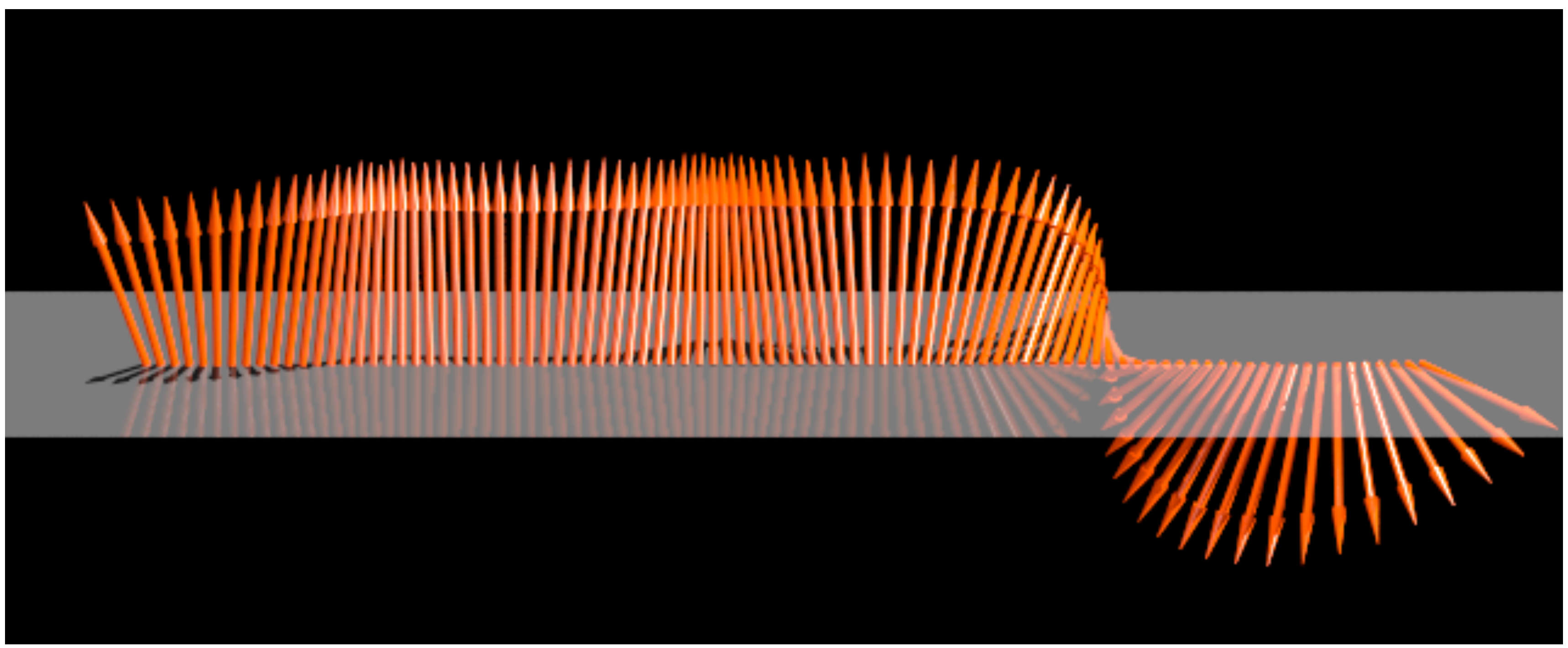
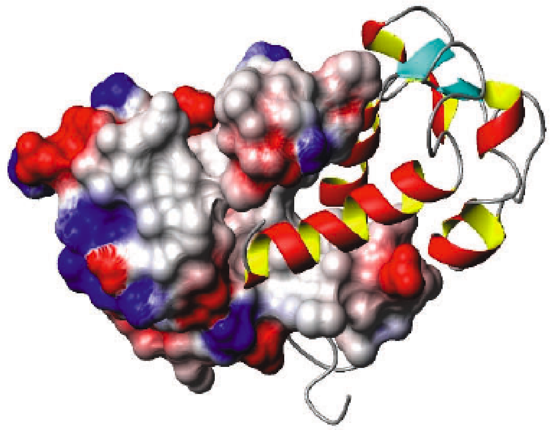
Spectroscopy & beyond
Optimal control of uncoupled and coupled spins

Steffen Glaser, TU München



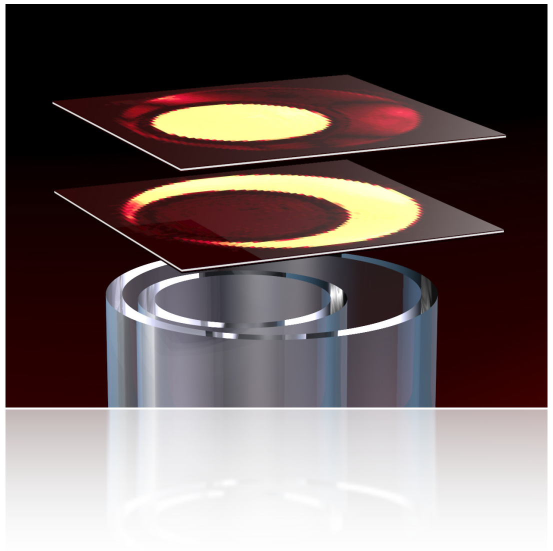
Optimal Control in Spin Systems

physical limits of spin dynamics



spectroscopy

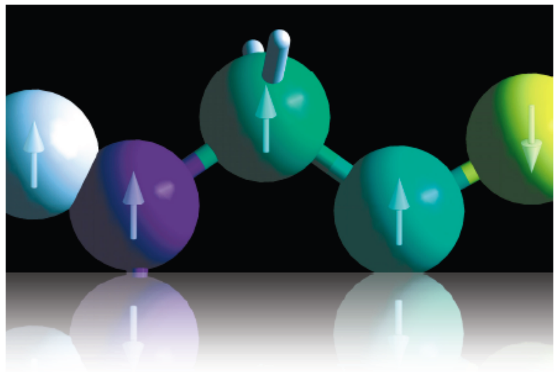
imaging



quantum computing

metrology

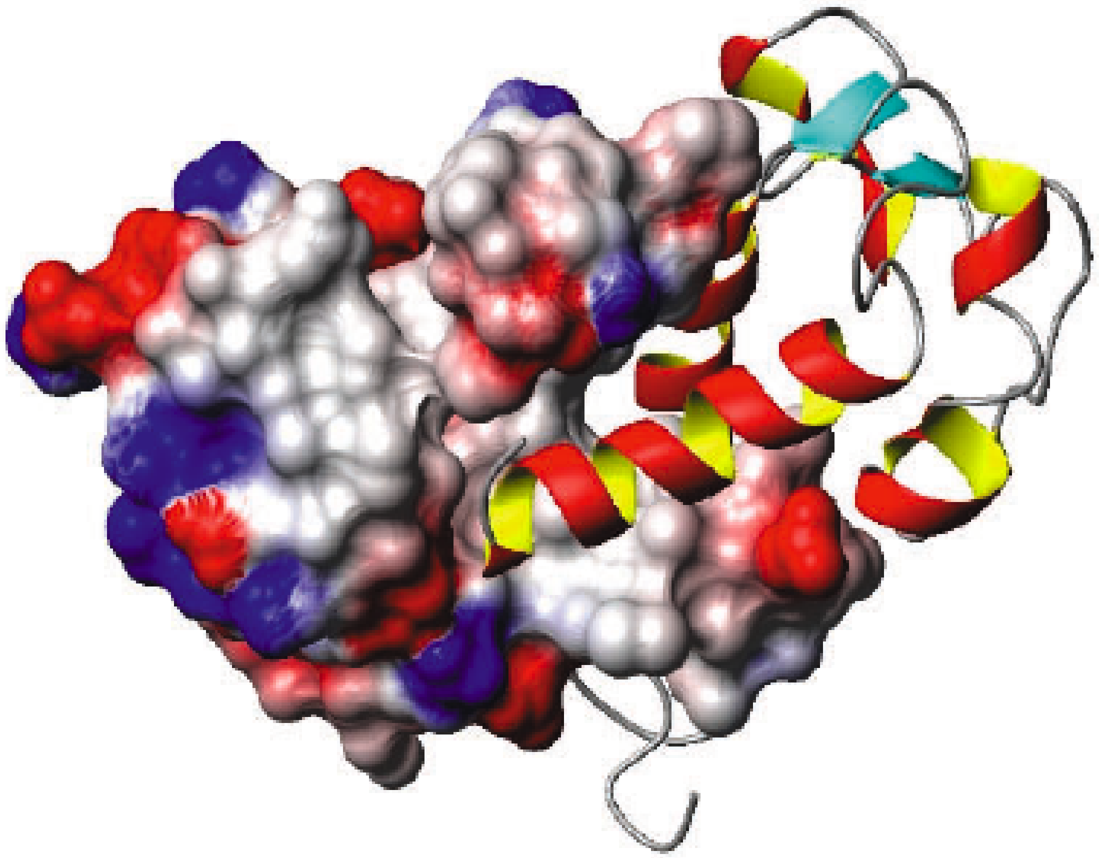
hyperpolarization

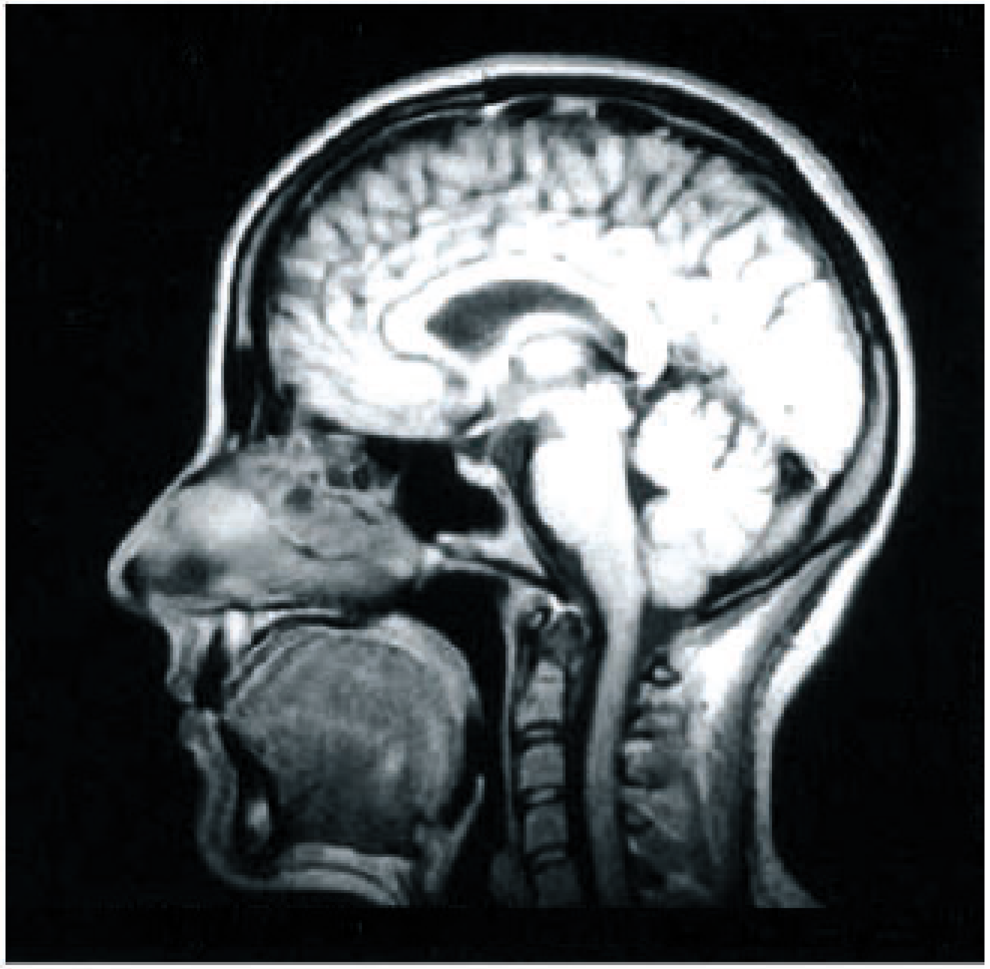


robust pulses

cooperative pulses

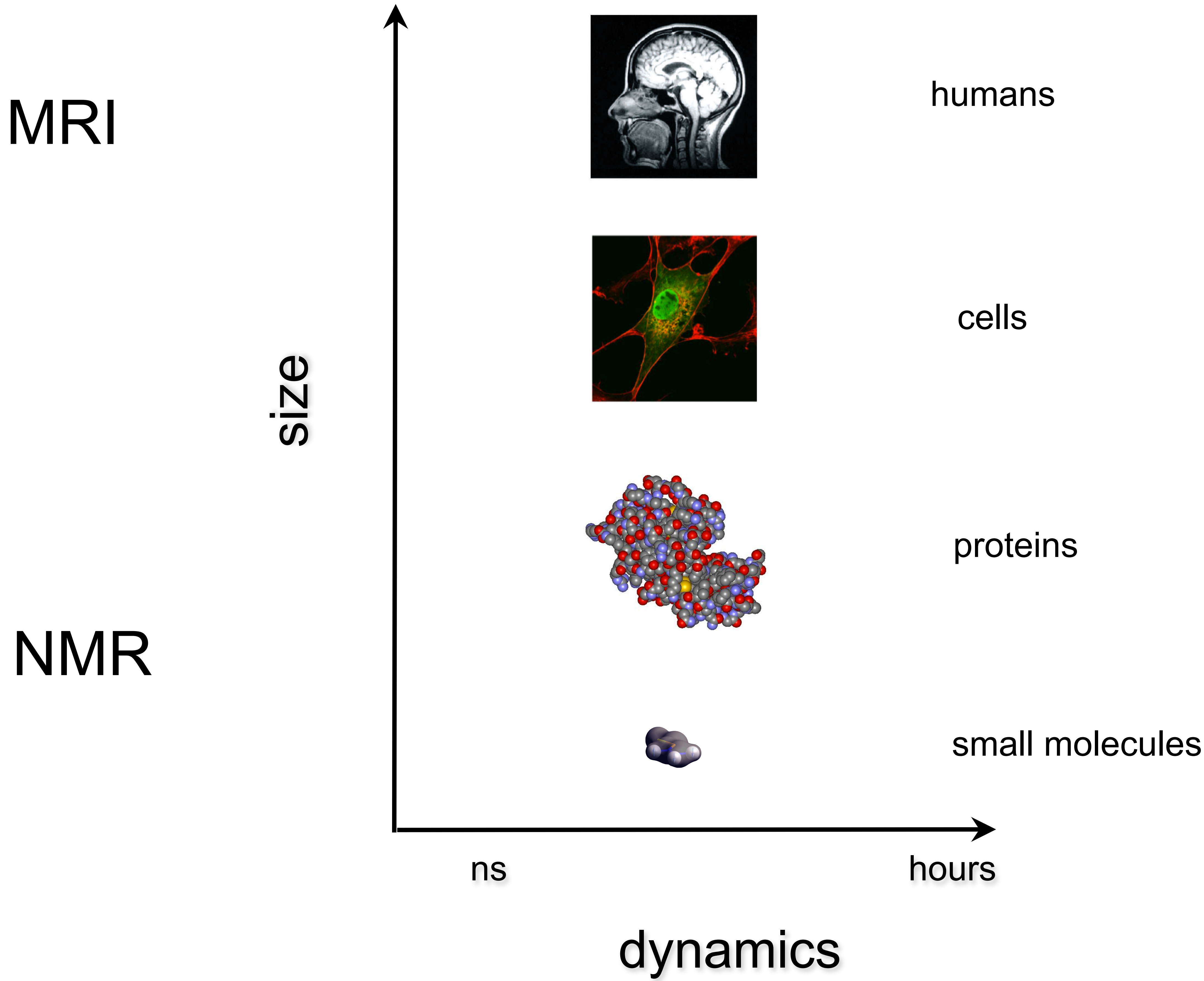
decoupling





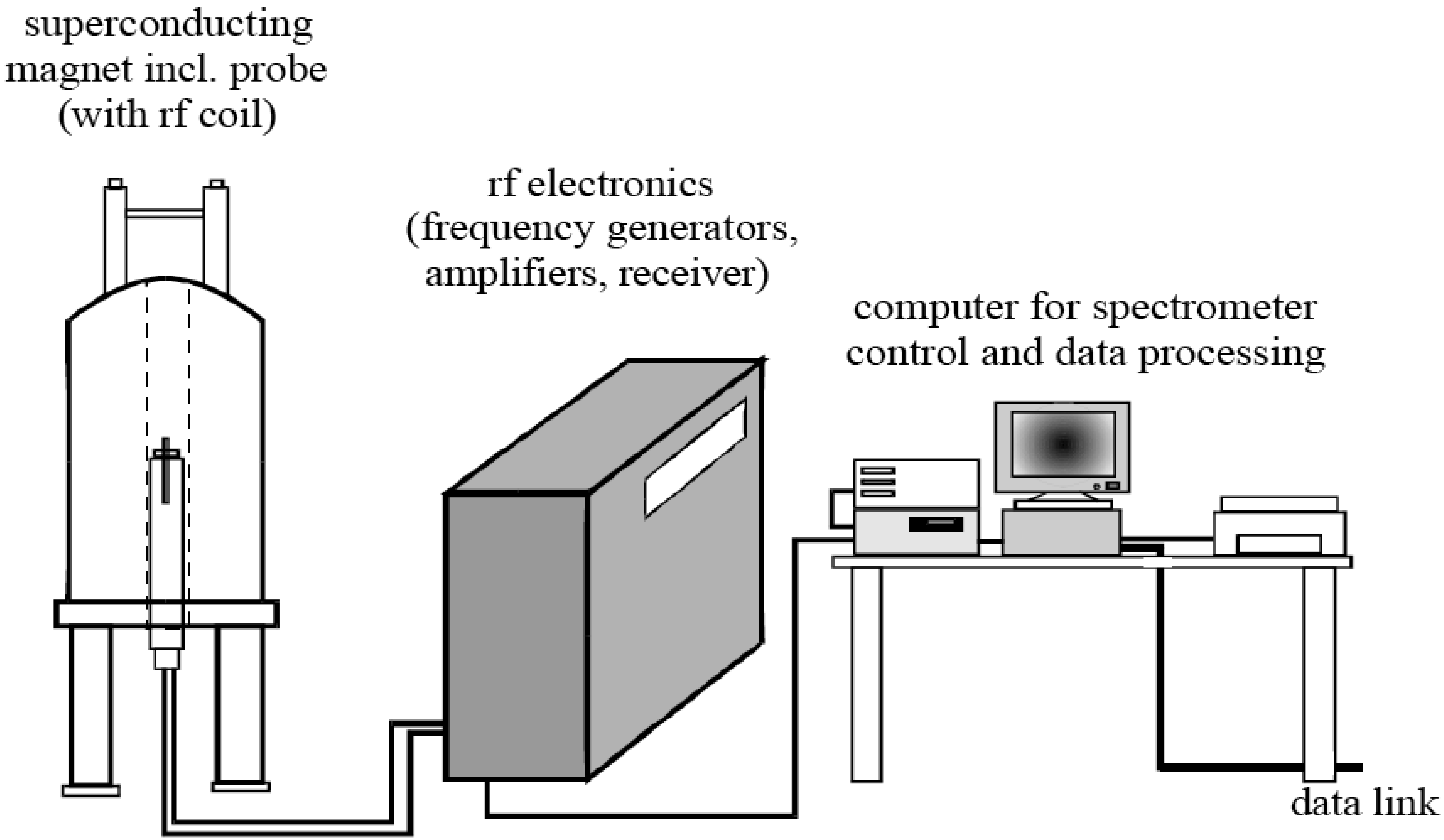
Nobel Prizes:

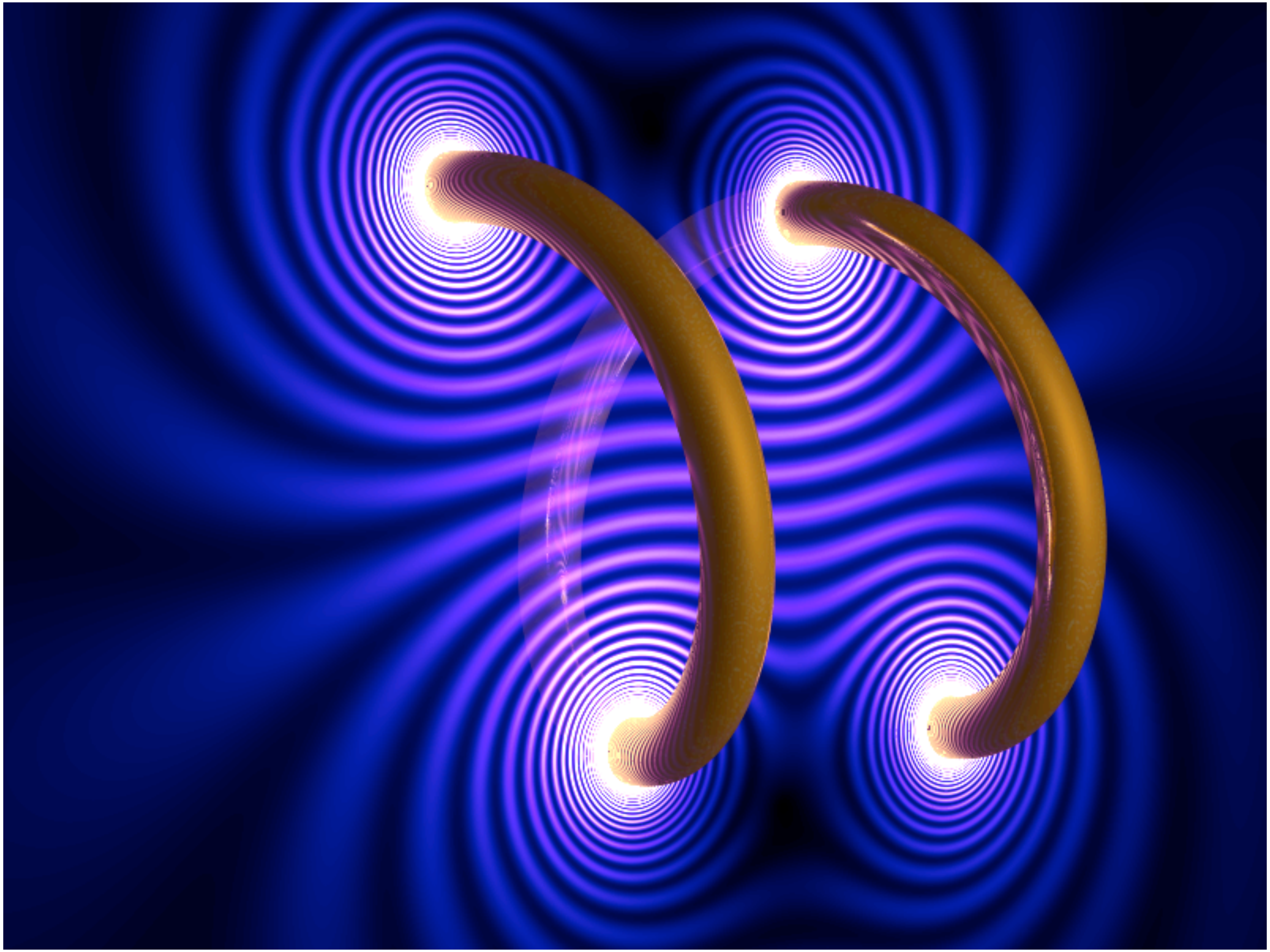
1952: Edward Purcell, Felix Bloch (Physics)
1991: Richard Ernst (Chemistry)
2002: Kurt Wüthrich (Chemistry)
2003: Paul Lauterbur, Peter Mansfield (Medicine)

