

Spin Glasses and Related Topics

Michael Cranston (University of California, Irvine),
Erwin Bolthausen (University of Zurich),
Dmitry Panchenko (Texas A&M University)

July 20–25, 2014

1 Overview of the conference

Spin glasses has been an active area of research in theoretical physics since about mid-seventies. On the mathematics side, it has been increasingly more active over the last two decades, and over the last ten years or so it has become a tradition to organize a conference on the mathematics of spin glasses once every one or two years. This year the conference took place at the Banff International Research Station, which allowed us to bring together many of the mathematicians and physicists working in the area of spin glass models and various related areas, and gave us all a chance to share recent results and ideas, as well as to have many wonderful discussions in a great collaborative environment that BIRS provides. As the organizers, we tried to ensure the success of the conference by careful planning.

First of all, we attempted to increase the diversity in a number of ways. First of all, although the main focus of the conference was on the mathematics of spin glasses and related areas, out of 35 participants we had 5 theoretical physicists as participants and speakers. The diversity of topics represented was also an important factor in making the conference successful. Among invited participants, we had experts working on various models of spin glasses, random matrices, various extremal processes, Brownian motion, optimization and algorithms, random combinatorial optimization problems, biological applications of spin glass models, and genealogy in populations dynamics. Furthermore, there were 5 female researchers (unfortunately, one of them could not make it at the very last moment due to lost passport). Finally, we also had a significant number of young researchers – 5 postdocs and 4 current Ph.D. students – with all of them giving a talk (except one beginning Ph.D. student).

In addition, we were encouraged by the BIRS recommendations to create a schedule that leaves plenty of time for discussions and collaboration, and we tried to do that. We had a relaxing schedule of only 28 talks that ended at 4 p.m. every day, and many participants worked in groups after that both before and after dinner. Moreover, we left Friday morning completely open, which resulted in a wonderful three-hour discussion session led by Giorgio Parisi.

This was a second spin glass meeting at BIRS in the last 6 years. We consider the meeting to have been a great success and we hope to be back in the future.

2 Overview of the field

Few materials in the history of solid state physics have been as intriguing and perplexing as certain alloys of ferromagnets and conductors, such as *AuFe* or *CuMg*, known as *spin glasses*. The attempts to study

these magnetic alloys theoretically gave rise to a class of *disordered spin models*, whose analysis by both physicists and mathematicians has grown into one of the most fascinating fields of statistical mechanics over the last 35 years. Probably, the two most famous models are the Edwards-Anderson model and its mean-field analogue, the Sherrington-Kirkpatrick (SK) model. Even the simpler SK model has proven to possess a very rich structure that was very hard to discover and took a long time to confirm mathematically. Theoretical physicists, starting with the groundbreaking work of Giorgio Parisi, have created a general theory of this model, mostly within the framework of the celebrated replica method, with the physical properties of the system described by such famous features as the “replica symmetry breaking” and “ultrametricity”. After more than two decades, the predictions of the physicists in the SK model have been now mostly confirmed mathematically in recent years, but many interesting problems remain.

Furthermore, the ideas developed by theoretical physicists in the setting of the SK model have found striking applications in unexpected areas, for example, in the models arising from optimization—such as the satisfiability of Boolean formulas, the independent set problem, the matching problem, the traveling salesman problem, the assignment problem, the graph partitioning problem—as well as in the models arising in biology, such as the Hopfield model of neural networks. In many of these models, the physicists came up with detailed predictions about the structure of the Gibbs measure, which relates to the structure of the solution space, so mathematicians have their hands full for many years to come.

Encouraging progress in various directions has been happening for a while now, and a number of interesting recent results were presented at this conference.

3 Presentation Highlights

Louis-Pierre Arguin (Université de Montreal) spoke on “Fluctuation bounds for interface free energies in spin glasses.” The quantitative dependence of free energy fluctuations on the quenched disorder is a fundamental thermodynamic property of all disordered systems. Bounds on such fluctuations have been used to prove uniqueness of the Gibbs state in a class of two-dimensional disordered systems, including the two-dimensional random field Ising model. Moreover, it has been noted by several authors that determining quantitative bounds on the interface free energy variance with respect to the couplings would allow a resolution of a long-standing open problem at the heart of the statistical mechanics of short-range spin glasses in finite dimensions: how many pure states (or at zero temperature, ground states) are present in the spin glass phase? Thus, one way to understand the structure of the Gibbs states of disordered systems is to get good bounds on the fluctuations of the free energy difference between two states. This approach has led to the proof of the absence of phase transition in the 2D Random Field Ising Model (RFIM) by Aizenman and Wehr. This talk explained a method to obtain lower bounds for the variance of the free energy difference of the Edwards-Anderson (EA) spin glass model on \mathbb{Z}^d between certain incongruent states (if they exist...). Unlike the RFIM, there is no dominance of the (+) and (−) states in the EA model. One interesting point of the method is to overcome this lack of monotonicity. The lower bound is also used to rule out particular structures of the Gibbs states in $d = 2$. This is joint work with C. Newman, D. Stein and J. Wehr.

