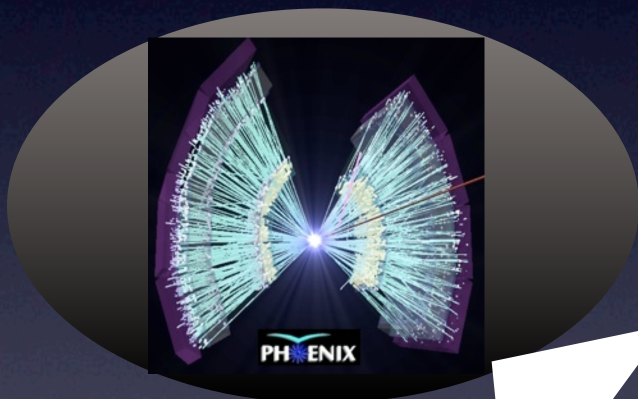
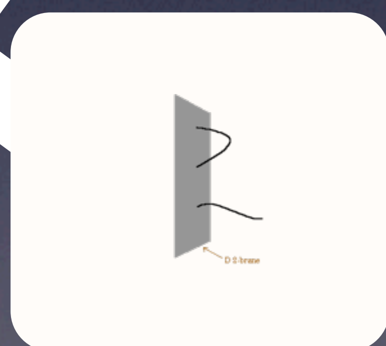
Hyperscaling Violation on Probe D-Branes

"Holography & Applied String Theory",
BIRS, Banff, Canada, February 14th 2013





[Fuini, MK, Yaffe xxxx.xxxx]

today: [Ammon, MK, Karch 1207.1726]

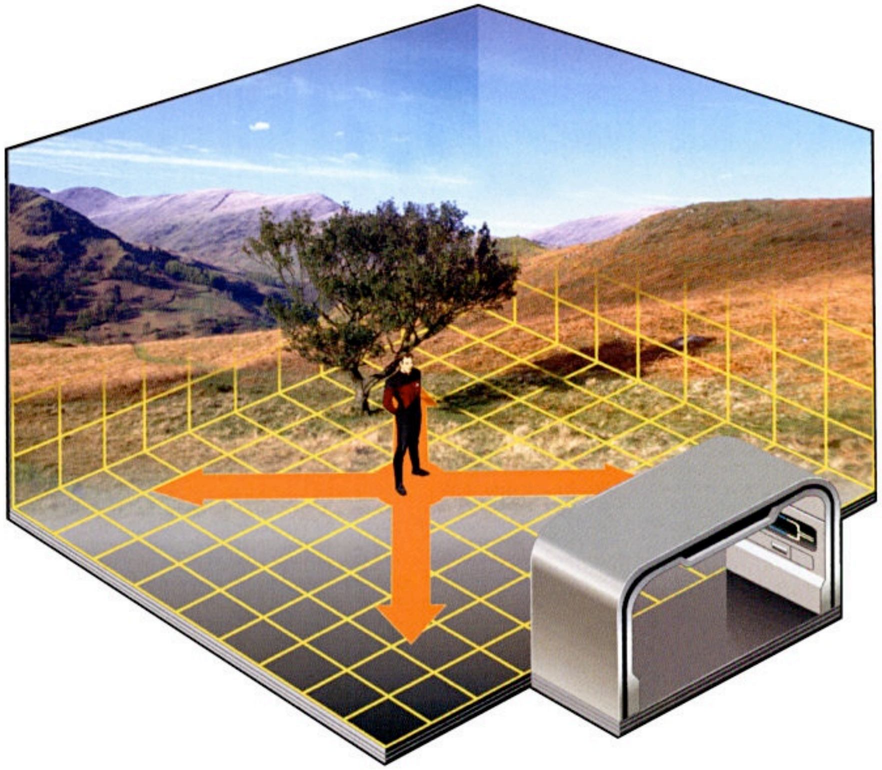
Hyperscaling violation

cf. [Kiritsis' talk]

volume scales not like (lenght)$^{\text{dimension}}$

Hyperscaling violation

cf. [Kiritsis' talk]

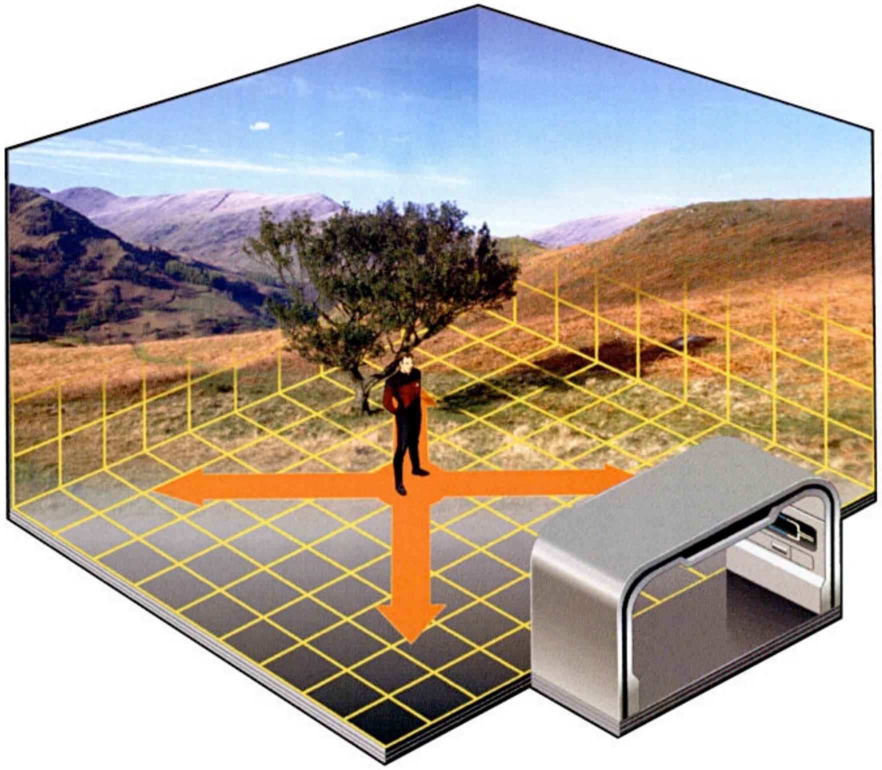


volume scales not like $(\text{lenght})^{\text{dimension}}$

bigger on the inside ...

