

# Operator algebras and dynamical systems from number theory

Nov 24 - Nov 29, 2013

## MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

## MEETING ROOMS

All lectures will be held in room 201 of the TransCanada Pipelines Pavilion (TCPL) LCD projector and blackboards are available for presentations. Ceiling-mounted video cameras are installed in the lecture room.

## SCHEDULE

### Sunday

**16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

**17:30–19:30** Buffet Dinner, Sally Borden Building

**20:00** Informal gathering in 2nd floor lounge, Corbett Hall (if desired)

Beverages and a small assortment of snacks are available on a cash honor system.

### Monday

**7:00–8:45** Breakfast

**8:45–9:00** Introduction and Welcome by BIRS Station Manager, TCPL 201

**9:00–10:00** Joachim Cuntz: *Periodic cyclic homology in positive characteristic / An alternative to Witt vectors*

**10:00–10:30** Coffee Break, TCPL

**10:30–11:30** Siegfried Echterhoff: *K-theory of crossed products by group actions on totally disconnected spaces and semi-group algebras*

**11:30–13:00** Lunch

**13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

**14:00–15:00** Astrid an Huef: *KMS states on the  $C^*$ -algebras associated to finite graphs*

**15:00–15:30** Coffee Break, TCPL

**15:30–17:30** TBA Lecture/Informal discussion, TCPL

**17:30–19:30** Dinner

**19:30–** Informal discussion, TCPL / Corbett Hall lounge

### Tuesday

**7:00–9:00** Breakfast

**9:00–10:00** Gunther Cornelissen: *Noncommutative dynamical systems and point counting*

**10:00–10:30** Coffee Break, TCPL

**10:30–11:30** Elias Katsoulis: *Piecewise conjugacy as an isomorphism invariant for operator algebras*

**11:30** Group Photo, on the steps outside TCPL (meet in the foyer).

**11:40–13:30** Lunch

**14:00–15:00** Aidan Sims: *KMS states on  $C^*$ -algebras associated to  $k$ -graphs*

**15:00–15:30** Coffee Break, TCPL

**15:30–16:00** Joachim Cuntz: *An alternative to Witt vectors*

**16:00–17:30** Informal discussion

**17:30–19:30** Dinner

**19:30–** Informal discussion, TCPL / Corbett Hall lounge

## Wednesday

- 7:00–9:00** Breakfast  
**9:00–10:00** Iain Raeburn: *Operator-algebraic dynamical systems associated to self-similar groups.*  
**10:00–10:30** Coffee Break, TCPL  
**10:30–11:30** Xin Li:  *$C^*$ -algebras, monoids, and dynamical systems*  
**11:30–13:30** Lunch  
Free Afternoon  
**17:30–19:30** Dinner  
**19:30–** Informal discussion, TCPL / Corbett Hall lounge

## Thursday

- 7:00–9:00** Breakfast  
**9:00–10:00** Nadia Larsen:  *$C^*$ -algebras associated to graphs, path spaces and equilibrium states*  
**10:00–10:30** Coffee Break, TCPL  
**10:30–11:30** Mak Trifkovic: *Bost-Connes systems and induction*  
**11:30–13:30** Lunch  
**14:00–15:00** Masoud Khalkhali: *A noncommutative view of zeta regularized determinants and analytic torsion*  
**15:00–15:30** Coffee Break, TCPL  
**15:30–16:30** Nicolai Stammeier:  *$C^*$ -algebras associated to irreversible algebraic dynamics*  
**17:30–19:30** Dinner  
**19:30–** Informal discussion, TCPL / Corbett Hall lounge

## Friday

- 7:00–9:00** Breakfast  
**9:00–10:00** Nathan Brownlowe: *Zappa-Szep products of semigroups and their  $C^*$ -algebras*  
**10:00–10:30** Coffee Break, TCPL  
**11:30–13:30** Lunch

**Checkout by  
12 noon.**

\*\* 5-day workshop participants are welcome to use BIRS facilities (BIRS 2nd floor lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

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## ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Astrid an Huef** (University of Otago)

Title: *KMS states on the  $C^*$ -algebras associated to finite graphs.*

Abstract: In 1984, Enomoto, Fujii and Watatani proved that if  $E$  is a strongly connected finite directed graph which is not a simple cycle, then the associated graph  $C^*$ -algebra admits a unique KMS state for the gauge action. I will give an introduction to KMS states on the  $C^*$ -algebras associated to finite graphs by proving the Enomoto-Fujii-Watatani Theorem using elementary methods. The main idea is the philosophy of Exel-Laca and Laca-Neshveyev that the Toeplitz algebra of the graph has a much richer supply of KMS states. This is joint work with Marcelo Laca, Iain Raeburn and Aidan Sims.