Antonio Auffinger and Wei-Kuo Chen (both are postdocs at the University of Chicago) gave two talks about their joint work on Parisi measures and bipartite spherical spin glass models. Chen’s talk focused on various important properties of Parisi measures, heuristically and numerically observed by physicists, and recently proved by the two of them mathematically. The most important one (this was quite a high impact result) is that the Parisi functional order parameter in the Sherrington-Kirkpatrick model (which determines all other parameters of the model) is unique. They proved this by showing that the Parisi functional is strictly convex. For this purpose they developed and applied techniques from stochastic control theory. Auffinger’s talk focused on their recent results about bipartite models. In particular, they proved a formula for the free energy based on the Crisanti-Sommers representation of the mixed spherical model and showed that the mean number of local minima at low levels of energy is exponentially large in the size of the system.

David Belius (Université de Montreal) spoke on “The subleading order of two dimensional cover times.” The cover time is the time it takes for a Markov process on a finite or compact state space to visit the whole state space. This time can be studied by considering the extrema of a correlated random field, namely the

field of occupation times, placing it in a similar category to many spin glass problems. In this talk Belius presented a recent result on the cover time of the two dimensional torus by Brownian motion, where the field of occupation times turns out to be log-correlated. He used techniques inspired by those for Branching Brownian Motion (BBM) to establish the exact subleading correction for the expectation of the cover time, which corresponds to the well-known $3/2$ -correction for BBM. The leading order had been established by Dembo, Peres, Rosen and Zeitouni [Ann. of Math., 160 (2004)]. This involves a multiscale analysis of the field of occupation times, and the identification of an approximate hierarchical structure. This work establishes deep connections with the asymptotics of BBM proved in the groundbreaking work of Maury Bramson which was motivated by a model for the propagation of genes in a population, and its relations with traveling waves for the KPP equation. This is joint work with Nicola Kistler.

Nathanael Berestycki (University of Cambridge) spoke on "Liouville Brownian Motion." A subject of considerable interest in recent years has been to develop a notion of random surface, understand its geometry and relate it to the theory developed by physicists in the context of Liouville quantum gravity. In particular conformal invariance properties are expected and the so-called KPZ relation is believed to hold. Consider a uniform random triangulation of a random surface with a large but finite number of faces, represented so that every triangle is equilateral. Such a triangulation can also be represented in the plane via the circle packing theorem. Either way, after rescaling, it is strongly conjectured that the density of vertices converges, to a measure with density with respect to Lebesgue measure given by the exponential of a constant times a Gaussian Free Field. Even defining this limiting measure ? is nontrivial and was one of the main results in in these two landmark papers (Duplantier-Sheffield, Rhodes-Vargas). In his talk, the speaker introduced and discussed a canonical notion of Brownian motion in the random geometry of Liouville quantum gravity, called Liouville Brownian motion which is supposed to be the scaling limit of a random walk on a random triangulation. He discussed some of its basic properties, for instance related to the time spent in the thick points of the Gaussian Free Field. He also discussed the construction of this process in the supercritical phase of Liouville quantum gravity and showed a certain duality with the subcritical phase.

Anton Bovier (Rheinische Friedrich-Wilhelms-Universität Bonn) spoke on "Extremal processes of Gaussian processes indexed by trees." Gaussian processes indexed by trees form an interesting class of correlated random fields where the structure of extremal processes can be studied. One popular example is Branching Brownian motion, which has received a lot of attention over the last decades, not the least reason being its connection to the KPP equation. The KPP equation was derived to model the spread of a gene in a population and has a long history. Analysis of this equation involves branching Brownian motion an approach initiated by McKean with a fundamental contribution by Bramson. More recent developments in the field are by Lalley and Sellke. In his talk Bovier reviewed the construction of the extremal process of BBM in the time to ∞ limit (with Arguin and Kistler) which has an interesting description as a decorated Poisson-Dirichlet point process. He also presented more recent results on variable speed BBM, obtained recently with Lisa Hartung.