Hyperscaling violation

cf. [Kiritsis' talk]



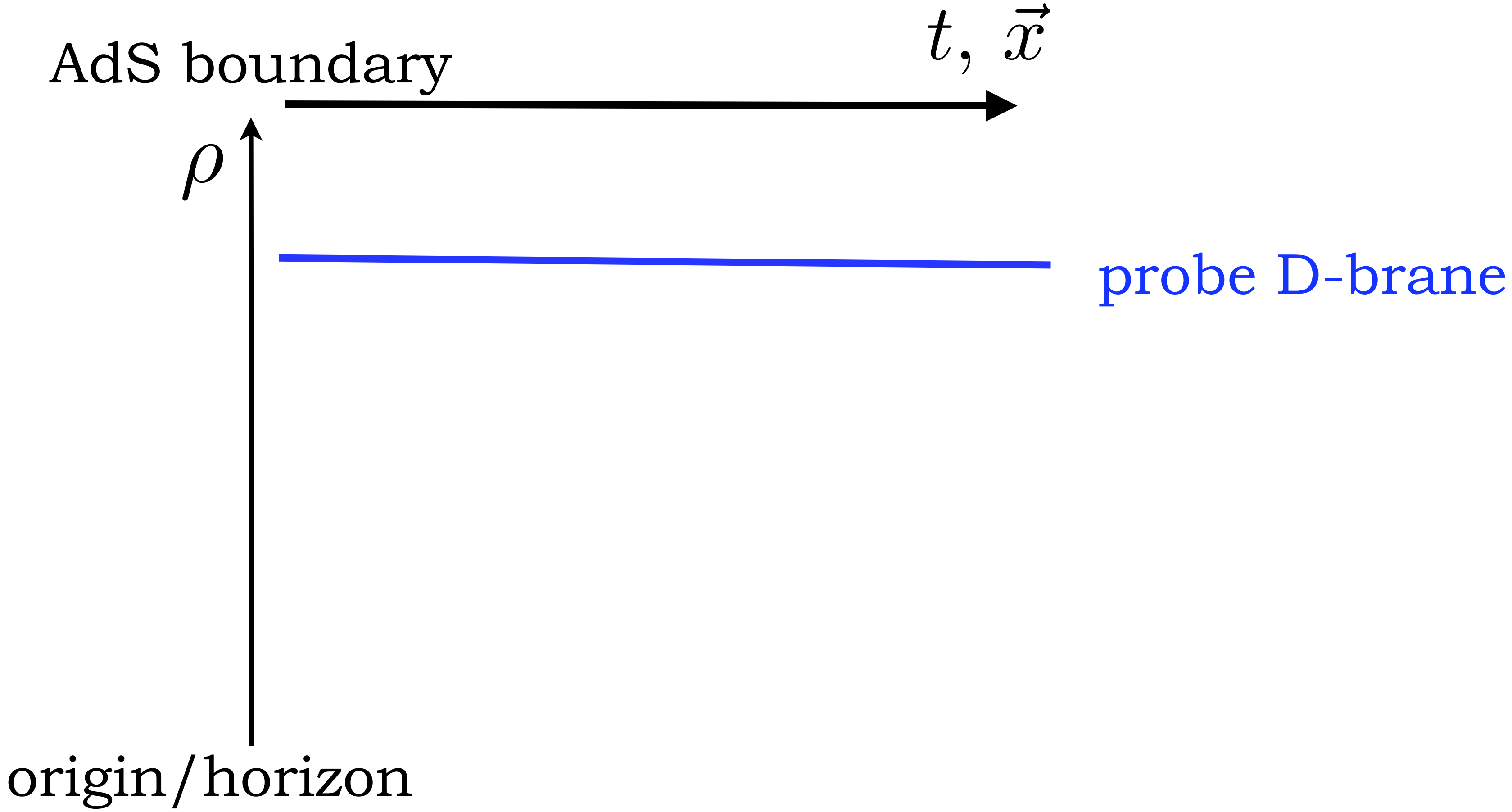
volume scales not like (lenght)$^{\text{dimension}}$

bigger on the inside ...

spatial volume scales with reduced dimension : $n - \theta$

hidden Fermi surfaces
cf. [Cremonini's talk] [AdS/CMT diss.]
compressible states
cf. [Iqbal's talk] [Ammon's talk]

Probe D-branes



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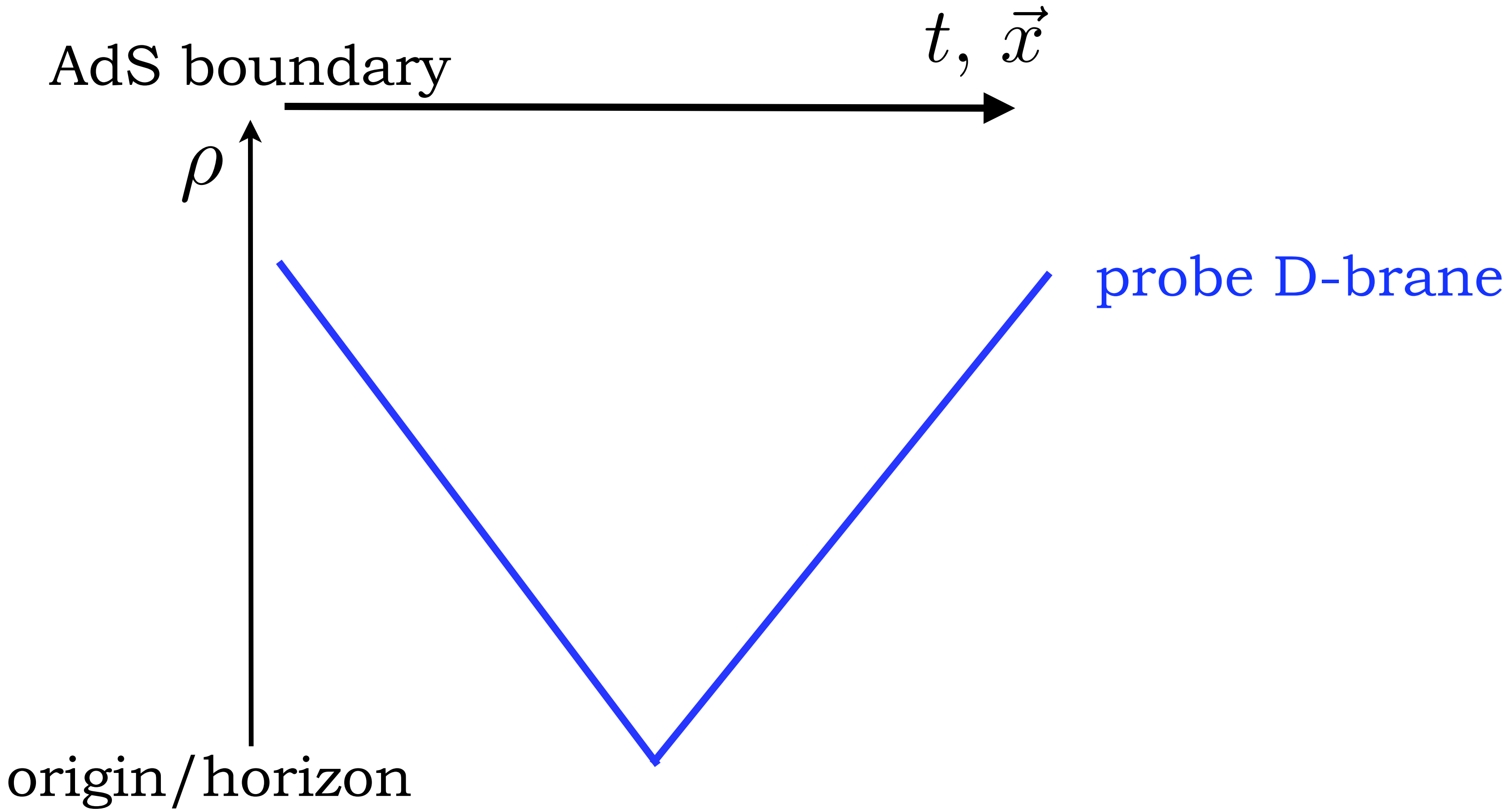