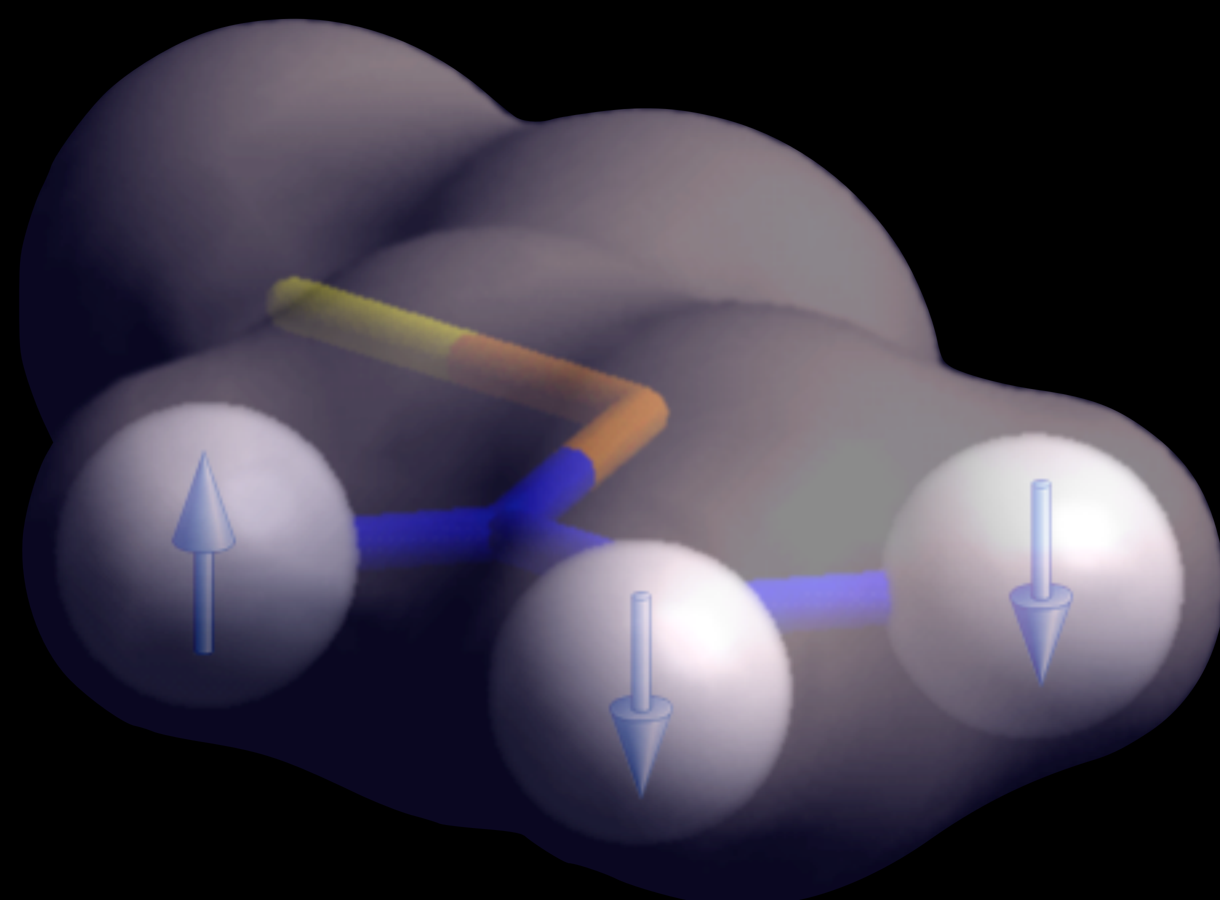

MRI
NMR
size
humans
cells
proteins
small molecules
ns
hours
dynamics

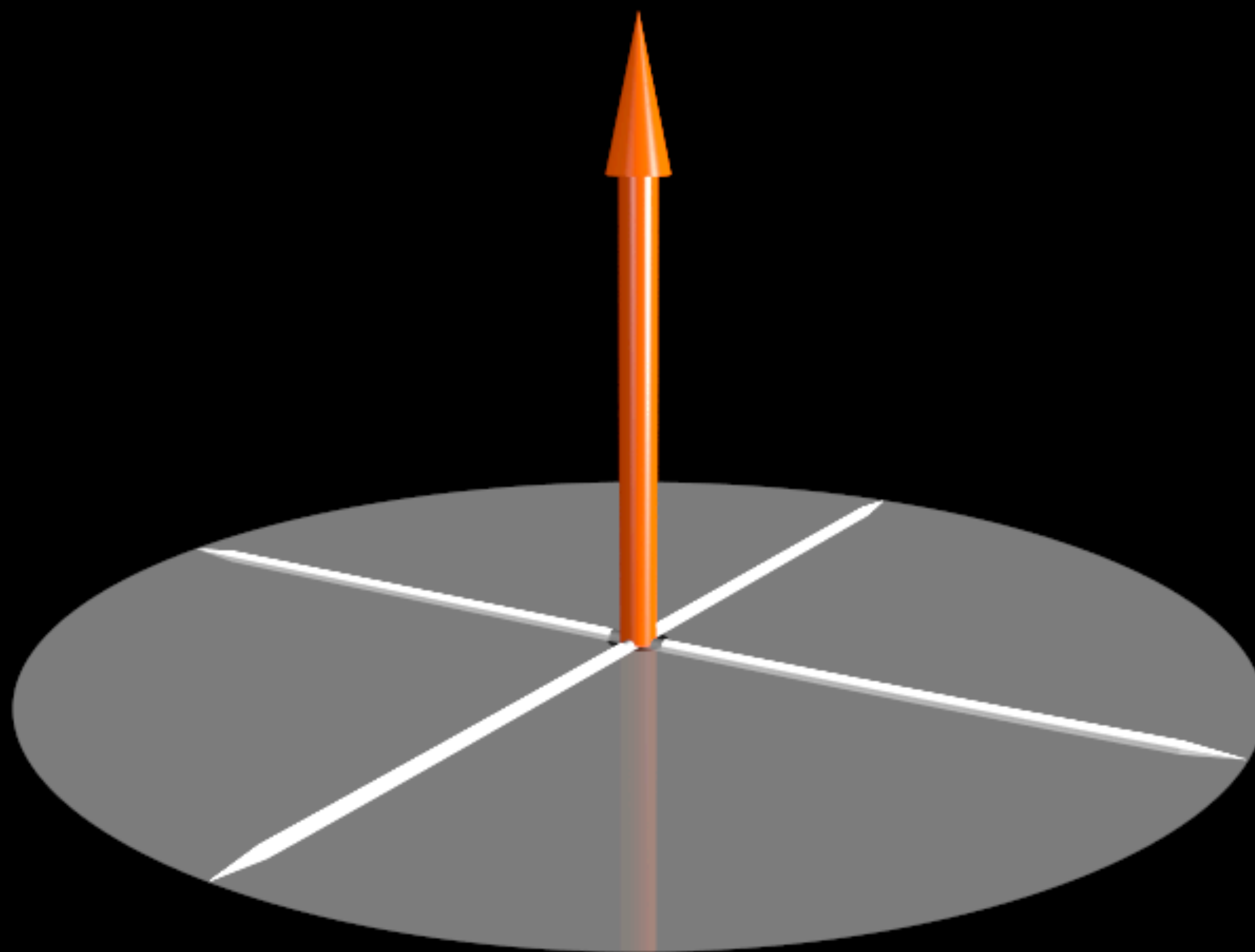

BRUKER
900
BRUKER

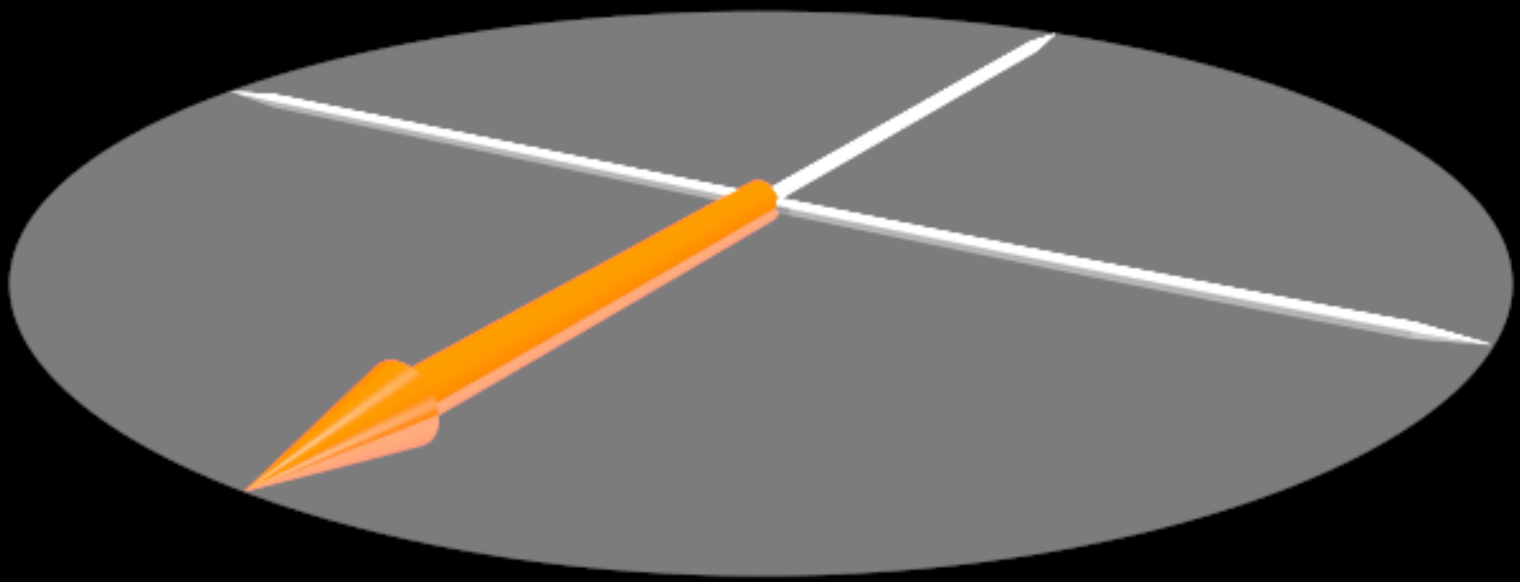
How do you measure an NMR signal?

