A Spatial Control Chart for FWI Monitoring

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Outline

Control Charts

Exponential Control Charts

Application to Fire Weather Index

Spatial Control Charts

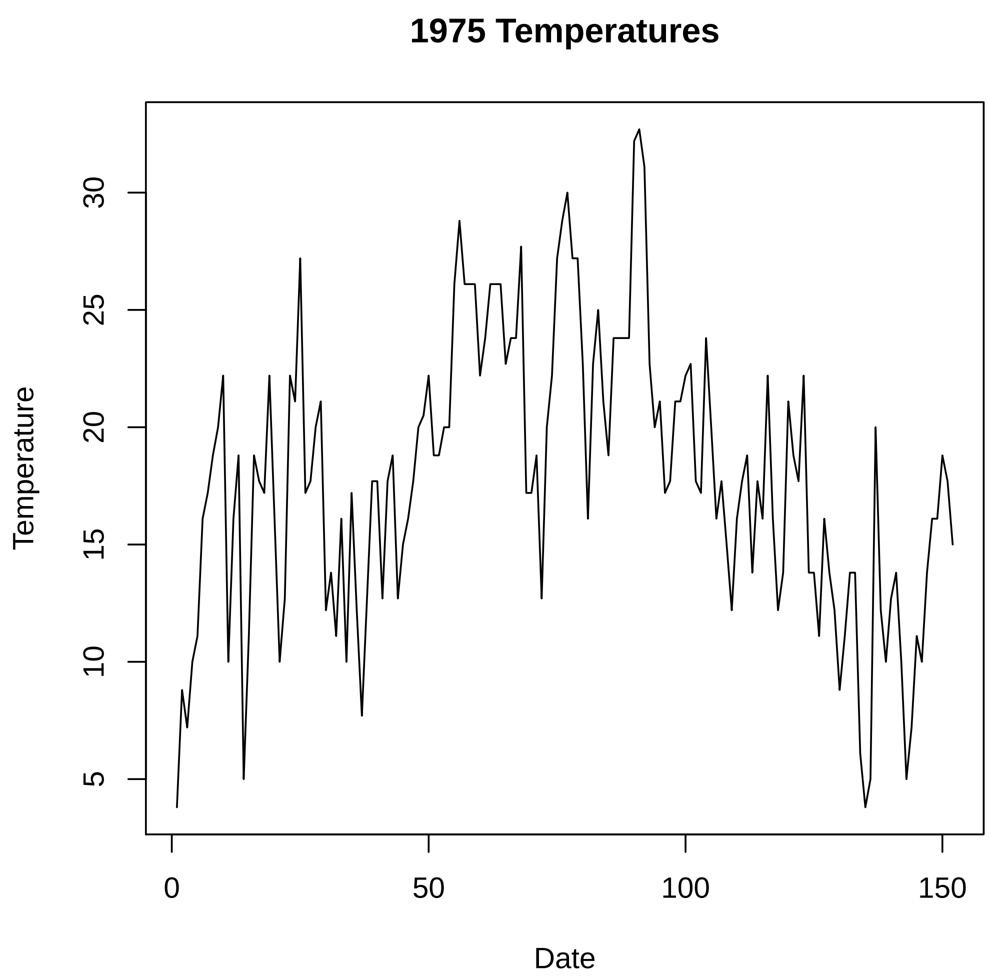
Control Charts

Popular in industry.

