



# Banff International Research Station

for Mathematical Innovation and Discovery

## Uncovering Transport Barriers in Geophysical Flows 23<sup>rd</sup>-27<sup>th</sup> September 2013

### MEALS

\*Breakfast (Buffet): 7:00 – 9:30 am, Sally Borden Building, Monday – Friday

\*Lunch (Buffet): 11:30 am – 1:30 pm, Sally Borden Building, Monday – Friday

\*Dinner (Buffet): 5:30 – 7:30 pm, Sally Borden Building, Sunday – Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

### MEETING ROOMS

All lectures will be held in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations. Ceiling-mounted video cameras are installed in the main lecture room of 201, TCPL. To enable both half workshops to use the recording facilities during the week, one group will be assigned to Room 201 during the mornings and to Room 202 in the afternoons. The Station Manager will confirm these details with you prior to your workshop. All sessions will be in TCPL 201.

### SCHEDULE

#### Sunday

- 16:00 Check-in begins (Front Desk – Professional Development Centre - open 24 hours)  
17:30-19:30 Buffet Dinner  
20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)  
Beverages and small assortment of snacks are available on a cash honor system.

#### Monday

- 7:00 Breakfast  
8:45 Introduction and Welcome by BIRS Station Manager, TCPL  
9:00 Morning session host: Shane Ross  
G. Froyland, “*Transfer operator methods for discovering transport barriers in geophysical flows*”  
9:30 G. Haller, “*Geodesic theory of transport barriers*”  
10:00 Coffee Break, TCPL  
10:30 S. Balasuriya, “*Nonautonomous control of invariant manifolds*”  
11:00 E. Bollt, “*Differential geometry perspective of shape coherence and curvature evolution by finite-time nonhyperbolic splitting*”  
11:30 D. Blazeovski, “*Lagrangian transport barriers in three-dimensional unsteady flows*”  
12:00 Lunch  
13:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall  
14:00 Afternoon session host: Irina Rypina  
C. Rowley, “*Coherent structure identification using flow map composition and spectral interpolation*”  
14:30 J.L. Thiffeault, “*Moving walls accelerate mixing*”  
15:00 Coffee Break, TCPL  
15:30-17:30 Tutorial: G. Froyland, “*A tutorial on numerical transfer operator methods*”  
17:30-19:30 Dinner

## Tuesday

- 7:00-9:00 Breakfast  
9:00 Morning session host: Shawn Shadden  
L. Pratt, “*Chaotic advection in a steady, three-dimensional, Ekman-driven eddy: Part I*”  
9:30 I. Rypina, “*Chaotic advection in a non-steady, three-dimensional, Ekman-driven Eddy: Part II*”  
10:00 Coffee Break, TCPL  
10:30 F.J. Beron-Vera, “*Geodesic analysis of oceanic flows*”  
11:00 P. Lermusiaux, “*Bayesian learning of stochastic dynamical model formulation and Lagrangian coherent structures*”  
11:30 M. Green, “*Practical concerns of using FTLE with experimental data*”  
12:00 Lunch  
13:15 Group photograph (TCPL Foyer)  
13:30 Tutorial: D. Blazeovski, “*LCS Tool: A variational computing platform for Lagrangian coherent structures*”  
15:00 **Free Afternoon**  
17:30 Dinner

## Wednesday

- 7:00 Breakfast  
9:00 Morning session host: Melissa Green  
I. Mezic, “*Coherent structure identification in fluid flows related to spectral properties of the Koopman operator*”  
9:30 N. Ouellette, “*Connecting spectral dynamics to coherent structures*”  
10:00 Coffee Break, TCPL  
10:30 J. Olascoaga, “*Clustering on the surface ocean*”  
11:00 D. Del Castillo-Negrete, “*Transport barriers and coherent structures in mean-field Hamiltonian systems*”  
11:30 M. Allshouse, “*Accounting for windage in the identification and classification of FTLE ridges*”  
12:00 Lunch  
13:30 Panel session: “*Strengths and challenges for different approaches to transport barriers*” (Hosts: D. Del Castillo-Negrete, S. Balasuriya)  
15:00 Coffee Break, TCPL  
15:30 Tutorial: J.L. Thiffeault, “*An introduction to Braidlab*”  
17:30 Dinner  
19:30 Drinks at Banff Avenue Brewing Company