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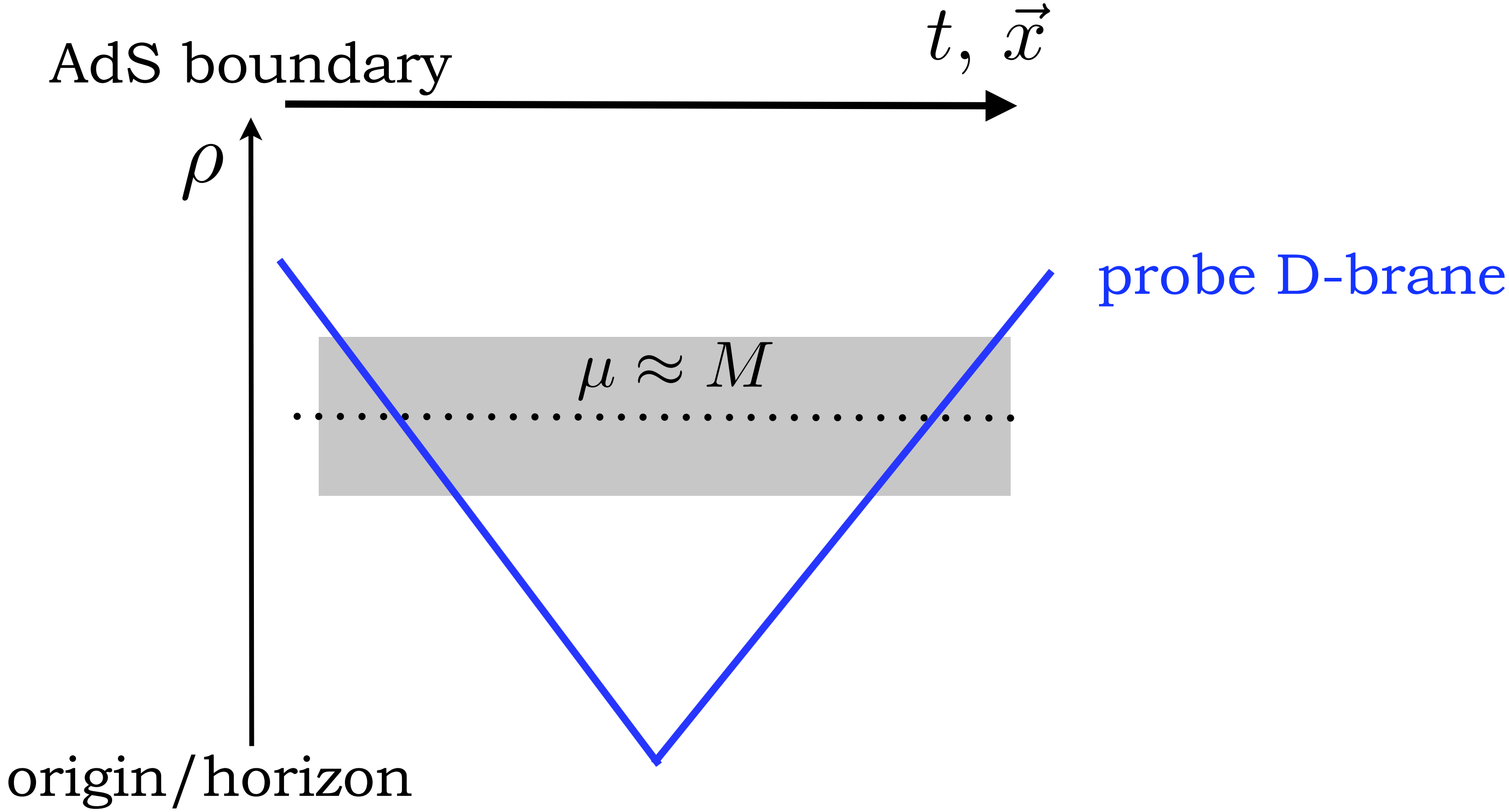

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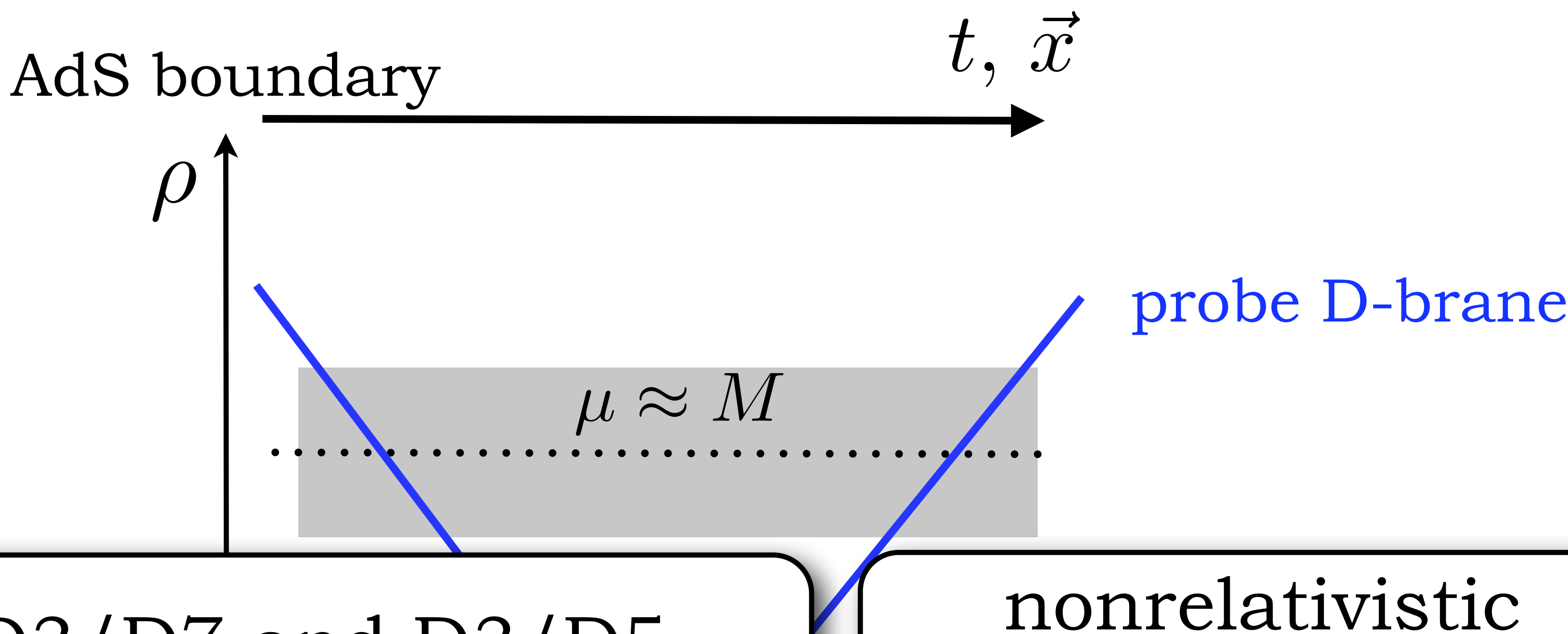

D3/D7 and D3/D5 result

$\theta = 1,\ z = 2$

$\alpha,\ \nu$

nonrelativistic region with hyperscaling violation near phase transition

cf. [AdS/CMT discussion] [Gauntlett's talk]