Amir Dembo (Stanford University) spoke on "Spin glasses on locally tree like graphs" a collaboration with Gerschenfeld and Montanari. This is work motivated by earlier results of Bray and Viana, 1985. Many problems of interest in computer science and information theory can be phrased in terms of a probability distribution over discrete variables associated to the vertices of a large (but finite) sparse graph. In recent years, considerable progress has been achieved by viewing these distributions as Gibbs measures and applying to their study heuristic tools from statistical physics. The model is a Gibbs measure on a finite sparse graph with n vertices. The Hamiltonian of the Gibbs measure involves only a coupling between adjacent vertices and an external field term. As n tends to infinity, the graph is assumed to locally converge to a tree. The main result is an existence of the free energy in the $n \rightarrow \infty$ limit for high temperatures.

Silvio Franz (Université de Paris-Sud 11) spoke on "Glassy critical points and the Random Field Ising Model." Research in recent years has emphasized the importance of fluctuations in understanding glassy phenomena in supercooled liquids. The present comprehension of long lived dynamical heterogeneities in these systems compares the growth of their typical size to the appearance of long range correlations at second order phase transition points. Unfortunately, in supercooled liquids, the theoretical study of these correlations beyond the mean field is just at an embryonic level. One of the difficulties lies in the fact that -with good physical reasons- the critical point corresponds to an unstable field theory. It turns out that one can cure the instability introducing appropriate physical constraints. In that case true critical points appear and they can

be analyzed theoretically through the replica method. In usual phase transition often there is a line of first order transitions that ends at a second order terminal critical point. The most popular case are ferromagnets: at low temperatures there is a first order transition when the magnetic field crosses zero (the magnetization has a discontinuity) and this transition line ends at the usual critical point. The same phenomenon happens for the gas liquid transition: it is a first order transition at low temperatures that ends in a second order transition at the critical point. A similar situation can occur for liquids undergoing a glass transition, where lines of discontinuous glass transitions can terminate in critical points. In this talk the focus was on these terminal points, where the glass transition becomes continuous and activation does not play a major role in establishing equilibrium. The speaker presented the field theory of these critical points and showed that the universality class of the Random Field Ising Model appears.

Yan Fyodorov (Queen Mary College, London) spoke on "High-dimensional random landscapes and random matrices." In this talk, Fyodorov discussed a picture of the "topology trivialization transition" (in the sense of an abrupt reduction of the number of stationary points and minima of the underlying energy landscape) which takes place at zero temperature in p -spin spherical model of spin glasses with increasing random magnetic field, as well as in related high-dimensional models not restricted to the sphere. After developing the general formalism based on the high-dimensional Kac-Rice formulae it is combined with the Random Matrix Theory (RMT) techniques to perform analysis of the random energy landscape of p -spin spherical spin glasses and a related glass model, both displaying a zero-temperature one-step replica symmetry breaking glass transition as a function of control parameters (e.g. a magnetic field or curvature of the confining potential). In particular, the emphasis was on the role of the "edge scaling" and the Tracy-Widom distribution of the largest eigenvalues of random matrices for providing some universal features of the above transition. For the simplest case $p = 2$, he discussed the large deviation function for the minimal energy extracted via a variant of the replica formalism. He also discussed how random matrix methods can be used to get insights into topology of random real algebraic varieties. The talk was based on the paper arXiv:1307.2379 as well as on joint works with Pierre Le Doussal.

David Gamarnik (Massachusetts Institute of Technology) talked about his joint work with Maghu Sudan related to the problem of constructing algorithms for solving randomly generated constraint satisfaction problems, such random K-SAT problem, and, more precisely, about the limitations of local algorithms. This is a very interesting work, because they establish a fundamental barrier on the power of local algorithms to solve such problems, despite the conjectures put forward in the past. In particular, they refute a conjecture regarding the power of so-called i.i.d factors to find nearly largest independent sets in random regular graphs. They also showed that a broad class of local algorithms, including the so-called Belief Propagation and Survey Propagation algorithms, cannot find satisfying assignments in the random NAE-K-SAT problem above a certain asymptotic threshold, below which even simple algorithms succeed with high probability. Their negative results are based on the analysis of the geometry of feasible solutions of random constraint satisfaction problems, which was first predicted by physicists heuristically and confirmed by rigorous methods. According to this picture, the solution space exhibits a clustering property whereby the feasible solutions tend to cluster according to the underlying Hamming distance. Their main idea was to show that success of local algorithms would imply violation of such a clustering property.