## Thursday

- 7:00 Breakfast  
9:00 Morning session host: Michael Allshouse  
S. Shadden, “*A software pipeline for LCS computation*”  
9:30 A. Surana, “*Dynamical system analysis of crowd videos*”  
10:00 Coffee Break, TCPL  
10:30 M. Budisic, “*Mesochronic analysis: computation and interpretation*”  
11:00 M. Leclair, “*Fractal challenges in FTLE ridge refinement?*”  
11:30 S. Ross, “*Geophysical transport structure and ecology: challenges and opportunities*”  
12:00 **Free afternoon**  
17:30 Dinner

## Friday

7:00 Breakfast  
9:00 Panel session: “*Physical applications and future directions*” (Hosts: L. Pratt, N. Ouellette)?  
10:00 Coffee Break, TCPL  
10:30 Checkout  
11:30-13:30 Lunch

### **Checkout by 12 noon.**

\*\* 5-day workshop participants are welcome to use BIRS facilities (BIRS 2<sup>nd</sup> floor lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*



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### ABSTRACTS

Allshouse, Michael (University of Texas, Austin)

#### *Accounting for windage in the identification and classification of FTLE ridges*

Understanding the transport of contaminants on the surface of the ocean is aided by identifying regions of large Lagrangian deformation. Given that the FTLE field measures the local deformation, its ridges represent the locally maximum repelling structures in the flow. We present a numerical approach to identifying and refining FTLE ridges that enables their advection and classification of local deformation. This technique is applied to the ocean surface near the Ningaloo Reef along the Western Australia coastline, which contains a large fringing reef and significant offshore natural resources. Because surface winds as well as ocean surface currents force oil, a hybrid ocean-wind velocity field is created and analyzed. The resulting hybrid LCSs provide critical information for the transport of the oil that differs significantly from analysis of the ocean surface currents alone.

Balasuriya, Sanjeeva (University of Adelaide)

#### *Nonautonomous control of invariant manifolds*

It is well known that the temporal evolution of stable and unstable manifolds plays a significant role in transport in nonautonomous dynamical systems. As a first step towards understanding how to control such manifolds, this talk addresses the control problem of determining the nonautonomous velocity perturbation in two dimensions required to ensure that a one-dimensional heteroclinic manifold splits into stable and unstable manifolds whose primary segments lie along specified time-varying curves in space.

Beron-Vera, F. Javier (University of Miami)

#### *Geodesic analysis of oceanic flows*

We describe recent applications of geodesic analysis to altimetry-derived velocity data in the Gulf of Mexico (drifting buoy dispersal) and the South Atlantic (Agulhas ring detection and transport estimates).

Blazevski, Daniel (ETH Zurich)

#### *Lagrangian transport barriers in three-dimensional unsteady flows*

Detecting transport barriers is a fundamental problem in the study of Lagrangian transport of fluids. In the context of unsteady, temporally aperiodic velocity fields, transport barriers are time-dependent material surfaces that either repel or shear nearby tracers. In particular the shear Lagrangian barriers we consider are way to define and accurately detect 3D Lagrangian vortices. We illustrate the theory with example of steady and unsteady velocity fields.

Bollt, Erik (Clarkson University)

*Differential geometry perspective of shape coherence and curvature evolution by finite-time nonhyperbolic splitting*

Mixing, and coherence are fundamental issues at the heart of understanding fluid dynamics and other non- autonomous dynamical systems. Only recently has the notion of coherence come to a more rigorous footing, and particularly within the recent advances of finite-time studies of nonautonomous dynamical systems. Here we define shape coherent sets which we relate to measure of coherence in differentiable dynamical systems from which we will show that tangency of finite time stable foliations (related to a forward time perspective) and finite time unstable foliations (related to a "backwards time" perspective) serve a central role. This perspective is agreeable with the recent theory of geodesics by Haller and colleagues derived from a variational principle of geodesics. We develop zero-angle curvers, meaning non-hyperbolic splitting, by continuation methods in terms of the implicit function theorem, from which follows a simple ODE description of the boundaries of shape coherent sets.

