

Evolution Equations of Physics, Fluids, and Geometry

Sep. 9 - 14, 2012

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday

- 16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30 Buffet Dinner, Sally Borden Building
20:00 Informal gathering in 2nd floor lounge, Corbett Hall
Beverages and a small assortment of snacks are available on a cash honor system.

Monday

- 7:00–9:00** Breakfast
9:00–9:15 Introduction and Welcome by BIRS Station Manager, TCPL
9:15–10:00 Edriss Titi, "On the Loss of Regularity for the Three-Dimensional Euler Equations"
10:00–10:30 Coffee Break
10:30–11:15 Gabriel Koch, "Blow-up of critical Besov norms at a Navier-Stokes singularity"
11:30–13:00 Lunch
13:00–14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–14:15 Group Photo; meet in foyer of TCPL (photograph will be taken outdoors).
14:15–15:00 Jeremie Szeftel, "On collapsing ring blow up solutions to the mass supercritical NLS"
15:00–15:30 Coffee Break
15:30–16:15 Nobu Kishimoto, "Construction of blow-up solutions for the Zakharov system on two dimensional torus"
16:25–17:10 Tristan Roy, "On Control Of Sobolev Norms Of Solutions Of Some Semilinear Wave Equations With Localized Data"
17:30–19:30 Dinner

Tuesday

- 7:00–9:00** Breakfast
9:00–9:45 Daniel Tataru, "The Maxwell-Klein-Gordon equations"
9:55–10:40 Ioan Bejenaru, "Global solutions for equivariant Schroedinger Maps"
10:40–11:00 Coffee Break
11:00–11:45 Paul Smith, "Low-regularity local wellposedness of Chern-Simons-Schroedinger for small data"
11:45–13:30 Lunch
14:00–14:45 Piotr Bizon, "Turbulent instability of anti-de Sitter space"
14:45–15:15 Coffee Break
15:15–16:00 Boris Ettinger, "Local well-posedness for the minimal surface equation in Minkowski space"
16:10–16:55 Arick Shao, "Null cones to infinity, curvature flux, and Bondi mass"
17:30–19:30 Dinner

Wednesday

- 7:00–9:00** Breakfast
9:00–9:45 Patrick Gerard, "Quasiperiodic solutions of the cubic Szegö equation"
9:55–10:40 Oana Pocovnicu, "Global behavior of the Gross-Pitaevskii equation and cubic-quintic nonlinear Schrödinger equations"
10:40–11:00 Coffee Break
11:00–11:45 Tadahiro Oh, "Invariant weighted Wiener measure for the derivative NLS"
11:45–13:30 Lunch
Afternoon Free
17:30–19:30 Dinner

Thursday

- 7:00–9:00** Breakfast
9:00–9:45 Vladimir Sverak, "On scale-invariant solutions of the Navier-Stokes equations"
9:55–10:40 Kay Kirkpatrick, "Bose-Einstein condensation, the NLS, and a central limit theorem"
10:40–11:00 Coffee Break
11:00–11:45 Magdalena Czubak, "Cosmic strings and abelian Higgs model"
11:45–13:30 Lunch
14:00–14:45 Dong Li, "Remarks on the quasi-geostrophic equation with supercritical dissipation"
14:45–15:15 Coffee Break
15:15–16:00 Pierre Germain, "A discrete to continuous spectrum limit for 2D NLS"
16:10–16:55 Anne-Sophie de Suzzoni, "On the propagation of weakly nonlinear random dispersive waves"
17:30–19:30 Dinner

Friday

- 7:00–9:00** Breakfast
9:00–11:30 Informal Discussions
11:30–13:30 Lunch
**Checkout by
12 noon.**

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Evolution Equations of Physics, Fluids, and Geometry

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ABSTRACTS (in alphabetic order by speaker surname)

Speaker: **Ioan Bejenaru** (University of California - San Diego)

Title: *Global solutions for equivariant Schroedinger Maps*

Abstract: I will discuss the problem of global well-posedness for equivariant Schroedinger Maps with energy below the natural threshold both in the focusing (maps to S^2) and defocusing case (maps to H^2).