[Huijse, Sachdev, Swingle; PRB (2012)]

[Dong et al]; (2012)

[Donos, Gauntlett, Sonner, Withers; (2012)]

[Gouteraux, Kiritsis; JHEP (2011)]

cf. [Karch's talk]

[Janiszewski, Karch; (2012)]

Outline

I. Critical scaling exponents

II. Our argument

III. Probe D-branes

IV. Conclusions

Continuous phase transitions

$\langle \text{order parameter} \rangle = 0$

disordered

$T = T_c$

$\langle \text{order parameter} \rangle \neq 0$

ordered

Critical exponents:

$$\xi \sim \tau^{-\nu}$$

$$\tau_{corr} \sim \xi^{z}$$

$$C \sim \tau^{-\alpha} \qquad \tau = \frac{T - T_c}{T_c}$$

Continuous phase transitions

$\langle \text{order parameter} \rangle = 0$

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Hyperscaling relation

$$n\nu = 2 - \alpha$$

Critical exponents:

$$\xi \sim \tau^{-\nu}$$

$$\tau_{corr} \sim \xi^{z}$$

$$C \sim \tau^{-\alpha} \qquad \tau = \frac{T - T_c}{T_c}$$

Continuous phase transitions

Violated hyperscaling relation

$$(n-\theta)\nu = 2-\alpha$$

[Fisher; PRL (1985)]

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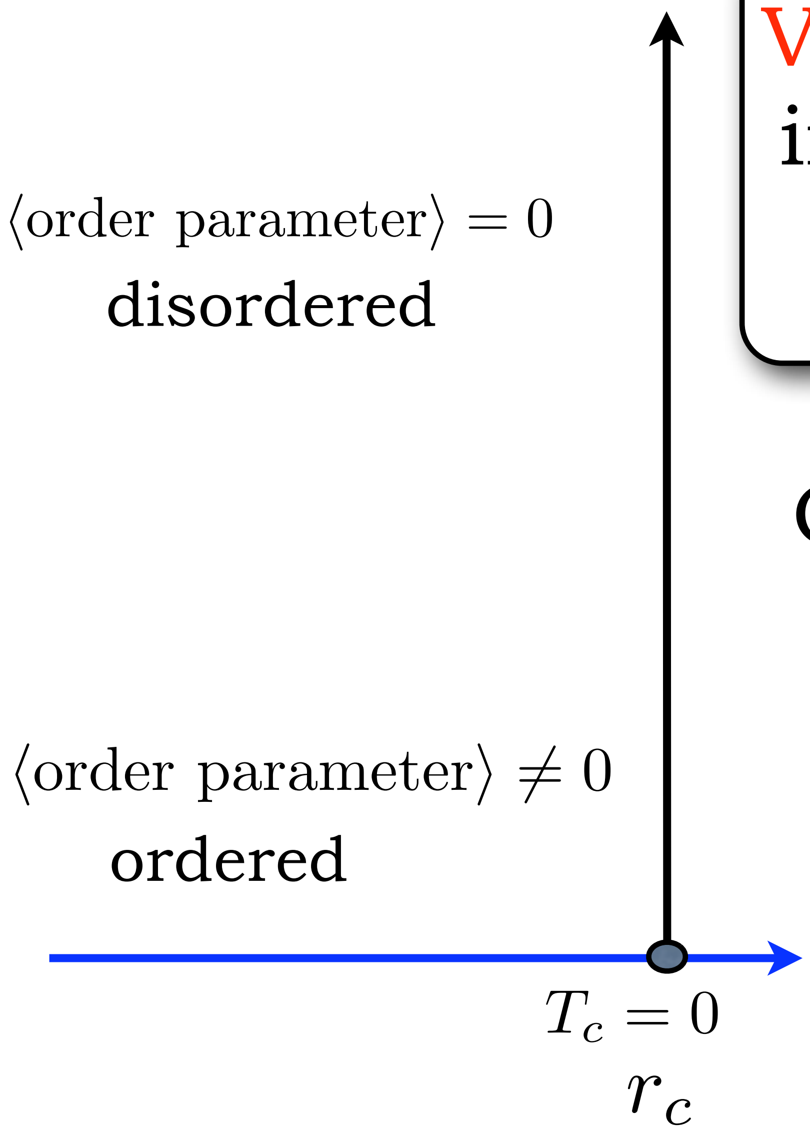

Violated hyperscaling relation in quantum phase transition

$$(n + z - \theta)\nu = 2 - \alpha$$

Critical exponents:

$$\xi \sim \tau^{-\nu}$$

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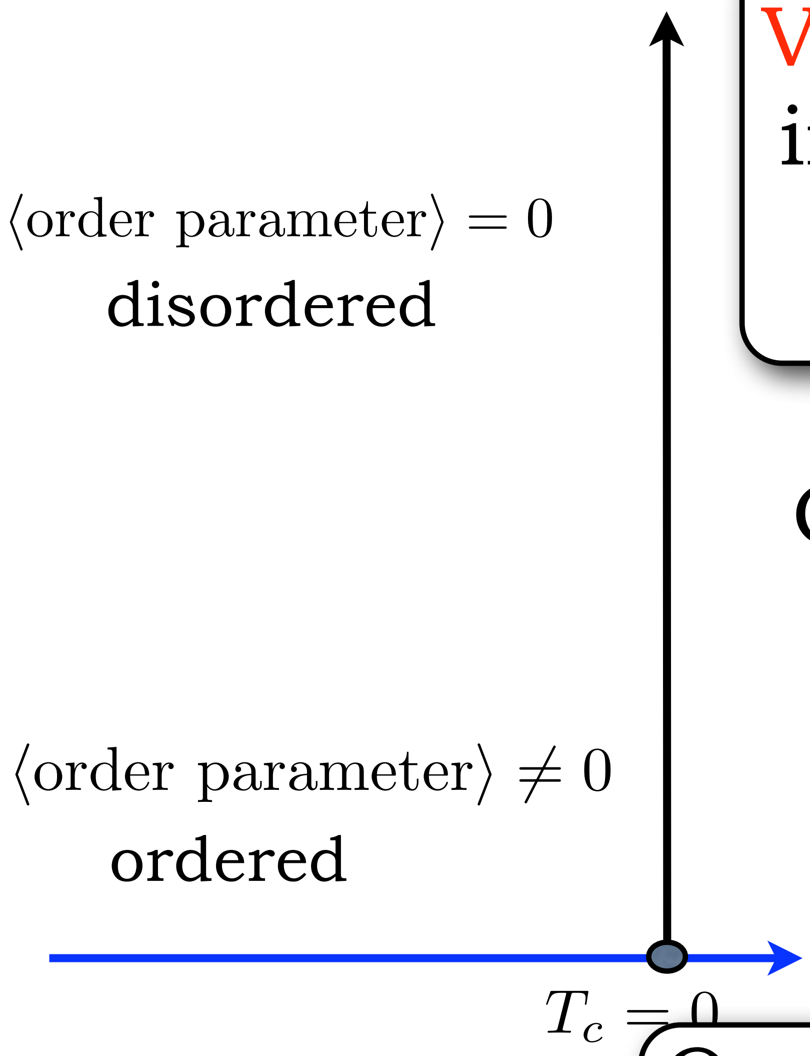
Continuous phase transitions



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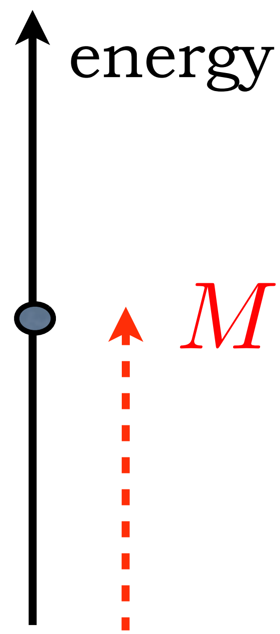
Organize *all possible* phases into universality classes.