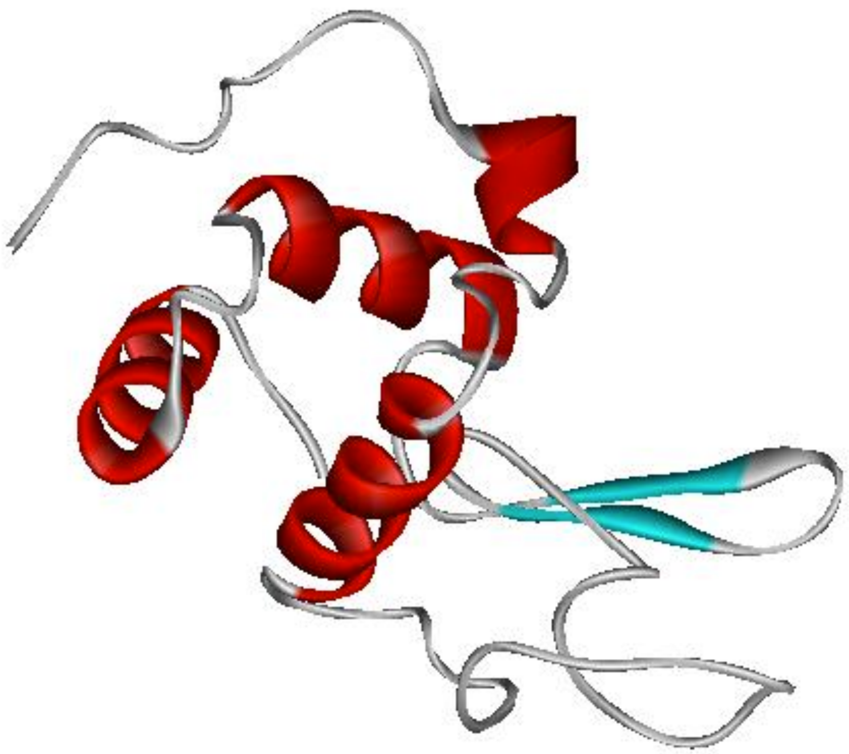


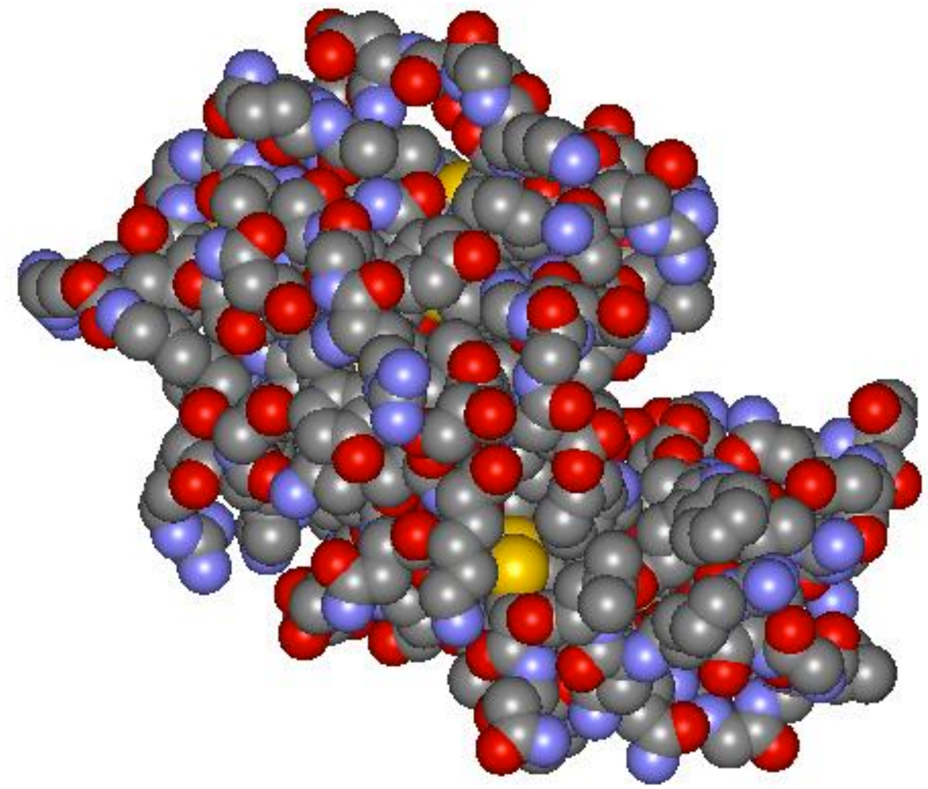


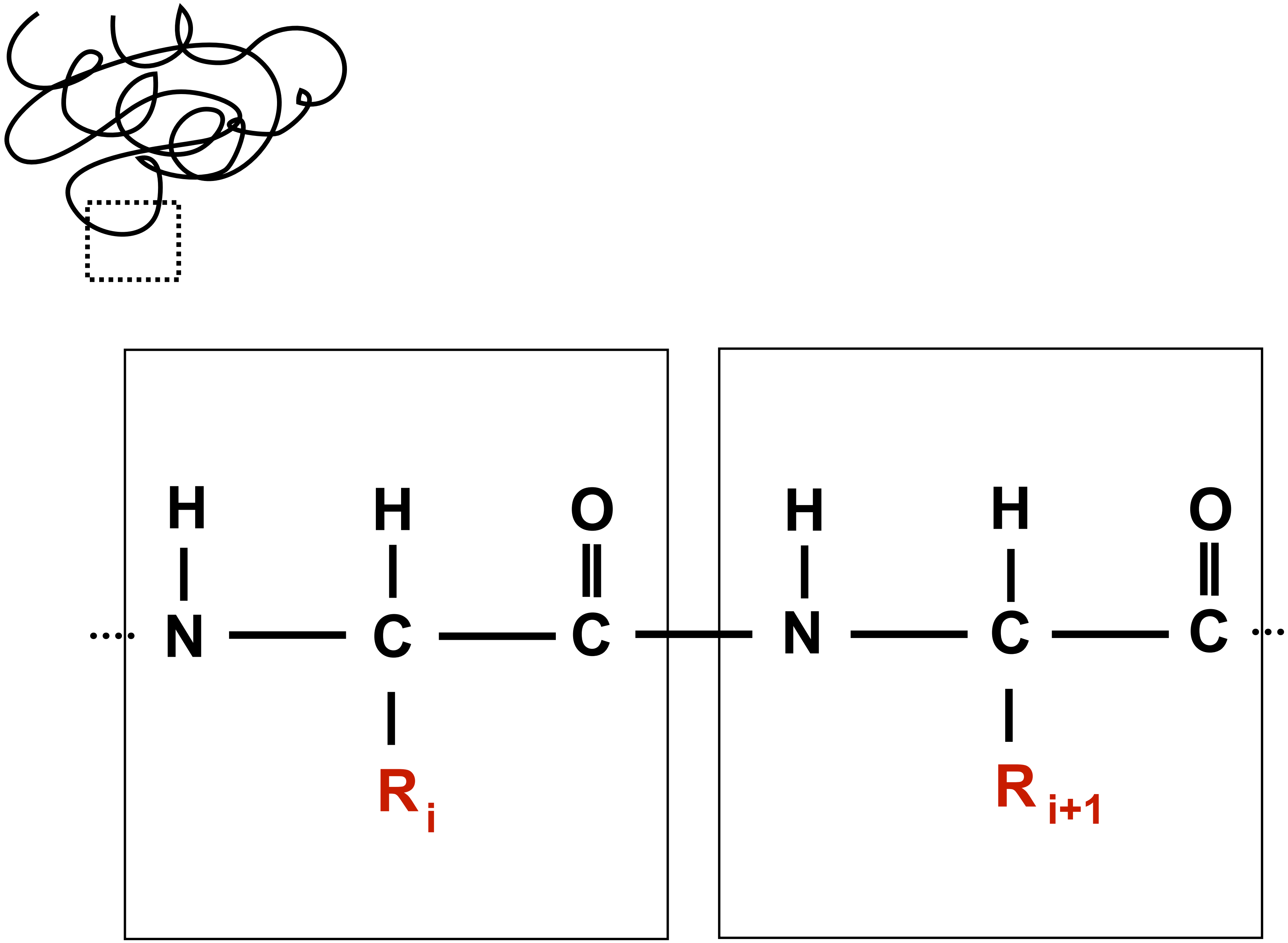




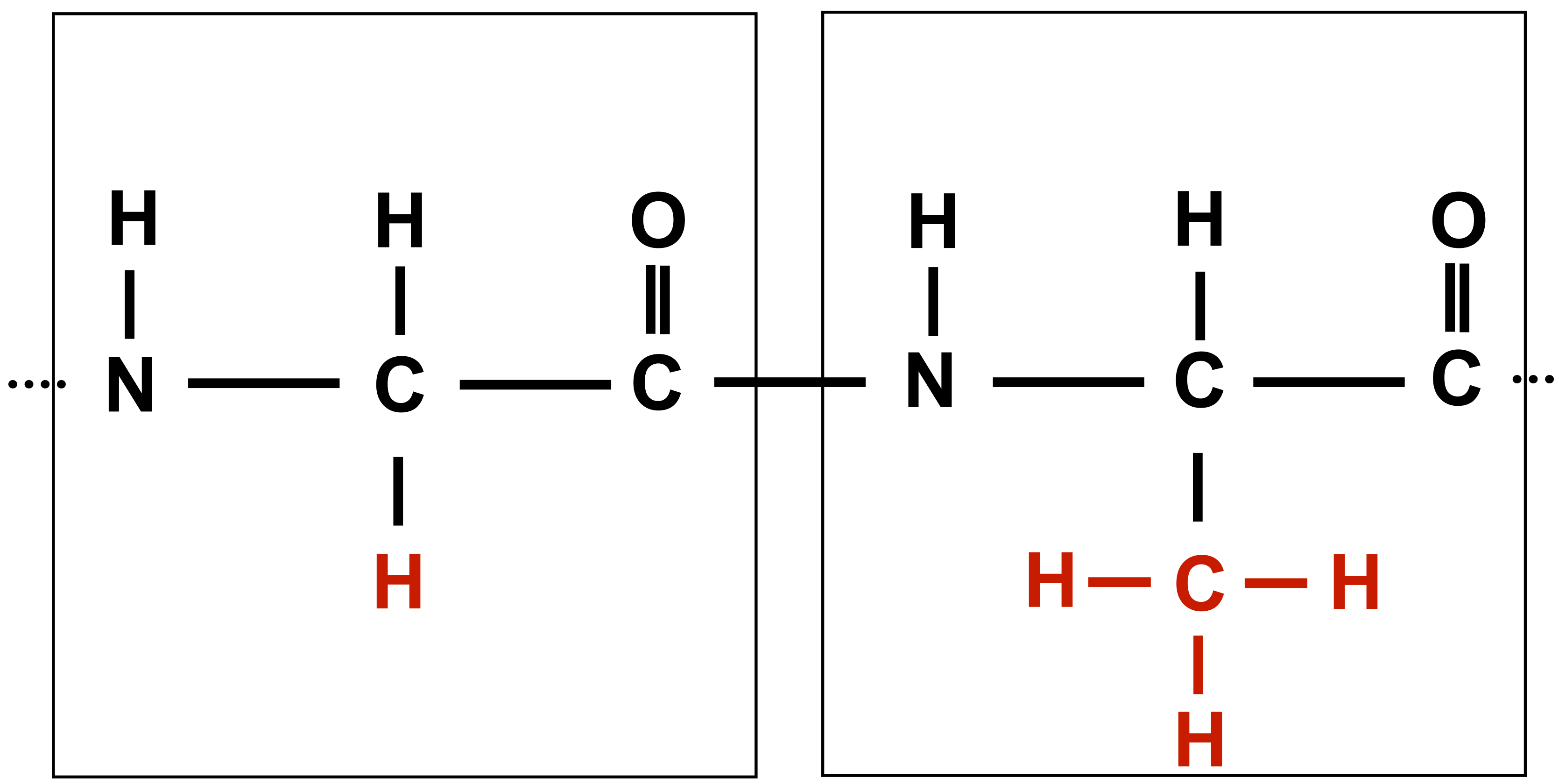






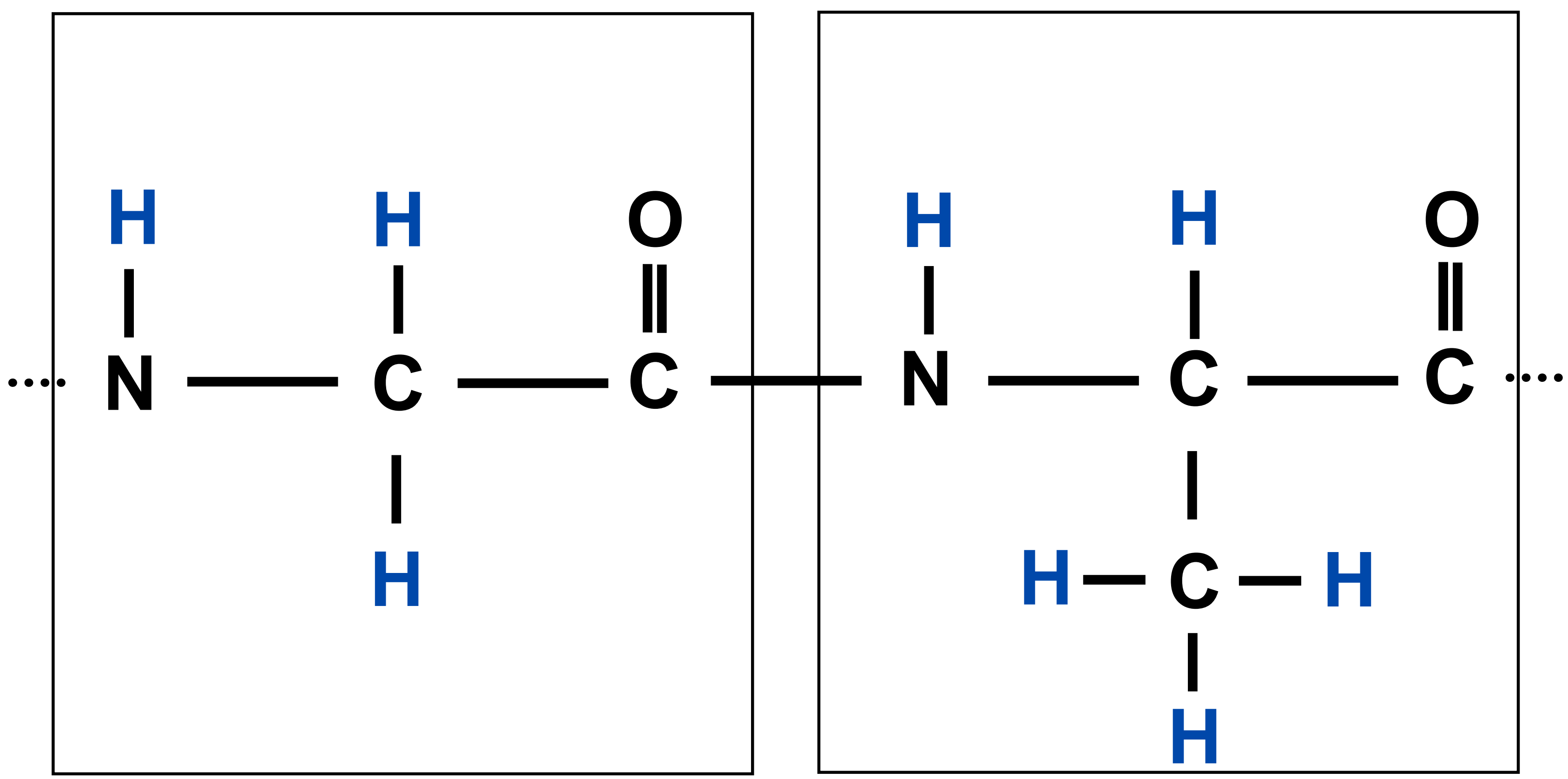

H H O H H O
N C C N C C
R i
R i+1

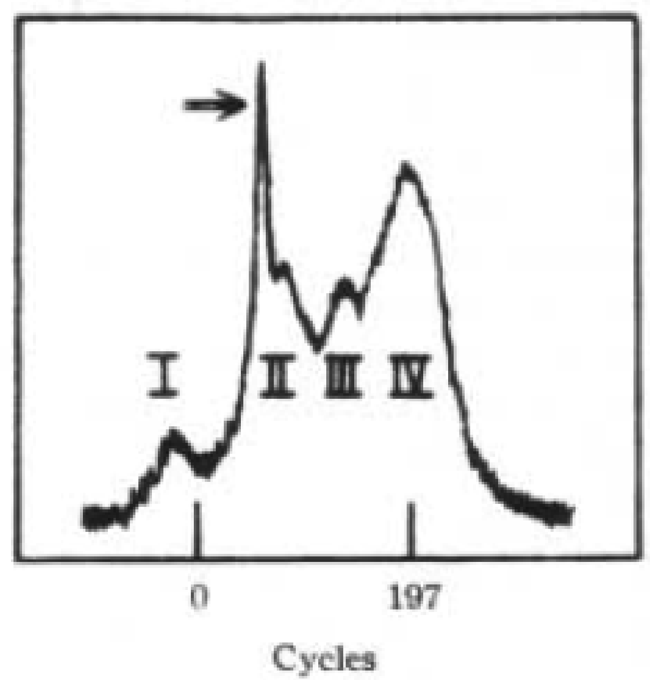
Glycine Alanine



Resonance frequencies at 14 Tesla: ^{1}H 600 MHz

chemical shift range: ± 3 kHz



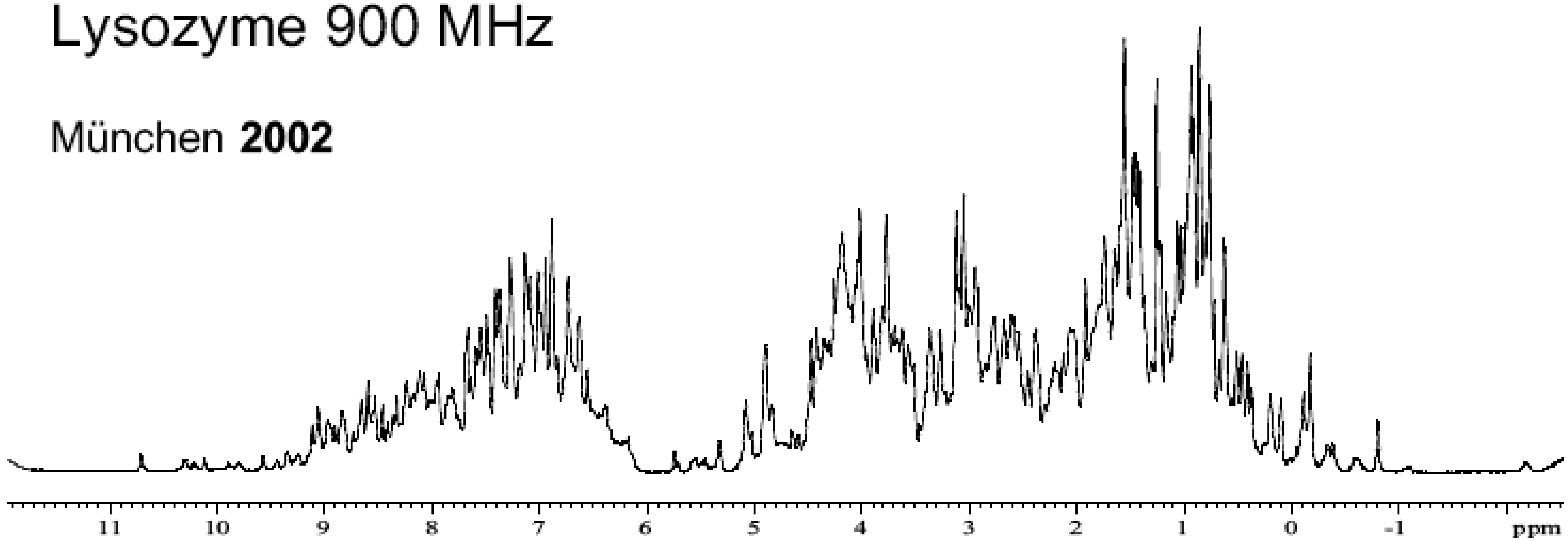


Ribonuclease
40 MHz

M. Saunders et al.
J.Amer.Chem.Soc. **1957**, *79*, 3289

Lysozyme 900 MHz

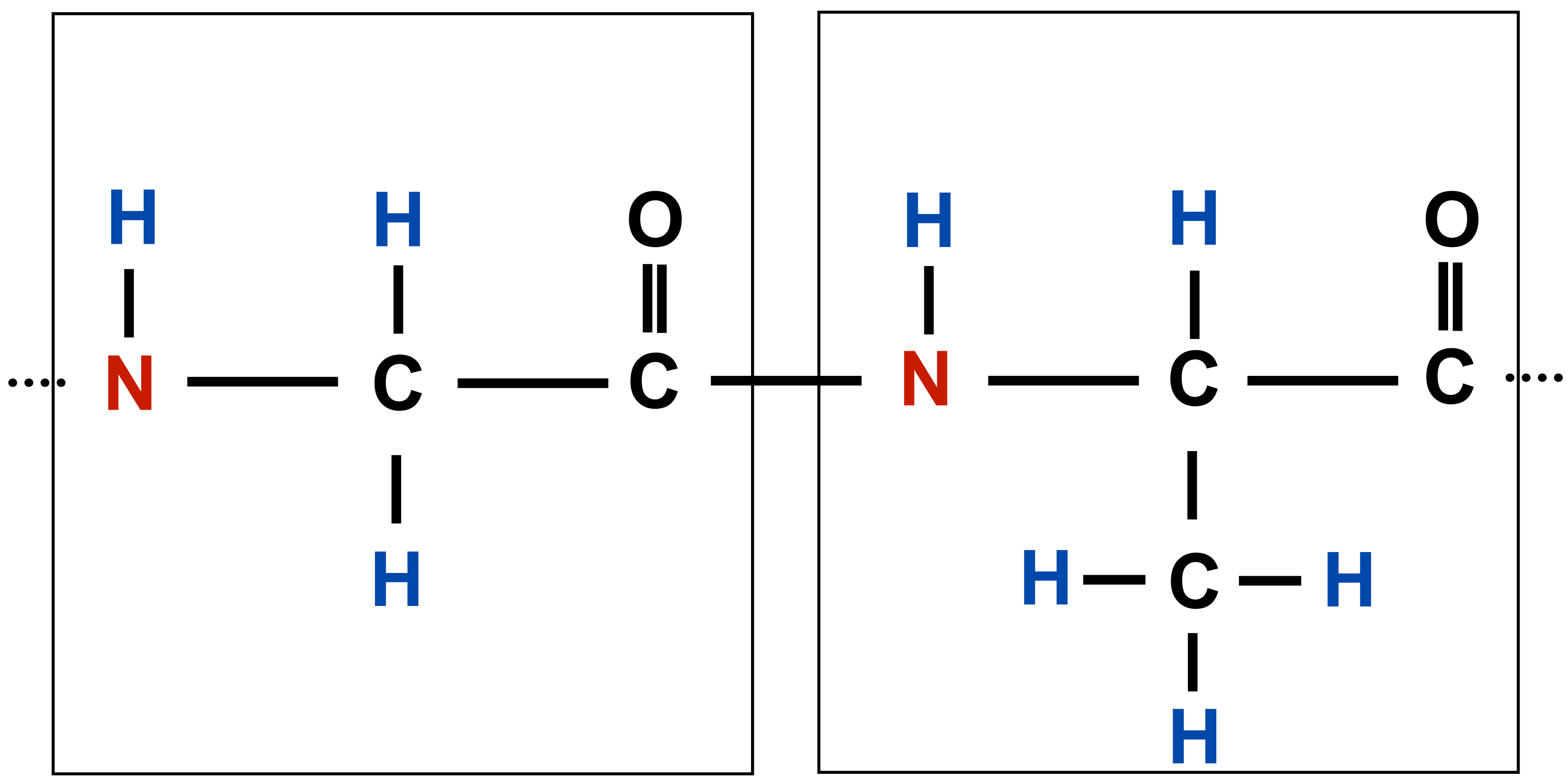
München **2002**



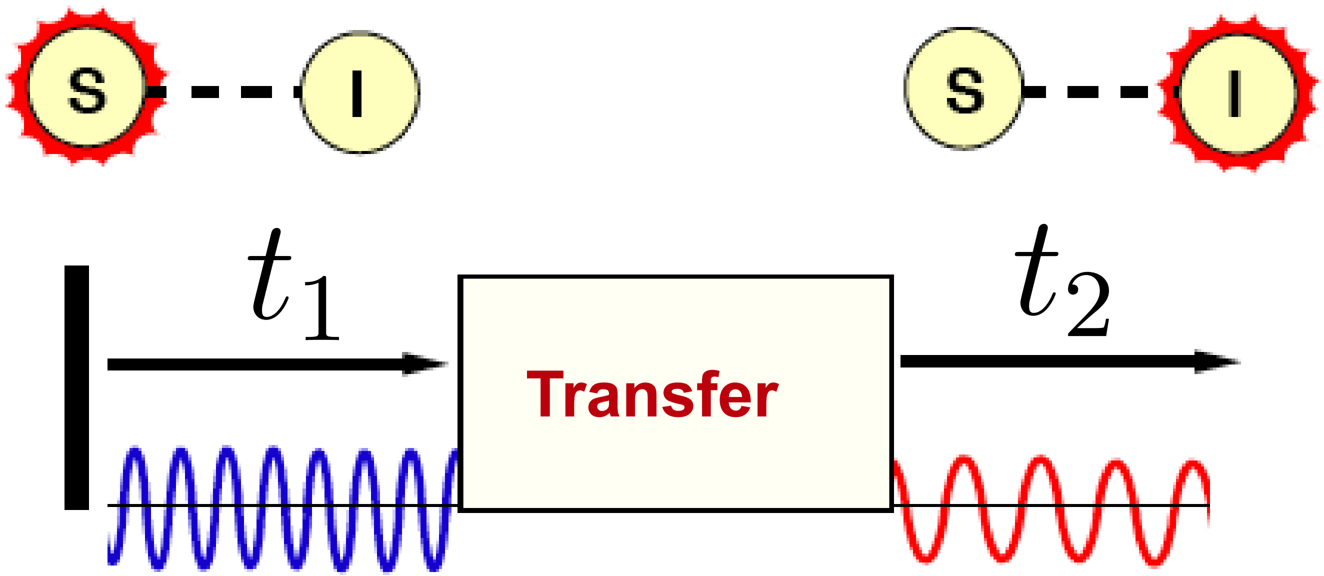
frequency dispersion: 10 kHz

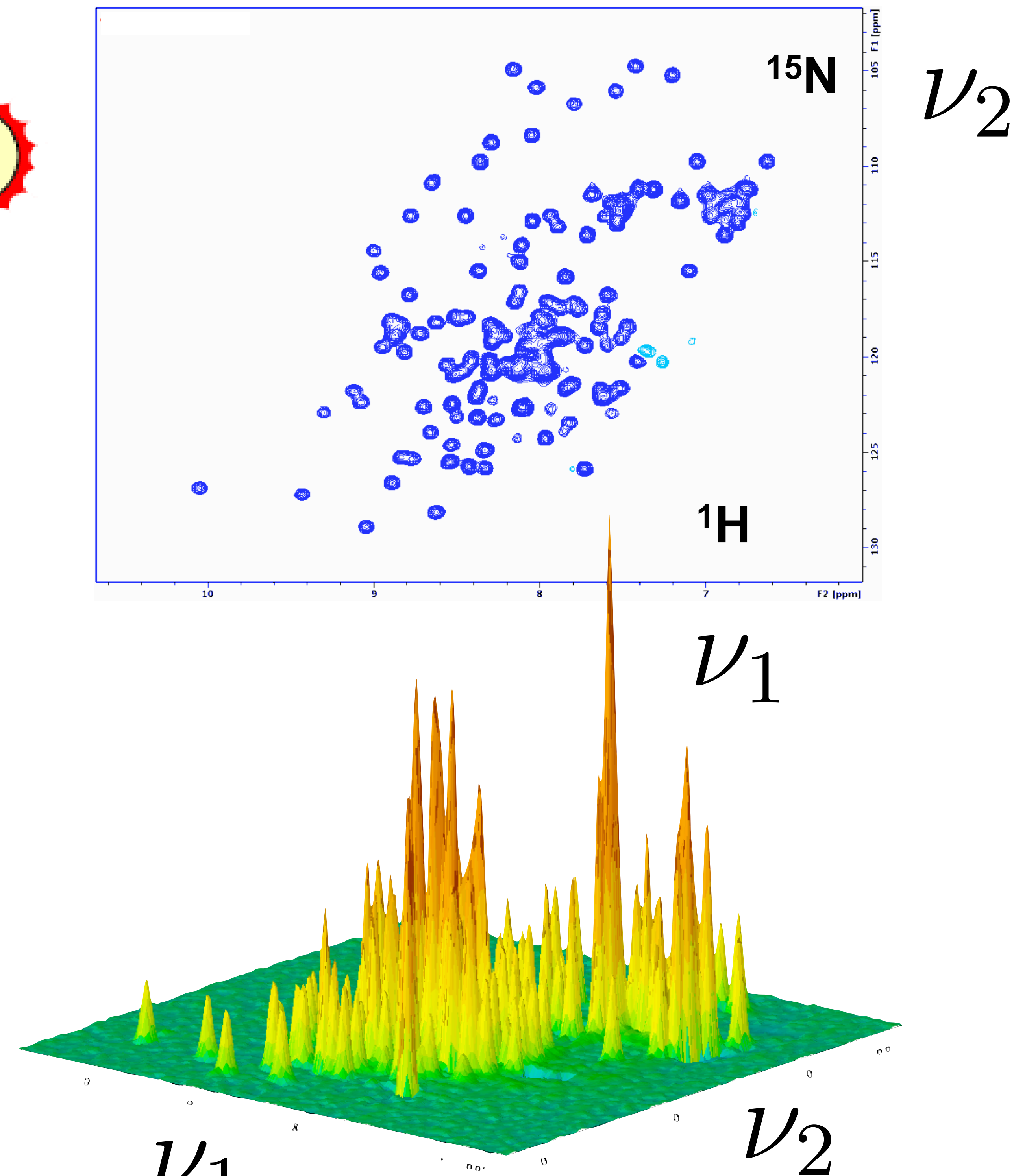
Resonance frequencies at 14 Tesla:

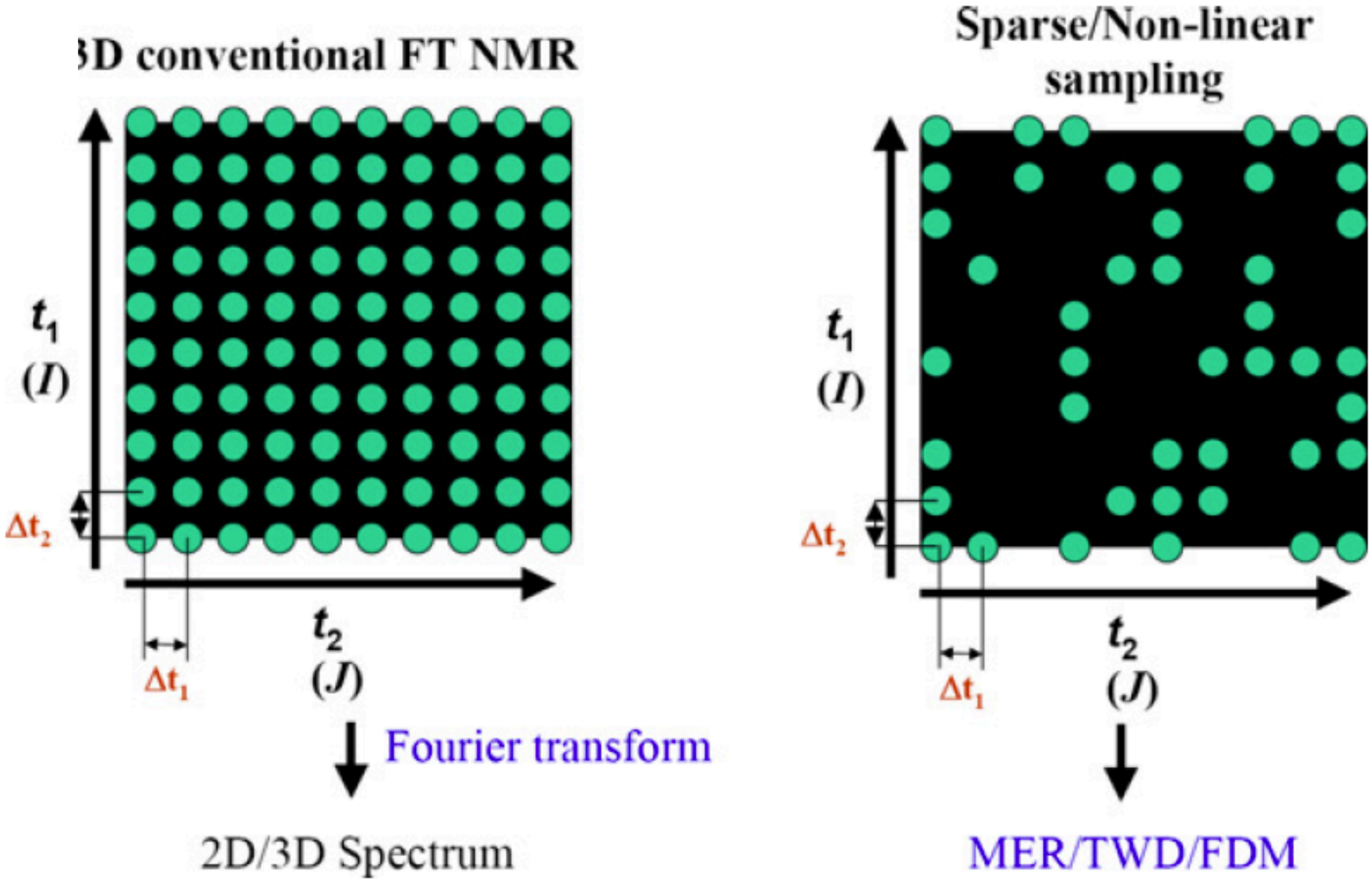
^{1}H	600 MHz
^{15}N	60 MHz



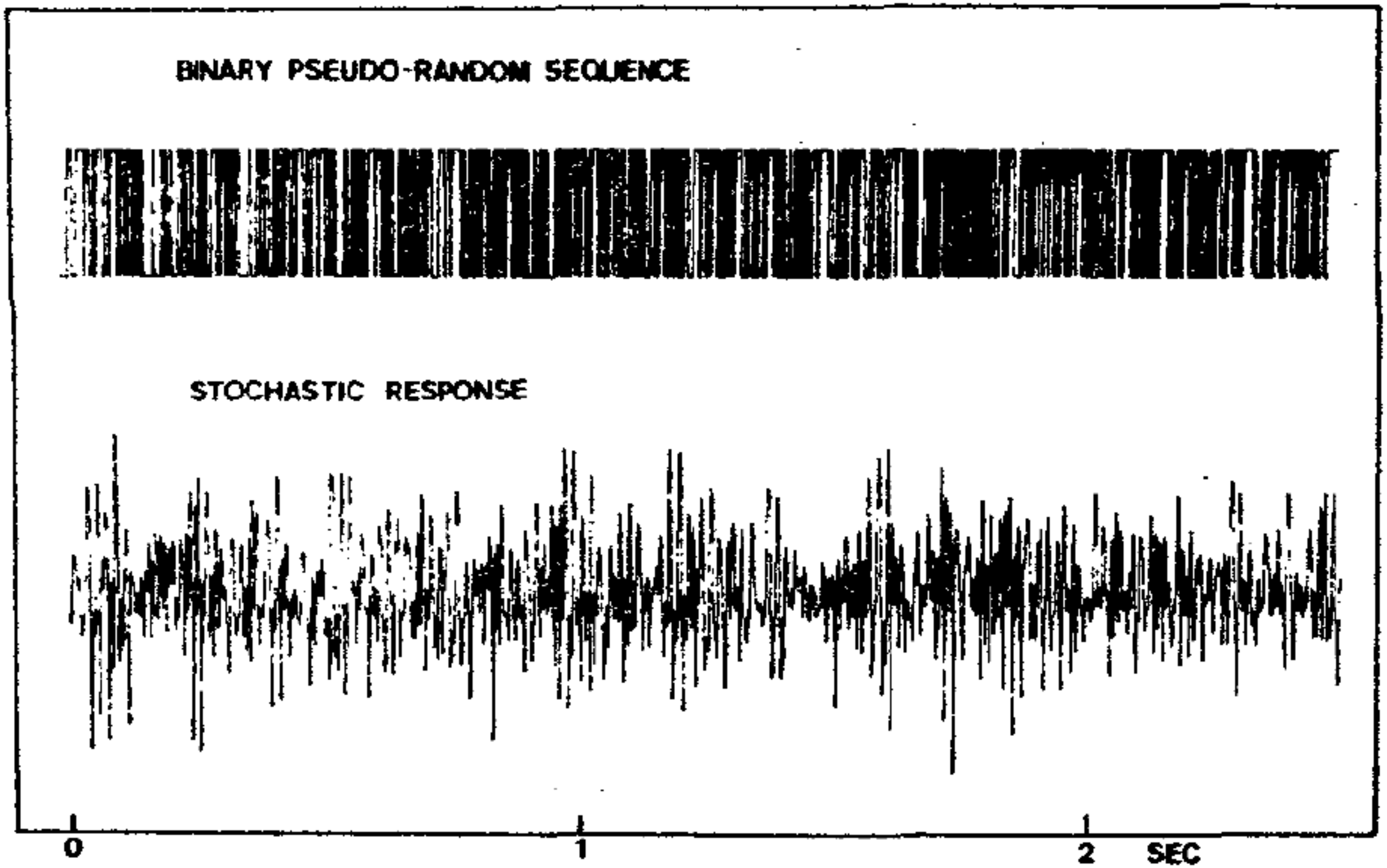
Two-dimensional spectroscopy






3D conventional FT NMR
t1
(I)
Δt2
Δt1
t2
(J)
Fourier transform
2D/3D Spectrum
Sparse/Non-linear
sampling
t1
(I)
Δt2
Δt1
t2
(J)
MER/TWD/FDM

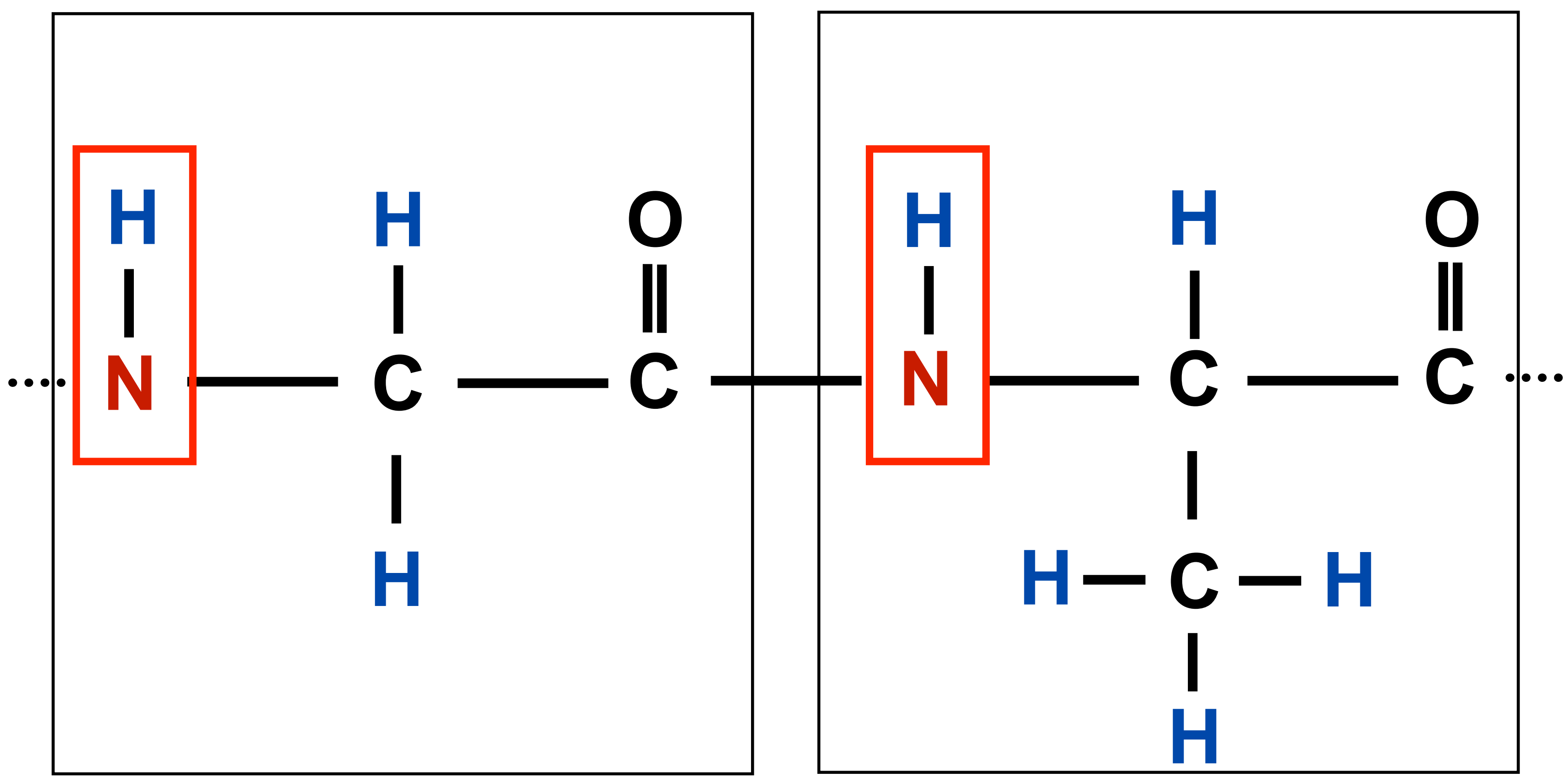
H. S. Atreya



H. Primas, *Helv. Phys. Acta* **34**, 36 (1961)

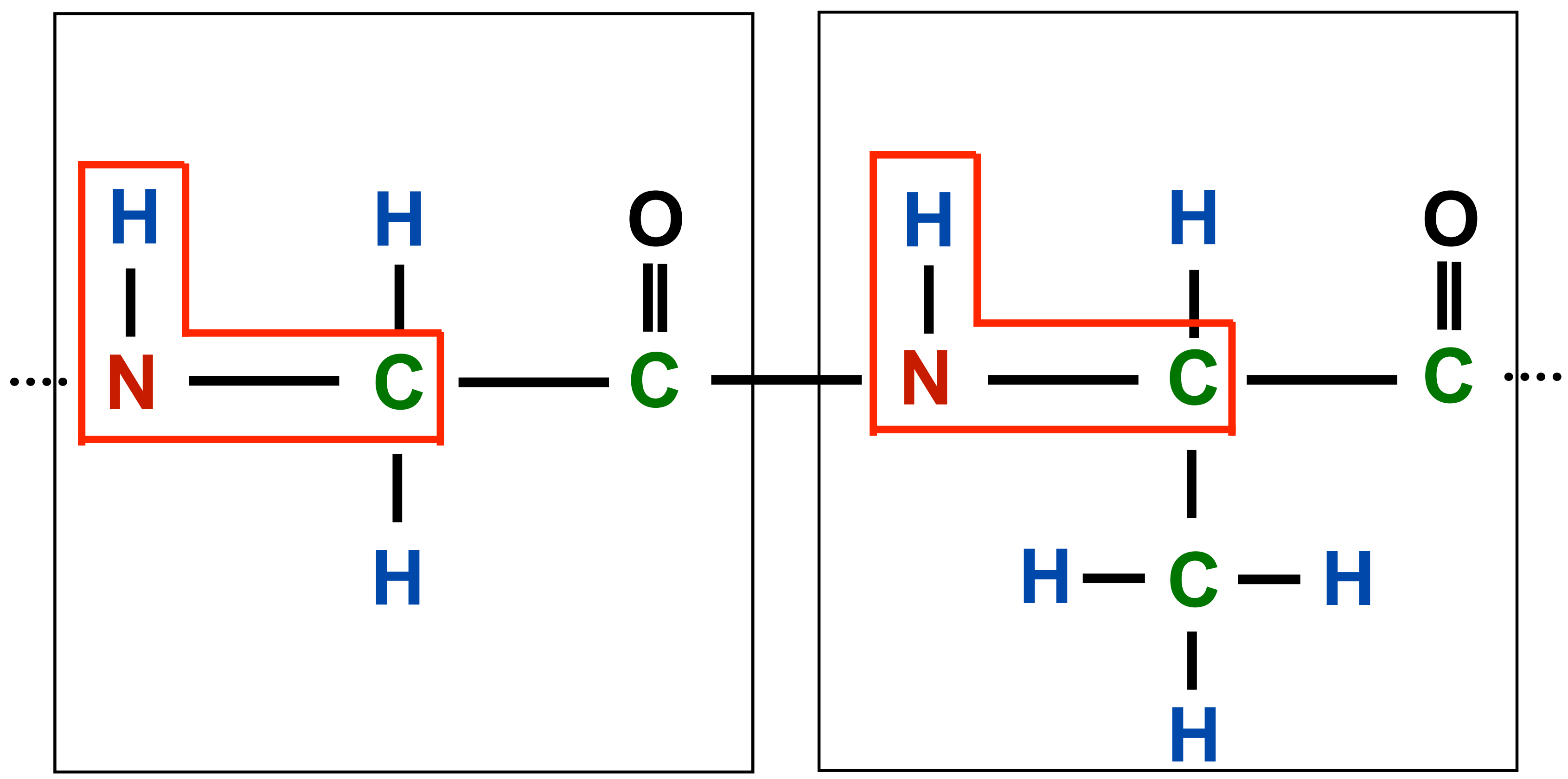
Resonance frequencies at 14 Tesla:

^{1}H	600 MHz
^{15}N	60 MHz



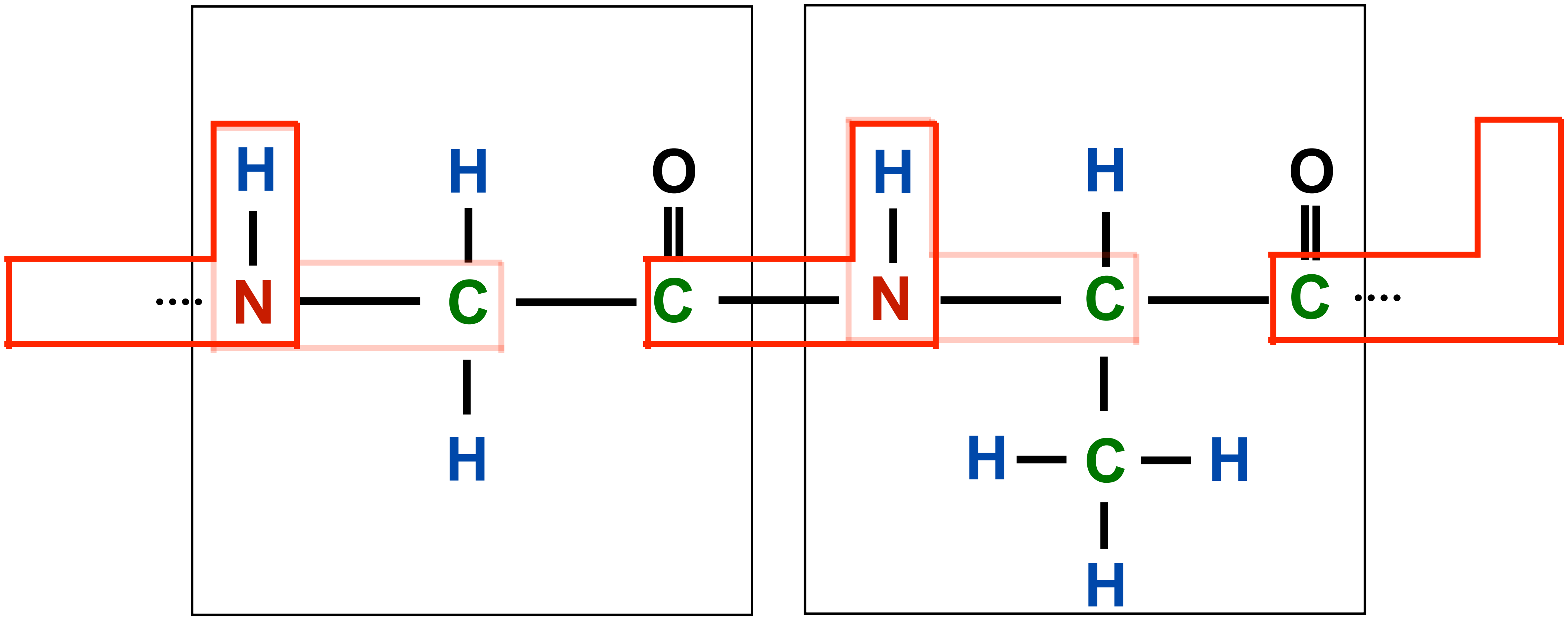
Resonance frequencies at 14 Tesla:

