

## Focused Research Group Report

### Hyperplane Arrangements, Wonderful Compactifications, and Tropicalization

Graham Denham, June Huh, and Alexander Suciú

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One of the main goals for this meeting was to attack long-standing problems about the topology of hyperplane arrangements and their Milnor fibers using methods of toric and tropical geometry. We assembled some experts in tropical geometry and combinatorial algebraic geometry. We started with some interactive discussions about recent progress in the area, then moved on to some intensive group work which, we hope, began some new collaborations as well as furthering ones which were already in progress.

Highlights from the week included the following. Alex Fink outlined the techniques from his joint paper with David Speyer, which interprets the Tutte polynomial of a matroid in terms of a class in  $K$ -theory. June Huh explained in detail some ideas from his recently completed thesis involving intersection-theoretic matroid constructions. This included carefully elucidating the key part of the argument in his paper with Eric Katz, which seemed to show the way to strengthening the result. Katz proposed that these two projects could be understood within the same framework, giving a lead for further developments in the area.

Alex Suciú explained his recent work with Stefan Papadima on the Milnor fibration of a hyperplane arrangement, and the group began a collective effort (as outlined in the proposal) to find a useful compactification of the Milnor fiber using ideas from toric geometry. As we expected, this turned out to be a relatively ambitious project. June Huh suggested using Mumford's notion of a semistable degeneration for this approach, and we attempted to do so explicitly with some small examples, restricting ourselves to a degeneration with toric varieties as fibers.

Michael Falk described work in progress with Eva Feichtner involving the study of resonance varieties for arrangements using tropicalization. The resonance varieties are also relevant to questions about the cohomology of the Milnor fiber, and so we explored as a group possible interplays between compactifications, tropicalization, and the Milnor fibration.

Spending a concentrated and highly intense week in a relatively small group allowed for in-depth and continuing conversations, in particular with new acquaintances. These opportunities (difficult to find at larger meetings) were enhanced by the diversity of backgrounds of the participants. There was general agreement that the focused research group created an effective and stimulating research atmosphere. The work initiated at BIRS is continuing now in several smaller research groups. The intense interactions within the research group gave rise to new projects, which should start bearing fruit in the not too distant future.

#### LIST OF PARTICIPANTS

- (1) Graham Denham, University of Western Ontario, Canada.
- (2) Michael Falk, Northern Arizona University, USA.

- (3) Eva Maria Feichtner, University of Bremen, Germany.
- (4) Alex Fink, Queen Mary University, UK.
- (5) June Huh, University of Michigan, USA.
- (6) Eric Katz, University of Waterloo, Canada.
- (7) Hal Schenck, University of Illinois at Urbana-Champaign, USA.
- (8) Alexander Suci, Northeastern University, USA.

## ORGANIZERS

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