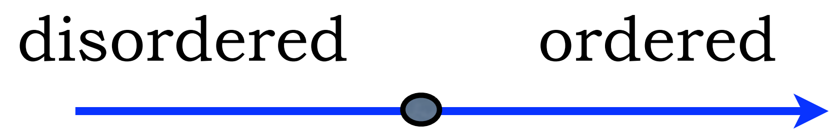
Desired theory at zero temperature

quantum phase transition: $T = 0$

$\mu \neq 0$



QFT with transition:



$$\mu = \mu_c = M$$

hyperscaling violating: $\theta, \ z$

spatial dimensions: n

charge density: d

Desired theory at zero temperature

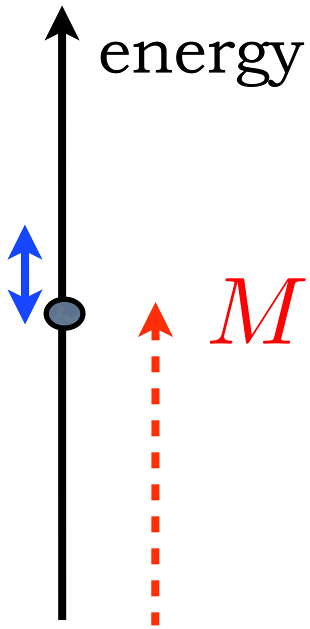
quantum phase transition: $T = 0$

$\mu \neq 0$

single dimensionful scale:

$\bar{\mu} = \mu - M$

$\bar{\mu} \ll M$



QFT with transition:

disordered | ordered

$\mu = \mu_c = M$

hyperscaling violating: $\theta, \ z$

spatial dimensions: n

charge density: d

$$e = \epsilon - \epsilon_{\rm rest} = -p + \mu d - M d = -p + \bar{\mu} d$$

Main argument

$$e = -p + \bar{\mu} d = \frac{n-\theta}{z} p$$

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$$[\bar{\mu}] = [\partial_t] = z$$

$$\Omega = e - \bar{\mu} d$$

$$[e] = z + n - \theta$$

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$$\Rightarrow \quad [d] = n - \theta$$

$$p = -\Omega = C_0 \bar{\mu}^{(n+z-\theta)/z}$$

single dimensionful scale: $\bar{\mu}$

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$$p = -\Omega = C_0 \bar{\mu}^{(n+z-\theta)/z}$$

$$d = \frac{\partial p}{\partial \bar{\mu}} = C_0 \frac{n-\theta+z}{z} \bar{\mu}^{\frac{n-\theta}{z}}$$

single dimensionful scale: $\bar{\mu}$

Main argument

$$e = -p + \bar{\mu} d = \frac{n-\theta}{z} p$$

Claim 1: $e = p$ (D3/D7 thermodynamics: $n = 3$)

$$\Rightarrow \quad 3 - \theta = z$$

Claim 2: $\omega \simeq \sqrt{\frac{\bar{\mu}}{M}} k$ (D3/D7 speed of sound)

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$[\omega] = z$ $\qquad$ $[\bar{\mu}]/2 + [k] = z/2 + 1$

$$\Rightarrow \quad z = 2 \quad \Rightarrow \quad \theta = 1$$

Main argument

$$e = -p + \bar{\mu} d = \frac{n-\theta}{z} p$$

Claim 1: $e = p$ $\quad e = \frac{p}{2}$ (D3/D7 thermodynamics: $n = 3$)
(D3/D5: $n = 2$)

$$\Rightarrow \quad 3 - \theta = z \qquad 2 - \theta = \frac{z}{2}$$

Claim 2: $\omega \simeq \sqrt{\frac{\bar{\mu}}{M}} k$ (D3/D7 speed of sound)

$[\omega] = z$ $\qquad [\bar{\mu}]/2 + [k] = z/2 + 1$

$$\Rightarrow \quad z = 2 \quad \Rightarrow \quad \theta = 1$$ identical for D3/D5

Friday, February 15, 13

Probe D7-branes

[Karch,Katz; JHEP (2002)]

cf. [Ammon's talk]

- N_c *D3-branes*
  dual to $\mathcal{N} = 4$ SYM with $SU(N_c)$

Probe D7-branes

[Karch,Katz; JHEP (2002)]

cf. [Ammon's talk]

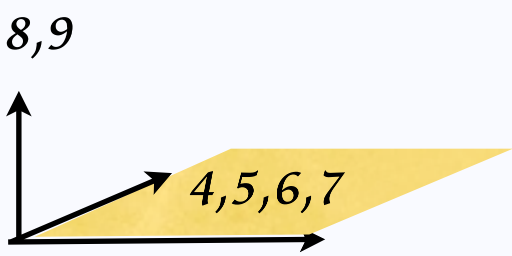
N_f D7-branes

dual to $\mathcal{N}=2$ $SU(N_f)$ flavor

	0	1	2	3	4	5	6	7	8	9
N_c D3	x	x	x	x						
N_f D7	x	x	x	x	x	x	x	x		
N_f D5	x	x	x		x	x	x			

N_c D3-branes

dual to $\mathcal{N}=4$ SYM with $SU(N_c)$



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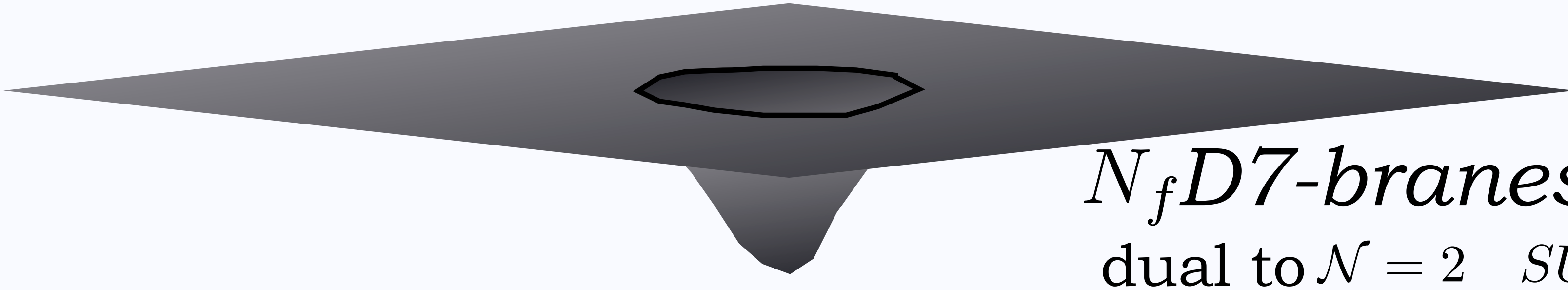