Budisic, Marko (University of Wisconsin, Madison)

*Mesochronic analysis: computation and interpretation*

Mesochronic analysis characterizes the deformation of material as it is advected by an incompressible flow. The central object of analysis is the mesochronic, i.e., time-averaged, velocity field, which is affinely related to the flow map over a time interval of finite, non-zero length. Locations of eigenvalues of the Jacobian of the mesochronic velocity field indicate whether the associated material transport is hyperbolic, elliptic, or parabolic. Application of mesochronic analysis to data collected in the aftermath of Deepwater Horizon Spill (Mezic et al., Science, 2010) demonstrated that the flow in areas where majority of the oil slick was transported showed mesochronic hyperbolicity. We now make further connections between planar mesochronic analysis and earlier efforts: Okubo-Weiss partition, Greene's stochasticity criterion, and Haller-Iacono hyperbolicity and shear indicators. Finally, we demonstrate how the planar analysis extends to three-dimensional flows. Our theoretical considerations are backed by numerical analysis of analytic, numeric, and experimental flows.

Del Castillo-Negrete, Diego (Oak Ridge National Laboratory)

*Transport barriers and coherent structures in mean-field Hamiltonian systems*

Understanding the formation and destruction of transport barriers is a problem of common interest to many areas of science and technology including geophysical fluid dynamics, chemical and mechanical engineering systems, and magnetically confined fusion plasmas, among others. Quite often progress in this important area of research is hindered by the inherent mathematical complexity of the underlying models that involve the self-consistent nonlinear coupling of a transported field,  $F$ , with an underlying advective flow. Two examples of interest to this talk are: vortex dynamics in shear flows and charged particles in plasmas. In the first case,  $F$  is the vorticity, the advection equation is the 2-D Navier-Stokes equation, and the self-consistent coupling is the vorticity-streamfunction relation. In the plasma case,  $F$  is the electron distribution function, the advection equation is the Vlasov equation, and the self-consistent coupling is the Poisson equation. We show that near marginal stability, the weakly nonlinear dynamics of these two systems can be described by the same universal mean-field Hamiltonian model known as the Single Wave Model (SWM). We present numerical and analytical results on the SWM in the  $N \rightarrow \infty$  kinetic limit and in the

finite-N limit, where N is the number of degrees of freedom. Using this model, we study the formation and persistence of coherent structures and transport barriers in the presence of self-consistent chaotic transport. Starting from the finite-N SWM we construct mean-field coupled symplectic map models of self-consistent transport. We discuss how these maps open the possibility of studying the role of coherent structures in transport in Hamiltonian systems with a large number of degrees-of-freedom.

Froyland, Gary (University of New South Wales)

*Transfer operator methods for discovering transport barriers in geophysical flows*

I will report on transfer operator methods for analyzing time-dependent flows over a finite time duration. Particular attention will be paid to the identification of optimally coherent sets. I will also demonstrate how the strongest transport barriers vary with time duration and make connections with the geometry of invariant manifolds. The methods can be seamlessly applied to dynamical systems with both advective and diffusive components.

Green, Melissa (Syracuse University)

*Practical concerns of using FTLE with experimental data*

Using Lagrangian techniques to find transport barriers and vortex boundaries in complex, aperiodic flows necessitates a careful consideration of the spatial and temporal resolution of the data to be analyzed. Using previous data from a direct numerical simulation of a fully turbulent channel, I will show that by simulating "experimental" data the resultant FTLE fields degrade in way that cannot be attributed to a simple spatial or temporal filter. Instead, fundamental flow physics are not being captured. However, with some physical knowledge of the flow fields of interest, some of the pitfalls maybe avoided. The simulated experiments are done by artificially degrading the time resolution of the velocity data or by only using a single two-dimensional slice of the velocity data from the fully 3D simulation (analogous to 2D particle image velocimetry data).

Haller, George (ETH Zurich)

*Geodesic theory of transport barriers*

We discuss a general global variational approach to finite-time material transport barriers in two-dimensional unsteady flows. The barriers turn out to be null-geodesics of appropriately defined Lorentzian metrics computed from the flow map. The different types of geodesics lead to a classification of transport barriers as hyperbolic, parabolic and elliptic barriers. We show by examples how these results enable the detection of generalized stable and unstable manifolds, generalized KAM tori and generalized jet cores in unsteady geophysical flow data.