Véronique Gayrard (Aix-Marseille Université and CNRS) spoke on "Aging in mean-field spin-glasses."

She presented recent results on the activated aging dynamics of mean-field spin glasses (REM, p -spin SK model, GREM-like trap model.) The key result is a very general criterion for the convergence of clock processes in random dynamics in random environments that is applicable in cases when correlations are not negligible: An important ingredient is based on a general criterion for convergence of sums of dependent random variables due to Durrett and Resnick [Ann. Probab. 6 (1978) 829-846]. The power of this criterion is demonstrated by applying it to the case of random hopping time dynamics of the p -spin SK model. It is proved that on a wide range of time scales, the clock process converges to a stable subordinator almost surely with respect to the environment. Also, the time-time correlation function converges to the arcsine law for this subordinator, almost surely. Another application of the general criterion is for the aging behavior of a truncated version of the Random Energy Model evolving under Metropolis dynamics. There, it is proved that the natural time-time correlation function defined through the overlap function converges to an arcsine law distribution function, almost surely in the random environment and in the full range of time scales and

temperatures for which such a result can be expected to hold. This establishes that the dynamics ages in the same way as Bouchaud's REM-like trap model, thus extending the universality class of the latter model.

Giuseppe Genovese spoke on "SK-spherical spin glass approximation for the Hopfield model with Gaussian patterns." By means of a mapping into an appropriate bipartite spin glass, it is possible to decompose the free energy of the Hopfield model with Gaussian patterns into the free energies of a SK model and a spherical one. We will discuss some results in this direction obtained in collaboration with A. Barra, F. Guerra and D. Tantari. In our paper we investigate the high storage regime of a neural network with Gaussian patterns. Through an exact mapping between its partition function and one of a bipartite spin glass (whose parties consist of Ising and Gaussian spins respectively), we give a complete control of the whole annealed region. The strategy explored is based on an interpolation between the bipartite system and two independent spin glasses built respectively by dichotomic and Gaussian spins: Critical line, behavior of the principal thermodynamic observables and their fluctuations as well as overlap fluctuations are obtained and discussed. Then, we move further, extending such an equivalence beyond the critical line, to explore the broken ergodicity phase under the assumption of replica symmetry and we show that the quenched free energy of this (analogical) Hopfield model can be described as a linear combination of the two quenched spin-glass free energies even in the replica symmetric framework.

Cristian Giardinà (University of Modena and Reggio Emilia) spoke on "Central limit theorems for Ising models on random graphs." A classical result of Newman established the central limit theorem for the magnetization over large blocks for the Ising model using the fact that the spins are associated random variables under the Gibbs measure in the ferromagnetic case. For various classes of random graphs with N vertices, we prove that the total spin of the Ising model defined on them satisfies a central limit theorem, provided it is centered by the total magnetization and rescaled by the square root of N . We consider both quenched and annealed measures in the one-phase region. We conjecture that when the vertex degrees do not fluctuate, the variance of the limiting Gaussian law is the same in the two settings. We substantiate this claim with the analysis of some configuration models. Joint work with C. Giberti, R. van der Hofstad, M.L. Prioriello.

Friedrich Götze (University of Bielefeld) gave a talk about "Asymptotic approximations for spectra of random matrices". From the fundamental contribution by E. Wigner [Annals of Math. Vol 67, 325-328] it is known that Hermitian $n \times n$ random matrices with independent entries have a $n \rightarrow \infty$ limit distribution of the eigenvalue statistics given by the so-called semi-circle distribution. If one drops the Hermitian assumptions, the situation becomes much more complicated, and eigenvalue statistics with an elliptic shape appear in the limit. It however turns out, as recently proved by Götze, together with Naumov and Tikhomirov, that products of such matrices in nearly all natural cases have a limit distribution of the density of states which is circular. Friedrich Götze gave an overview on the recent progresses in this field, as well as on very precise new results concerning finite n error bounds.