Speaker: **Nathan Brownlowe** (University of Wollongong)

Title: *Zappa-Szep products of semigroups and their  $C^*$ -algebras.*

Abstract: We examine a class of Zappa-Szep products of semigroups which generalise both the self-similar actions of Nekrashevych and the quasi-lattice ordered groups of Nica. We consider the  $C^*$ -algebras of these products in the sense of Li, and we give an alternative presentation involving isometric representations of the semigroups. We discuss examples including self-similar actions of groups and semigroups, the semigroups  $\mathbb{N} \rtimes \mathbb{N}^\times$  and  $\mathbb{Z} \rtimes \mathbb{Z}^\times$ , the Baumslag-Solitar groups, and products of self-similar groups. We also define a quotient  $C^*$ -algebra we call the boundary quotient. This is joint work with Jacqui Ramagge, David Robertson and Michael Whittaker.

Speaker: **Gunther Cornelissen** (Utrecht University)

Title: *Noncommutative dynamical systems and point counting*

Abstract: Counting the points of a curve over finite fields doesn't determine the curve up to isomorphism; this is analogous to the famous fact that you cannot "hear the shape of a drum". I will show how to remedy this by counting points in a weighted way (namely, using characters). This weighted point counting arises naturally from considering the KMS-states of a noncommutative dynamical system associated to the curve.

Speaker: **Joachim Cuntz** (University of Münster)

Title: *Periodic cyclic homology in positive characteristic / An alternative to Witt vectors*

Abstract: The ring of Witt vectors associated to a ring  $R$  is a classical tool in algebra. We give a construction which is much simpler than the usual one. This is also of interest in view of a possible version of periodic cyclic homology for algebras over finite fields. This is joint work with Christopher Deninger.

Speaker: **Siegfried Echterhoff** (University of Münster)

Title: *K-theory of crossed products by group actions on totally disconnected spaces and semi-group algebras*

Abstract: In this lecture we give a report on joint work with Joachim Cuntz and Xin Li on the computation of the K-theory for crossed products by certain actions of groups on totally disconnected spaces. We apply the results to the computation of the K-theory for certain semi-group  $C^*$ -algebras. In particular, we obtain explicit computations for the  $ax + b$ -semigroups  $R \rtimes R^*$ , where  $R$  is the ring of integers in a number field.

Speaker: **Elias Katsoulis** (East Carolina University)

Title: *Piecewise conjugacy as an isomorphism invariant for operator algebras*

Abstract: The concept of piecewise conjugacy for classical multivariable dynamical systems arose from the work of Davidson and Katsoulis on non-selfadjoint operator algebras and was used recently by Cornelissen

and Marcolli. In this talk we will review several results from the original work of Davidson and Katsoulis on piecewise conjugacy for classical dynamical systems. We will also present a recent generalization of piecewise conjugacy for multivariable  $C^*$ -dynamical systems, due to Kakariadis and Katsoulis. We will show that this "generalized" piecewise conjugacy continues to be an isomorphism invariant for certain operator algebras associated with such systems. We will also discuss the completeness of piecewise conjugacy as an isomorphism invariant.

Speaker: **Masoud Khalkhali** (University of Western Ontario)

Title: *A noncommutative view of zeta regularized determinants and analytic torsion*

Abstract: I shall first recall the classical theory of Ray-Singer analytic torsion, and conformal anomaly, for families of elliptic operators. I will mostly focus on families of elliptic operators on Riemann surfaces. The methods used here are based on ideas of spectral geometry and hence stand a chance of extension to a noncommutative setting. The extensions, when possible, are however quite nontrivial and involve many new elements and difficult computations. I shall then look at some known examples of noncommutative Riemann surfaces, the noncommutative elliptic curves equipped with curved metrics, and sketch the progress made in the last few years in understanding their conformal and spectral geometry. Scalar curvature can be defined by study of special values of spectral zeta functions. In particular I shall explain a formula for scalar curvature obtained in my joint work with F. Fathizadeh (and independently by Connes and Moscovici). This formula plays an important role for further study of noncommutative spectral geometry of noncommutative tori.