Leclair, Matthieu (MIT)

*Fractal challenges in FTLE ridge refinement?*

We present an overview of a methodology used to calculate FTLE ridges, including a hybrid parallel structure: particles are distributed over the computer's cores but each core has to store the entire velocity data. This code has made possible the treatment of large datasets and the calculation of high-resolution FTLE fields. In trying to refine the resolution of FTLE ridges in a case study we observe some seemingly fractal behavior. This makes the clear identification of ridges more complex but also raises the question of the significance of such small-scale features, particularly if sub-grid scale diffusive processes are taken into account.

Lermusiaux, Pierre (MIT)

*Bayesian learning of stochastic dynamical model formulation and Lagrangian coherent structures*

In this presentation, we first highlight recent results by our MSEAS group, including high-order Finite-Element schemes for biogeochemical ocean dynamics and exact path planning for swarms of ocean vehicles using new level-set equations. We then address a holistic challenge in ocean Bayesian estimation: i) predict the probability distribution functions (pdfs) of large nonlinear ocean systems using stochastic partial differential equations, ii) assimilate data using Bayes' law with these pdfs, iii) rank the known and learn the new model formulations themselves. Overall, we allow the joint inference of the state, equations, geometry, boundary conditions and initial conditions of dynamical models. The Bayesian model inference is illustrated by the estimation of obstacle shapes and of biogeochemical reaction equations. Possibilities of the combination of this approach with the computation of LCS in uncertain ocean flows are discussed.

Mezic, Igor (University of California at Santa Barbara)

*Coherent structure identification in fluid flows related to spectral properties of the Koopman operator*

I will review the approaches to coherent structure identification in fluid flows related to spectral properties of the Koopman operator. I will pay special attention to the theory of ergodic partition and its uses in such identification, while the talks of M. Budisic and A. Surana will present results of mesohyperbolicity theory, which I will introduce as well. I will discuss the issue of frame indifference of approaches to structure identification and relate the current approaches to earlier attempts to understand dynamics of two-dimensional conservative maps.

Olascoaga, M. Josefina (University of Miami)

*Clustering on the surface ocean*

We apply recent results for finite-size (inertial) particles to explain the observed tendency of floating material (drifting buoys, sargassum, phytoplankton) to cluster within coherent material vortices revealed from altimetry-derived velocity data.

Ouellette, Nicholas (Yale University)

*Connecting spectral dynamics to coherent structures*

A broad range of tools have been developed over the past few decades to identify coherent structures in fluid flows, ranging from simple metrics based on thresholding the velocity gradients to more modern Lagrangian approaches. But in most cases, these tools take advantage only of kinematic information; that is, they define structures based only on the structure of the flow field or the motion of fluid particles. Turbulent flows, however, are highly dynamical, and the fluxes of energy and momentum in space and particularly between motion on different length and time scales play key roles in their evolution. By using so-called filter-space techniques, I will present results on the spatiotemporal localization of the energy flux between scales in an experimental quasi-two-dimensional flow, and will show how these fluxes can be related to coherent structures.

Pratt, Larry (Woods Hole Oceanographic Institution)

*Chaotic advection in a steady, three-dimensional, Ekman-driven eddy: part I*

We investigate the existence and stability of transport barriers in a fully three-dimensional Navier-Stokes flow in a rotating cylinder. This idealization of an isolated ocean eddy is driven from above by a surface stress and has both horizontal swirl and overturning. The invariant tori that act as material barriers in the steady, axisymmetric case start to break when a symmetry-breaking disturbance is added. Chaos is induced either by resonance or by the breakup of the central axis streamline into stable and unstable manifolds in 3D. We identify several distinct regimes of Lagrangian behavior and map these out in terms of the Ekman and Rossby numbers. We also use Eulerian constraints such as the Taylor-Proudman theorem to motivate the differences. We calculate the stirring rate due to the chaos for different flow regimes and find some surprising trends. A formula for the resonance width is derived, and this and a version of the KAM theorem are used to interpret our findings. This talk will concentrate on steady behavior while a follow-up talk by Rypina will discuss the time-periodic case.