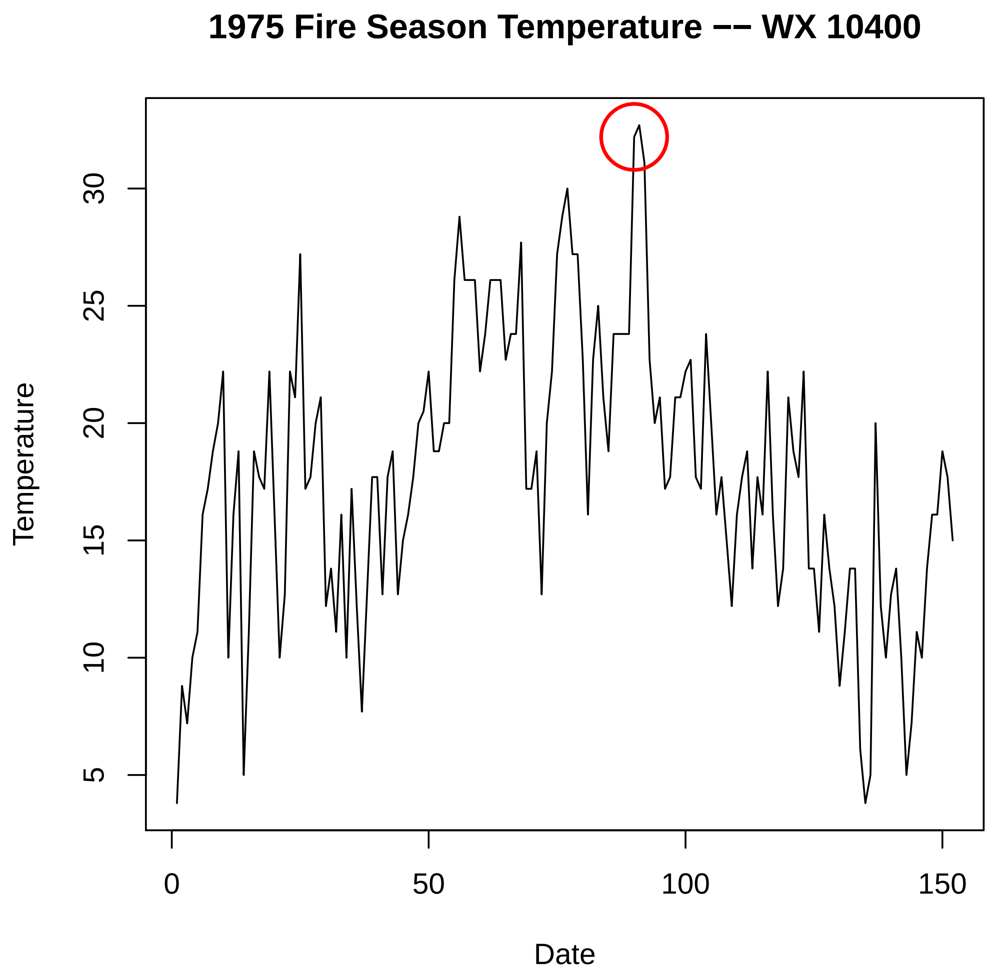
Useful for determining if a given measurement is unusual, or a result of "chance" variation.

Could be used (more?) in forest management practice.

Daily temperatures at weather station 10400 (Ontario):



Control Charts



Is the temperature on July 29 ($32.2°$) unusual?

Individuals Control Charts

Let μ be the mean, and σ be the standard deviation of the measurements.

Assuming that the measurements are independent and normally distributed, they should lie within the following limits with probability .0027.

Upper Control Limit (UCL): $\mu + 3\sigma$

Lower Control Limit (LCL): $\mu - 3\sigma$

Individuals Control Charts

They should lie within the following warning limits with probability .0455.