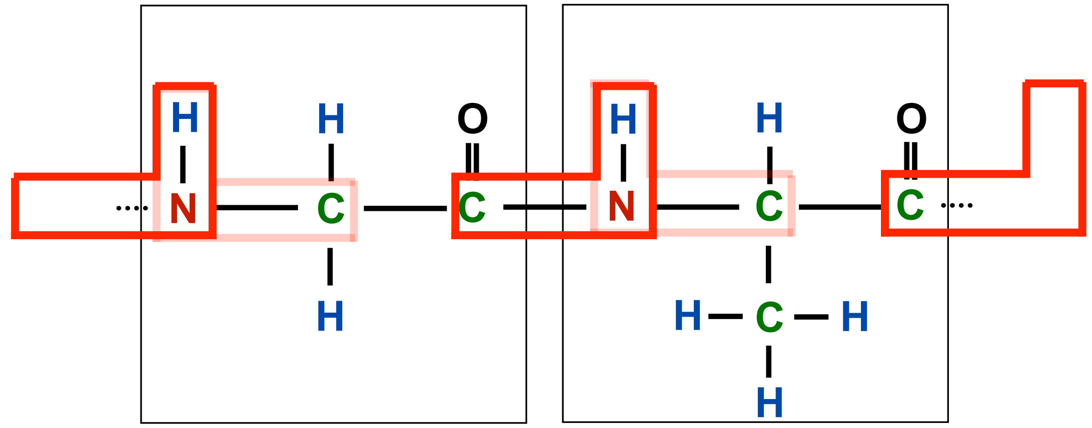
^{1}H	600 MHz
^{15}N	60 MHz
^{13}C	150 MHz

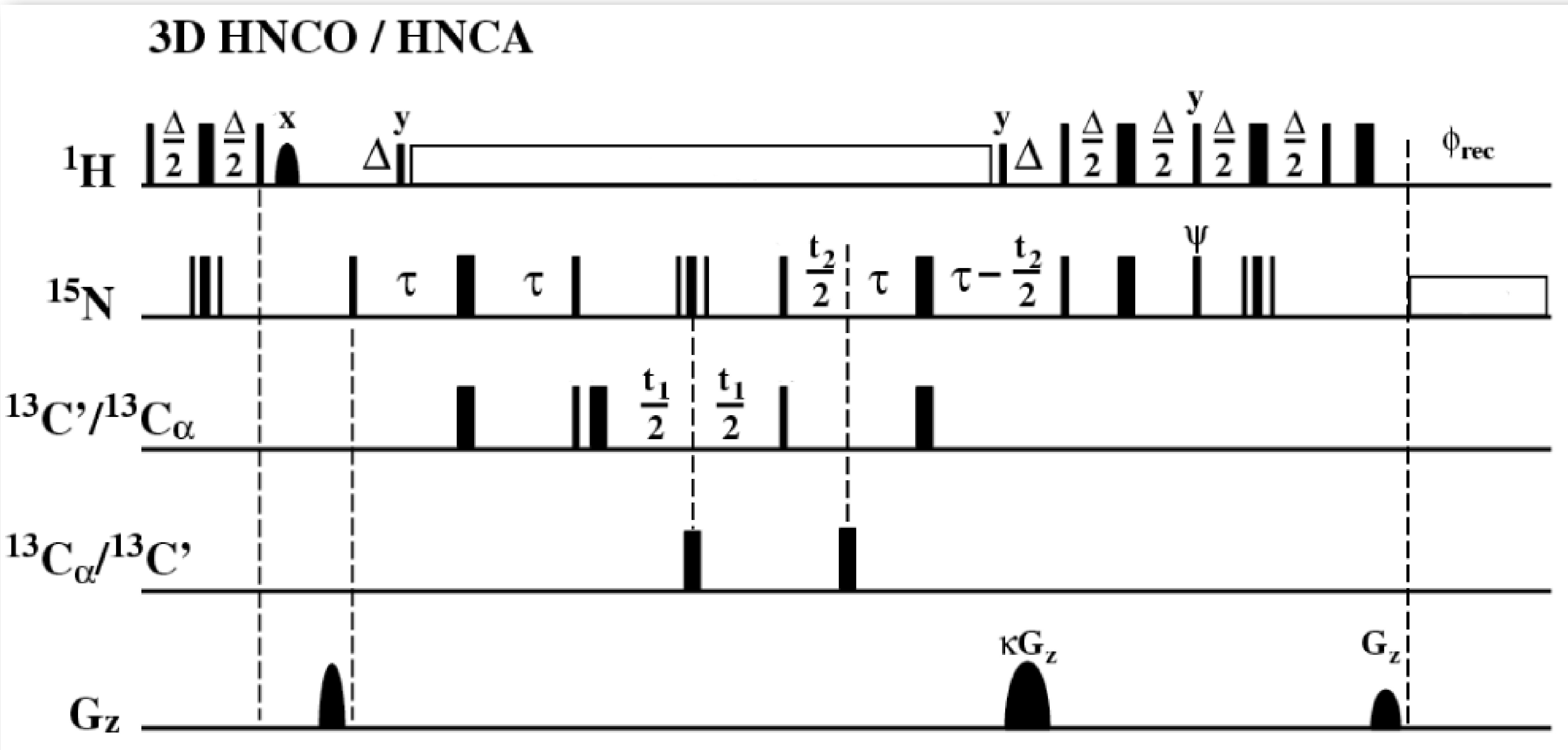


Resonance frequencies at 14 Tesla:

^{1}H	600 MHz
^{15}N	60 MHz
^{13}C	150 MHz



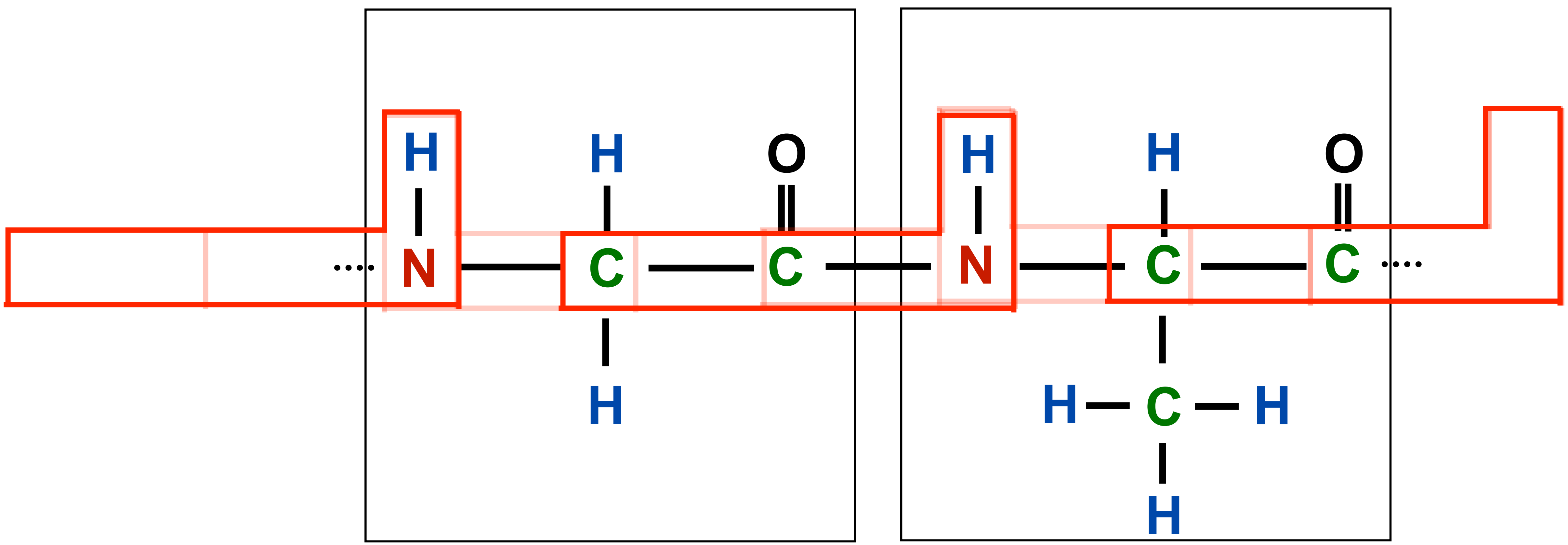




3D HNCO / HNCA
1H
15N
13C'/13Cα
13Cα/13C'
Gz
Δ/2
x
Δ
y
φrec
ψ
τ
t1/2
t2/2
τ − t2/2
κGz
Gz

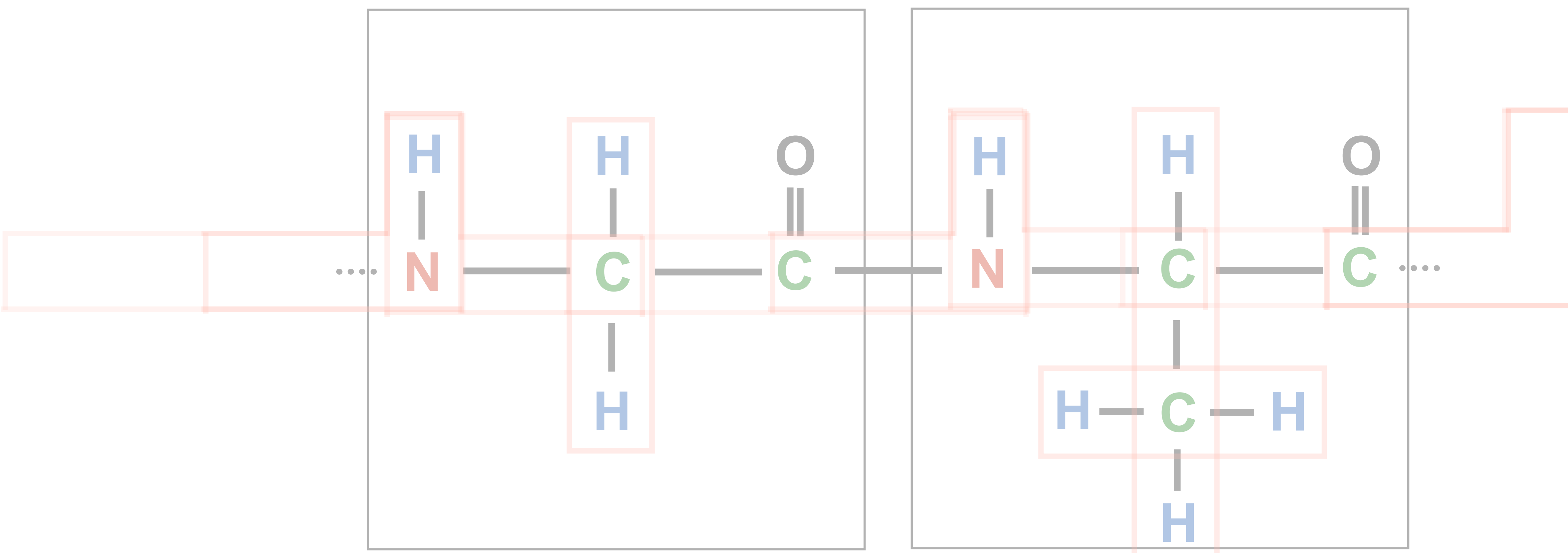
Resonance frequencies at 14 Tesla:

^{1}H	600 MHz
^{15}N	60 MHz
^{13}C	150 MHz



Resonance frequencies at 14 Tesla:

^{1}H	600 MHz
^{15}N	60 MHz
^{13}C	150 MHz



exp. correlations + known sequence of amono acids

⟶ assignment of all resonances to individual atoms

Structure calculation

- The NOE intensities measured in a NOESY spectrum are calibrated and used to derive proton/proton distance restraints (NOE ~ $1/r^6$)
- These are applied in a restrained molecular dynamics / simulated annealing (MD/SA) calculation.
- Different and/or randomized starting structures are used. The result is an ensemble of structures that is consistent with the experimentally derived distance restraints.

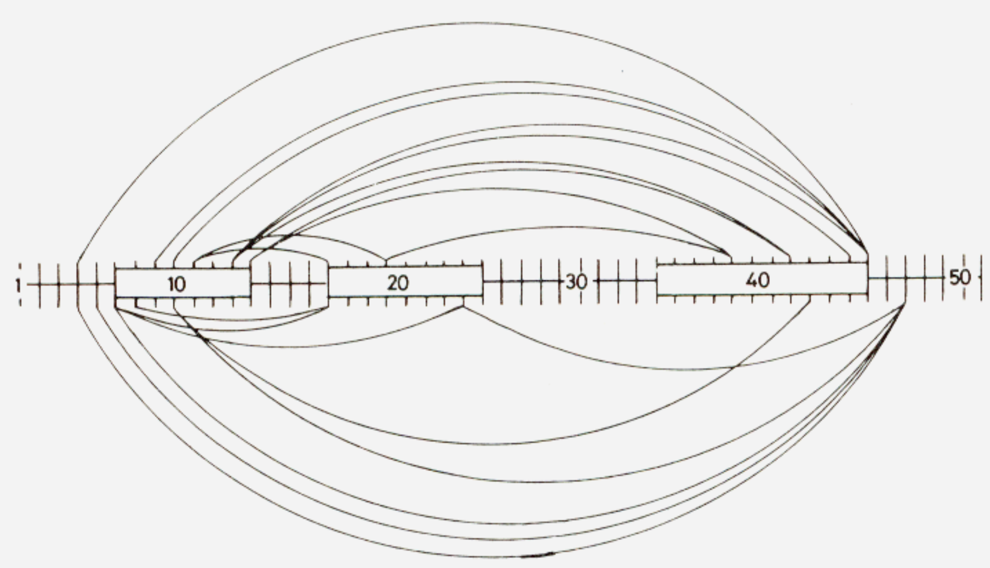
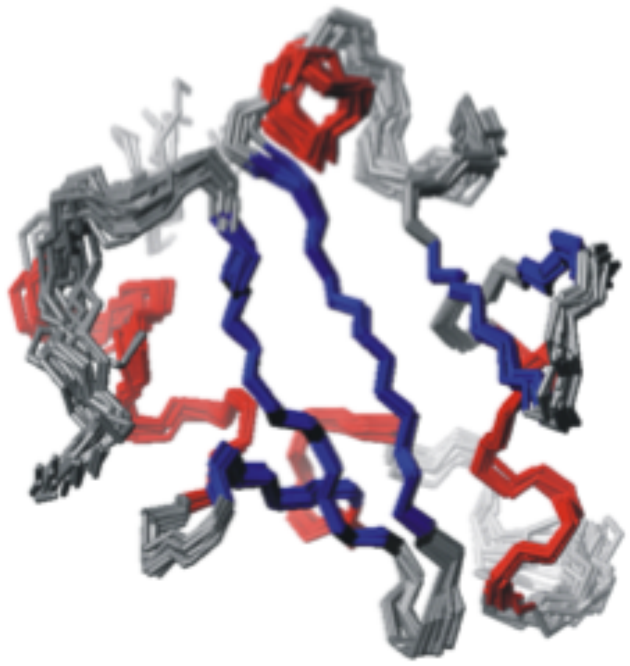


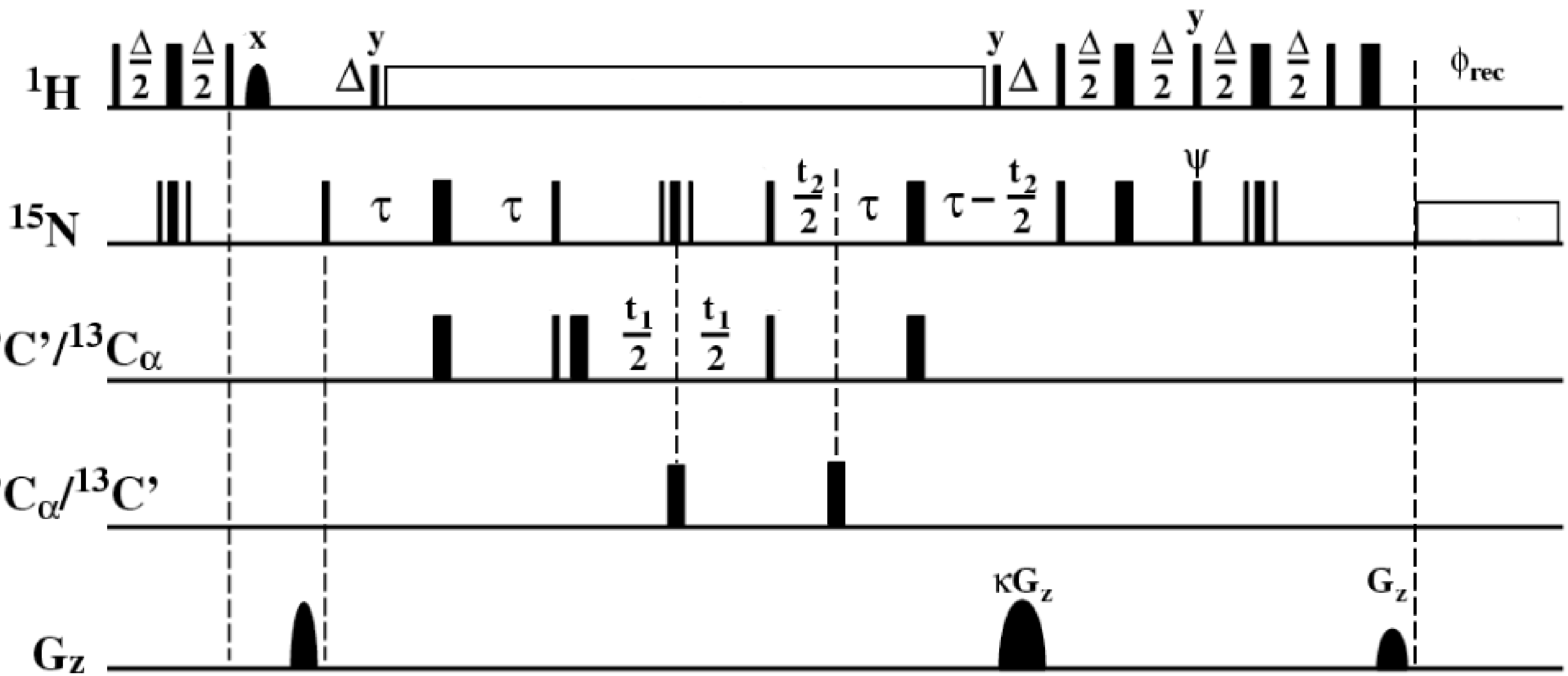
Figure 10.2. Schematic presentation of the amino acid sequence of *lac* headpiece, with three boxes identifying α-helical regions. The curved lines connect residues between which one or several long-range NOE's were observed (from Zuiderweg et al., 1984b).



An ensemble of NMR structures obtained from a restrained MD/SA calculation

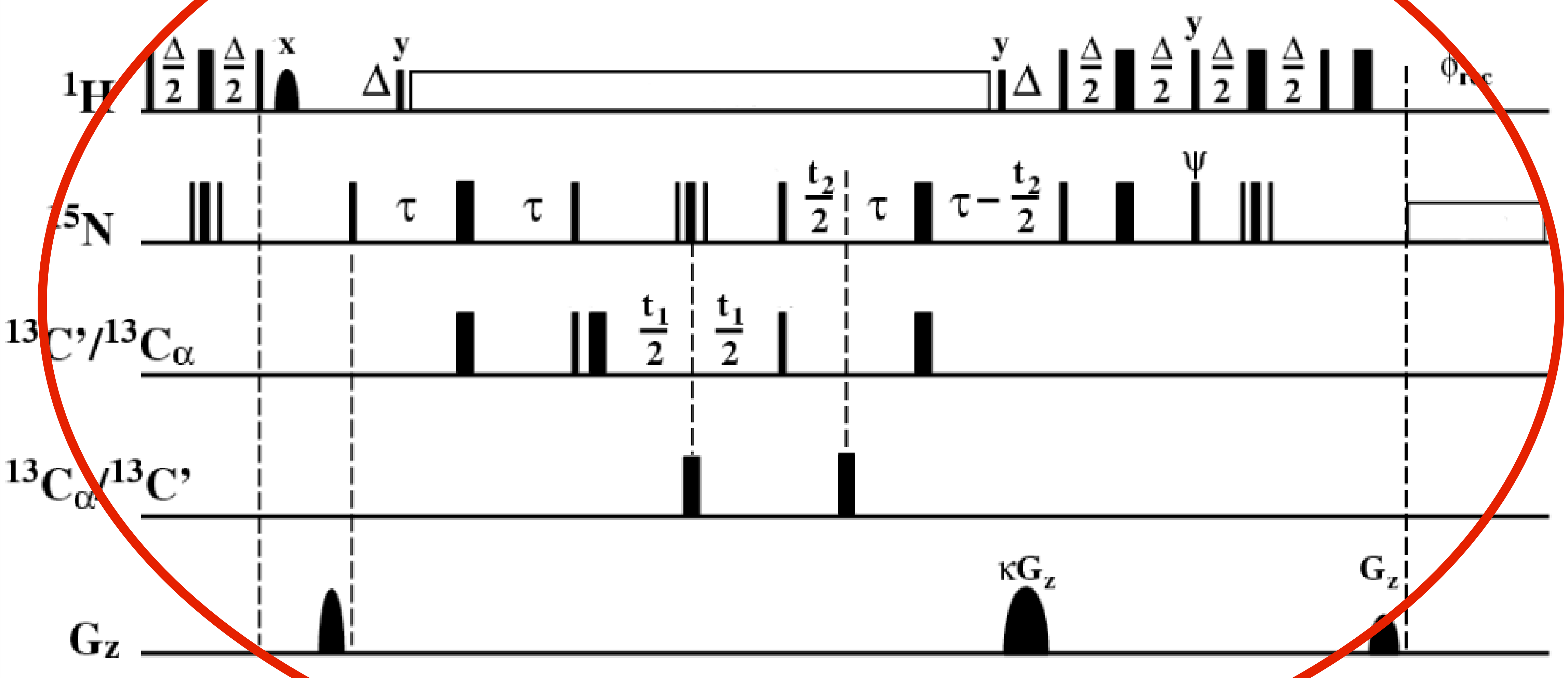
M. Sattler

3D HNCO / HNCA

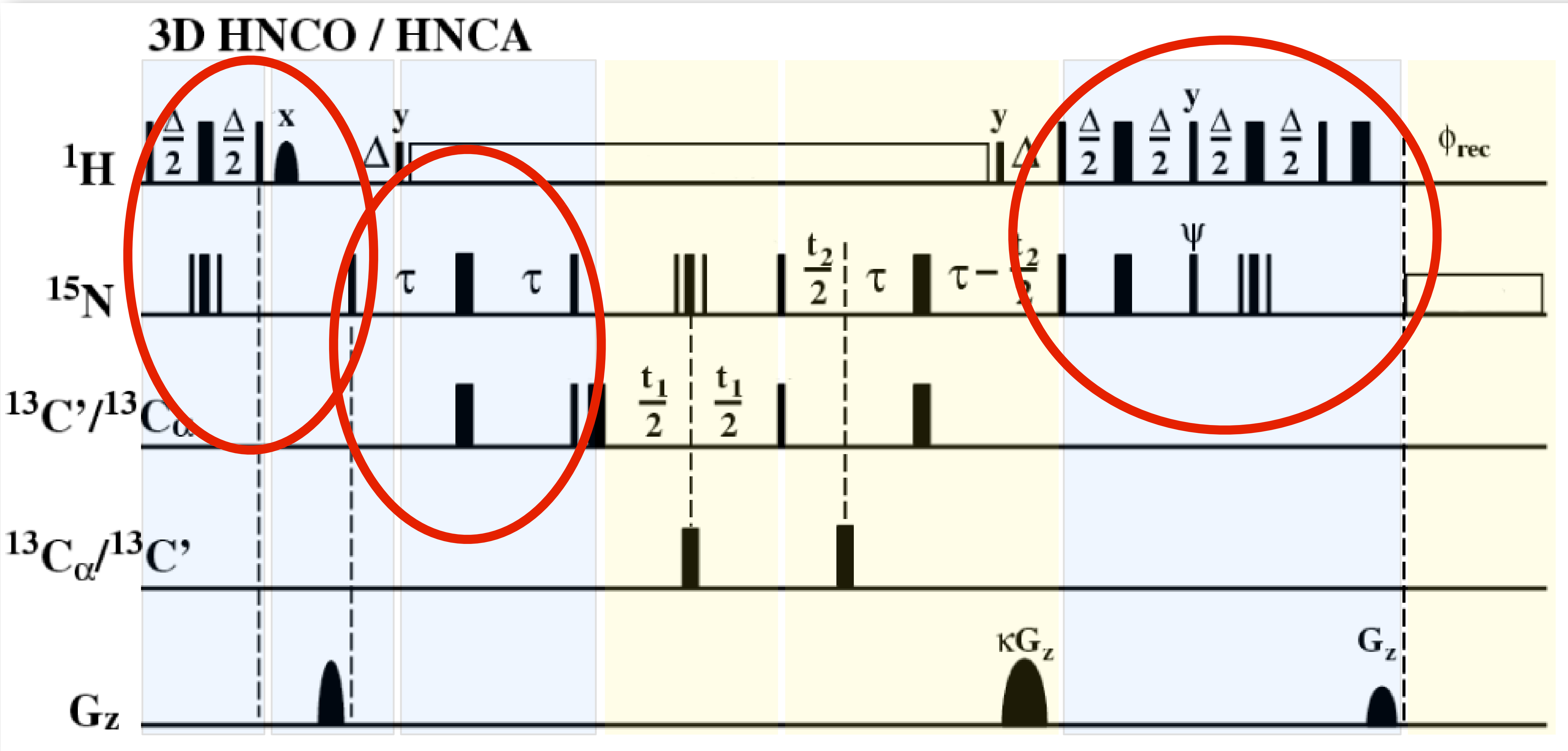


^{1}H
^{15}N
$^{13}C'/^{13}C_{\alpha}$
$^{13}C_{\alpha}/^{13}C'$
G_z
$\frac{\Delta}{2}$
x
y
Δ
τ
$\frac{t_2}{2}$
$\tau - \frac{t_2}{2}$
$\frac{t_1}{2}$
ψ
ϕ_{rec}
κG_z
G_z

