Titles and Abstracts

Q & A Session

Description. The conference will bring together people with varying backgrounds. Also, some participants are rather young. As a consequence, concepts familiar to some and important in various talks can not be expected to be universally understood in the audience. We have therefore scheduled an informal Questions-and-Answers session where the more elementary questions (hopefully) get answered. Questions arising during the first two days of the conference can be passed on to Kai-Uwe Bux who will collect the questions. The organizers will then ask experts to volonteer answers during the Q&A session. So, if you have a question during a talk that you don't feel comfortable asking right away (maybe, its too broad), please contribute it for the Q&A session.

Mikhail Belolipetsky (IMPA, Brazil) *Arithmetic hyperbolic reflection groups*

Abstract. A group of isometries of the hyperbolic n-space is called a reflection group if it has a finite generating set which consists of reflections in hyperplanes. The study of hyperbolic reflection groups has a long and remarkable history going back to the papers of Makarov and Vinberg. In recent years there has been a wave of activity in this area which has led to a solution to the open question of the finiteness of these groups and to some quantitative results towards their classification. In the talk I will review the results of the last ten years and discuss the main open problems concerning arithmetic hyperbolic reflection groups.

Vladimir Chernousov (University of Alberta) *A finiteness theorem for the genus*

Abstract. In the talk we shall present some results related to the following conjecture:

Let G be an absolutely almost simple simply connected algebraic group over a finitely generated field K of characteristic zero (or of "good" characteristic relative to G). Then there exists a finite collection $G_1, \ldots, G_r$ of K-forms of G such that if H is a K-form of G having the same isomorphism classes of maximal K-tori as G, then H is K-isomorphic to one of the G_i's.

Ted Chinburg (University of Pennsylvania) *Generating arithmetic groups by small subgroups using Lefschetz Theorems*

Abstract. This talk will be about work with Matt Stover on using Lefschetz Theorems to generate large arithmetic groups by small arithmetic subgroups. One consequence is that the simple factors of the Albanese varieties of arithmetic quotients of the complex hyperbolic plane are isogenous to simple quotients of the Jacobians of arithmetic Fuchsian curves.

Brian Conrad (Stanford University) *Pseudo-reductive groups and their arithmetic applications* (mini-course)

Abstract. Natural arithmetic questions about linear algebraic groups over local and global function fields cannot be readily reduced to the case of reductive groups, and the notion of pseudo-reductivity (initiated by Borel and Tits in the late 1970's) provides a substitute for reductivity that

is better-suited to problems over imperfect fields. However, this is only useful if one can establish a rich structure theory for pseudo-reductive groups.

In these two lectures, I will present an overview of recent developments in the theory of pseudo-reductive groups that have led to a very satisfying structure theory in the general case (Cartan subgroups, root systems, rational conjugacy theorems), going far beyond the initial discoveries of Borel and Tits. We will illustrate the general theory with a variety of interesting examples and counterexamples, as well as discuss applications to problems whose formulation does not involve pseudo-reductivity, and give indications about the techniques of proof. This is joint work with O. Gabber and G. Prasad.

Vincent Emery (Stanford University) *Bounds for torsion homology of arithmetic groups*

Abstract. I will present results that give upper bounds for the torsion in homology of non-cocompact arithmetic lattices. And I will also explain how together with recent results of Calegari-Venkatesh this can be used to obtain upper bounds on K_2 of the ring of integers of number fields.

Lizhen Ji (University of Michigan) *Outer automorphism groups of free groups and tropical geometry*

Abstract. Motivated by results on locally symmetric spaces and interaction with moduli spaces of Riemann surfaces, I will discuss the moduli space of tropical curves and the tropical Jacobian varieties, and their application towards constructing complete invariant geodesic metrics on outer spaces associated with $\mathrm{Out}(F_n)$. I will also discuss a description of compactifications of the moduli space of tropical curves in terms of Satake compactifications of locally symmetric spaces.

Benjamin Klopsch (Royal Halloway University of London, UK) *Representation growth of irreducible lattices in higher rank semisimple groups*

Abstract. In my talk I will present and discuss joint work with N. Avni, U. Onn and C. Voll. Consider an arithmetic group $\Gamma = \mathbf{G}(O_S)$, where $\mathbf{G}$ is a connected, simply connected absolutely almost simple algebraic group defined over a number field K and O_S denotes the ring of S-integers of K for a finite set of places S. For every natural number n, let $R_n(\Gamma)$ denote the number of irreducible complex representations of Γ up to dimension n. By a result of Lubotzky and Martin, $R_n(\Gamma)$ grows at most polynomially in n if and only if Γ has the weak Congruence Subgroup Property. Our main result is that, if Γ has the weak Congruence Subgroup Property, then the precise degree of representation growth of Γ depends only on the absolute root system of the underlying algebraic group $\mathbf{G}$. This relates to a conjecture of Larsen and Lubotzky on the representation growth of irreducible lattices in higher rank semisimple groups.