Aukosh Jagannath (a Ph.D. student of Gérard Ben Arous at New York University) gave a talk about his recent work on the approximate ultrametricity of Gibbs measures in spin glass models. The main idea of his work was the following. In a number of spin glass models, the famous Parisi ultrametric ansatz was proved rigorously in the work of Panchenko (one of the organizers of the conference). This is a statement about the geometry of the Gibbs measure in the thermodynamic limit when the size of the system goes to infinity. Jagannath's work was to show that this picture also holds in an approximate sense for systems of finite but large size. His idea was to use the information about the geometry of the sample from the Gibbs measure to carefully reconstruct the nested sequence of clusters in a controlled way which showed that they satisfy the Parisi ansatz approximately.

Kay Kirkpatrick talked about "Non-normal asymptotics of the mean-field Heisenberg model". This work is about spin models of magnets and superconductors which are of XY -type, i.e. spin states given by the circle, or of Heisenberg type where the spin state is a d -dimension sphere, $d \geq 2$. These models have interesting phase transitions. Together with Elizabeth Meckes, there are recent results on a non-Gaussian scaling limit of the mean-field models exactly at critical temperature. This is an extension of work done by Ellis-Newman [J. of Stat. Physics, Vol. 19, 149–161 (1978)] for the Curie-Weiss model.

Emanuele Mingione (a Ph.D. student of Pierluigi Contucci at the University of Bologna) talked about his work (with collaborators) on a multi-species version of the Sherrington-Kirkpatrick model. They introduced

a new model where the spins of the system are divided into several groups (called species) and interactions between spins depend on the species they come from. Under a certain assumption (which essentially means that the interactions within species are stronger than inter-species interactions) they suggested an analogue of the Parisi formula for the free energy and showed, using Guerra's interpolation method, that it actually gives an upper bound on the free energy. The formula they suggested was very interesting because it suggested that, when comparing likely realizations of such a system, the inner-species similarity between these two populations will be synchronized in all the species. This was later proved in the work of Panchenko, completing the proof of the free energy formula in this model.

Charles Newman (New York University) spoke on "Statistical mechanics and the Riemann hypothesis." In this talk Newman reviewed a number of old results concerning certain statistical mechanics models and their possible connections to the Riemann Hypothesis. A standard reformulation of the Riemann Hypothesis (RH) is: The (two-sided) Laplace transform of a certain specific function Ψ on the real line is automatically an entire function on the complex plane; the RH is equivalent to this transform having only pure imaginary zeros. Also Ψ is a positive integrable function, so (modulo a multiplicative constant C) is a probability density function. A (finite) Ising model is a specific type of probability measure \mathbb{P} on the points $S = (S_1, \dots, S_N)$ with each $S_j = \pm 1$. The Lee-Yang theorem implies that for non-negative a_1, \dots, a_N , the Laplace transform of the induced probability distribution of $a_1 S_1 + \dots + a_N S_N$ has only pure imaginary zeros. The big question here is whether it's possible to find a sequence of Ising models so that the limit as N tends to ∞ of such distributions has density exactly $C\Psi$? He discussed some hints as to how one might try to do this.

Dmitry Panchenko (professor at the University of Toronto, and one of the organizers) talked about his recent work on diluted spin glass models. The main motivation for his work was the conjectured Mézard-Parisi formula for the free energy in the diluted spin glass models, such as the random K -sat model, or diluted p -spin models. One possible approach to proving this formula is to show that the structure of Gibbs measure in these models is described by the Mézard-Parisi ansatz, and Panchenko proved recently that this ansatz holds in all cases when the distance between two spin configurations drawn from the Gibbs measure takes finitely many values in the thermodynamic limit (the called finite-replica symmetry breaking case). The general case remains an open problem.

Giorgio Parisi (professor at the University of Rome La Sapienza) talked about his work (with collaborators) on fractal free energy landscapes in structural glasses. Realistic models for glasses seem intractable, so he described a model which is a simplification, but its solution and behaviour are still very complicated. Using theory and numerical simulation, they showed that the landscape is much rougher than is classically assumed, and they determined analytically critical exponents for the basin width, the weak force distribution and the spatial spread of quasi-contacts near jamming. Their value was found to be compatible with numerical observations.

Nicholas Read (professor at Yale University) talked about the the famous Edwards-Anderson short range spin glass model and, in particular, about the metastate interpretation of replica symmetry breaking in these models. The main idea of his talk was to show how the replica symmetry breaking scheme of Parisi can be described in terms of a non-trivial metastate picture proposed by Newman and Stein.

