Water waves: computational approaches for complex problems BIRS Workshop Report

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1 Overview

The workshop was highly successful: BIRS provides an almost unique environment for a weeklong intensive high-level research workshop. The workshop participants spanned a range of interests from direct applications to theoretical questions. The focus of the workshop was on numerical methods which tend often to pervade this range of interests. By their very nature the numerical solutions of wave problems in fluid flows is extremely complex and often new methods need to borrow from both theory (e.g. Dirichlet to Neumann maps etc...) as well as from "what works" in practice (e.g. optimal Boussinesq parameters etc...). Whilst the central focus was water waves there were also several invitees whose main interests are more broadly in "waves in fluids": this includes mainly internal waves in stratified flows (i.e. internal atmospheric or oceanic waves) and planetary scale waves. The workshop was particularly timely within the framework of the 2013 "Mathematics of Planet Earth" initiative.

In the short summaries below we have classified the workshop topics into four subareas and described some highlights in each area. It is evident that there is strong cross-fertilisation between areas.

2 Focus Area Summaries

2.1 Numerical and Computational

Central participants: Beale, Siegel, Nicholls, Choi, Wang, Athanassoulis, Akers, Vanden-Broeck, Wilkening, Milewski, Nachbin, Ambrose, Trichtchenko, Kutz, Parau, Blyth.

Within this focus area, important new results were presented, for example, in the dynamics of focussing packets of capillary-gravity waves (Wang) and hydroelastic behaviour (Parau and Blyth), as well as discussions of general issues in numerical computations. In particular, these included methods for computing singular integrals in complex three-dimensional geometry (Beale), issues of well-posedness of approximate models which are often used numerically (Ambrose), and issues of computational efficiency and speed (Siegel and Wilkening). The presentation by Ambrose on well-posedness was particularly relevant at the interface of theory and numerics as it points to possible flaws in certain widely used models.

2.2 Theoretical

Central participants: Ablowitz, Lannes, Beale, Camassa, Murashige, Ambrose, Henry, Barros.

Within this focus area, several theoretical results of interest to modelling and computational aspects of waves in fluids were presented. This included the ubiquitous occurrence of multi-stem Kadomtsev-Petviashvilli on nature (Ablowitz) and the interesting apparent paradox of horizontal momentum conservation in stratified flows (Camassa).

2.3 Math Modelling and Wave Behavior in the Environment

Central participants: Ablowitz, Zhang, Deconinck, Duncan, Cooker, Chumakova, Rosales, Akylas, McLaughlin, Tabak, Guyenne, Bush, Milewski, Henderson, Segur, Nachbin, Kasimov.

Modelling was another topic strongly represented at the workshop, as it is usually the first step to address physical phenomena that then drive the development of numerical methods. Novel models included the leakylid approximation to atmospheric wave motions (Chumakova and Rosales), interaction of internal wave beams in the ocean (Akylas), the modelling of mixing at shocks in internal waves (Tabak and Milewski), a discussion of possible dissipative effects in surface waves (Henderson and Segur), the interaction of waves under floating ice sheets (Parau and Blyth) and the modelling of surface wave propagation in networks of channels (Nachbin). Research on these topics have direct impact on applications of waves such as climate and tsunami modelling.

2.4 Experiments

Central participants: Zhang, Duncan, McLaughlin, Bush, Henderson, Kasimov.

Progress in understanding Fluid Dynamics has traditionally been driven by experiments, and it is important that the mathematical questions pertaining to waves in fluids remain aligned with the reality and complexity brought forth by experiments. The experimentalists presented a fascinating range of phenomena including a hydrodynamic quantum analogue in a Faraday-bouncing droplet experiment (Bush), the stability and dynamics of surface tension driven hydraulic jumps (Kasimov), experiments in the details of wave breaking (Duncan).

3 Conclusion and Future Considerations

The workshop was a success at fostering an important exchange of ideas between members of the community in the various aspects of wave propagation as described above. Workshops of this type break the isolation of researchers along the theory – numerical – modelling – experiment spectrum revitalising existing, and creating new collaborative projects. In the closing days of the workshop, several of the participants concluded that the experimental – mathematical collaboration is particularly interesting and that there should be follow up workshops under this remit.