Ross, Shane (Virginia Tech)

*Geophysical transport structure and ecology: challenges and opportunities*

Techniques uncovering transport barriers and structures in environmental flows are poised to make a considerable impact on the field of ecology. Here we discuss some results relevant for real-time analysis and prediction of geophysical transport structures, which have arisen from applications to fluid-borne microbial ecology. Focusing on aeroecology, we consider challenges involved in forecasting atmospheric Lagrangian coherent structures, including the effect of subgrid turbulence and ensemble averaging. We also consider notions that may be relevant in an ecological context, such as persistent barriers, which may lead to the most diverse populations (in terms of origins) sampled sequentially at a geographically fixed location.

Rowley, Clancy (Princeton University)

*Coherent structure identification using flow map composition and spectral interpolation*

We propose an efficient method for approximating the long-time flow map associated with an uncertain, time-varying velocity field. The method can be used to efficiently compute Finite-Time Lyapunov Exponents, or to approximate the evolution of probability density functions for systems with uncertain initial conditions or parameters. Short-time flow maps are approximated using polynomial basis functions, and are then composed to form a long-time flow map. Thus, the degree of the polynomial approximation grows exponentially in the number of compositions, while the number of coefficients needed to represent the short-time flow maps grows only linearly in the number of compositions. The long-time flow map is then used to compute stochastic quantities, which are shown to be correlated to coherent structures in the velocity field.

Rypina, Irina (Woods Hole Oceanographic Institution)

*Chaotic advection in a non-steady, three-dimensional, Ekman-driven eddy: part II*

We consider a 3d fluid flow arising in a rotating cylinder driven by a surface stress. This flow is an idealization of an isolated ocean eddy. The previous talk by Pratt covered the existence and stability of transport barriers in a steady case. In this talk we focus on the behavior of the

system under the influence of a time-dependent, symmetry-breaking disturbance. Consistent with the KAM theorem, we illustrate the existence in the non-steady system of both regular motion on KAM tori, and chaotic motion due to the resonances and break-up of resonant tori. A methodology is derived that describes the resonant condition, the geometry of the flow in the vicinity of the resonances, and the resonance widths.

Shadden, Shawn (Illinois Institute of Technology)

*A software pipeline for LCS computation*

The utility of Lagrangian coherent structure computation for advective transport analysis is well established. Broader adoption and refinements of this method are facilitated by development of robust and flexible software elements that accommodate wide-ranging application areas, and modularity that is easily extended or modified. We discuss the development of a software pipeline for object-oriented LCS computation, and integration with flow visualization softwares. We will also discuss GPU and multicore architecture implementations for efficient parallel computation and optimization strategies for better utilization.

Surana, Amit (United Technologies Research Center)

*Dynamical System Analysis of Crowd Videos*

Behavior analysis of crowded scenarios from videos is a challenging computer vision problem. Recently, new insights have been obtained by treating dense crowd motion as a fluid flow driven by optical flow field extracted from video frames. In this talk we describe how geometric, statistical and spectral concepts from dynamical systems theory can be used to analyze such flow fields, and thereby the crowd behavior. In particular we use Finite Time Lyapunov Exponents, Perron Frobenius Operator and Koopman Operator based analysis, and demonstrate their application in crowd segmentation and group anomalous behavior detection problems.

Thiffeault, Jean-Luc (University of Wisconsin, Madison)

*Moving walls accelerate mixing*

Mixing in viscous fluids is challenging, but chaotic advection in principle allows efficient mixing. In the best possible scenario, the decay rate of the concentration profile of a passive scalar should be exponential in time. In practice, several authors have found that the no-slip boundary condition at the walls of a vessel can slow down mixing considerably, turning an exponential decay into a power law. This slowdown affects the whole mixing region, and not just the vicinity of the wall. The reason is that when the chaotic mixing region extends to the wall, a separatrix connects to it. The approach to the wall along that separatrix is polynomial in time and dominates the long-time decay. However, if the walls are moved or rotated, closed orbits appear, separated from the central mixing region by a hyperbolic fixed point with a homoclinic orbit. The long-time approach to the fixed point is exponential, so an overall exponential decay is recovered, albeit with a thin unmixed region near the wall. This is joint work with Emmanuelle Guillard and Olivier Dauchot.