Upper Warning Limit (UWL): $\mu + 2\sigma$

Lower Warning Limit (LWL): $\mu - 2\sigma$

Individuals Control Charts

Measurements within year are

- **dependent**
- **seasonal (nonconstant mean, standard deviation)**

$\rightsquigarrow$ **control chart is not valid, without adjustment**

Individuals Control Charts

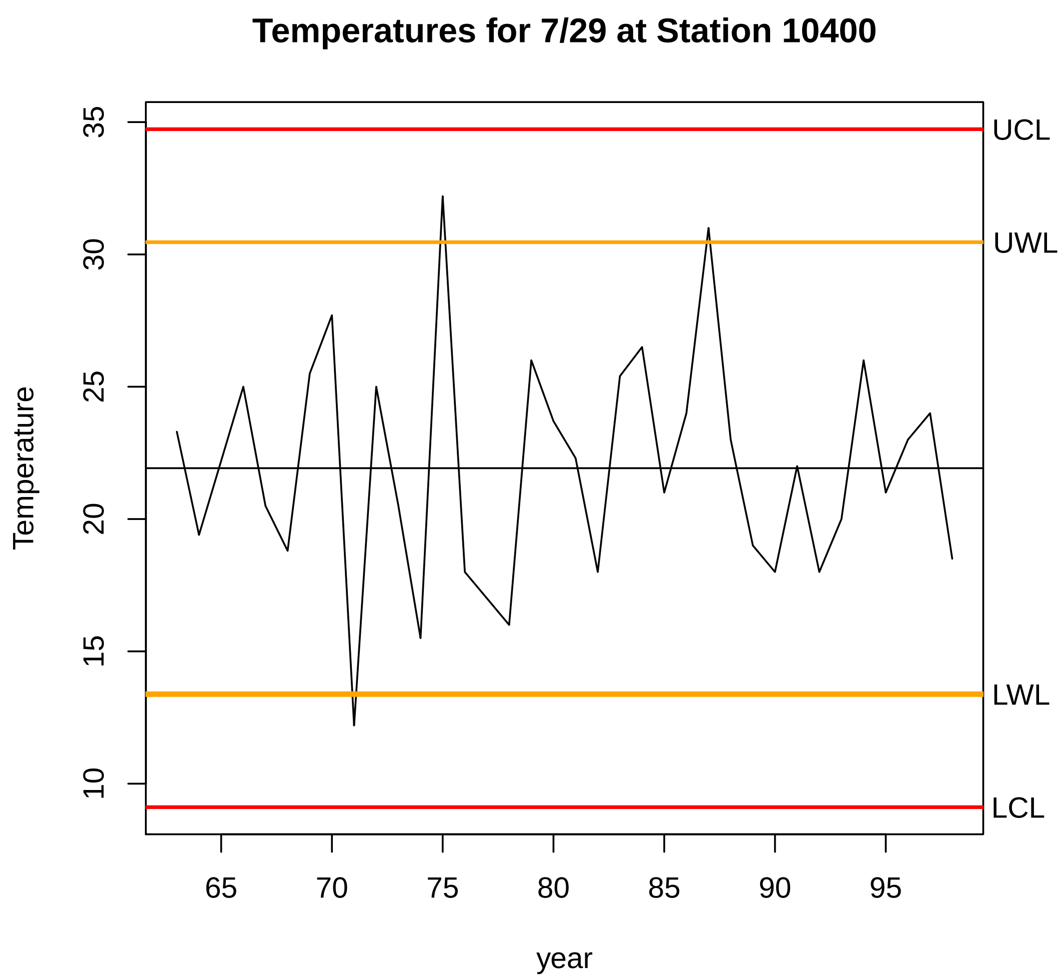
Measurements across years, for fixed day, are approximately

- **independent**

- **nonseasonal (constant mean, standard deviation)**

$\rightsquigarrow$ **control chart is okay, without adjustment**

Individuals Control Charts



The 1975 temperature is unusual but not shocking.

What if the Data aren't Normal?

This is often ignored in practice.

Consequence: for right-skewed data, the lower limits are useless and the upper limits are not very useful.

An Exponential Individuals Chart

If T is exponentially distributed with mean μ, then

$$P(T > t) = e^{-t/\mu}$$

We can use this to find new upper control and warning limits.

e.g. Find UCL so that $e^{-UCL/\mu} = .0027$.