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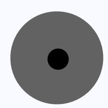
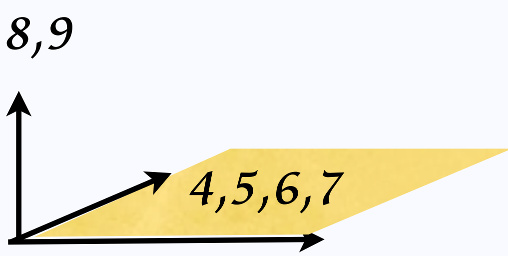
N_f D5	x	x	x		x	x	x			



N_c D3-branes

dual to $\mathcal{N}=4$ $\quad$ SYM with $SU(N_c)$



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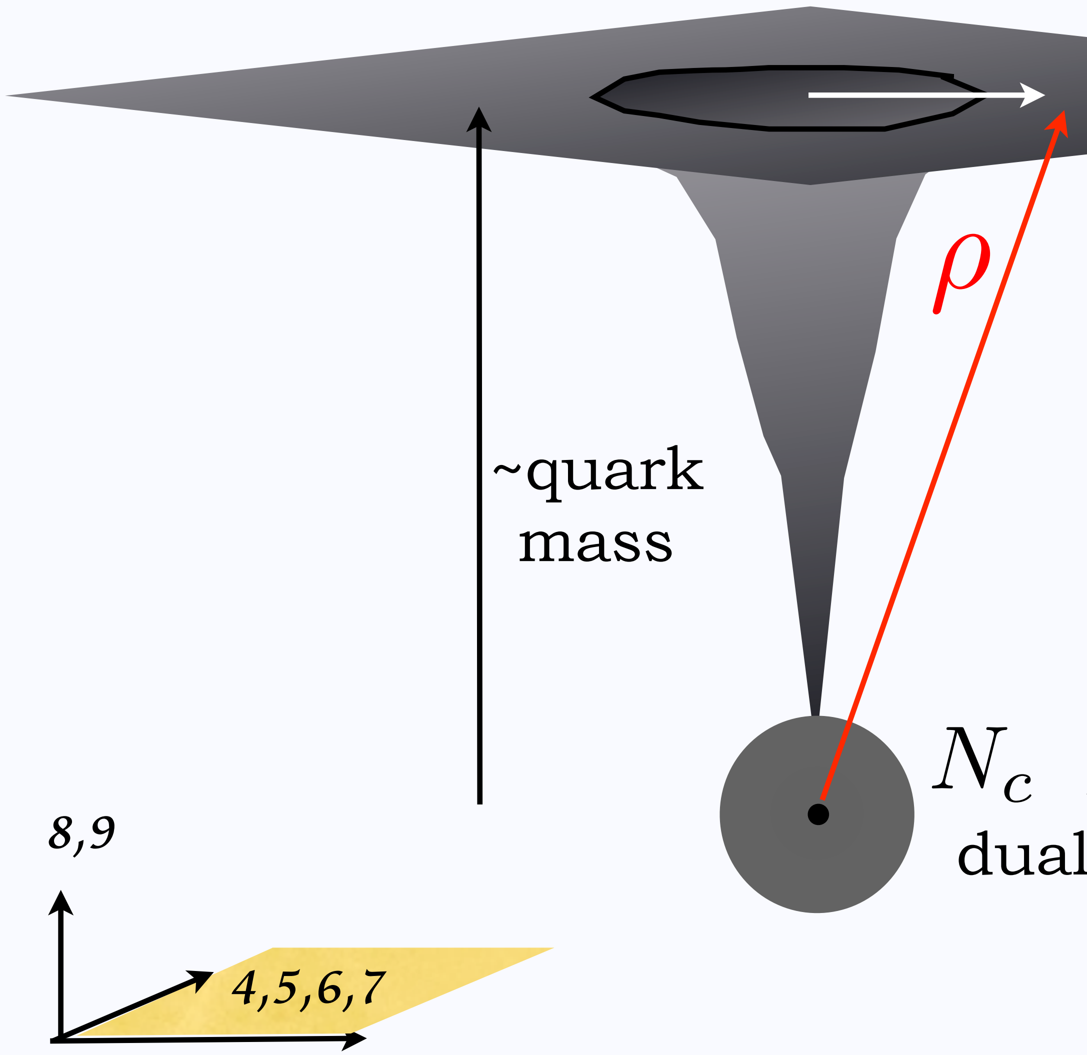

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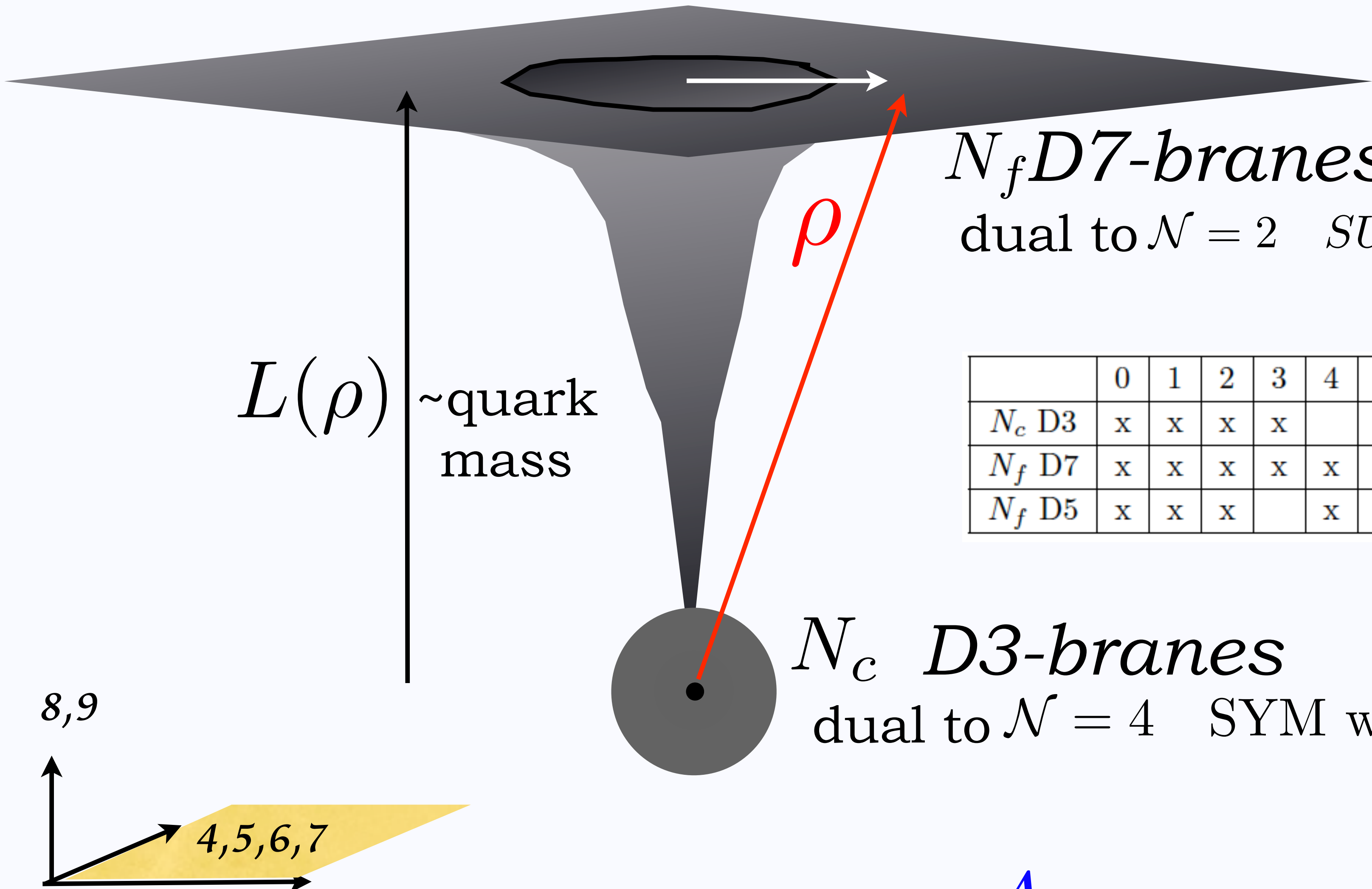

