

ALEXANDROV GEOMETRY

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1 Overview of the Field

Alexandrov Geometry has been much developed in the last 20 years. Perelman’s solution of the Poincaré conjecture has drawn wide attention to the theory of spaces with lower curvature bounds. Spaces with upper curvature bounds have turned out to interact extensively with other branches of mathematics such as geometric analysis and geometric group theory, as well as having applications to such varied fields as dynamical systems in mechanics, and robotics.

2 Overview of the Project

The purpose of our research stay was to work on our book “Alexandrov Geometry”. Our book is a comprehensive text and reference work on the fields of curvature bounded below and curvature bounded above. Although these two fields developed quite independently, they have many similar guiding intuitions and technical tools. Our approach is novel in its attention to the interrelatedness of the two fields, and its emphasis on the way each illuminates the other.

We should emphasize that this is truly a research project. The book will contain a lot of new material. In addition, almost every theorem will appear in an improved form and in the correct generality.

In addition to all the basic material in both fields, the book includes all the important advanced material on spaces of curvature bounded below. This material is unavailable in any book, and not all of it is in the literature. For spaces of curvature bounded above, we are emphasizing topics and proofs inspired by considering the two contexts simultaneously, and again include material not in the literature.

3 Progress Made

At present, our draft is about 368 pages; more than half available online. The final version should be about 500 pages. As a result of our workshop, three new chapters now available online there were also number of improvements in all the parts of the book.

We were working on the book for more than half a year without face-to-face contact. During this time, a number of issues accumulated. We were able to get through the complete list of them and make very substantial progress in just one week at Banff.

Since the last meeting we were mostly working on the chapter on volume (currently chapter 30). This was one of the hardest chapters to write. It is used a lot everywhere in the book and it took us quite some

time to make a comprehensive presentation suitable for all chapters in the book. The main part of the chapter is written and checked carefully by all of us.

An outline and timetable were drawn up for remaining topics in curvature bounded above.

There were countless corrections and improvements made, even in chapters written a while ago. A number of proofs became more transparent. These findings improve the book's coherence and elegance.