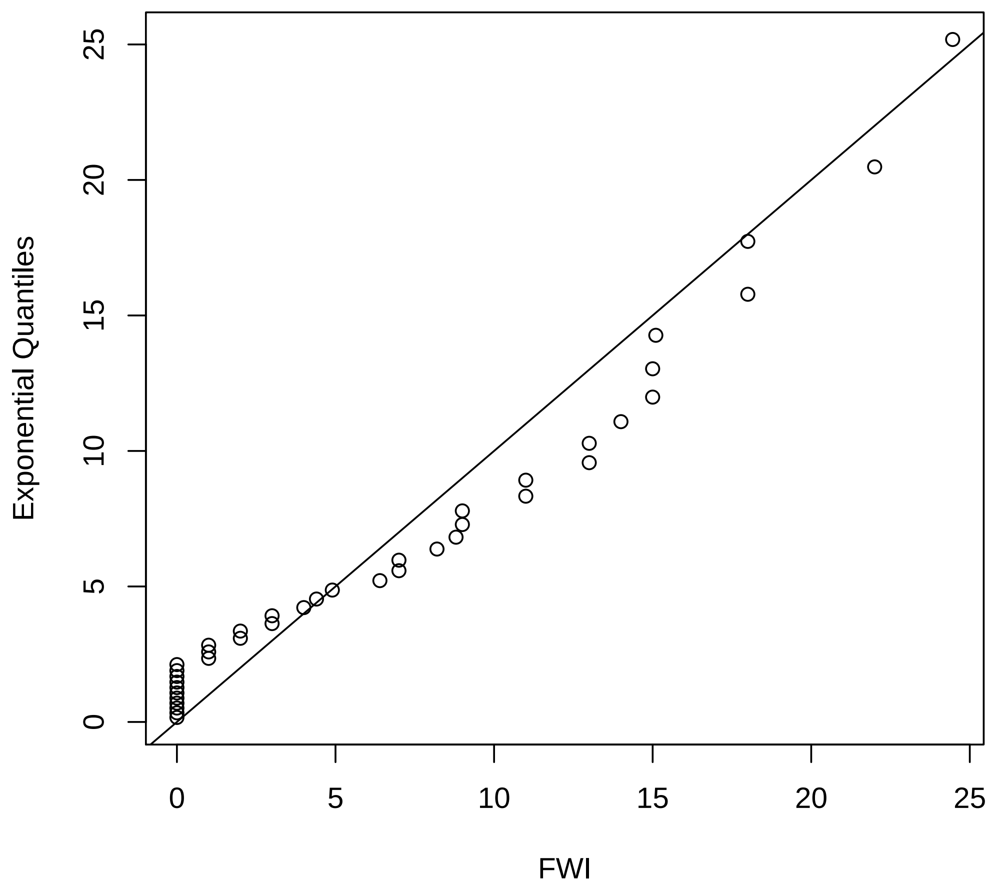
Result:

