions). This invariant is the same as that arising from the variation of the Willmore energy. We find higher order submanifold invariants that generalise that curvature quantity found by ACF. The construction also leads to a route for manufacturing large classes of other conformal submanifold invariants.

This is joint work with Andrew Waldron

14. HAFNER, Dietrich

Title: Asymptotic completeness for superradiant Klein-Gordon equations and applications to the De Sitter Kerr metric

Abstract: Superradiance appears on space times which have no global timelike Killing vector field. In this situation there is no positive conserved energy for the wave equation and natural positive energies can grow in time even for the linear wave equation. The most famous example is that of the (De Sitter) Kerr metric which describes rotating black holes. We present in this talk an abstract framework for this phenomenon and show an asymptotic completeness result for the Klein-Gordon equation within this framework. Applications to the Klein-Gordon equation on the De Sitter Kerr metric with small angular momentum of the black hole are given. For this equation we obtain asymptotic completeness for fixed angular momentum of the field. This is joint work with Vladimir Georgescu and Christian Grard, see arXiv:1405.5304.

15. HASSELL, Andrew

Title: Resolvent, spectral measure, spectral multipliers and Strichartz estimates on asymptotically hyperbolic manifolds

Abstract: I will discuss results obtained recently by my PhD student Chen Xi, some of which are collaborations with me and others of which are his alone. These results concern the microlocal nature of the resolvent kernel (on the spectrum) and the spectral measure on asymptotically hyperbolic, nontrapping spaces. Applications include results on spectral multipliers and global-in-time Strichartz estimates on asymptotically hyperbolic spaces. These generalize results of mine in collaboration with Vasy, Wunsch, Guillarmou, Sikora and Zhang on asymptotically conic spaces (as well as recent work of Vasy, Melrose and Sá Barreto on perturbations of hyperbolic space), but with key differences due to the negative curvature at infinity.

16. HUNSICKER, Eugenie

Title: A Hodge Theorem for Intersection Space Cohomology

Abstract: Intersection space cohomology is a new family of (co)homologies defined on singular spaces, defined by Markus Banagl. This talk will introduce the theory and its relationship to mirror symmetry, and will present a relationship between the theory and the now more classical theory of intersection (co)homology as well as a Hodge theorem relating the spaces to spaces of harmonic forms. This is joint work with Markus Banagl.

17. INGREMEAU, Maxime

Title: Distorted plane waves in chaotic scattering

Abstract: Distorted plane waves, sometimes called Eisenstein function, are a family of generalized eigenfunctions of a Schrödinger operator, which are not square integrable. We will study distorted plane waves in the semiclassical limit, on manifolds which are euclidean or hyperbolic near infinity, with a compactly supported potential or metric perturbation. We make the hypothesis that the classical dynamics is hyperbolic on the trapped set, and that a certain topological pressure is negative. We will be able to write distorted plane waves as a convergent sum of Lagrangian states, associated to Lagrangian leaves which are very close to the unstable manifold of the trapped set. In particular, this allows us to describe the semiclassical measures associated to these distorted plane waves.

18. KOTTKE, Chris

Title: Gluing monopoles at infinity

Abstract: In this joint work with M. Singer, we give a gluing construction for $SU(2)$ magnetic monopoles on R^3 . In contrast to Taubes' original result on the subject, our construction allows the gluing of monopole 'clusters' of arbitrary charge, and more importantly is uniform up to infinity, giving a partial compactification of the monopole moduli space and the asymptotic expansion of the hyperkahler metric there. The full compactification will be as a smooth manifold with corners, and it is believed that the asymptotics of the metric thereon will provide a means of tackling the conjectures of Sen on the L^2 cohomology of monopole moduli space.

19. LASSAS, Matti

Title: Scattering problems and the inverse problems for non-linear wave equations

Abstract: We show how the analysis of scattering problems can be applied to study an inverse problem for non-linear hyperbolic equations. More precisely, we study a non-linear hyperbolic equation with a time-dependent metric tensor on a manifold. Considering the non-linear interaction of waves we show that for generic incoming waves scattering can be observed in generic directions. To study the inverse problem we define the concept of

light observation sets and show that the scattering observations determine these sets that further the conformal class of the metric. The results have been done in collaboration with Yaroslav Kurylev and Gunther Uhlmann.

20. MARZUOLA, Jeremy

Title: Dispersive estimates for Schrodinger operators with potential on H^d (joint work with David Borthwick).