N_f*D7-branes*

dual to $\mathcal{N}=2$ $\quad SU(N_f)$ flavor

	0	1	2	3	4	5	6	7	8	9
N_c D3	x	x	x	x						
N_f D7	x	x	x	x	x	x	x	x		
N_f D5	x	x	x		x	x	x			

N_c *D3-branes*

dual to $\mathcal{N}=4$ SYM with $SU(N_c)$

Chemical potential: A_t

Exact solutions

Fixed metric

$$ds^2 = H(r)\,\eta_{\mu\nu}\,dx^\mu dx^\nu + H^{-1}(r)\left(d\rho^2 + \rho^2 ds^2_{S^n} + dy^2 + \sum_{i=1}^{4-n} dz_i^2\right)$$

$$r^2 = \rho^2 + y^2 + \sum_{i=1}^{4-n} z_i^2 \qquad\qquad H(r) = r^2/R^2$$

DBI-action

$$S_{D(2n+1)} = -N_f T_{D(2n+1)} \int d^{2n+2}\xi \sqrt{-det(g_{ab} + 2\pi\alpha' F_{ab})}$$

Exact solutions

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$$s_{D(2n+1)} \equiv \int d\rho\,\mathcal{L} = -\mathcal{N}_n \int d\rho\rho^n \sqrt{1 + y'(\rho)^2 - (2\pi\alpha')^2 A_t'(\rho)^2}$$

Two constants of motion

$$\frac{\delta\mathcal{L}}{\delta y'(\rho)} = -c\,, \qquad\qquad \frac{1}{(2\pi\alpha')}\frac{\delta\mathcal{L}}{\delta A_t'(\rho)} = d\,.$$

Exact solutions

$$y(\rho) = \frac{c}{\sqrt{d^2-c^2}}\,\rho\,{}_2F_1\left(\frac{1}{2n},\frac{1}{2};1+\frac{1}{2n};-\frac{\mathcal{N}_n^2\rho^{2n}}{d^2-c^2}\right),$$

$$A_t(\rho) = \frac{1}{(2\pi\alpha')}\frac{d}{\sqrt{d^2-c^2}}\,\rho\,{}_2F_1\left(\frac{1}{2n},\frac{1}{2};1+\frac{1}{2n};-\frac{\mathcal{N}_n^2\rho^{2n}}{d^2-c^2}\right)$$

$\rightarrow$ $\Omega = -s_{ren}$

Exact solutions

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Exact solutions

relate to physical parameters

$$y(\rho) = \frac{c}{\sqrt{d^2-c^2}}\,\rho\,{}_2F_1\left(\frac{1}{2n},\frac{1}{2};1+\frac{1}{2n};-\frac{\mathcal{N}_n^2\rho^{2n}}{d^2-c^2}\right),$$

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$$\Longrightarrow \qquad \Omega = -s_{ren}$$

D-brane thermodynamics I

Grandcanonical potential

D3/D7 case

$$\Omega = -s_{ren} = -\frac{\Gamma(1/3)\Gamma(7/6)}{4\sqrt{\pi}\mathcal{N}_3^{1/3}}(d^2 - c^2)^{2/3} = -\frac{1}{4}\mathcal{C}_3(\mu^2 - M^2)^2$$

$$\Rightarrow \; p = \frac{1}{4}\mathcal{C}_3(\mu^2 - M^2)^2 \qquad\qquad f = \epsilon - sT$$

$$\Rightarrow \; f = \Omega + \mu d = \frac{1}{4}\mathcal{C}_3(\mu^2 - M^2)(3\mu^2 + M^2)$$

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$$\Rightarrow f = \Omega + \mu d = \frac{1}{4}\mathcal{C}_3(\mu^2 - M^2)(3\mu^2 + M^2)$$

Nonrelativistic thermodynamic quantities:

$$\bar{\mu} = \mu - M$$

$$\bar{\mu} \ll M$$

$$\Omega = -p = -\mathcal{C}_3 M^2 \bar{\mu}^2$$

$$\epsilon = f = 2\mathcal{C}_3 M^3 \bar{\mu}$$

$$c = d = 2\mathcal{C}_3 M^2 \bar{\mu}$$

$$e = \epsilon - d\, M = \mathcal{C}_3 M^2 \bar{\mu}^2$$

Observe nonrelativistic regime near transition $\Rightarrow$

Claim 1

$$e = p$$

D-brane thermodynamics II

D3/D7 case

Speed of sound

$$v_s{}^2 = \frac{\partial p}{\partial \epsilon}$$

Pressure and energy density

$$v_s{}^2 = \frac{(\partial p/\partial \mu)}{(\partial \epsilon/\partial \mu)} = \frac{\mu^2 - M^2}{3\mu^2 - M^2} \approx \frac{\bar{\mu}}{M}$$

$\Rightarrow$

Claim 2

$$\omega = k\sqrt{\bar{\mu}/M} + \mathcal{O}(k^2, \bar{\mu}^2)$$

Zero sound

(confirms Claim 2)

Dynamical derivation of claim 2 (fluctuations)

D3/D7 case

$$\mathcal{A}_t(t,x,\rho) = A_t(\rho) + a_t(t,x,\rho)\,,$$
$$\mathcal{A}_x(t,x,\rho) = a_x(t,x,\rho)\,,$$
$$\mathcal{A}_\rho(t,x,\rho) = a_\rho(t,x,\rho)\,,$$
$$\mathcal{A}_i(t,x,\rho) = 0\,.$$

Field/coordinate redefinitions, Fourier trafo

$$\ddot{E} + \left(\frac{2}{z} + \frac{\dot{g}}{g} + \frac{1}{hf_5}\frac{k^2\dot{f}_1}{k^2-\omega^2 f_1}\right)\dot{E} + \frac{f_8 f_3}{z^4}(\omega^2 f_1 - k^2)E - \frac{\dot{f}_7}{hf_5}\frac{k^2-\omega^2}{k^2-\omega^2 f_1}\dot{X} = 0\,,$$
$$\ddot{X} + \left(\frac{2}{z} + \frac{\dot{g}}{g} + \frac{\dot{f}_5}{hf_5}\frac{k^2-\omega^2}{k^2-\omega^2 f_1}\right)\dot{X} + \frac{f_8 f_3}{z^4}(\omega^2 f_1 - k^2)X + \frac{\dot{f}_7}{hf_5}\frac{k^2}{k^2-\omega^2 f_1}\dot{E} = 0\,.$$