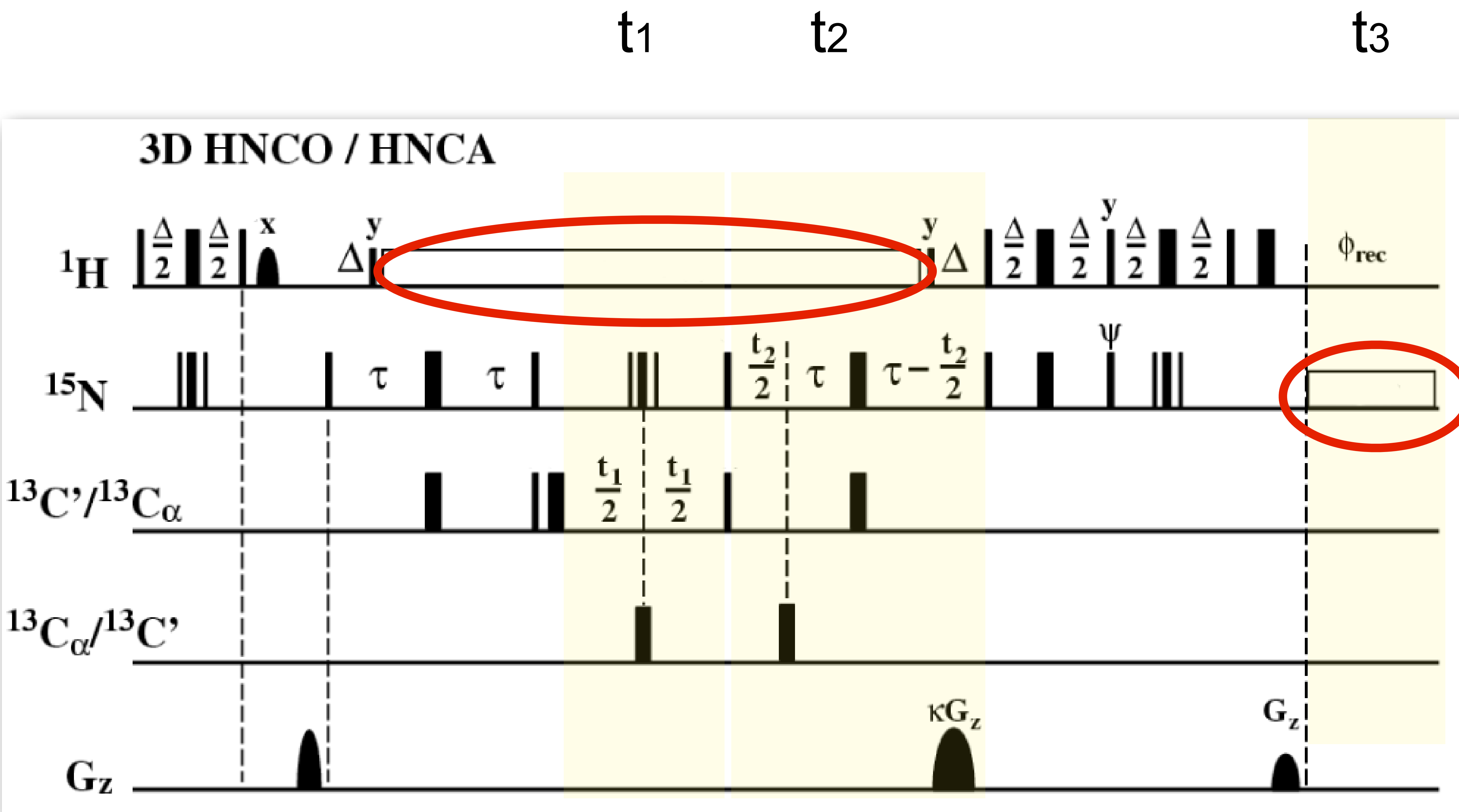
3D HNCO / HNCA



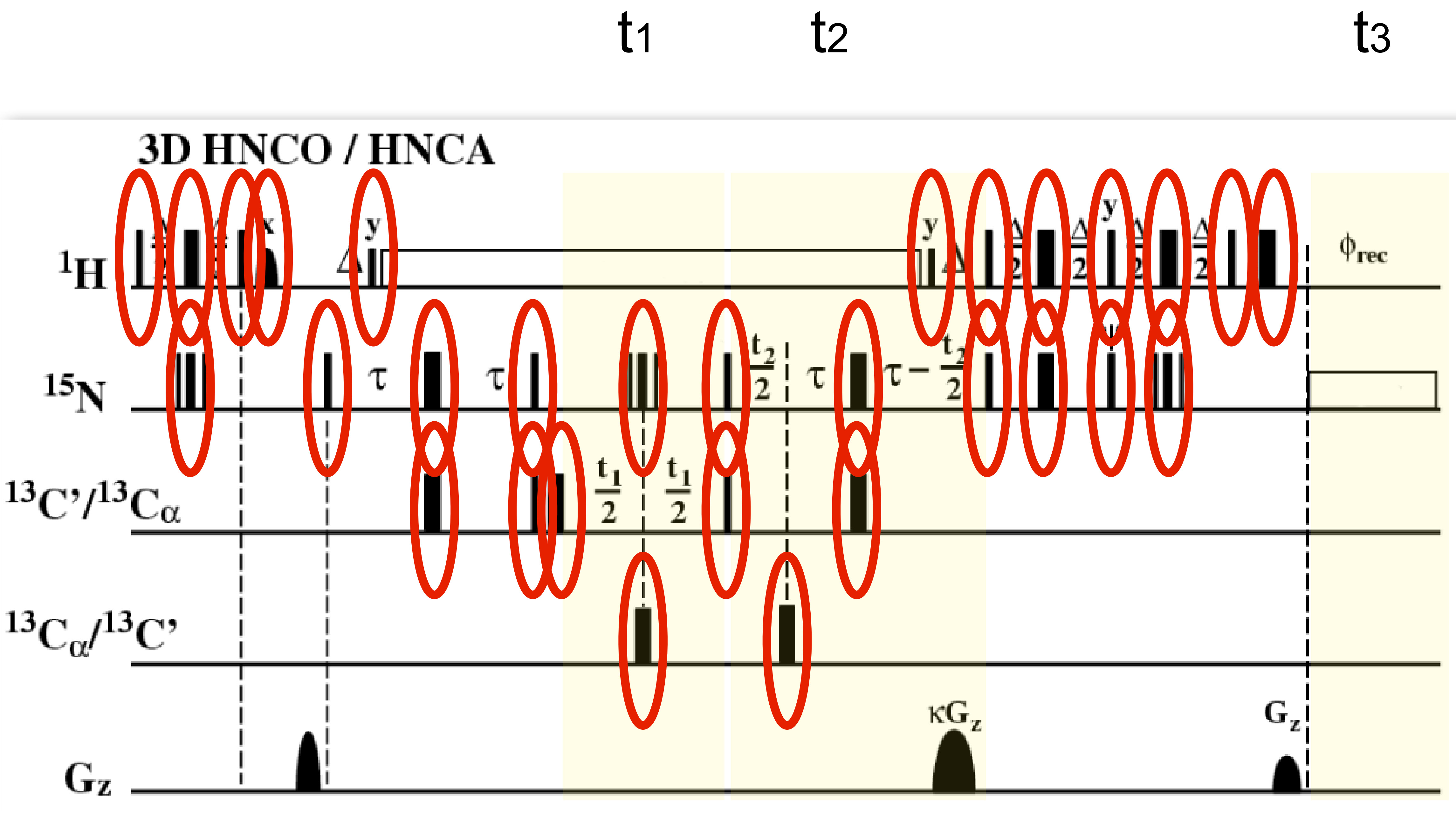
t1 t2 t3



3D HNCO / HNCA
1H
15N
13C'/13Cα
13Cα/13C'
Gz
x
y
ϕrec
ψ
κGz
Gz
τ
Δ/2
t1/2
t2/2

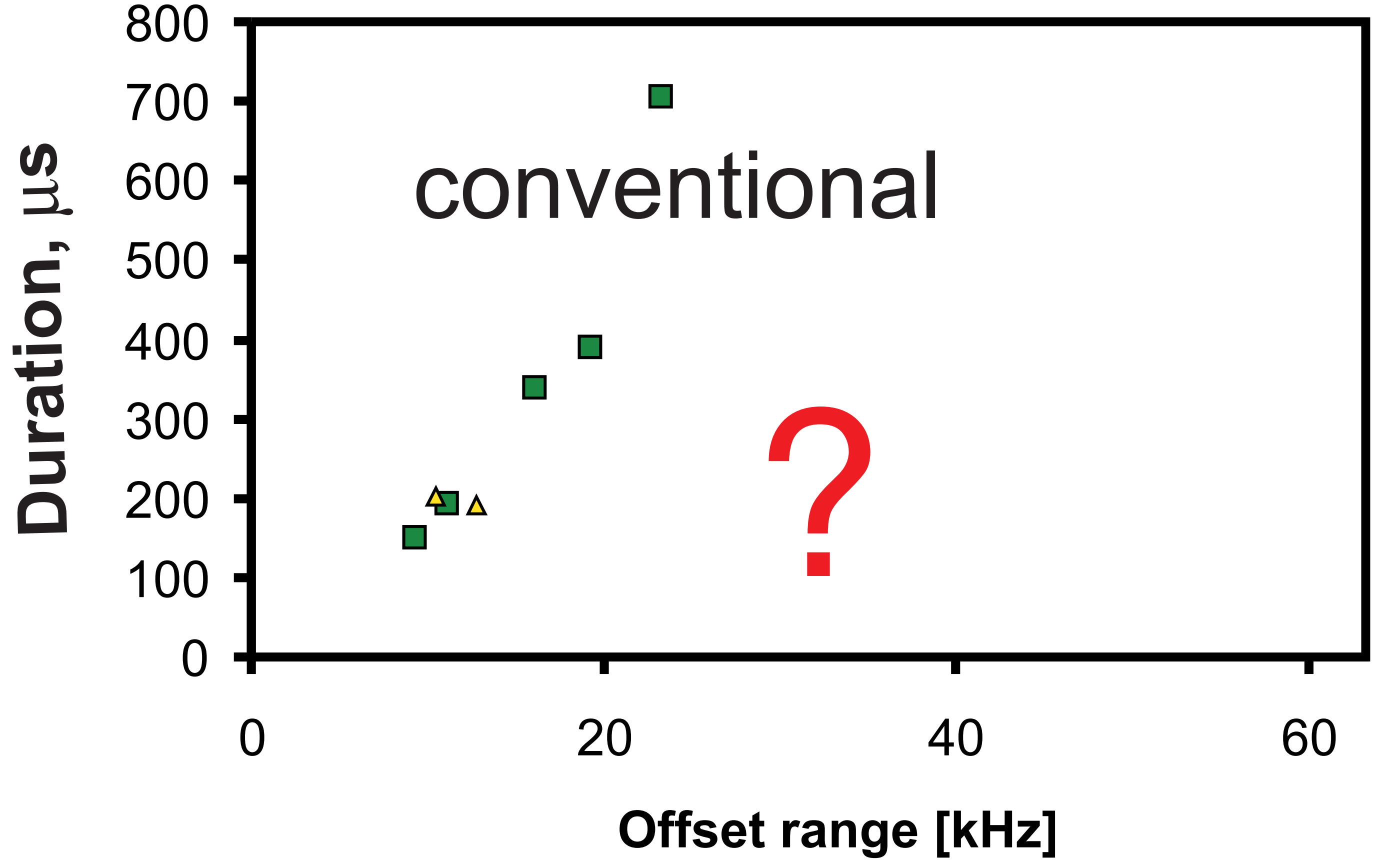


t1
t2
t3
3D HNCO / HNCA
1H
15N
13C'/13Cα
13Cα/13C'
Gz
Δ/2
x
y
Δ
ψ
τ
τ − t2/2
t1/2
t2/2
φrec
κGz



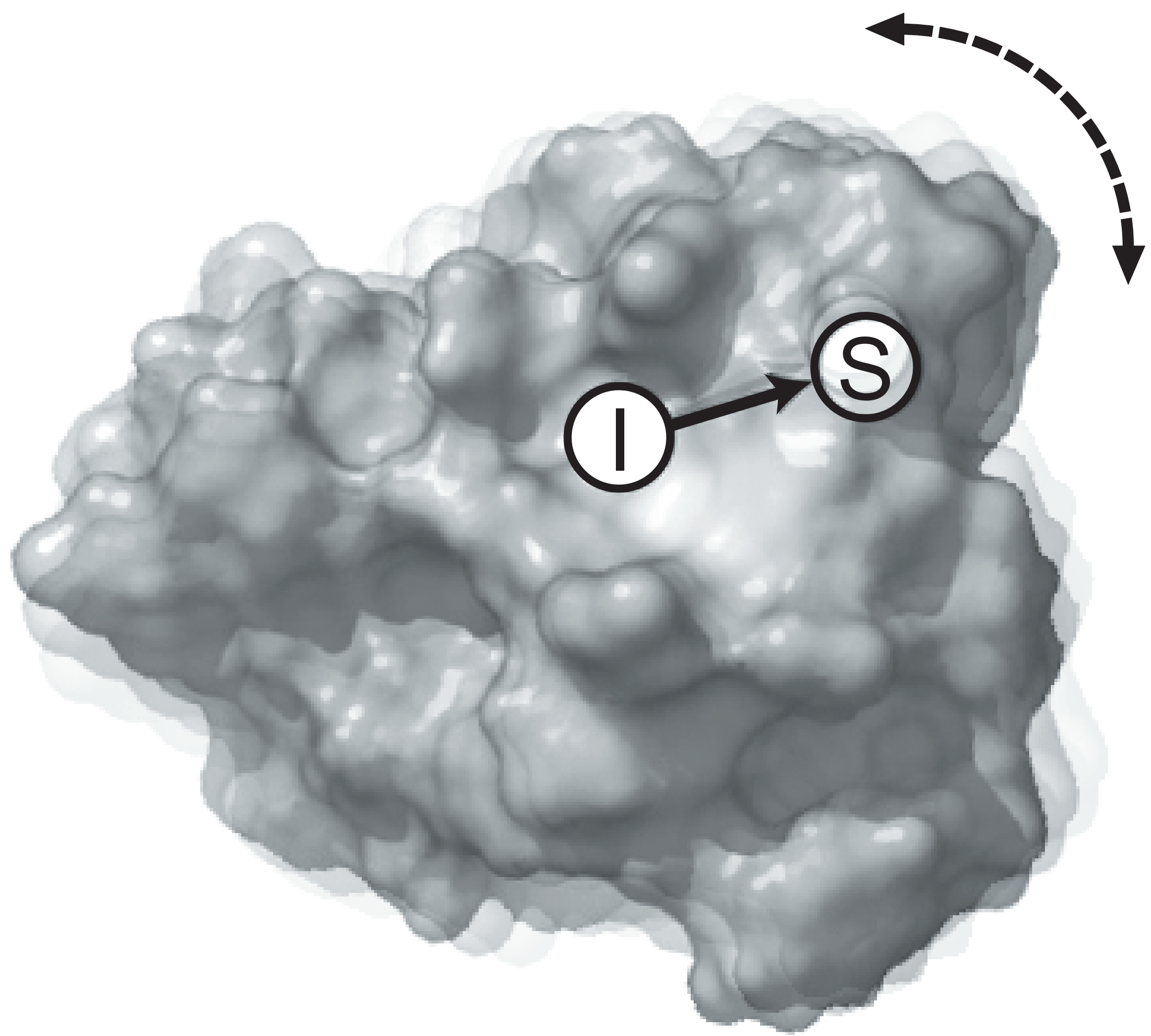
t1
t2
t3
3D HNCO / HNCA
1H
15N
13C'/13Cα
13Cα/13C'
Gz
φrec
κGz
Gz

Performance of conventional composite pulses for broadband (robust) excitation

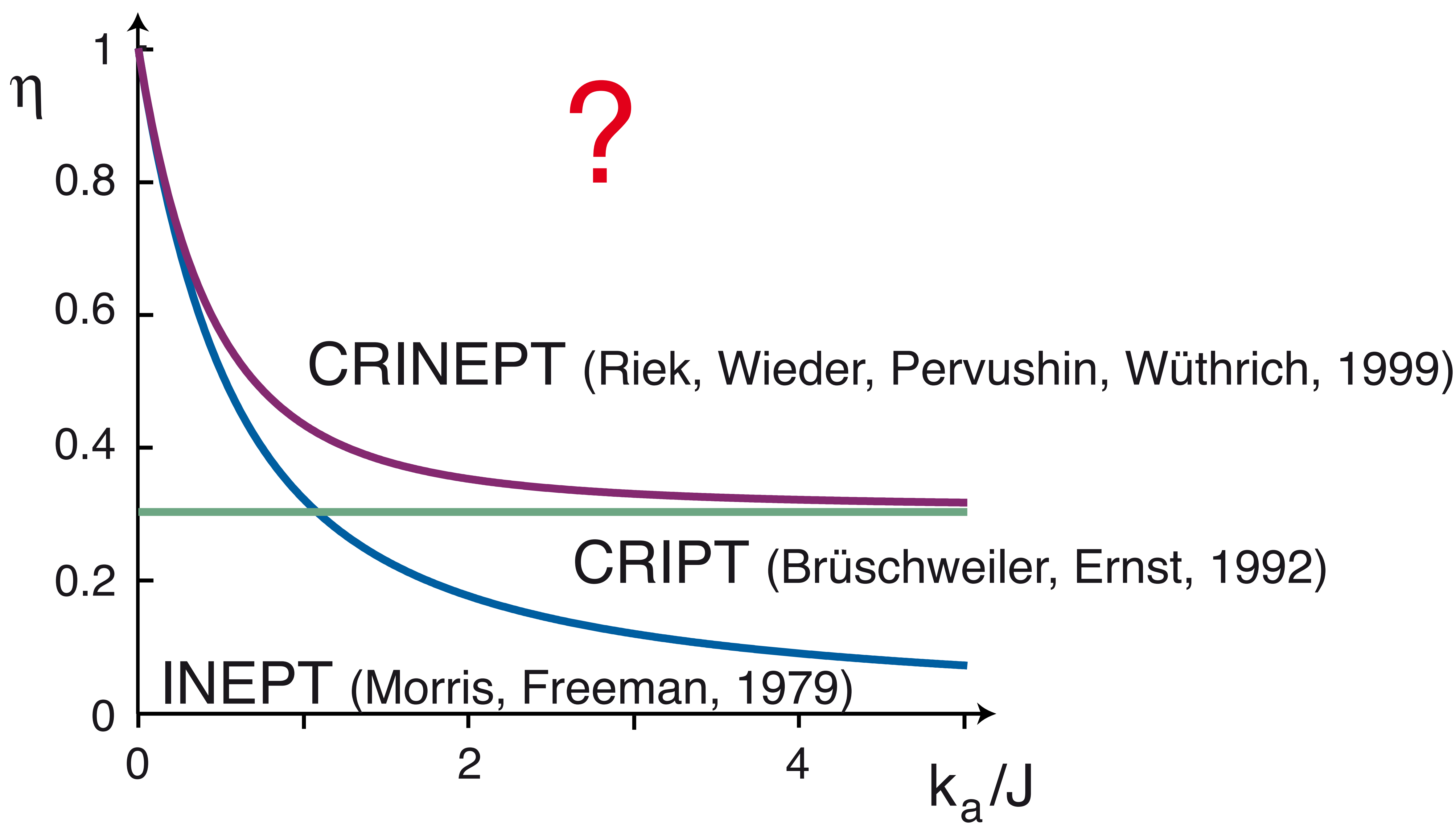


(excitation efficiency: 98%, max. rf amplitude: 10 kHz, no rf inhomogeneity)

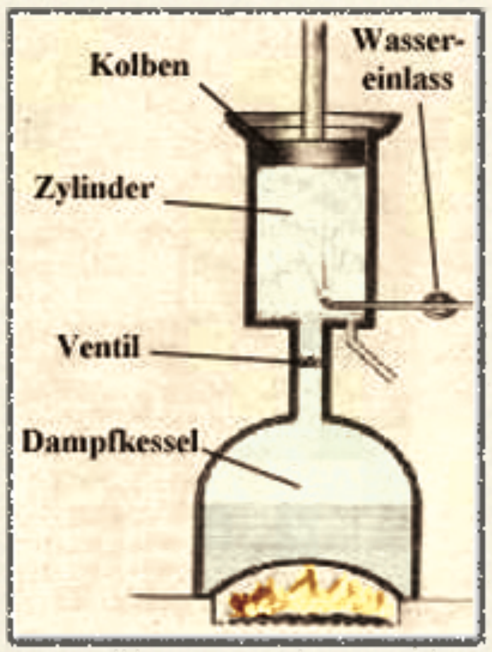
Relaxation rates k increase with molecular weight

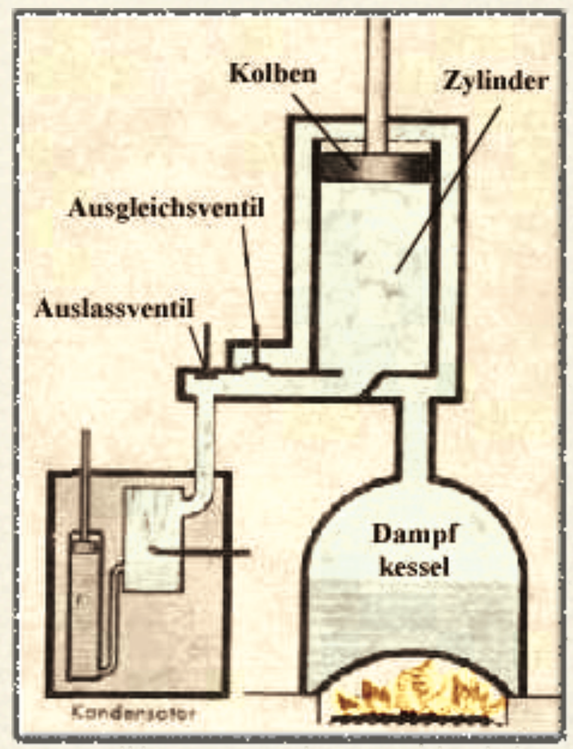


Transfer Efficiency $I_x \rightarrow 2I_z S_y$



Steam Engine





?

1697
D. Papin

1712
T. Newcomen

1765
J. Watt

Steam Engine

„The theory of its operation is rudimentary and attempts to improve its performance are still made in an almost haphazard way.“

1824

RÉFLEXIONS

SUR LA

PUISSANCE MOTRICE

DU FEU

ET

SUR LES MACHINES

PROPRES A DÉVELOPPER CETTE PUISSANCE.

PAR S. CARNOT,

ANCIEN ÉLÈVE DE L'ÉCOLE POLYTECHNIQUE.

A PARIS,

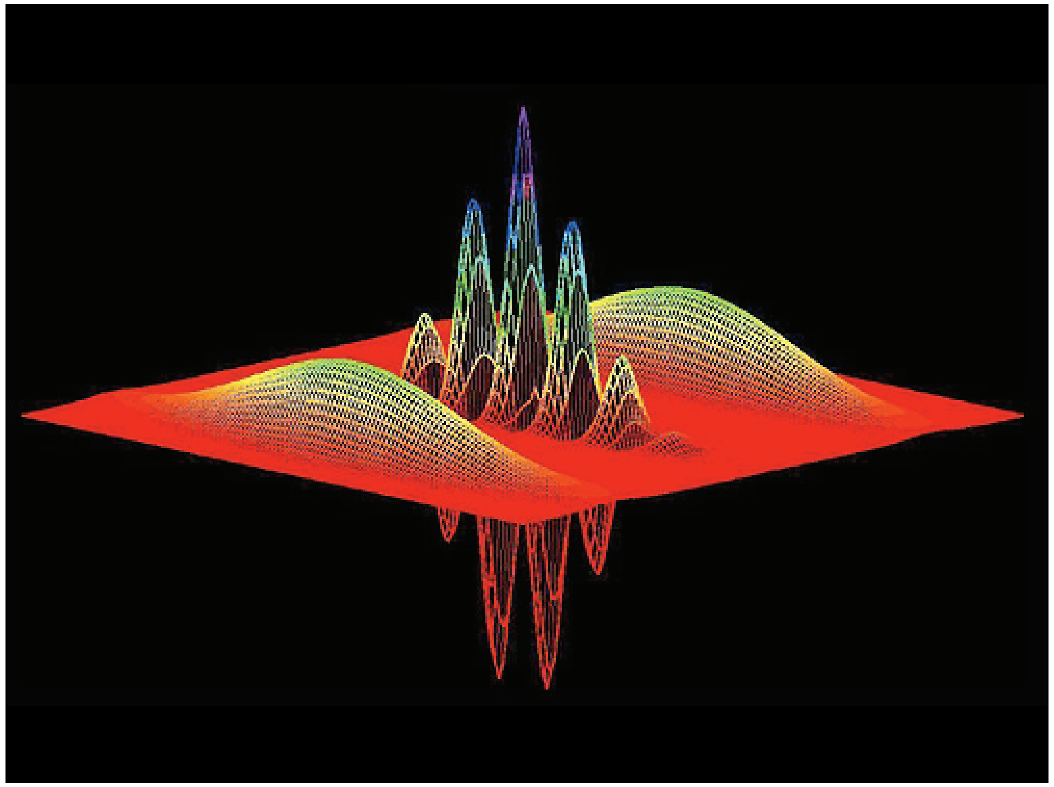
CHEZ BACHELIER, LIBRAIRE,

QUAI DES AUGUSTINS, N°. 55.

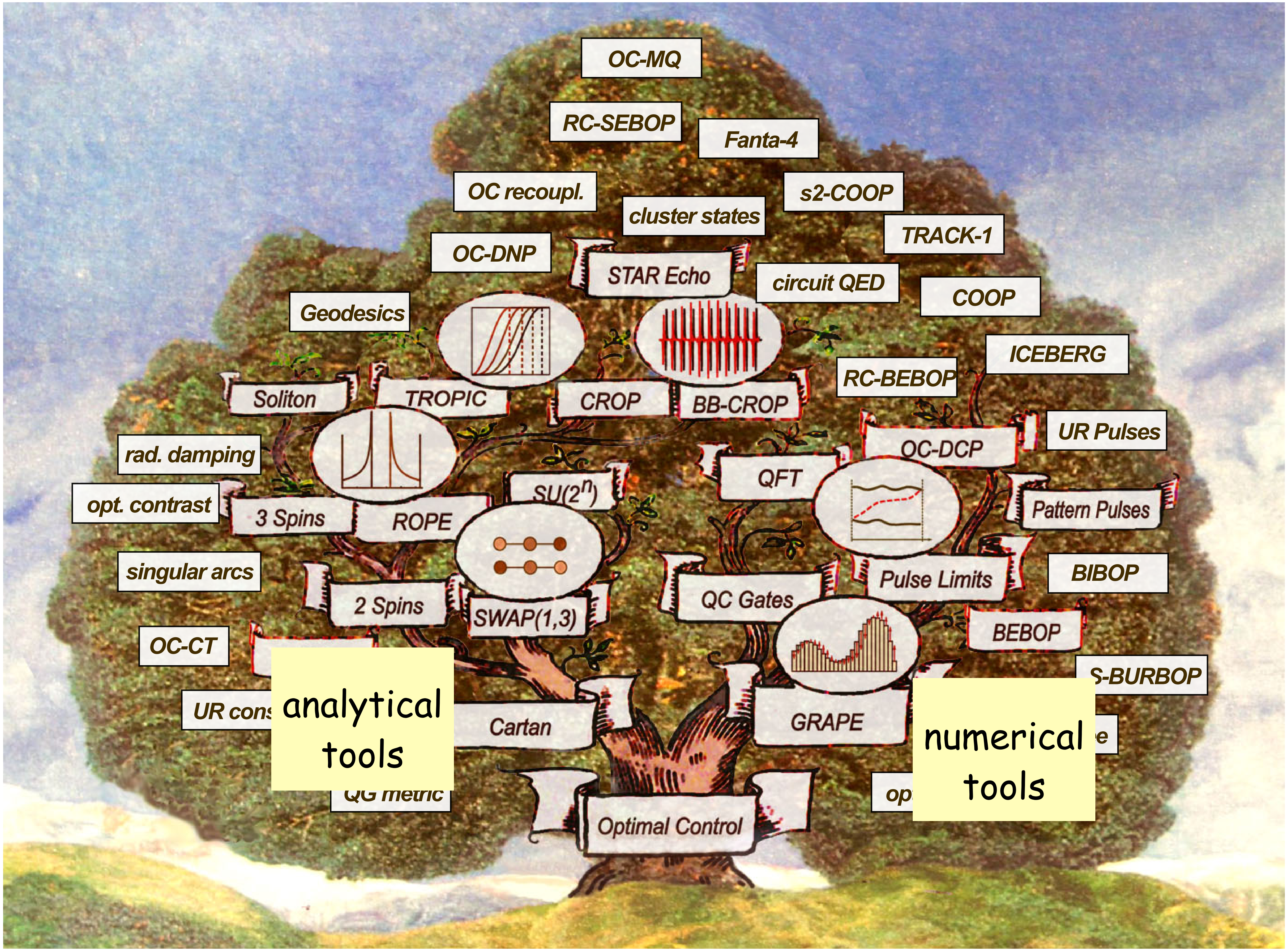
1824.