Ralf Köhl (University of Gießen, Germany) *Towards an arithmetic Kac-Moody theory* (mini-course)

Abstract. Kac-Moody theory provides a rich pool of examples and counterexamples for geometric group theory: By work of Rémy, Kac-Moody groups over sufficiently large finite fields are examples of lattices in locally compact groups; affine Kac-Moody groups over finite fields are in fact S-arithmetic, with S consisting of two rational points of the projective line over a finite field, as earlier observed by Abramenko and in the tree-case by Serre. Non-affine two-spherical Kac-Moody groups over sufficiently large finite fields provided the first class of examples

of infinite finitely presented discrete groups that are both simple and Kazhdan, by a combination of work by Abramenko and Mühlherr, by Caprace and Rémy, and by Dymara and Januszkiewicz. Also, Kac-Moody groups over finite fields allowed Ershov to construct Golod-Shafarevich groups with Kazhdan's property (T), thus providing counterexamples to a conjecture of Zelmanov's. Furthermore, affine and non-affine two-spherical topological Kac-Moody groups over local fields are also known to be Kazhdan as observed by Hartnick and Köhl, thus providing one of the few known classes of examples of non-locally compact compactly generated topological Kazhdan groups.

In mathematical physics, non-affine Kac-Moody theory is the key mathematical theory underlying recent and current investigations in string theory and M-theory. For instance, geometric object called the Kac-Moody symmetric space of split real type E_{10} — i.e. the homogenenous space consisting of a topological split real Kac-Moody group of type E_{10} modulo the fixed-point subgroup with respect to a Cartan-Chevalley involution — is conjectured to describe the universe in timewise vicinity of a cosmological/spatial singularily, i.e., right after a big bang or just before a big crunch.

The purpose of this series of two talks is to provide an introduction to Kac-Moody theory, with a focus on topological split Kac-Moody groups over local fields and their S-arithmetic subgroups. A key obstruction is that non-spherical topological Kac-Moody groups over local fields are not locally compact and that the existence of a non-zero sigma-finite translation invariant Borel measure is unknown. Nevertheless, the theory of buildings and, in particular, the building-theoretic local-to-global principle allow one to apply the classical theory of semisimple Lie groups and their S-arithmetic subgroups to non-affine Kac-Moody theory over local fields. I intend to illustrate this via a discussion of the Kazhdan property of topological Kac-Moody groups over local fields and of rigidity properties of Kac-Moody groups over the integers, notably strong rigidity and superrigidity.

C. S. Rajan (Tata Institute of Fundamental Research, India) *Characteristic equivalence and commensurability of arithmetic lattices*

Abstract. Gopal Prasad and Rapinchuk defined a notion of weakly commensurable lattices in a semisimple group, and gave a classification of weakly commensurable Zariski dense subgroups. A motivation was to classify pairs of locally symmetric spaces isospectral with respect to the Laplacian on functions. For this, in higher ranks, they assume the validity of Schanuel's conjecture.

We observe that if we use the notion of representation equivalence of lattices, then Schanuel's conjecture can be avoided. Further, the results are applicable in a S-arithmetic setting.

We introduce a new relation 'characteristic equivalence' on the class of arithmetic lattices, stronger than weak commensurability. This simplifies the arguments used by Prasad and Rapinchuk to deduce commensurability type results.

Andrei Rapinchuk (University of Virginia) *On the congruence subgroup problem*

Abstract. I will survey the results pertaining to the congruence subgroup problem

Igor Rapinchuk (Yale University) *On the conjecture of Borel and Tits for abstract homomorphisms of algebraic groups*

Abstract. The conjecture of Borel-Tits (1973) states that if G and G' are algebraic groups defined over infinite fields k and k', respectively, with G semisimple and simply connected, then given any abstract representation $\rho\colon G(k) \to G'(k')$ with Zariski-dense image, there exists a commutative finite-dimensional k'-algebra B and a ring homomorphism $f\colon k \to B$ such that ρ can essentially be written as a composition $\sigma \circ F$, where $F\colon G(k) \to G(B)$ is the homomorphism induced by f and $\sigma\colon G(B) \to G'(k')$ is a morphism of algebraic groups. We prove this conjecture

in the case that G is either a universal Chevalley group of rank ≥ 2 or the group $\mathbf{SL}_{n,D}$, where D is a finite-dimensional central division algebra over a field of characteristic 0 and $n \geq 3$, and k' is an algebraically closed field of characteristic 0. In fact, we show, more generally, that if R is a commutative ring and G is a universal Chevalley-Demazure group scheme of rank ≥ 2, then abstract representations over algebraically closed fields of characteristic 0 of the elementary subgroup $E(R) \subset G(R)$ have the expected description. We also give applications to deformations of representations of $E(R)$.