$$\textbf{UCL} = -\log(.0027)\mu$$

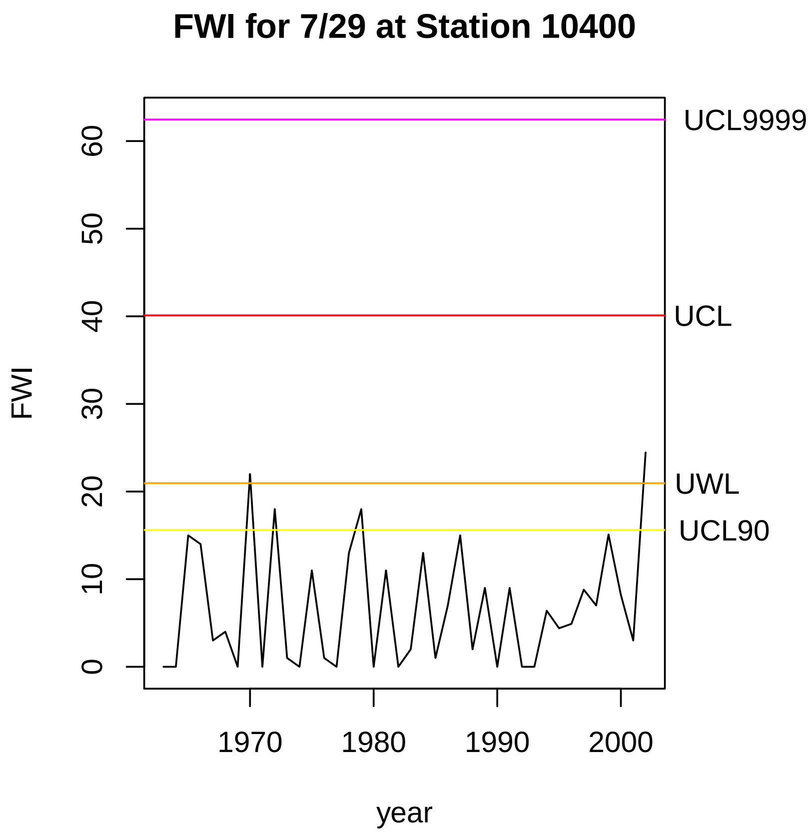
$$\textbf{UWL} = -\log(.0455)\mu$$

FWI are Approximately Exponential

An exponential QQ-plot for July 29 at station 10400:



An Exponential Individuals Chart Applied to FWI Data



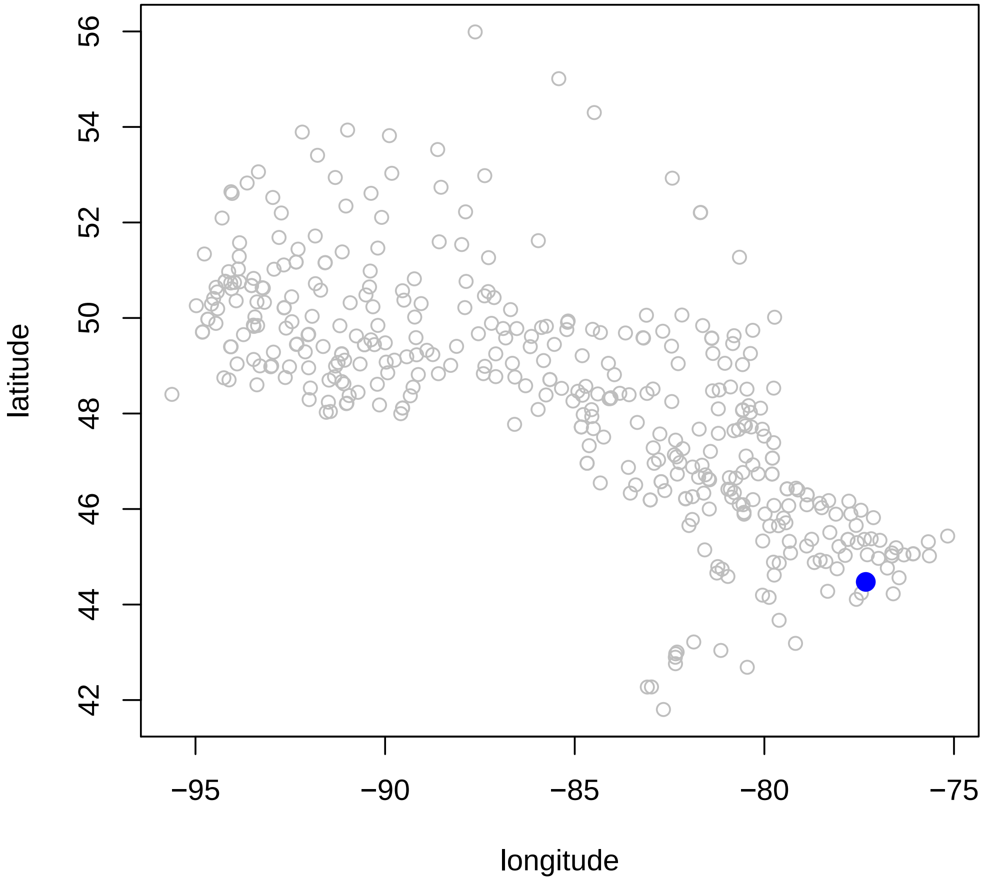
A Spatial Control Chart

Estimate the mean measurement for the given day of year at each weather station.

Calculate estimated control limits and warnings limits.

Plot the locations on a map, using a colour code corresponding to the magnitude of the measurement relative to the limits.

Example: April 1, 1963



Colour Coding

- **Blue: the current FWI is less than the median at the site.**
- **Grey: the current FWI is more than the median at the site.**
- **Green: the current FWI is more than the upper quartile at the site.**
- **Yellow: the current FWI is more than the 90th percentile at the site.**
- **Orange: the current FWI is more than the warning limit.**
- **Red: the current FWI is more than the control limit.**

A grey circle corresponds to a station for which there is no current data.