Optimal Control of Spin Systems

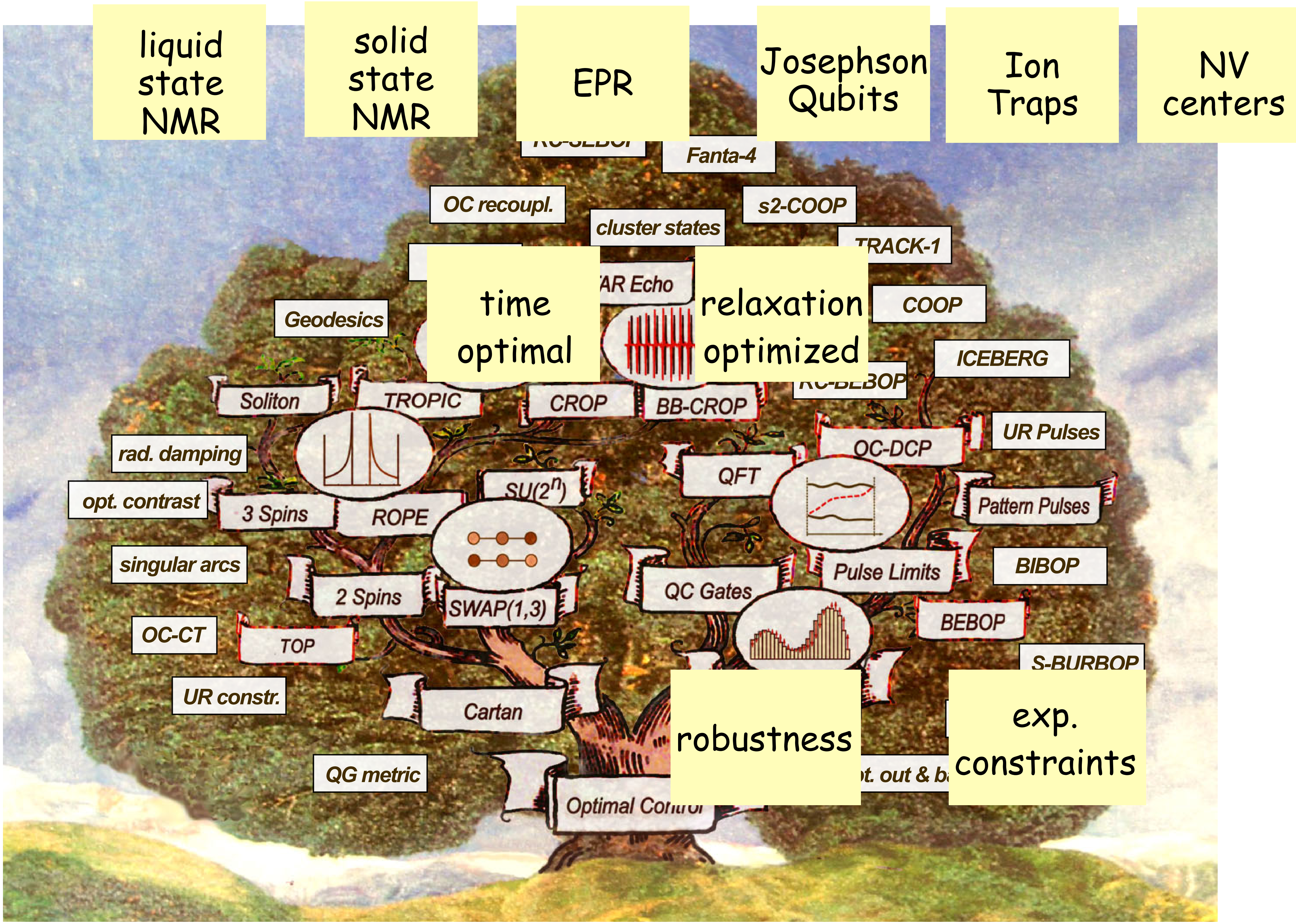
Optimal Control Theory



Quantum Mechanics

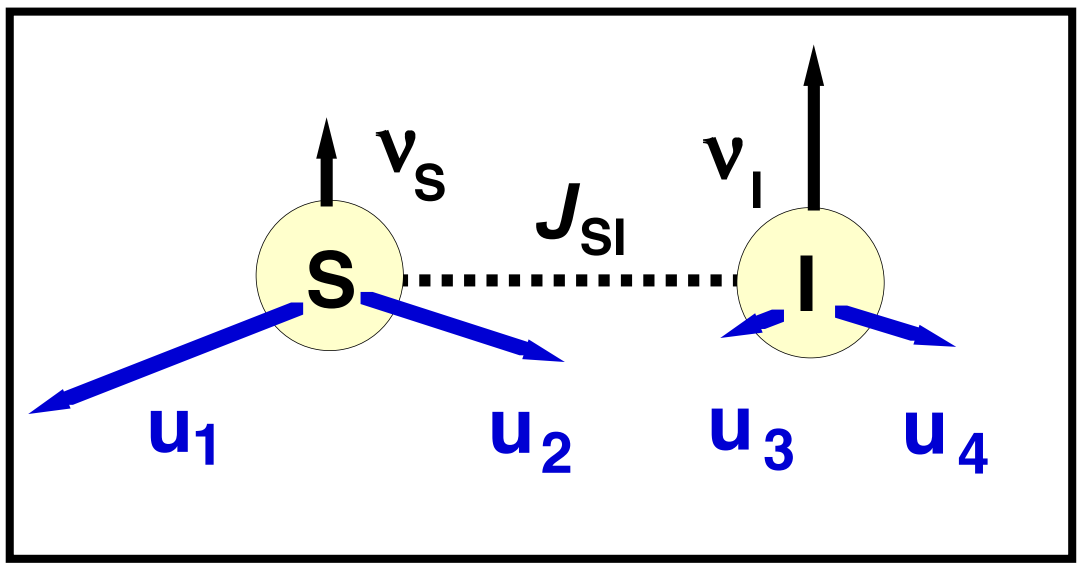


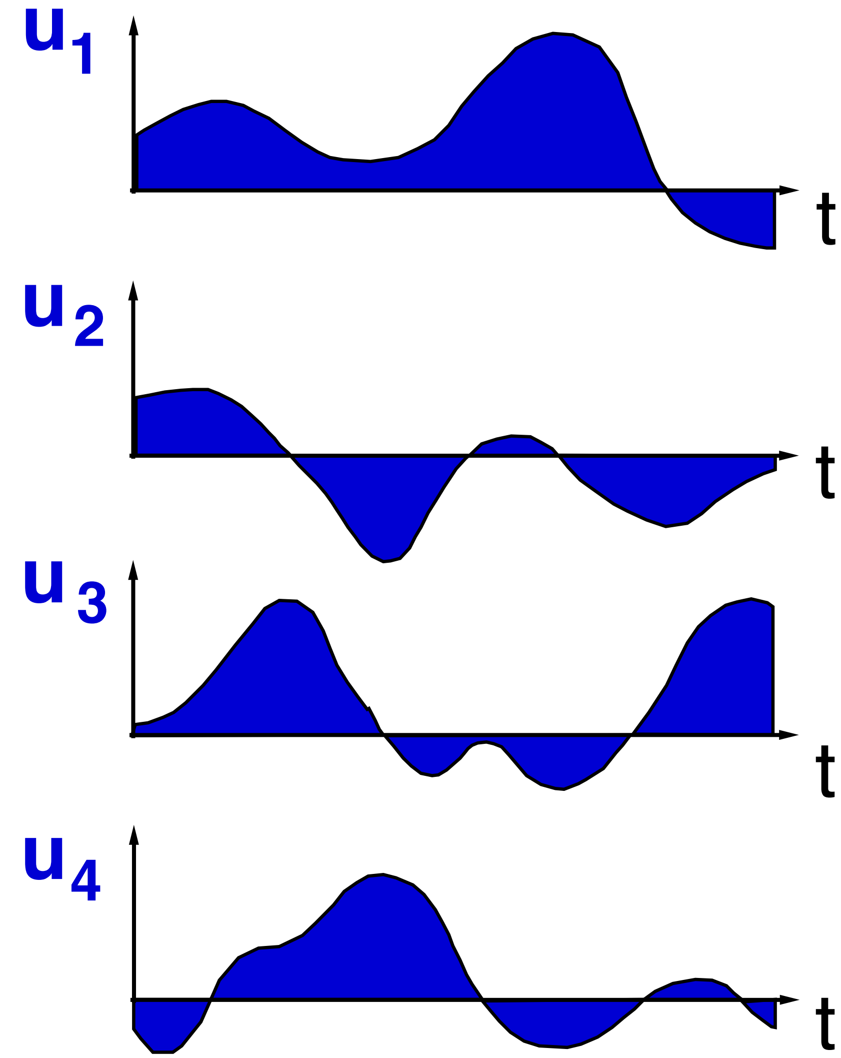
N. C. Nielsen, C. Kehlet, S. J. Glaser, N. Khaneja, Encyclopedia of Nuclear Magnetic Resonance (2010).



N. C. Nielsen, C. Kehlet, S. J. Glaser, N. Khaneja, Encyclopedia of Nuclear Magnetic Resonance (2010).

Control Parameters $u_k(t)$

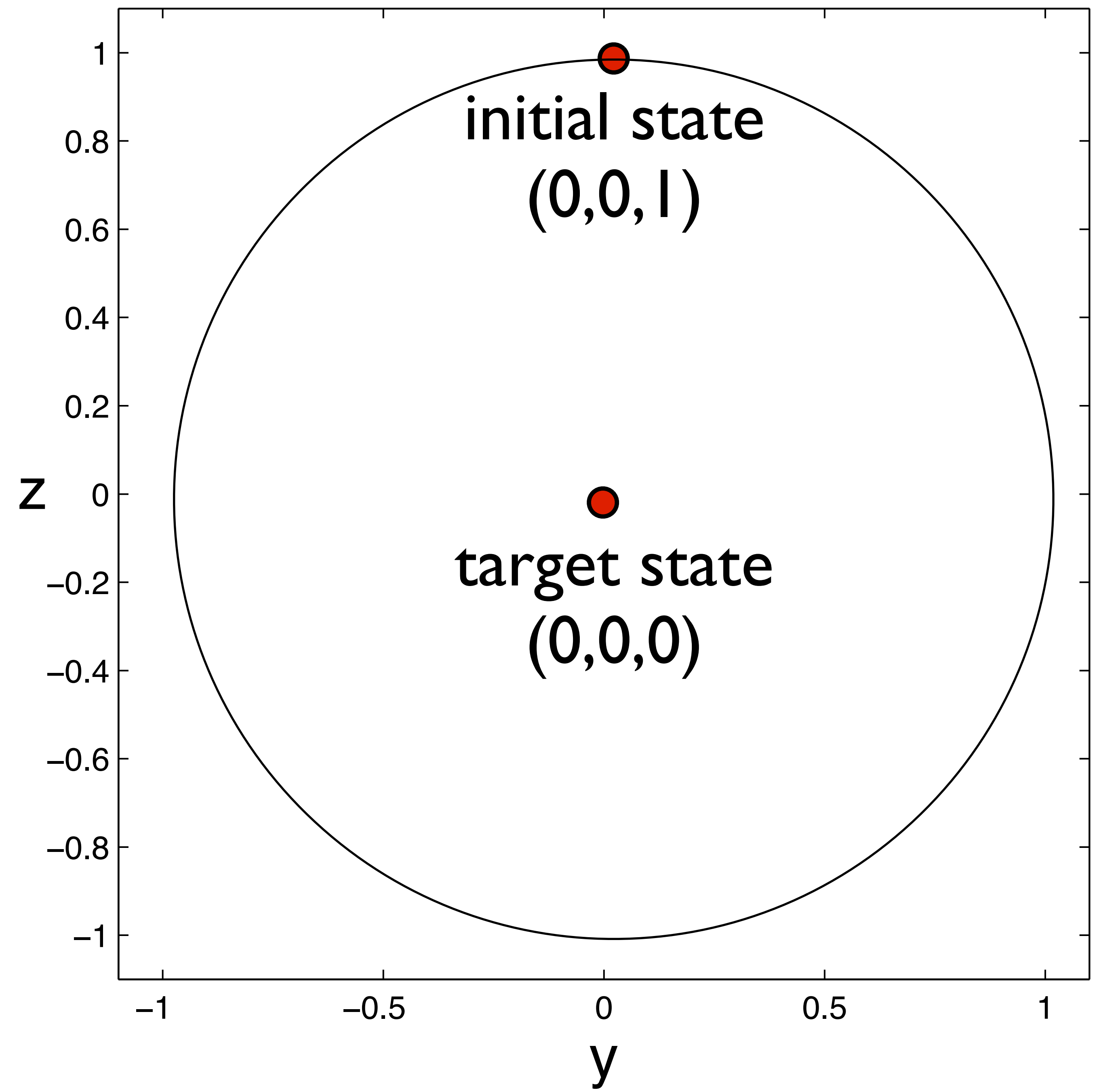




$$H_0 + \sum_k u_k(t)\, H_k$$

Time-optimal control of a spin 1/2 with relaxation

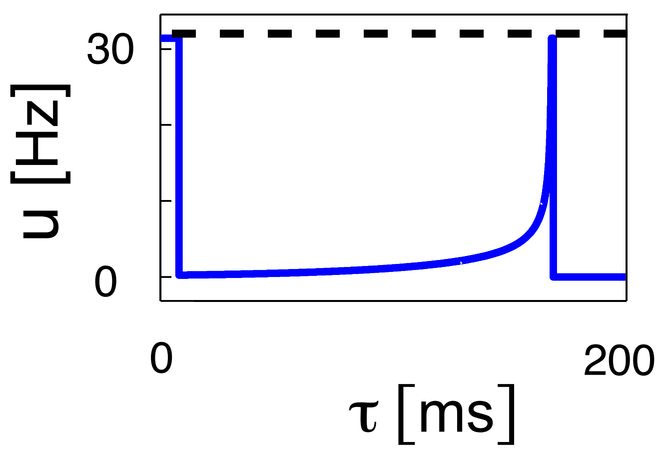
$T_1 = 740$ ms $\quad T_2 = 60$ ms



Time-optimal control of a spin 1/2 with relaxation

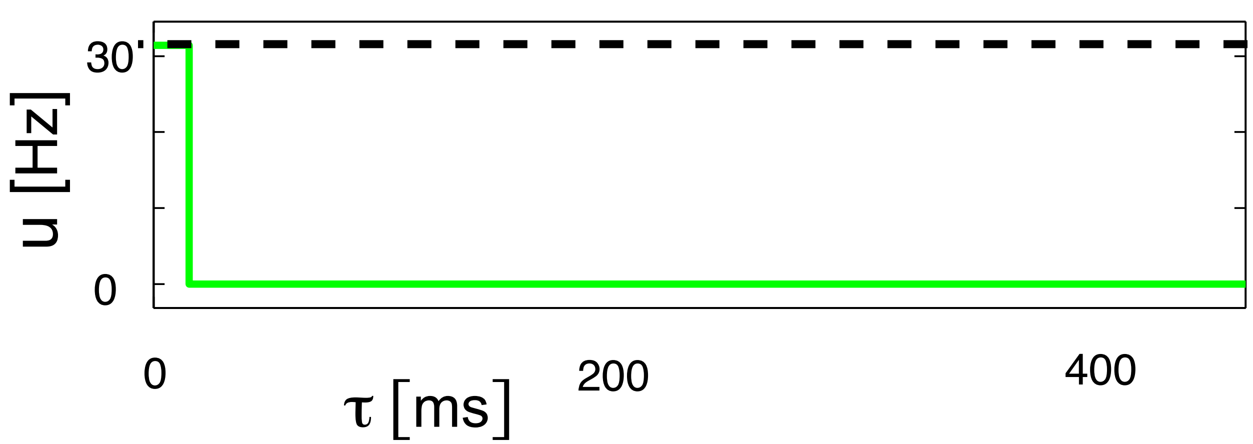
$T_1 = 740$ ms $T_2 = 60$ ms

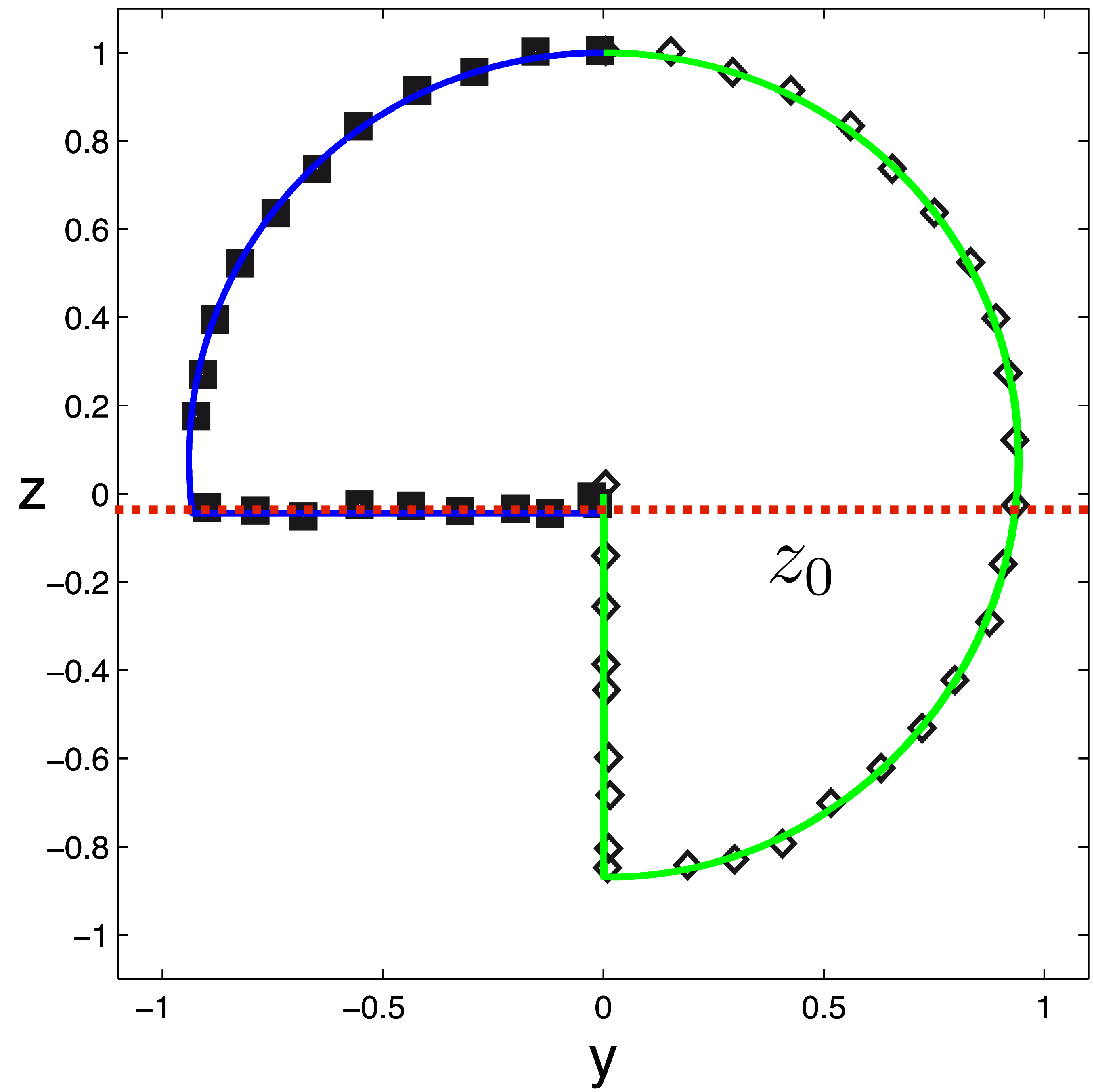
time-optimal pulse sequence



$u_{max} = 32.3$ Hz

inversion recovery



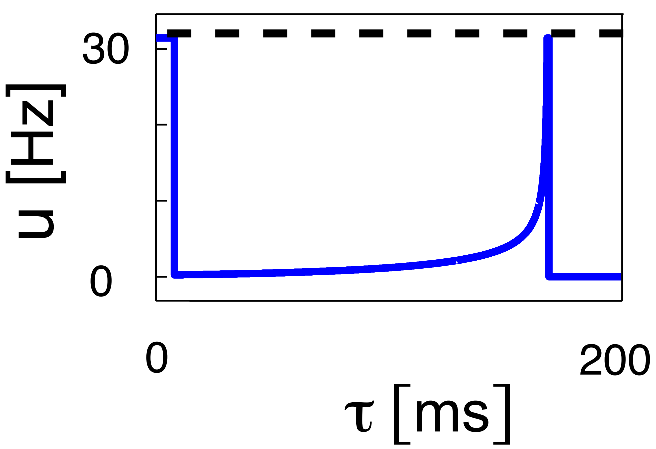


Lapert, Zhang, Braun, Glaser, Sugny, PRL 104 (2010)

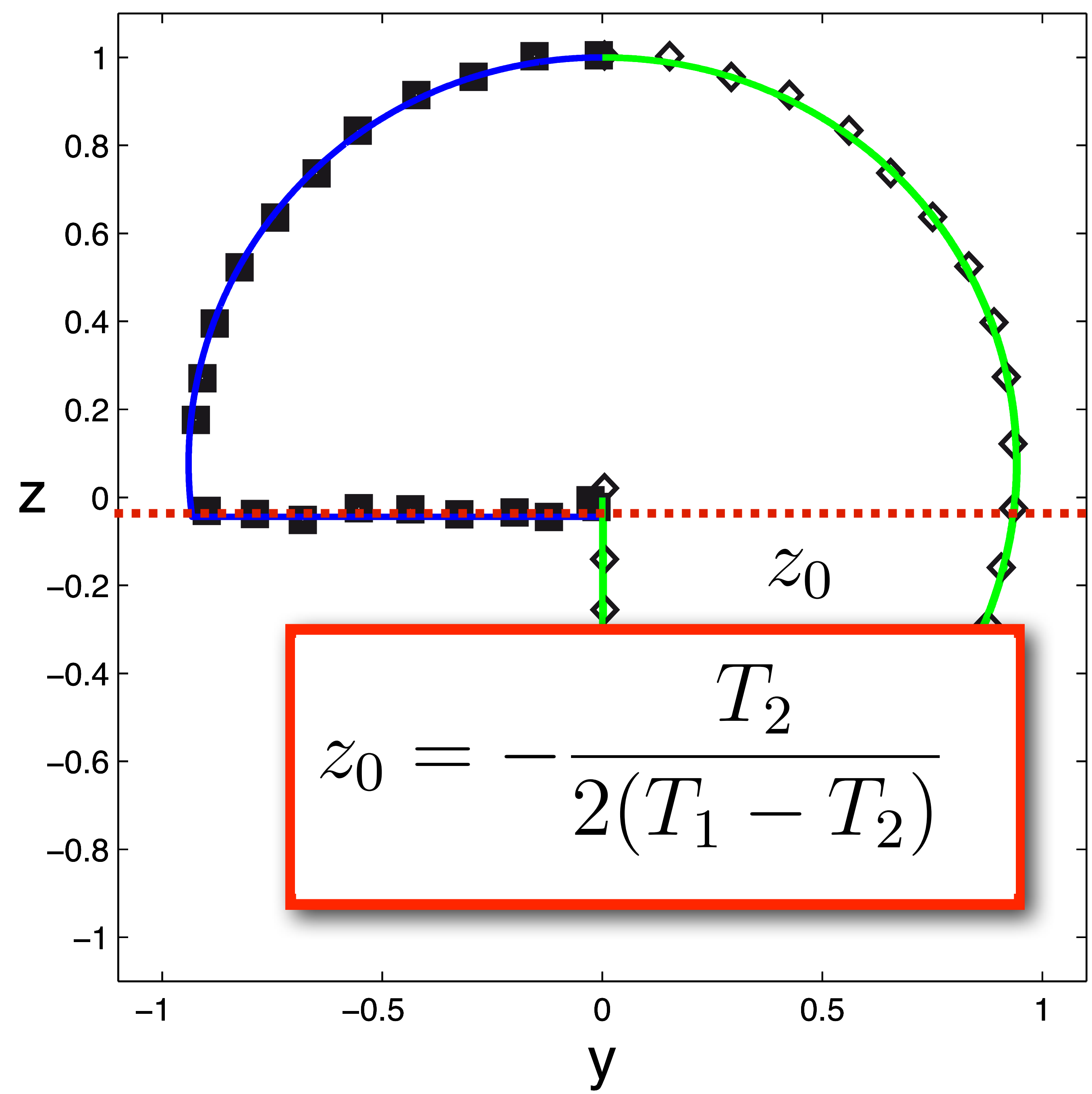
Time-optimal control of a spin 1/2 with relaxation