Speaker: **Piotr Bizon** (Jagiellonian University)

Title: *Turbulent instability of anti-de Sitter space*

Abstract: Our recent joint work with Andrzej Rostworowski on nonlinear evolution of a weakly perturbed anti-de Sitter (AdS) space indicates that AdS is unstable under arbitrarily small perturbations. We conjectured that this instability is due to the turbulent transfer of energy from low to high frequencies until eventually a singularity forms. I will present numerical, analytical and heuristic evidence supporting this conjecture.

Speaker: **Magdalena Czubak** (Binghamton University)

Title: *Cosmic strings and abelian Higgs model*

Abstract: We give a rigorous description of the dynamics of the Nielsen-Olesen vortex line. In particular, given a worldsheet of a string, we construct initial data such that the corresponding solution of the abelian Higgs model will concentrate near the evolution of the string. Moreover, the constructed solution stays close to the Nielsen-Olesen vortex solution. This is a joint work with Robert Jerrard.

Speaker: **Anne-Sophie de Suzzoni** (Université de Cergy-Pontoise)

Title: *On the propagation of weakly nonlinear random dispersive waves*

Speaker: **Boris Ettinger** (University of California - Berkeley)

Title: *Local well-posedness for the minimal surface equation in Minkowski space*

Abstract: The minimal surface equation for timelike surfaces of the Minkowski space is a quasi-linear wave equation. The nonlinear part of the equation exhibits a cancellation known as the null condition. We replicate the strategy of Smith and Tataru of constructing a wave-packet parametrix, which coupled with a space-time estimates for the null form allows us to lower the regularity compared to the general result for the quasilinear wave equation obtained by Smith and Tataru.

Speaker: **Patrick Gerard** (Université Paris-Sud)

Title: *Quasiperiodic solutions of the cubic Szegő equation*

Abstract : The cubic Szegő equation is a 1D Hamiltonian evolution equation which arises as the first order resonant normal form of some nonlinear wave equation. I shall describe how the Lax pairs associated to this equation allow to prove that all the solutions of this equation with rational initial data are quasiperiodic and are therefore bounded in all Sobolev spaces.

Speaker: **Pierre Germain** (Courant Institute)

Title: *A discrete to continuous spectrum limit for 2D NLS*

Abstract: I will report on work in progress, in collaboration with Zaher Hani and Erwan Faou. We start from 2D NLS set on the torus, and show that, in a certain weakly nonlinear regime, one can derive an

asymptotic model set in the whole space. In other words, one passes from a discrete to a continuous spectrum. This seems to have implications to weak turbulence questions.

Speaker: **Kay Kirkpatrick** (University of Illinois)

Title: *Bose-Einstein condensation, the NLS, and a central limit theorem*

Abstract: Near absolute zero, a gas of quantum particles can condense into an unusual state of matter, called Bose-Einstein condensation (BEC), that behaves like a giant quantum particle. The rigorous connection has recently been made between the physics of the microscopic many-body dynamics and the mathematics of the macroscopic model, the cubic nonlinear Schrodinger equation (NLS). I'll discuss recent progress with Gerard Ben Arous and Benjamin Schlein on a central limit theorem for the quantum many-body systems, a step towards large deviations for Bose-Einstein condensation.