Speaker: **Nadia Larsen** (University of Oslo)

Title:  *$C^*$ -algebras associated to graphs, path spaces and equilibrium states*

Abstract: Given a directed graph  $E$ , the graph algebra is the universal  $C^*$ -algebra for a generating family of partial isometries and projections satisfying the Cuntz-Krieger relations. There are other  $C^*$ -algebras associated to  $E$ , such as the Toeplitz graph algebra. We view graph  $C^*$ -algebras as crossed products where the free group on the edge set acts by partial homeomorphisms on a space of boundary paths of the graph. For a function on the edge set that induces a time evolution on a graph  $C^*$ -algebra, equilibrium (or KMS) states therefore can be characterised using a general theorem of Exel and Laca. Guided by their work on KMS states for Toeplitz-Cuntz-Krieger type algebras associated to infinite matrices, we describe in case of graph  $C^*$ -algebras the convex sets of KMS states of finite type and of KMS states of infinite type whose associated measures are supported on recurrent infinite paths. This is joint work with Toke M. Carlsen (NTNU, Norway).

Speaker: **Xin Li** (Queen Mary University)

Title:  *$C^*$ -algebras, monoids, and dynamical systems*

Abstract: This talk is about semigroup  $C^*$ -algebras on the one hand, and the interplay between  $C^*$ -algebras and dynamical systems on the other hand. I will highlight the connections between these topics and the themes of the workshop.

Speaker: **Iain Raeburn** (University of Otago)

Title: *Operator-algebraic dynamical systems associated to self-similar groups.*

Abstract: We discuss the notion of a self-similar group action, illustrating with examples from dynamics. Nekrashevych has shown that each such action has a Cuntz-Pimsner algebra, and hence it has a Toeplitz algebra too. Both algebras carry natural dynamics (that is, actions of the real line), and hence it makes sense to study the KMS states for these dynamics. We will discuss joint work on this topic carried out with Marcelo Laca, Jacqui Ramagge and Mike Whittaker.

Speaker: **Aidan Sims** (University of Wollongong)

Title: *KMS states on  $C^*$ -algebras associated to  $k$ -graphs*

Abstract: The  $k$ -graphs introduced by Kumjian and Pask in 2000 are  $k$ -dimensional analogues of directed

graphs. The  $C^*$ -algebra of a  $k$ -graph is generated by copies of the Cuntz-Krieger algebras of its coordinate graphs subject to commutation relations encoded by its path structure; the Toeplitz algebra of the  $k$ -graph is assembled from the Toeplitz algebras of the coordinate graphs in a similar fashion. I will describe the KMS states for 1-parameter subgroups of the gauge action on the Toeplitz algebra of a finite  $k$ -graph, including an analogue of Enomoto, Fujii and Watatani's theorem that the Cuntz-Krieger algebra of an irreducible directed graph admits a unique KMS state.

Speaker: **Nicolai Stammeier** (University of Münster)

Title:  *$C^*$ -algebras associated to irreversible algebraic dynamics*

Abstract: An irreversible algebraic dynamics is a triple  $(G, P, \theta)$  consisting of an infinite, countable, discrete, amenable group  $G$ , a countably generated, free abelian monoid  $P$  together with an action  $\theta$  of  $P$  on  $G$  by injective group endomorphisms satisfying certain regularity assumptions. This dynamical system is studied in terms of the semidirect product  $G \rtimes_{\theta} P$ , which semigroup turns out to satisfy the left-Ore condition. The structure of the family of constructible right ideals of  $G \rtimes_{\theta} P$  will enable us to deduce results about the full semigroup  $C^*$ -algebra  $C^*(G \rtimes_{\theta} P)$  and its canonical quotient  $\mathcal{O}(G \rtimes_{\theta} P)$ . In this talk, we will mainly focus on  $\mathcal{O}(G \rtimes_{\theta} P)$ . The spectrum of its diagonal will be identified as a completion of  $G$  with respect to  $(P, \theta)$ . The core of  $\mathcal{O}(G \rtimes_{\theta} P)$  is shown to be simple and in the case of finite cokernels, the core is a generalized Bunce-Deddens algebra in the sense of Orfanos. This implies, that it has many regularity properties. Using these results, we will indicate how to prove that  $\mathcal{O}(G \rtimes_{\theta} P)$  is a unital UCT Kirchberg algebra.

Speaker: **Mak Trifkovic** (University of Victoria)

Title: *Bost-Connes systems and induction*

Abstract: We show that the Bost-Connes system of a number field  $K$  can be obtained from a Hecke algebra naturally associated with  $K$  by an induction from the group of totally positive principal ideals to the whole group of ideals. For an extension  $L/K$  of number fields we show that the Bost-Connes system for  $L$  embeds into a system obtained from the Bost-Connes system for  $K$  by an induction from the group of ideals in  $K$  to the group of ideals in  $L$ . This gives a correspondence from the Bost-Connes system for  $K$  to that for  $L$ , and thus defines a functor from number fields to  $C^*$ -dynamical systems with equivariant correspondences as morphisms. This is joint work with Marcelo Laca and Sergey Neshveyev.