$T_1 = 740$ ms $\quad T_2 = 60$ ms

time-optimal pulse sequence

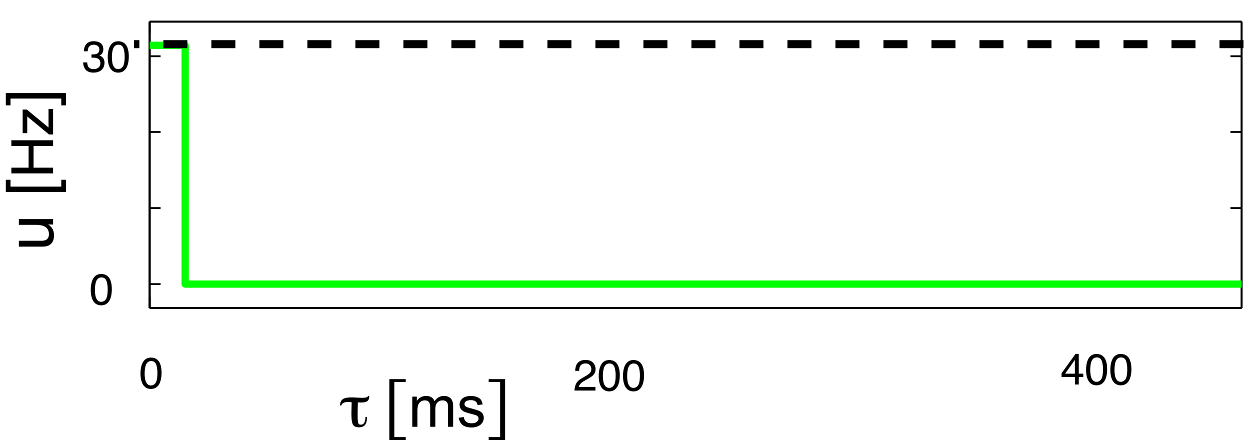


$u_{max} = 32.3$ Hz

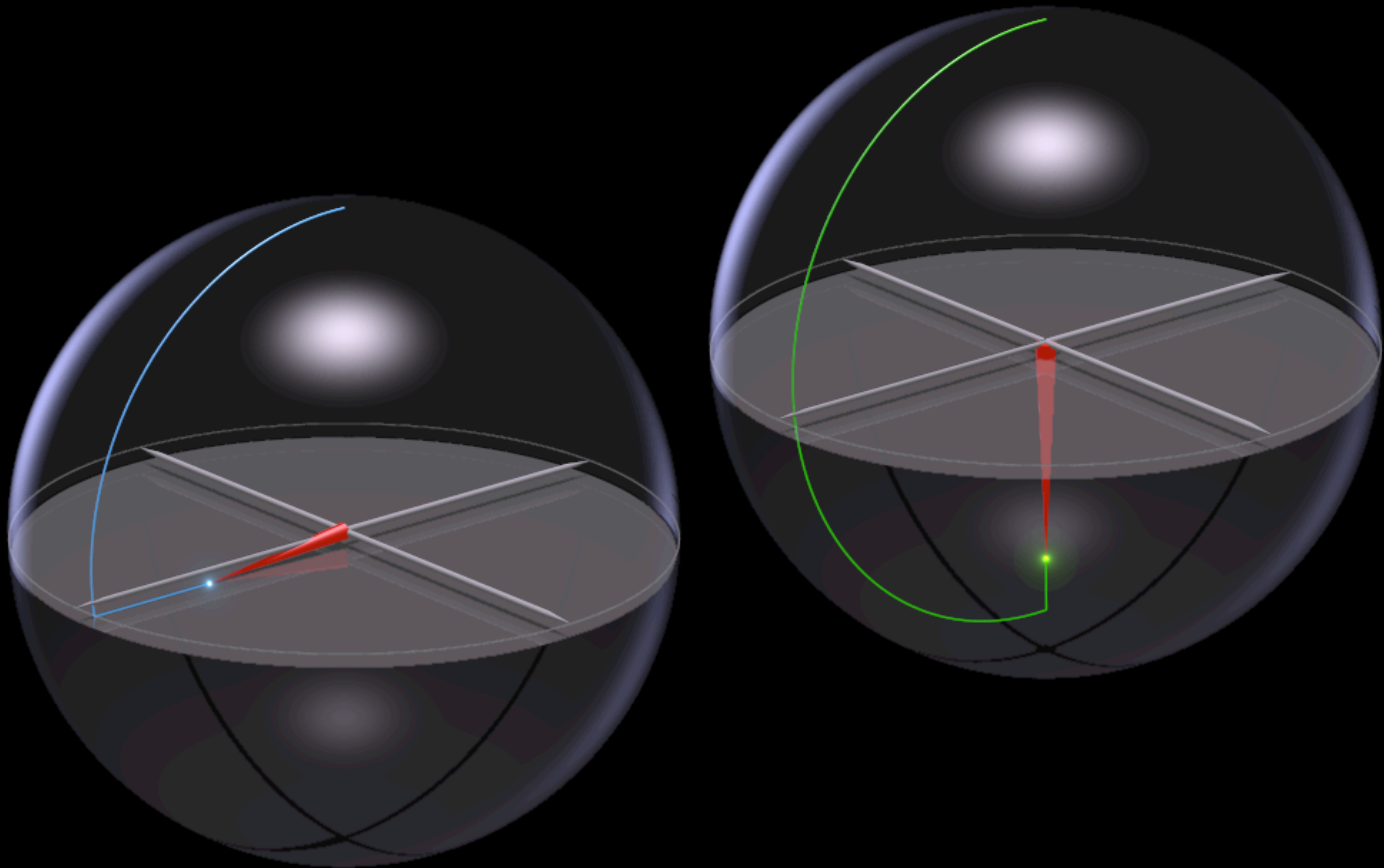


$$z_0 = -\frac{T_2}{2(T_1 - T_2)}$$

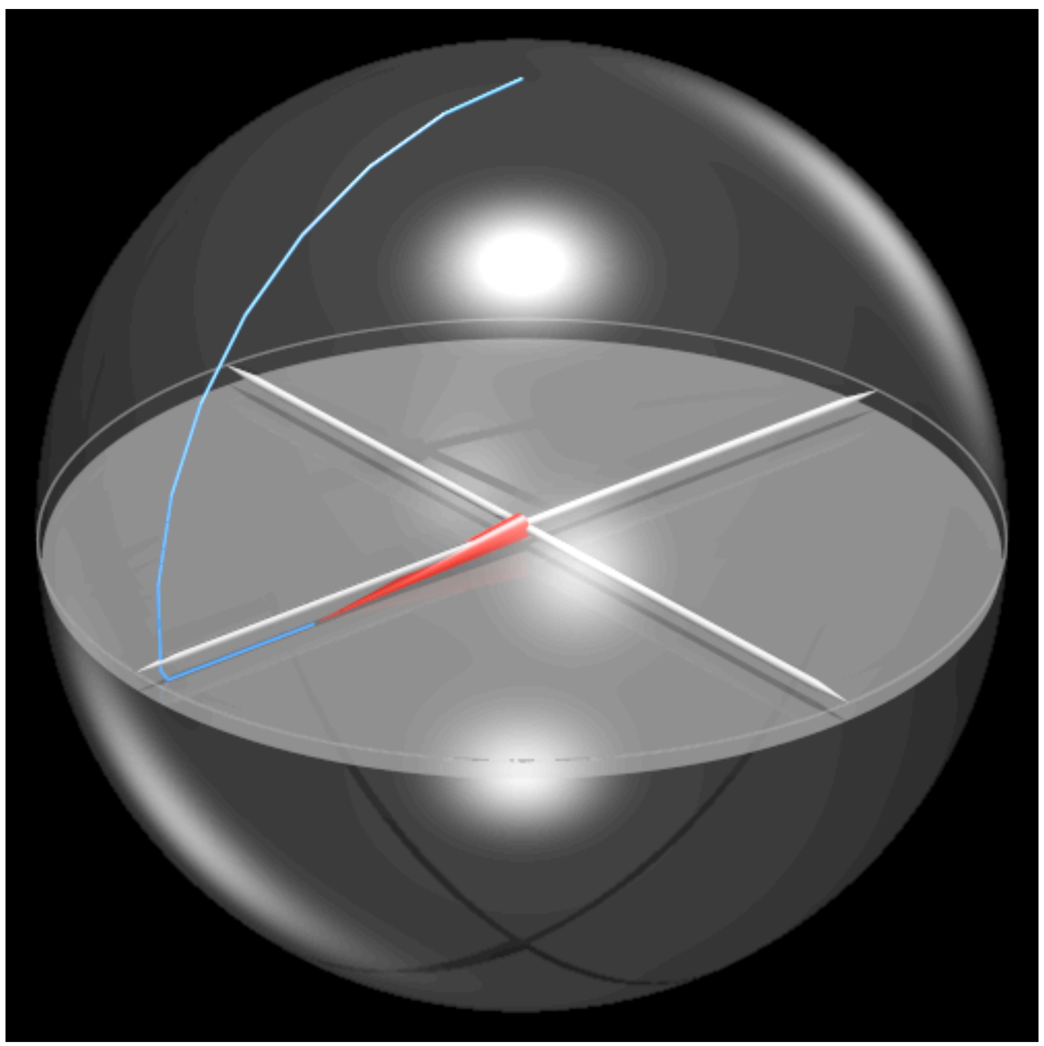
inversion recovery



Lapert, Zhang, Braun, Glaser, Sugny, PRL 104 (2010)



Time-optimal trajectories between any initial and target state can be determined in the presence of relaxation and bounds on the rf amplitude



extensions:

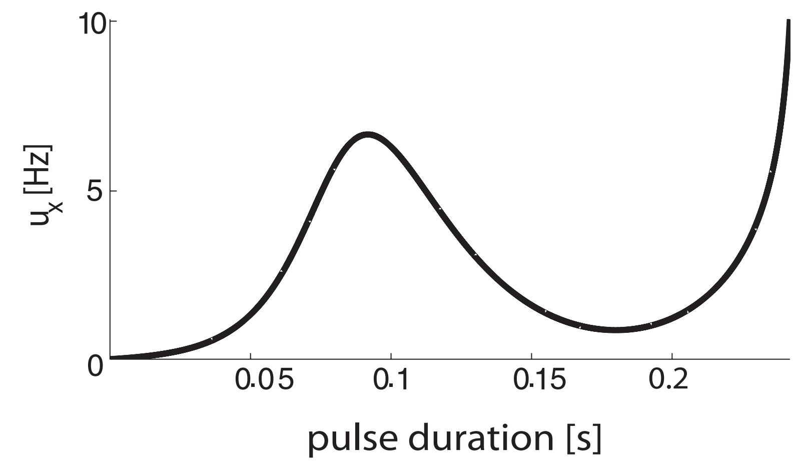
robustness
minimum energy
radiation damping
optimal contrast

Lapert, Zhang, Braun, Glaser, Sugny, PRL 104 (2010)

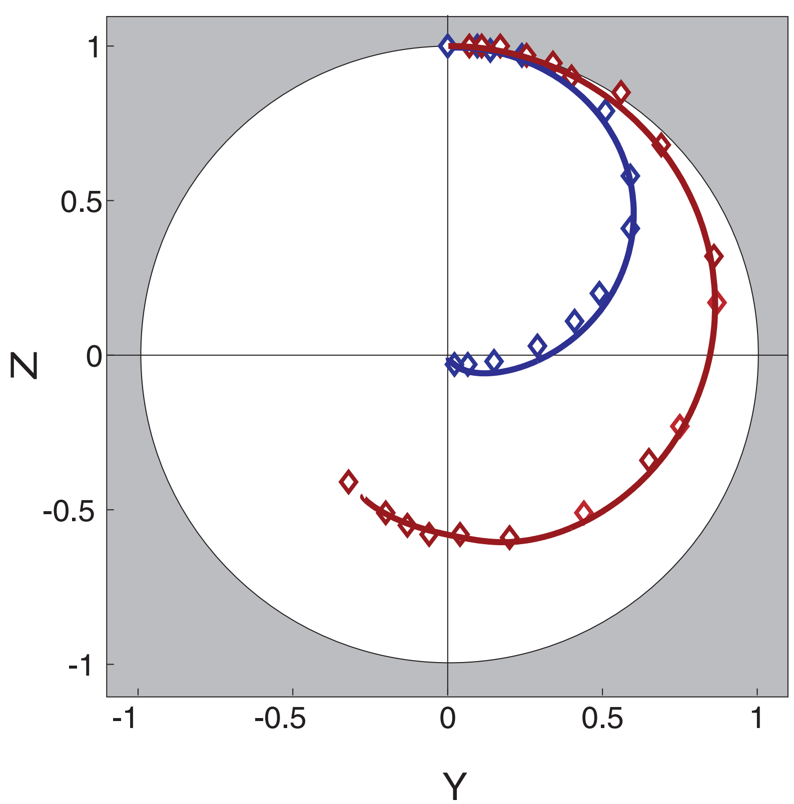
Zhang, Lapert, Sugny, Braun, Glaser, J. Chem. Phys. 134, 054103 (2011)

Lapert, Zhang, Glaser, Sugny, J. Phys. B 44, 154014 (2011)

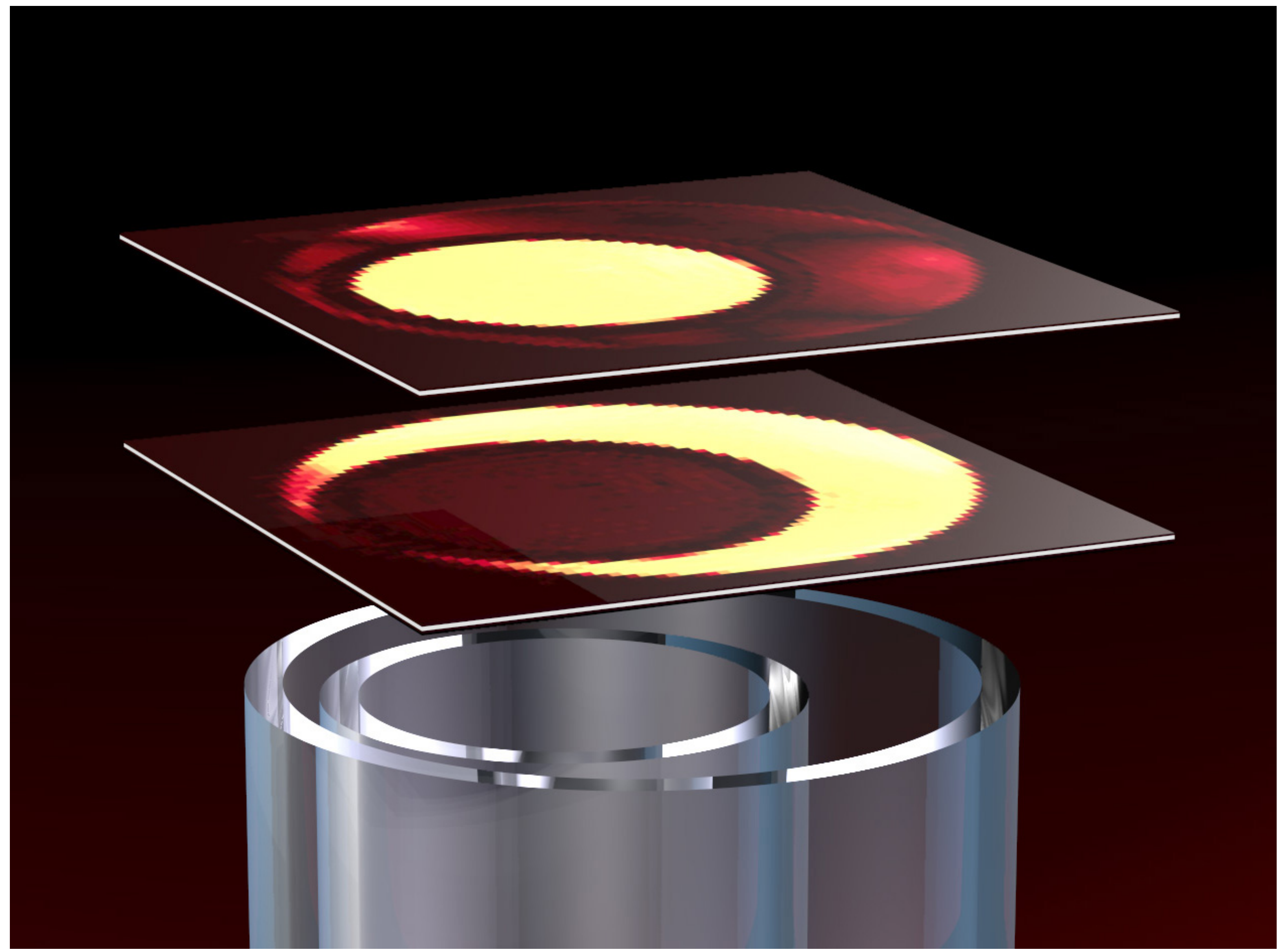
Optimal imaging contrast

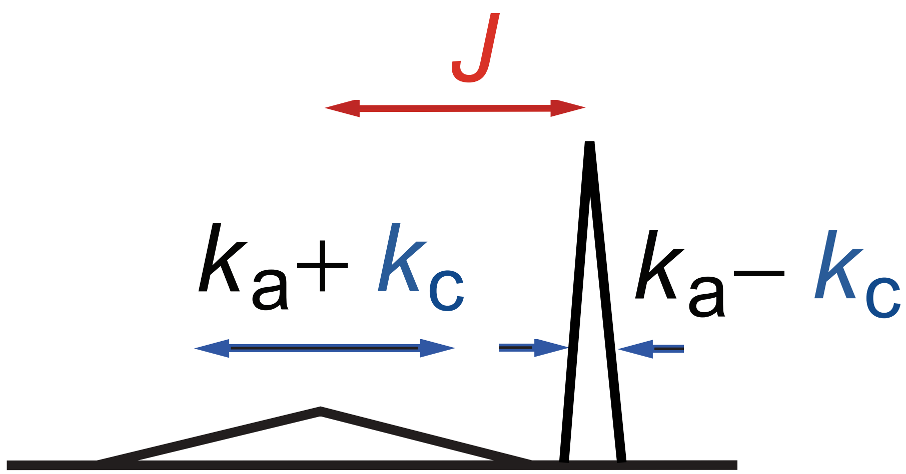


(a)

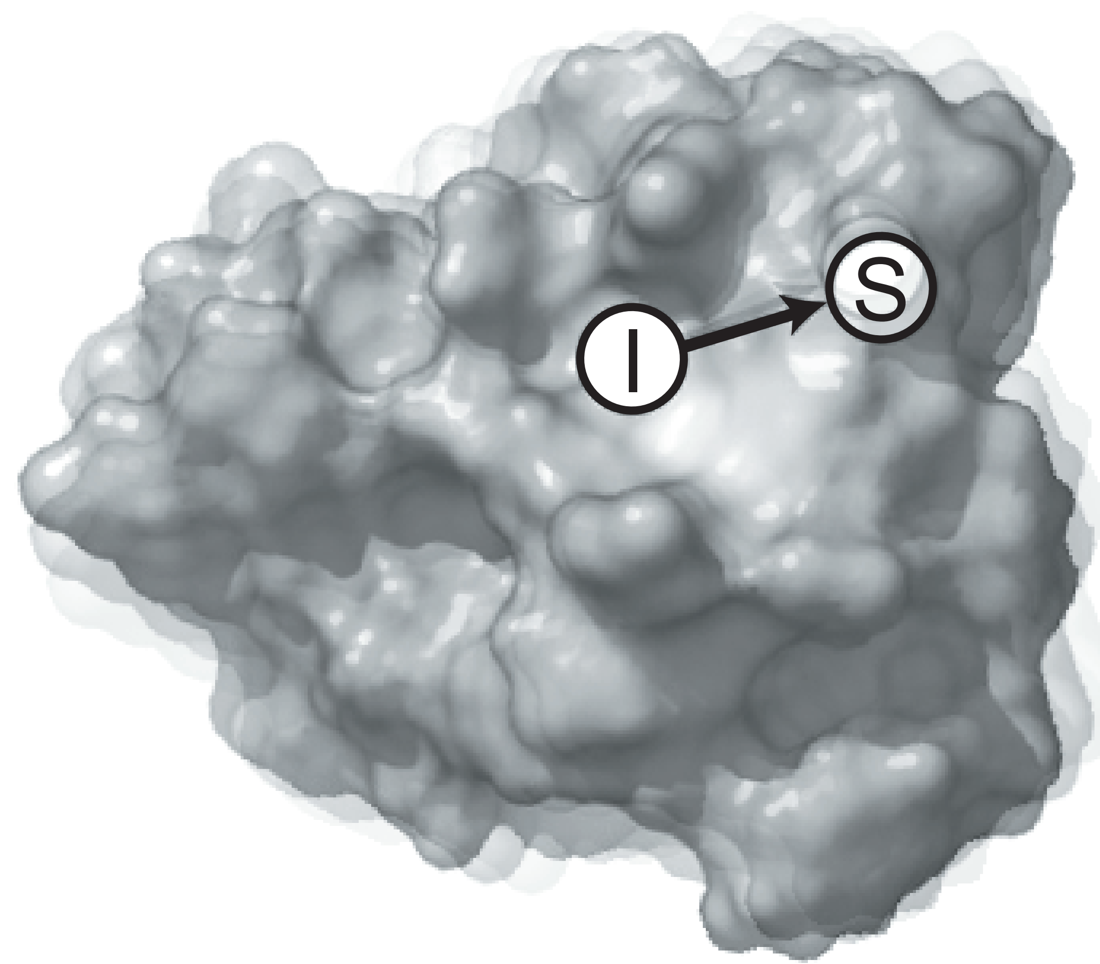


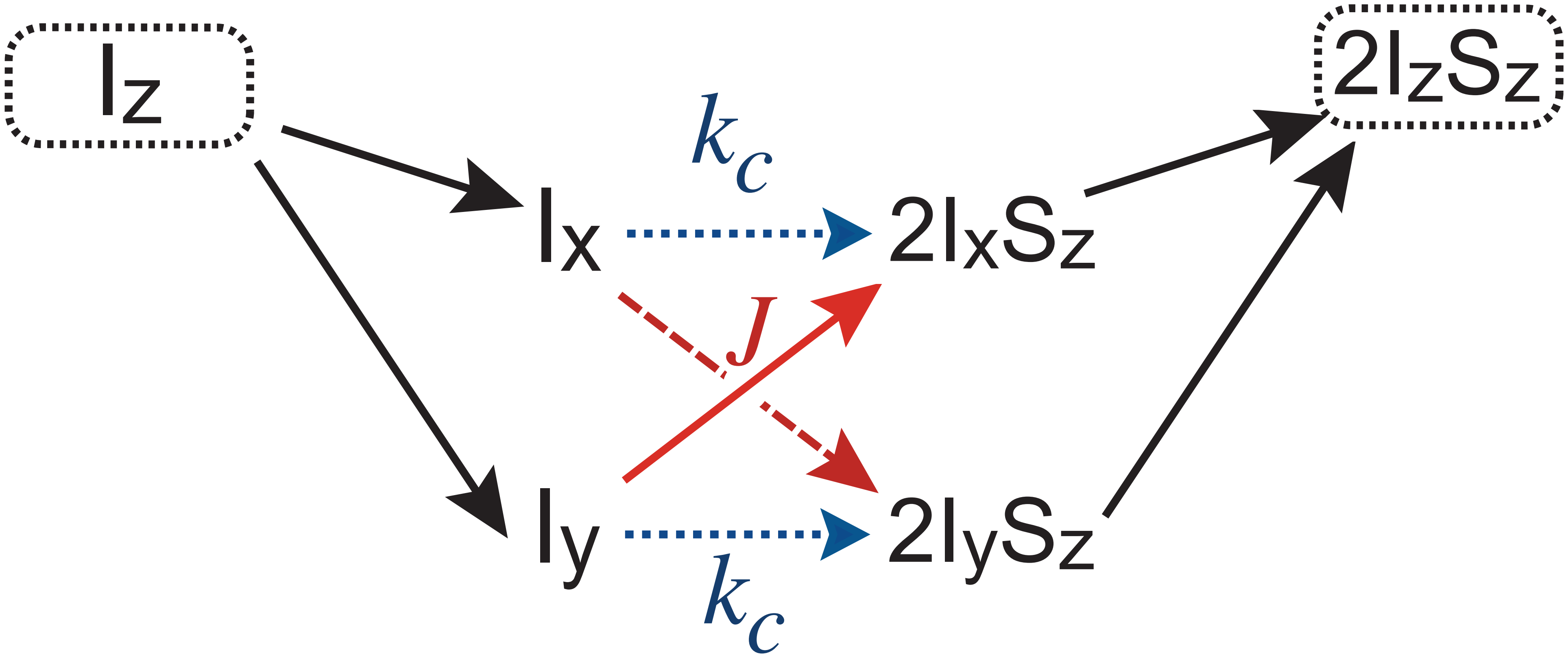
(b)





Multiplet of Spin I




Iz
2IzSz
kc
Ix
2IxSz
J
Iy
2IySz
kc

Optimal transfer efficiency η from I_z to $2 I_z S_z$:

$$\eta = \sqrt{1 + \xi^2} - \xi$$

with $\xi^2 = \frac{k_a^2 - k_c^2}{J^2 + k_c^2}$

Khaneja, Luy, Glaser, *Proc. Natl. Acad. Sci.* (2003)

maximum transfer efficiency:

$$\eta = \sqrt{1 + \xi^2} - \xi$$

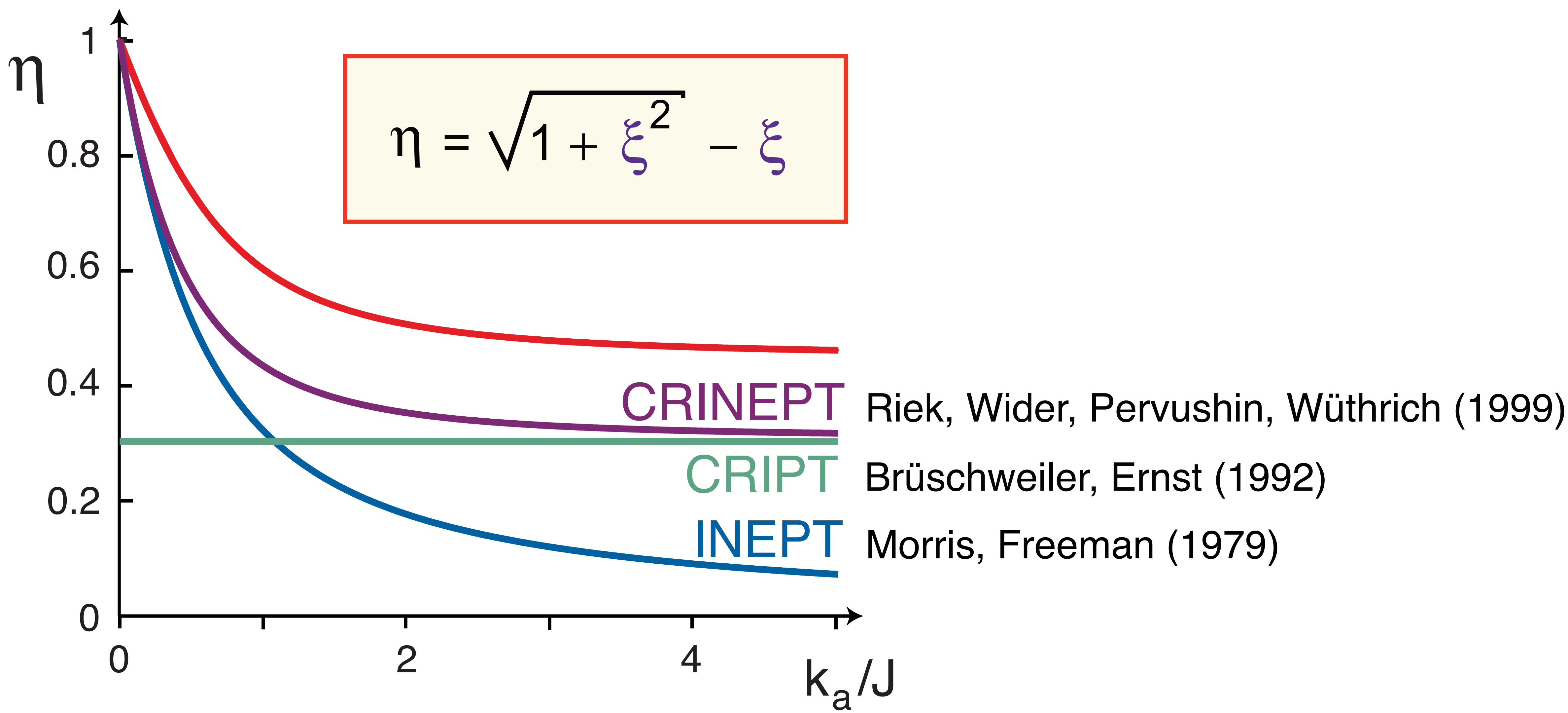
formal proof (based on principles of optimum control theory):