Jason Schweinsberg talked about "The genealogy of a population undergoing selection". Consider a population of constant size N in which each individual dies at rate 1 and during lifetime each experiences mutations at rate r . Mutations are assumed to be beneficial, so that the fitness of an individual increases (linearly) in the number of mutations. When an individual dies, a replacement is chosen at random from the population with probability proportional to the individual's fitness. It is shown that the genealogy of this population is given by the Bolthausen-Sznitman coalescent, confirming non-rigorous predictions of Neher and Hallatschek and Desai, Walczak, and Fisher. The Bolthausen-Sznitman coalescent arises in connection with spin glass models as an alternative description of Ruelle's probability cascades, and was recently also shown to describe the genealogy of branching Brownian motion with absorption.

Allan Sly (professor at the University of California, Berkeley) talked about his joint work with Jian Ding and Nike Sun (another participant) about the maximum independent sets in random d -regular graphs. In a very impressive work, they found explicit formulas for the leading order and logarithmic correction terms in the case when d is large enough and showed that the remaining fluctuations are of order one. This confirmed the so called one-step replica symmetry breaking predictions of the physicists.

Shannon Starr (University of Alabama, Birmingham) spoke "About eigenvectors for random matrices." In analogy with the random overlap structure for spin glasses, it seems useful to know the distribution of the eigenvector inner-products for models of random matrices, especially non-Hermitian models such as Ginibre's ensemble. Starr described an attempt to determine these, and to find reported values in the literature.

Daniel Stein (New York University) spoke on "Predictability in non equilibrium discrete spin dynamics." In this talk, he considered a dynamical many-body system with a random initial state subsequently evolving through stochastic dynamics. The question addressed is what is the relative importance of the initial state (nature) vs. the realization of the stochastic dynamics (nurture) in predicting the final state? We discuss this question and present both old and new results for low-dimensional homogeneous and disordered Ising spin systems.

Nike Sun (a Ph.D. student of Amir Dembo at Stanford University) described her work (with collaborators) on the Potts and independent set models on d -regular graphs. They proved that the replica symmetric (Bethe) prediction applies for all parameter values in the ferromagnetic Potts model on typical d -regular graphs, and the independent set model on typical bipartite d -regular graphs. Interestingly, these results are in contrast with the anti-ferromagnetic Potts model and the independent set model at high fugacity on non-bipartite graphs, where the replica symmetric prediction is known to fail.

Daniele Tantari (University of Rome La Sapienza) spoke on "Parallel retrieval in multitasking associative networks." Recently multitasking associative networks have been introduced in the statistical mechanics community to mimic the parallel processing capabilities of the immune system, the latter being thought of as a network of interacting B and T lymphocytes. In this talk, after a streamlined introduction to fundamentals of theoretical immunology, Tantari introduced these models (mainly bipartite spin glasses embedded on a finite connectivity topology) and spoke about their phase diagrams, and applied replica tricks and/or cavity techniques, in order to distinguish between a ferromagnetic region -where the system performs extensive parallel retrieval- and a spin glass one, where the amount of interferences among the patterns increases: Tantari gave a proof that this clonal cross-talk diminishes the multitasking features of these networks. Further, she showed that a second order phase transition occurs when varying the level of load (number of memorized patterns), network's dilution and fast noise: from low to high load, from fully connected to finite connectivity regimes.

Olivier Zindy (Université Pierre et Marie Curie, Paris) talked about "Poisson-Dirichlet statistics for the extremes of log-correlated Gaussian fields". Gaussian fields with logarithmically decaying correlations, such as branching Brownian motion, the two-dimensional Gaussian free field, the occupation field of two-dimensional random walks, and many others, are conjectured to form a new universality class of extreme value statistics (notably in works by Carpentier and Ledoussal, and Fyodorov and Bouchaud). This class is the borderline case between the class of i.i.d. random variables, and models where correlations start to affect the statistics, the latter being the case for the low-temperature SK-model. In his talk, Zindy describes a general approach based on rigorous works in spin glass theory to describe features of the Gibbs measure of these Gaussian fields. It is shown that at low temperature, the normalized covariance of two points sampled from the Gibbs measure is either 0 or 1. This is used to prove that the joint distribution of the Gibbs weights converges in a suitable sense to that of a Poisson-Dirichlet variable. (Joint work with Louis-Pierre Arguin).