Speaker: **Nobu Kishimoto** (Kyoto University)

Title: *Construction of blow-up solutions for the Zakharov system on two dimensional torus*

Abstract: We consider the Zakharov system (Z) on two dimensional torus. Local well-posedness of the Cauchy problem in the energy space is already known, and it is combined with conservation laws and a sharp Gagliardo-Nirenberg inequality to imply global well-posedness for initial data with L^2 norm less than that of the ground state of the cubic NLS on \mathbb{R}^2 . We prove that the L^2 norm of the ground state is actually the threshold for global solvability, namely, that there exists a finite-time blow-up solution to (Z) on 2d torus with the L^2 norm greater than but arbitrarily close to that of the ground state. To this end, we localize the explicit blow-up solution of (Z) on the whole space, which was constructed by Glangetas- Merle (1994), and then solve the equation for perturbation. The same approach was taken by Ogawa-Tsutsumi (1990) and Burq-Gérard-Tzvetkov (2003) for the 1d quintic and 2d cubic NLS, respectively. However, in the case of (Z) several difficulties arise due to the loss of derivative in (Z) and the lack of exponential decay of Glangetas-Merle's blow-up solution away from the blow-up point. We exploit the finite propagation speed for the wave part in (Z) and introduce a modified energy to overcome these difficulties. This is joint work with Masaya Maeda (Tohoku University)

Speaker: **Gabriel Koch** (University of Sussex)

Title: *Blow-up of critical Besov norms at a Navier-Stokes singularity (with I. Gallagher and F. Planchon)*

Abstract: In this talk we describe a generalization of the result of Escauriaza-Seregin-Sverak on blow-up of the L^3 norm at a Navier-Stokes singularity by establishing the blow-up of any weaker critical Besov norm with finite third index as well. Following previous joint works with C. Kenig and with I. Gallagher and F. Planchon respectively, we use the "dispersive-type" method of concentration compactness and critical elements developed by C. Kenig and F. Merle.

Speaker: **Dong Li** (University of British Columbia)

Title: *Remarks on the quasi-geostrophic equation with supercritical dissipation*

Speaker: **Tadahiro Oh** (Princeton University)

Title: *Invariant weighted Wiener measure for the derivative NLS*

Abstract: In this talk, we consider the dynamics of derivative NLS (DNLS) on T. In particular, we construct a weighted Wiener measure, globally defined flow almost surely with respect to this measure, and finally invariance of the measure. Bourgain '94, (i) used a conservation law to construct invariant measures for the finite dimensional ODE flows and (ii) constructed almost sure global-in-time PDE flow and an invariant measure for the PDE (such as NLS, mKdV, Zakharov, etc.) For DNLS, the finite dimensional ODE approximations do not conserve (the finite dimensional analogue of) the energy. In place of (i), we show that such finite dimensional energies are "almost conserved" in the spirit of the I-method developed by Colliander-Keel-Staffilani-Takaoka-Tao, and hence that the corresponding finite dimensional measures are "almost invariant". As a result, we obtain an almost sure global flow and an invariant measure for DNLS. This is a joint work with A. Nahmod (UMASS), L. Rey-Bellet (UMASS), and G. Staffilani (MIT).

Speaker: **Oana Pocovnicu** (Princeton University)

Title: *Global behavior of the Gross-Pitaevskii equation and cubic-quintic nonlinear Schrödinger equations*

Abstract: In the first part of the talk we consider the Gross-Pitaevskii equation on R^4 with non-vanishing boundary condition at spatial infinity. By viewing it as a perturbation of the energy-critical Schrödinger equation, we prove that it is globally well-posed in its energy space. We present an analogous result for the cubic-quintic Schrödinger equation on R^3 with non-vanishing boundary condition. An extra difficulty is added by the fact that the energy is not sign definite. In the second part of the talk we discuss scattering for the cubic-quintic Schrödinger equation on R^3 (with vanishing boundary condition). The talk presents joint work with R. Killip, T. Oh, and M. Visan.

Speaker: **Tristan Roy** (Kyoto University)

Title: *On Control Of Sobolev Norms Of Solutions Of Some Semilinear Wave Equations With Localized Data*

Abstract: We establish new bounds of the Sobolev norms of solutions of some semilinear wave equations for data lying in the H^s , $s \geq 1$, closure of compactly supported data inside a ball of radius R , with R a positive number. In order to do that we perform an analysis in a neighborhood of the cone, using an almost Shatah-Struwe estimate, an almost conservation law and some estimates for localized functions: this allows to prove a decay estimate and establish an estimate of the low frequency component of the position of the solution. Then, in order to establish an estimate of the high frequency component of the solution and the velocity, we use this decay estimate and another almost conservation law.