Example

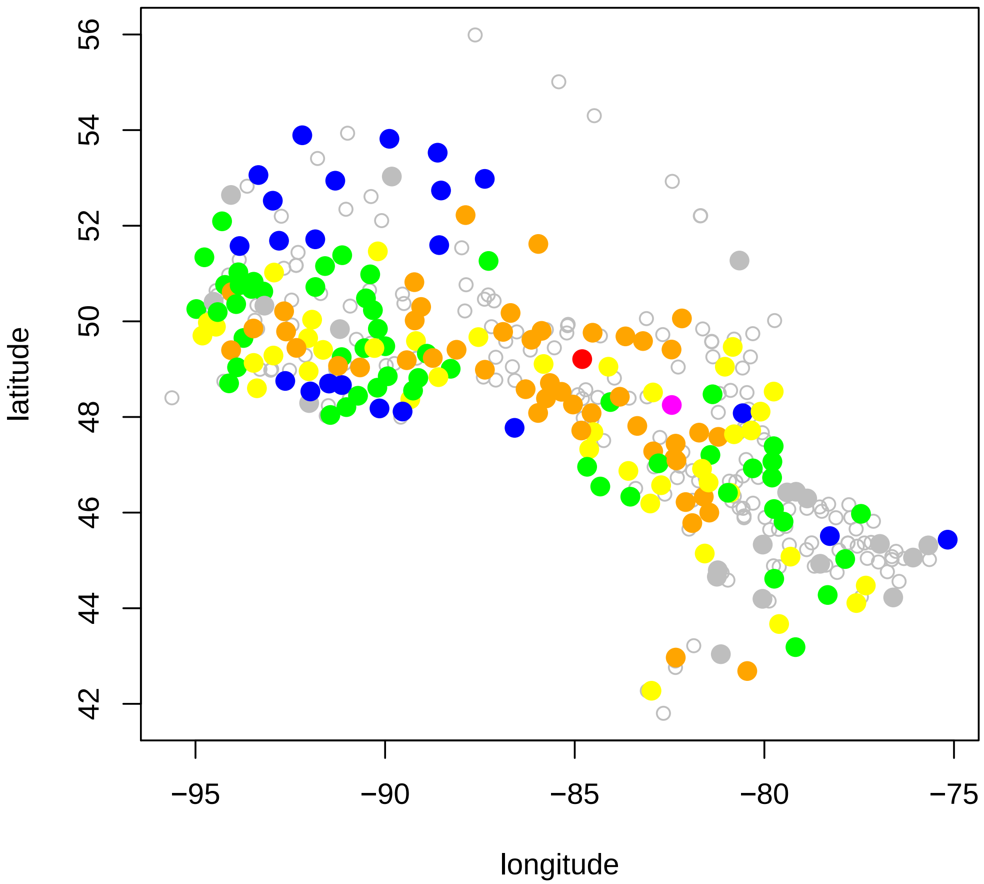
In the following plot, we see that there are a few sites with elevated FWI beyond their normal values.

The majenta coloured point is very interesting: an FWI of 214.