Limits $\rho \approx 0, \quad \omega/\rho \ll 1 \quad \ldots$

6 pages of appendix in [Ammon, MK, Karch; JHEP 2012]
related analysis [Ammon, Erdmenger, Lin, Muller, Shock; JHEP (2011)]

$$\Rightarrow \boxed{\omega = \sqrt{\frac{M^2-\mu^2}{M^2-3\mu^2}}\,k + \beta_3 k^2 + \ldots} \Rightarrow \quad \omega \approx k\sqrt{2\bar{\mu}/M}$$

Zero sound

(confirms Claim 2)

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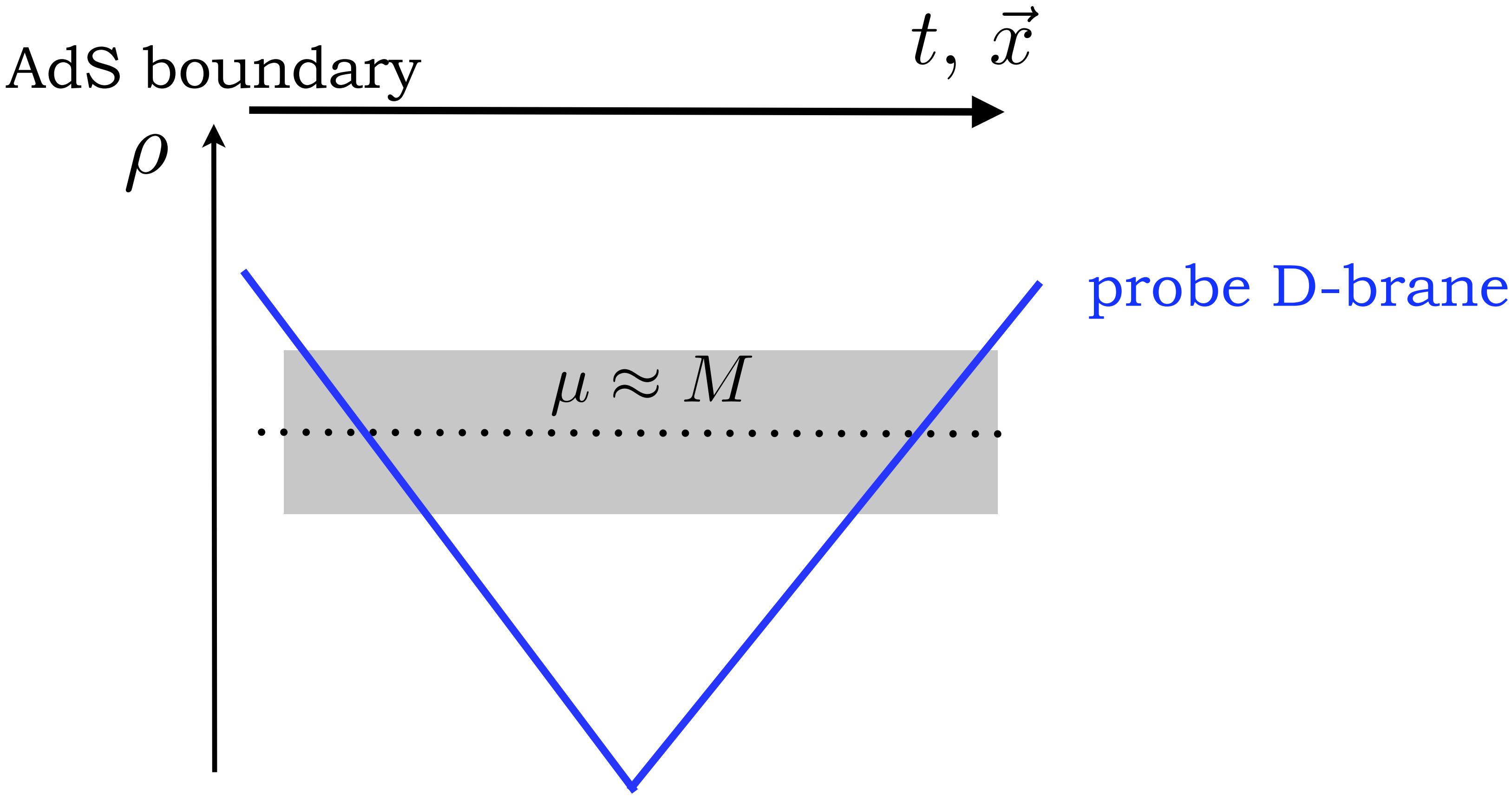
sound attenuation

Scaling emerges at intermediate scale



D3/D7 and D3/D5 result
$\theta = 1, \; z = 2$
$\alpha, \; \nu$

nonrelativistic region with hyperscaling violation near phase transition

Finite temperature

From critical phenomena

$$f \sim |\delta|^{2-\alpha} g\left(\frac{T}{|\delta|^{\nu z}}\right)$$

On D-branes

$$f(\mu, M, T) = f(\mu, M, T=0) + \pi d T + \mathcal{O}(T^2)$$

$$f_{non-rel.}(\mu, M, T) = \frac{1}{4}\mathcal{C}_3(\mu^2 - M^2)(3\mu^2 + M^2 + 4\pi\mu T - 4M\mu) + \mathcal{O}(T^2)$$

$$f_{non-rel.}(\bar{\mu}, M, T) = \mathcal{C}_3 M^2 \bar{\mu}^2 \left[1 + \frac{2\pi T}{\bar{\mu}}\right]$$

$$\Rightarrow \boxed{z\nu = 1\,, \qquad \alpha = 0}$$

$$\xi \sim \tau^{-\nu}$$

$$C \sim \tau^{-\alpha}$$

Conclusions

✓ hyperscaling violation from probe branes

✓ exact field theory known

➡ hidden Fermi surface; correlators, conductivity

➡ non-relativistic hydrodynamics

➡ non-equilibrium

➡ topological phases

Conclusions

✓ hyperscaling violation from probe branes

✓ exact field theory known

➡ hidden Fermi surface; correlators, conductivity
use techniques from [Ammon, Erdmenger, MK, O'Bannon; JHEP (2011)]

➡ non-relativistic hydrodynamics
in progress: [MK, Moroz, (...)]

➡ non-equilibrium

➡ topological phases

COMMERCIAL BREAK

1st Karl Schwarzschild Meeting KSM2013

July 22. - 26., 2013 at FIAS Frankfurt, Germany

http://fias.uni-frankfurt.de/ksm2013/

S. Gubser
A. Karch
R. Mann
R. Myers
L. Susskind
H. Verlinde
R. Wald
...

Weekend Meeting: QCD Phase Diagram & Holography

EUROPEAN SCIENCE FOUNDATION

July 27./28., 2013 at FIAS Frankfurt, Germany

http://fias.uni-frankfurt.de/holography/

S. Gubser *
A. Karch
K. Landsteiner *
...

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Helmholtz Graduate School for Hadron and Ion Research