Speaker: **Arick Shao** (University of Toronto)

Title: *Null cones to infinity, curvature flux, and Bondi mass*

Abstract: In this presentation, we revisit the problem, first tackled by S. Klainerman and I. Rodnianski, of controlling the geometry of null cones in an Einstein-vacuum spacetime by their curvature flux. These results can be viewed as low-regularity estimates, which are essential in establishing improved breakdown/continuation criteria for the Einstein equations and are also closely related to the L^2 -curvature conjecture. The first part of this current work involves simplifying and generalizing techniques for obtaining product and trace estimates on evolving geometries with low regularity. Next, we examine a variation of the above null cone problem, but with geodesically foliated null cones extending "toward infinity". Finally, an ongoing project is to connect the above estimates to the notion of Bondi mass in general relativity. Part of this work is joint with Spyridon Alexakis.

Speaker: **Paul Smith** (University of California - Berkeley)

Title: *Low-regularity local wellposedness of Chern-Simons-Schroedinger for small data*

Abstract: We consider the initial value problem for the Chern-Simons-Schroedinger model in two space dimensions. This is a covariant NLS-type problem which is L^2 critical. For this equation we introduce a so-called heat gauge, and prove that, with respect to this gauge, the problem is locally well-posed for initial data that is small in H^s , $s > 0$.

Speaker: **Vladimir Sverak** (University of Minnesota)

Title: *On scale-invariant solutions of the Navier-Stokes equations*

Abstract: We consider the Cauchy problem for the 3d incompressible, Navier-Stokes equations and prove that for any scale-invariant initial data which is Holder continuous away from the origin the Cauchy problem has a classical solution. We will motivate a conjecture that these solutions are not unique and will also discuss possible implications of the conjecture for the uniqueness of the Leray-Hopf solutions.

Speaker: **Jeremie Szeftel** (Ecole Normal Supérieure)

Title: *On collapsing ring blow up solutions to the mass supercritical NLS*

Abstract: We consider the nonlinear Schrödinger equation in dimension larger than 2 and in the mass super critical and energy subcritical range. For initial data in the energy space and with radial symmetry,

we prove a universal upper bound on the blow up speed. We then prove that this bound is sharp and attained on a family of collapsing ring blow up solutions first formally predicted in a work by Fibich et al. This is joint work with Frank Merle and Pierre Raphael.

Speaker: **Daniel Tataru** (University of California - Berkeley)

Title: *The Maxwell-Klein-Gordon equations*

Speaker: **Edriss Titi** (University of California - Irvine and The Weizmann Institute of Science)

Title: *On the Loss of Regularity for the Three-Dimensional Euler Equations*

Abstract: A basic example of shear flow was introduced by DiPerna and Majda to study the weak limit of oscillatory solutions of the Euler equations of incompressible ideal fluids. In particular, they proved by means of this example that weak limit of solutions of Euler equations may, in some cases, fail to be a solution of Euler equations. We use this shear flow example to provide non-generic, yet nontrivial, examples concerning the immediate loss of smoothness and ill-posedness of solutions of the three-dimensional Euler equations, for initial data that do not belong to $C^{1,\alpha}$. Moreover, we show by means of this shear flow example the existence of weak solutions for the three-dimensional Euler equations with vorticity that is having a nontrivial density concentrated on non-smooth surface. This is very different from what has been proven for the two-dimensional Kelvin-Helmholtz problem where a minimal regularity implies the real analyticity of the interface. Eventually, we use this shear flow to provide explicit examples of non-regular solutions of the three-dimensional Euler equations that conserve the energy, an issue which is related to the Onsager conjecture. In addition, we will use this shear flow to provide a nontrivial example for the use of vanishing viscosity limit, of the Navier-Stokes solutions, as a selection principle for uniqueness of weak solutions of the 3D Euler equations.