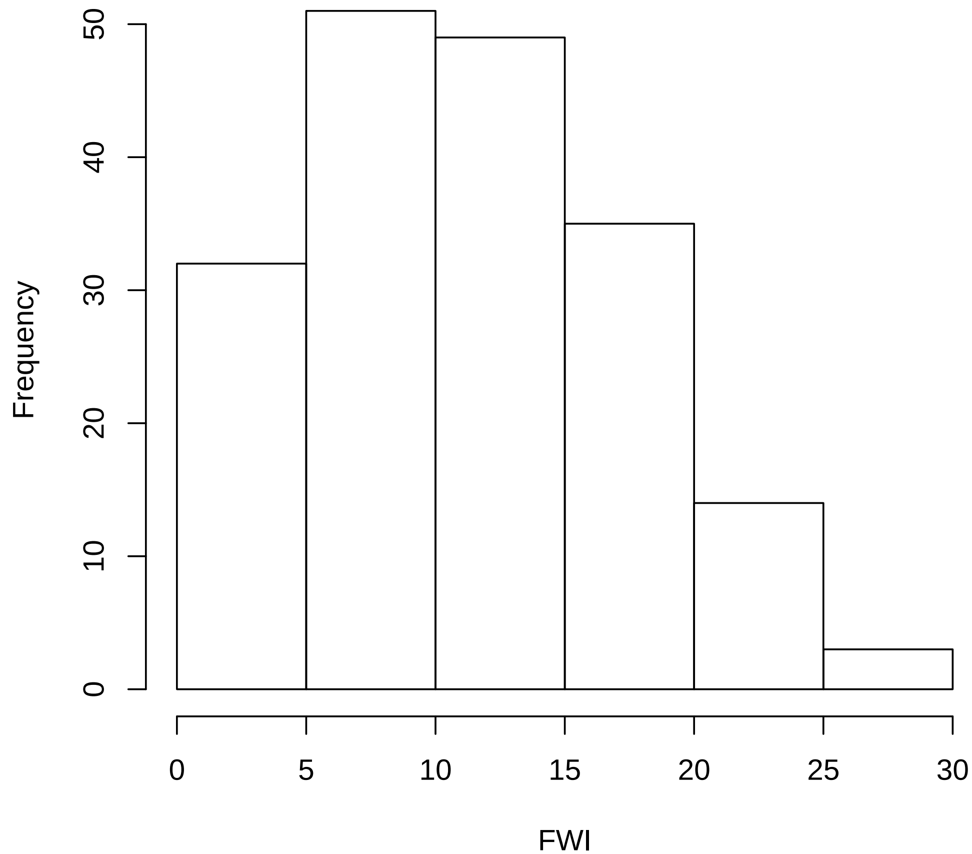
This is likely a mistake in the data set (21.4?), and it is easily caught in this kind of plot.

The blue and grey points indicate areas where it is likely cloudy and wet.