Alan Reid (University of Texas at Austin) *All finite groups are involved in Mapping Class Groups*

Abstract. Let Γ_g denote the orientation-preserving Mapping Class Group of the genus $g \geq 1$ closed orientable surface. In this talk we will explain ideas in the proof that for fixed g, every finite group occurs as a quotient of a finite index subgroup of Γ_g.

Alireza Salehi Golsefidy (University of California, San Diego) *Expansion properties of linear groups*

Abstract. In this talk first I will define expanders and their connections with discrete subgroups of Lie groups. Then I will describe the recent approach toward this kind of problem (starting from a work of Bourgain and Gamburd) and give an almost survey of the recent results. I will finish by a more detailed argument in the case of square-free modulus or powers of primes.

Matthew Stover (University of Michigan) *Counting ends of rank one arithmetic orbifolds*

Abstract. Let X be a real rank one symmetric space. Given $k > 0$, does X admit a quotient with k ends? Examples are abound in low dimensions, e.g., punctured hyperbolic surfaces and hyperbolic link complements. I will explain why, for any $k > 0$ the arithmetic rank one orbifolds with k ends fall into finitely many commensurability classes. For example, one-ended arithmetic hyperbolic n-orbifolds do not exist for n greater than 30.

T. N. Venkataramana (Tata Institute of Fundamental Research, India) *Monodromy and arithmetic groups*

Abstract. We consider monodromy associated to differential equations of hypergeometric type over the thrice punctired projective line, and prove in many cases that the monodromy is an arithmetic subgroup of a symplectic group.

Christopher Voll (University of Bielefeld, Germany) *Representation growth of arithmetic groups*

Abstract. A central aim in this area is to understand the distributions of the numbers of irreducible, finite-dimensional representations of arithmetic groups. For important classes of such groups, Dirichlet generating functions are an appropriate tool to study these numbers. If, for instance, the groups in question satisfy the Congruence Subgroup Property, these representation zeta functions have Euler products, indexed by places of number fields. Powerful tools from geometric representation theory (viz. the Kirillov orbit method) and algebraic geometry (such as p-adic integration) are available to study both the individual factors and the analytic properties

of their Euler products. I will survey some of recent results on representation zeta functions, and explain their implications for representation growth of arithmetic groups.

Stefan Witzel (University of Münster, Germany) *Bredon-finiteness properties of solvable arithmetic groups*

Abstract. The homological finiteness properties of a group are closely related to free actions of that group. Bredon-finiteness properties relate in the same way to proper actions. I describe a family of of groups of separating examples for classical and Bredon-finiteness properties which extends groups studied by Herbert Abels and others. This is joint work with Martin Fluch.

Kevin Wortman (University of Utah) *Homological finiteness properties of arithmetic groups in positive characteristic* (mini-course)

Abstract. For the first talk: I'll talk about what is known for "positive" results about the finiteness properties of arithmetic groups over function fields, for example, I'll talk about which arithmetic groups are finitely generated, or finitely presented. The main result in this direction is the theorem of Bux-Kohl-Witzel which states that any arithmetic group, G, defined over a function field is of type F_n if n is less than Euclidean rank of G.

Second talk: I'll talk about what is known for negative results about the finiteness properties of arithmetic groups over function fields, for example, I'll talk about which arithmetic groups are known to have infinitely generated cohomology in some dimension. This problem has not been completely solved, and I'll explain which cases remain open.

Pavel Zalesski (University of Brasilia, Brazil) *Profinite topology on arithmetic groups*

Abstract. We shall start with a discussion of residual properties of groups and their interpretation in connection with the profinite completion. According to J.–P. Serre a group G is *good* if the cohomology groups of G and its profinite completion $\widehat{G}$ are naturally isomorphic on finite coefficients. The talk will be about residual properties, goodness, and the congruence kernel for arithmetic groups and groups of geometric nature.