Abstract: With David Borthwick, we study resolvent estimates, spectral theory and dispersive properties of scalar and matrix perturbed Schrödinger-type operators on HS^{n+1} for $n \geq 1$. The primary techniques involve analysis of scalar and matrix resolvent expansions, as well as some spectral theory in the hyperbolic setting. The results show that the expected dispersive decay rate of $t^{-3/2}$ arises in all dimensions considered, which we hope will have implications for nonlinear stability theory analysis in future work.

21. MENDOZA, Gerardo

Title: Elliptic operators of variable order

Abstract: Sections of vector bundles of anisotropically varying regularity appear naturally as boundary values of elements in the maximal domain of certain elliptic operators on manifolds with fibered boundary. I will briefly discuss the statement of the boundary value problem to motivate the main topic of the talk, namely the introduction of the right kind of Sobolev spaces and of pseudodifferential operators. The talk is based on joint work with T. Krainer.

22. MULLER, Werner

Title: Analytic torsion of locally symmetric spaces of finite volume.

Abstract: Analytic torsion is a spectral invariant of a compact Riemannian manifold. The study of analytic torsion of locally symmetric spaces has interesting applications to the cohomology of arithmetic groups. It is very desirable to extend the existing methods and results to the non-compact case. In the talk I will present some recent results and discuss some related problems.

23. NAKAMURA, Shu

Title: Microlocal properties for scattering matrices

Abstract: In this paper (<http://arxiv.org/abs/1407.8299>) we prove the scattering matrix is a pseudodifferential operator on the energy surface and give an explicit expression of the principal symbol for a class of operators, which includes discrete Schrodinger operators with short range potentials.

24. NAUD, Frederic

Title: Sharp resonances on hyperbolic manifolds

Abstract: Sharp resonances are the closest to the unitary axis and are related to the metastable states who live the longest. In the case of Laplace resonances on Hyperbolic manifolds with infinite volume, there are some precise conjectures about the distribution and location of sharp resonances. We will review these conjectures and state some partial results, with elements of proof. We will address issues of spectral gap and fractal Weyl laws. Part of this work is joint with Dmitry Jakobson.

25. ROCHON, Frederic

Title: An index theorem via resolvent estimates

Abstract: Using known facts about the meromorphic continuation of the resolvent of the Laplacian on manifolds with asymptotically cylindrical ends, we obtain a local index formula for certain families Dirac-type operators which are not Fredholm, but have finite L^2 kernels and cokernels forming vector bundles over the parameter space of the family. The key step is to obtain decay estimates for the corresponding family of heat kernels. We will then indicate how such formula naturally comes up when one considers the moduli space of asymptotically cylindrical Calabi-Yau manifolds and its associated Weil-Petersson metric. This is a joint work with Ronan Conlon and Rafe Mazzeo.

26. SA BARRETO, Antonio

Title: Inverse problem on asymptotically hyperbolic manifolds with partial data

Abstract: see <http://arxiv.org/abs/1307.8402>

27. STROHMAIER, Alexander

Title: Low energy scattering on asymptotically Euclidean and asymptotically globally symmetric spaces and analyticity properties of the resolvent and scattering matrix at zero

28. VASY, Andras

Title: Non-elliptic Fredholm problems

Abstract: In this talk I will describe a framework for the Fredholm analysis of non-elliptic problems both on manifolds without boundary and manifolds with boundary. The latter include spaces such as ‘Lorentzian scattering spaces’, which generalize asymptotically Minkowski spaces, as well as generalizations of asymptotically de Sitter spaces and Kerr-de Sitter spaces, but also spaces with non-Lorentzian signature.

29. ZHU, Xuwen

Title: Resolution of the canonical fiber metrics for a Lefschetz fibration

Abstract: We consider the constant scalar curvature fiber metric in the case of a Lefschetz fibration with genus ≥ 1 , which arises naturally as the singular behavior across the divisors introduced in the Deligne-Mumford

compactification of the moduli space of Riemann surfaces. Starting from a local model of plumbing variety, we construct a resolution of the total space by doing three steps of blow-ups, and consider the solution to the curvature equation on this resolved space. We give a refinement of the result of Obitsu-Wolpert that the constant scalar curvature fiber metric has a complete polyhomogeneous expansion with respect to the plumbing metric. Joint work with R. Melrose.