Example: June 4, 1994

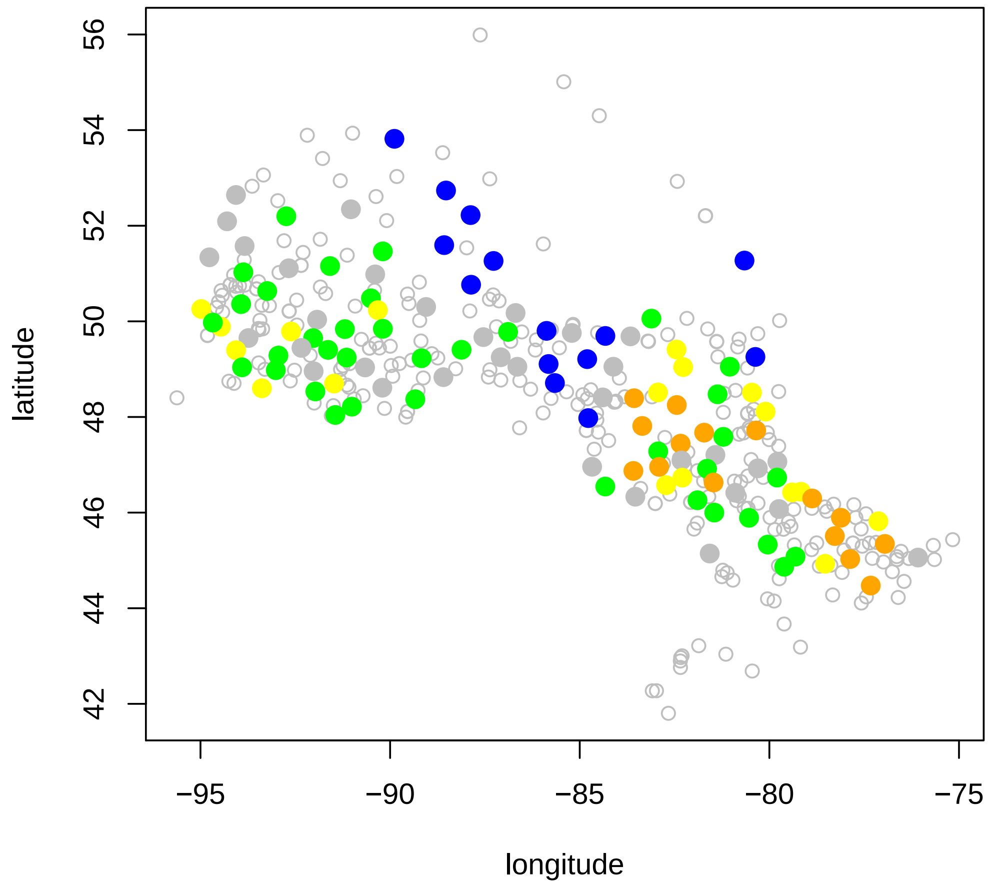


FWI Summary for June 4, 1994

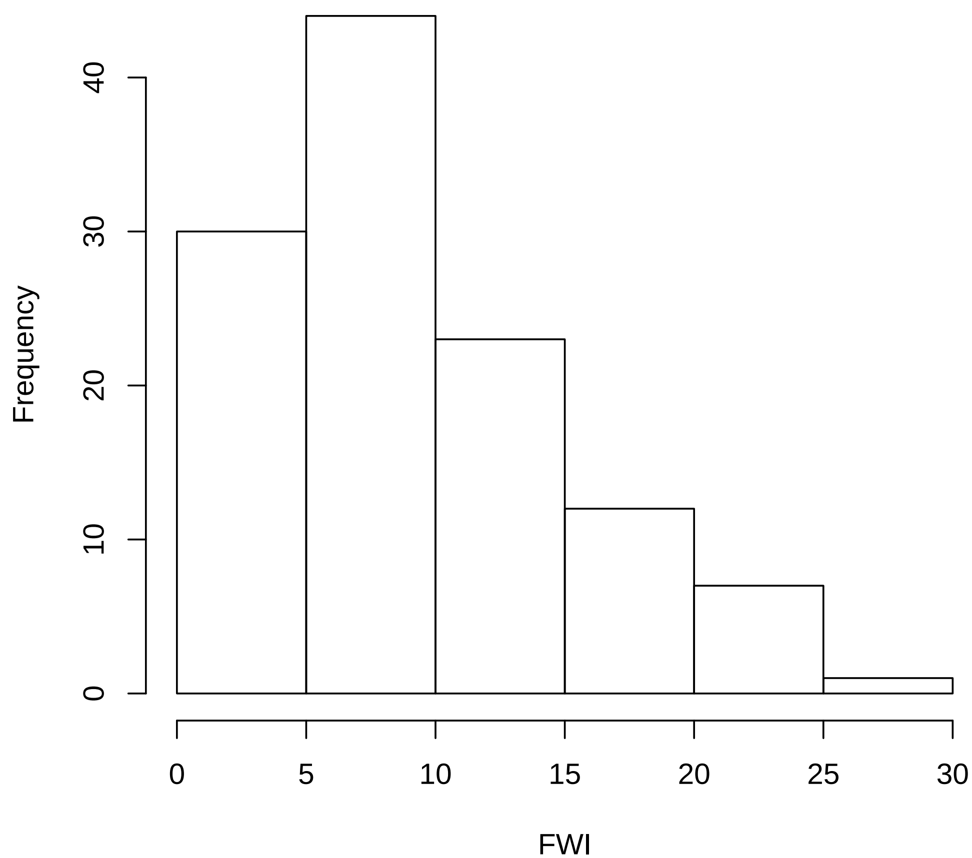


(excluding outlier)

Example: June 4, 1984



FWI Summary for June 4, 1984



Summary and Future Work

The proposed spatial control chart can be used for short term planning.

A plot of the FWI percentiles can be used for longer term planning.

Model the UCL at each station against day of year. A graph of this curve could be used to track daily FWI at that site.

Longer term goal: model spatial dependence.