optimal return function $V(r_1, r_2)$

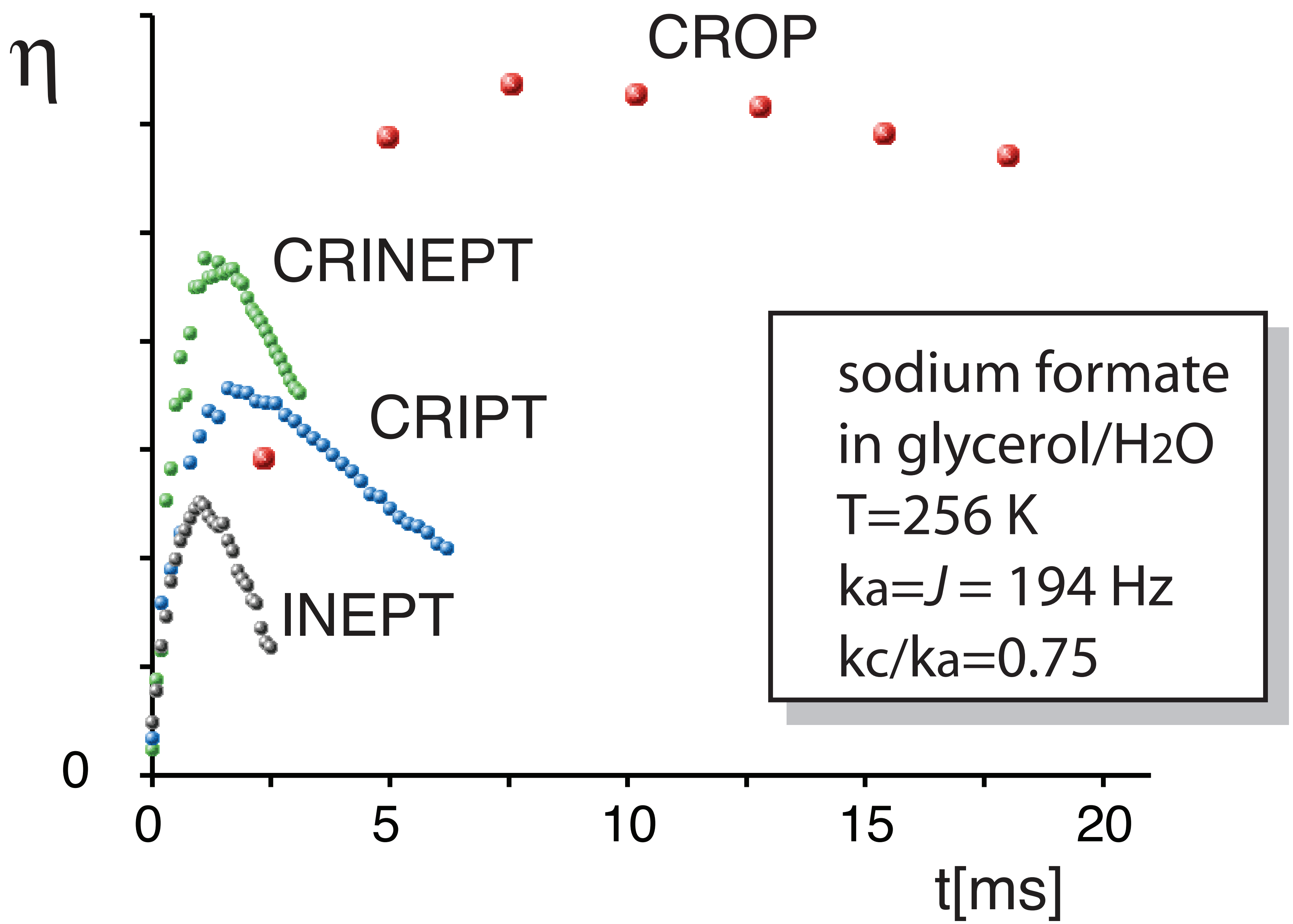
Hamilton-Jacobi-Bellman equation

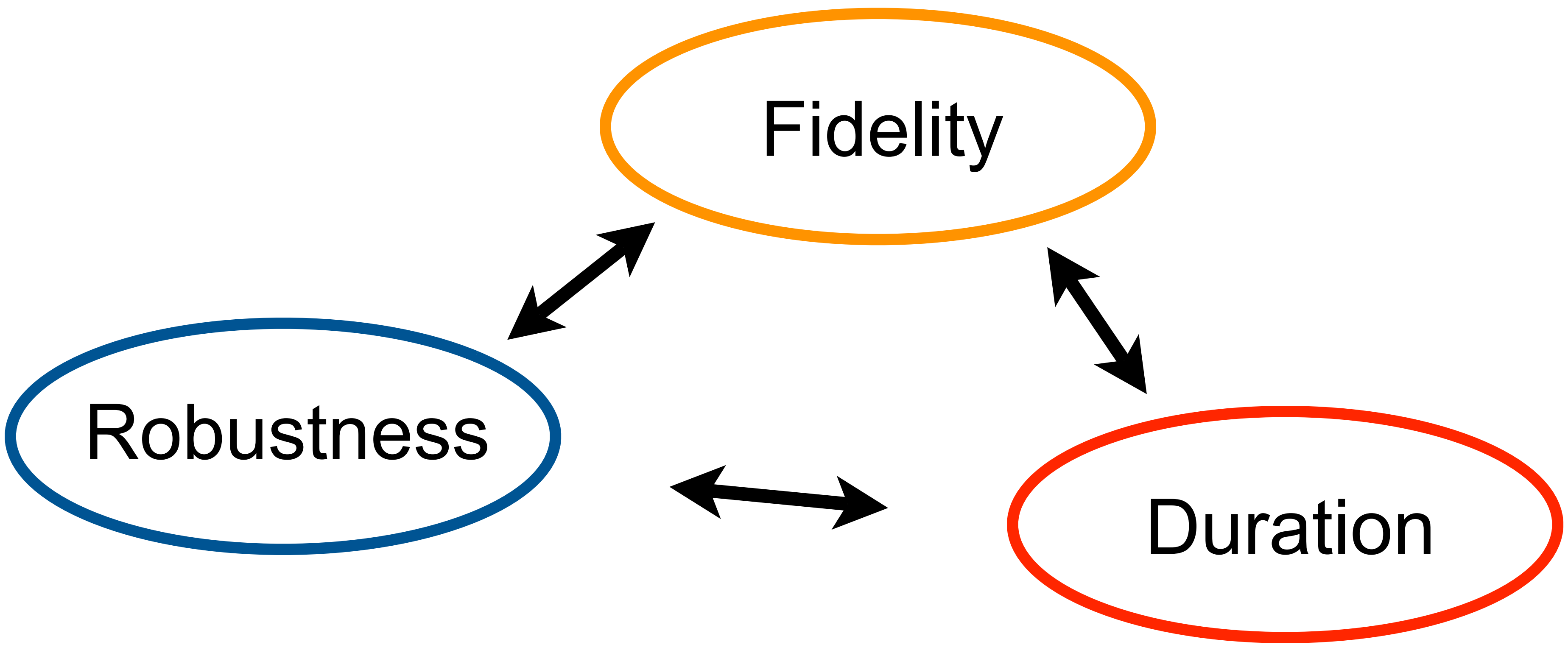
$$\max_{u_1, u_2} \left[\frac{\partial V}{\partial r_1} \delta r_1 + \frac{\partial V}{\partial r_2} \delta r_2 \right] = 0$$

Transfer Efficiency η for $k_c / k_a = 0.75$

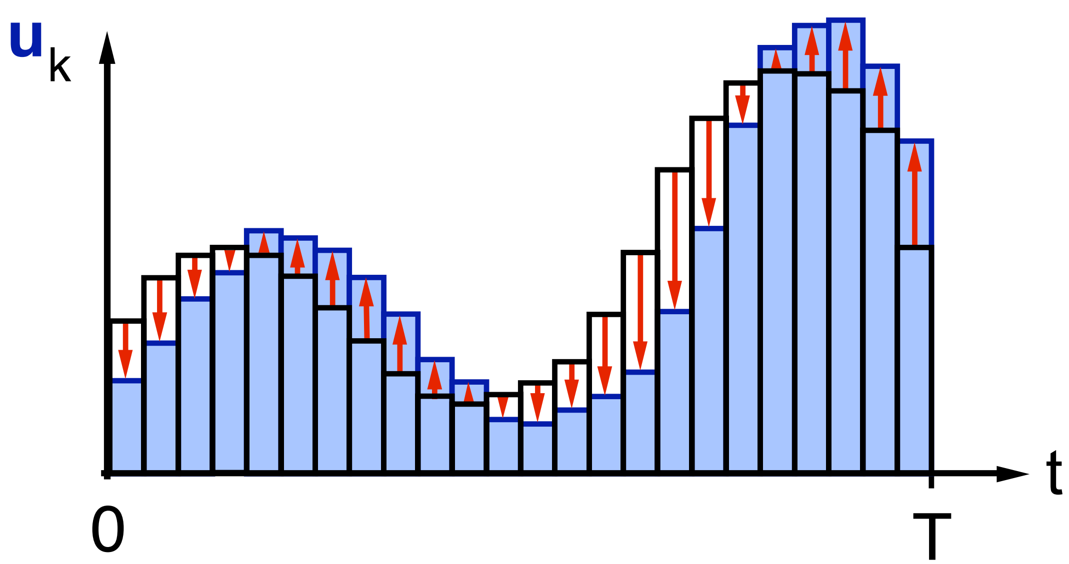


Experimental Transfer Functions




Fidelity
Robustness
Duration

GRAPE (Gradient Ascent Pulse Engineering)



desired transfer: $A \longrightarrow C$

performance: $\langle C \mid \rho(T) \rangle$

$\rho(0) = A$

$\lambda(T) = C$

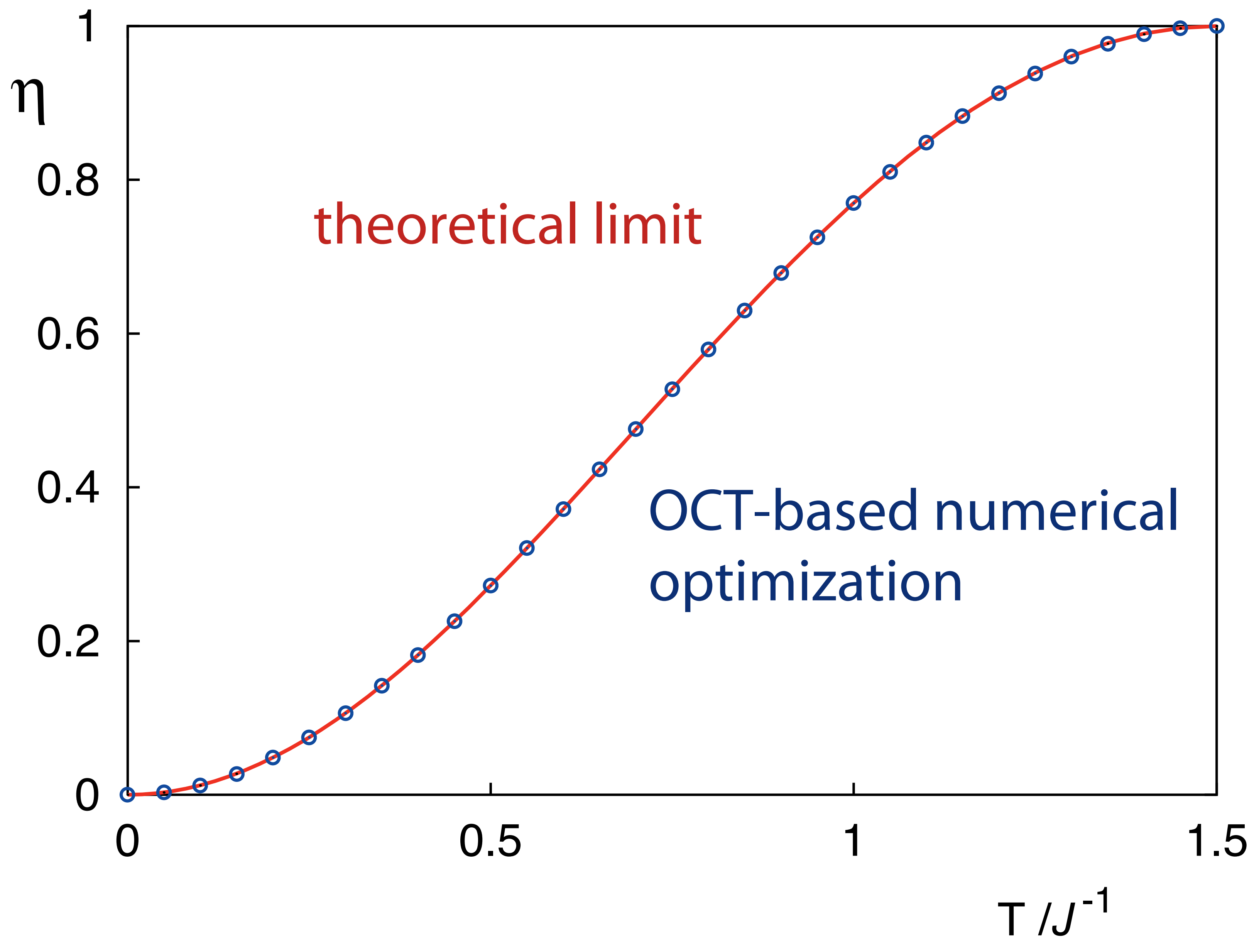
$$u_k(t) \longrightarrow u_k(t) + \varepsilon \langle \lambda(t) \mid [-i H_k, \rho(t)] \rangle$$

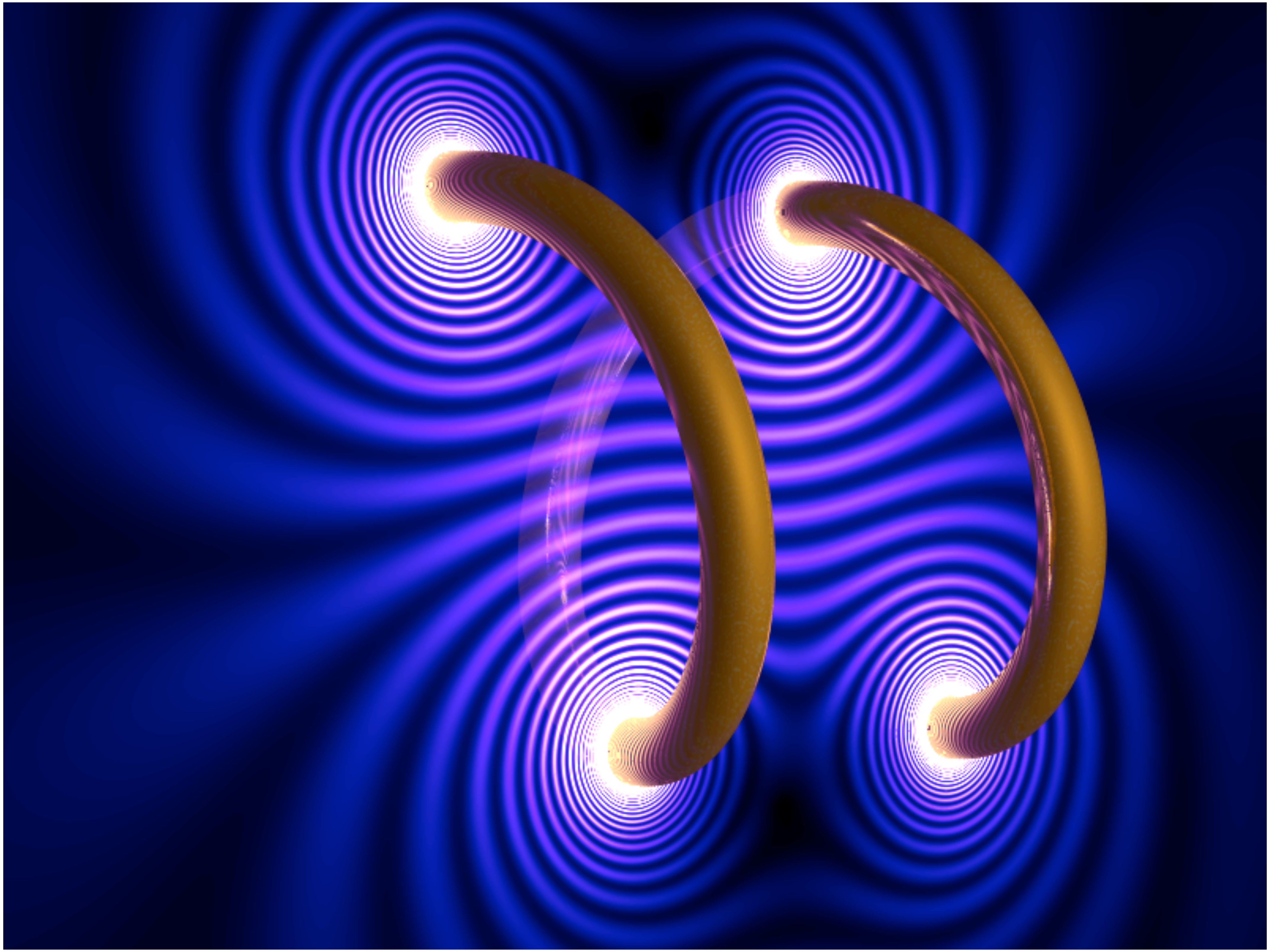
Khaneja, Reiss, Kehlet, Schulte-Herbrüggen, Glaser, J. Magn. Reson. 172, 296-305 (2005)

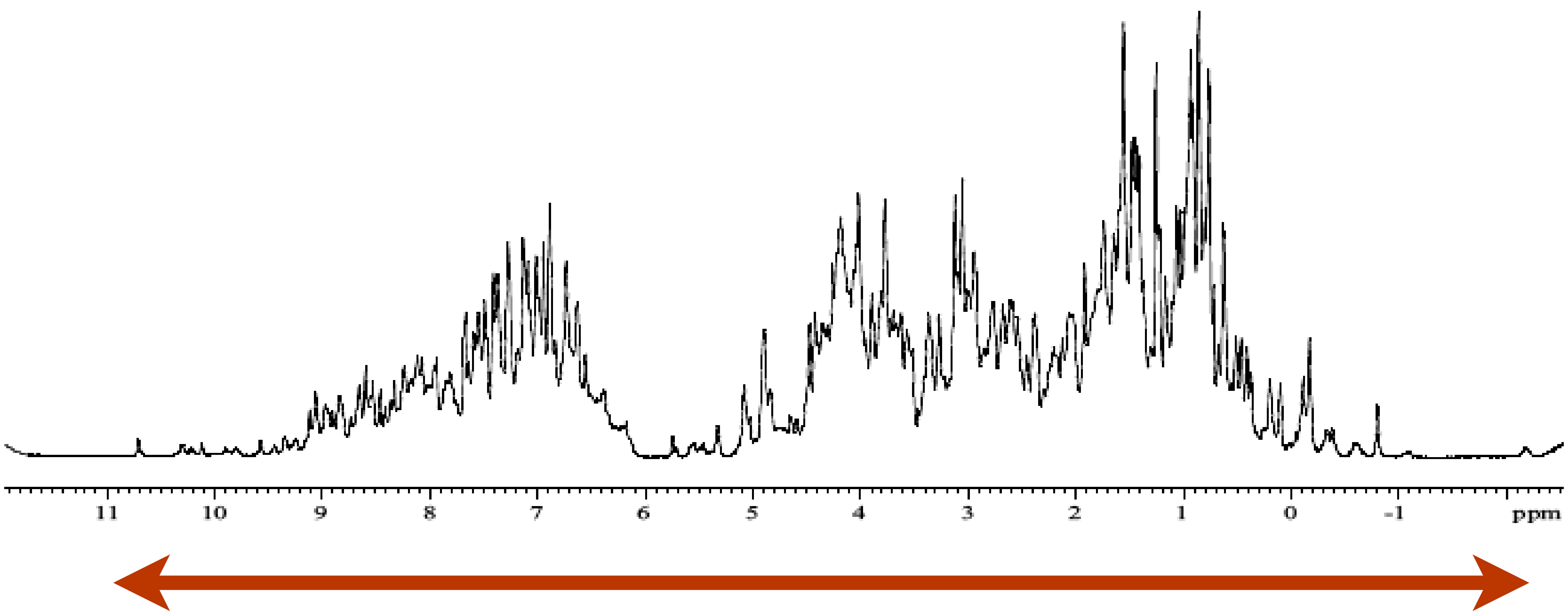
Machnes, Sander, Glaser, de Fouquieres, Gruslys, Schirmer, Schulte-Herbrüggen, Phys. Rev. A 84, 022305 (2011)

de Fouquieres, Schirmer, Glaser, Kuprov, J. Magn. Reson. 212, 412-417 (2011)

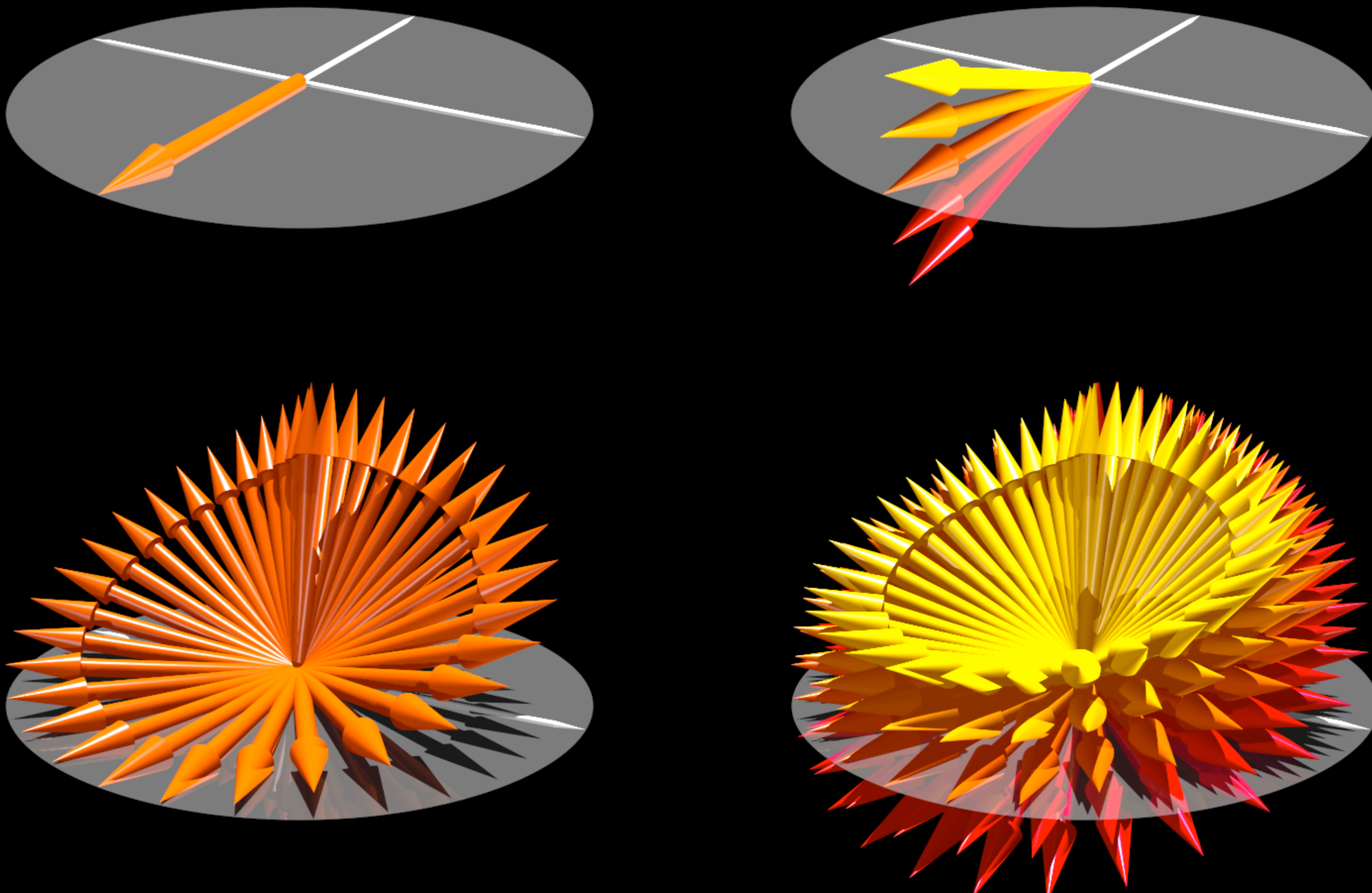
Numerical OCT-based Algorithm finds theoretical limits

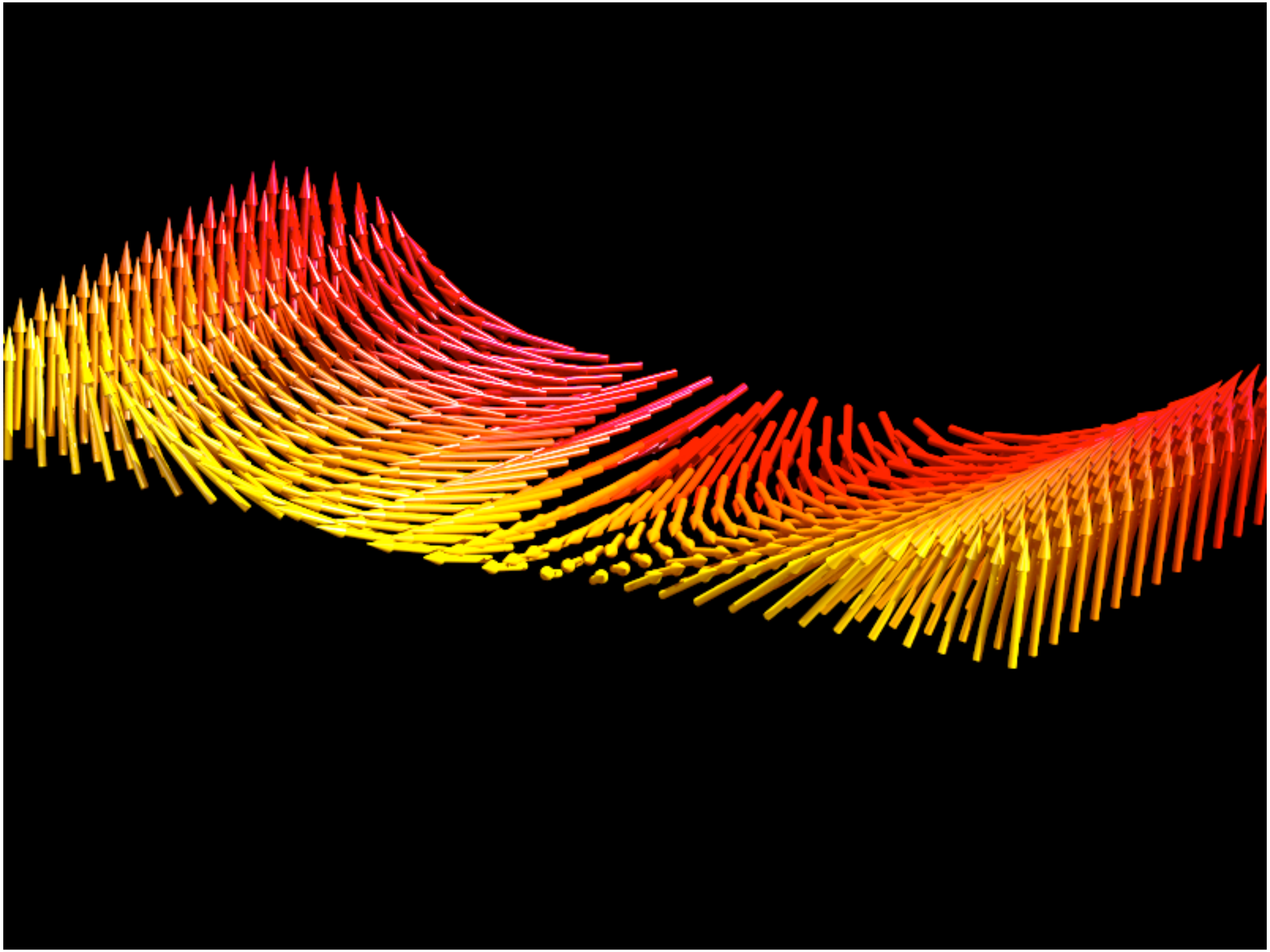


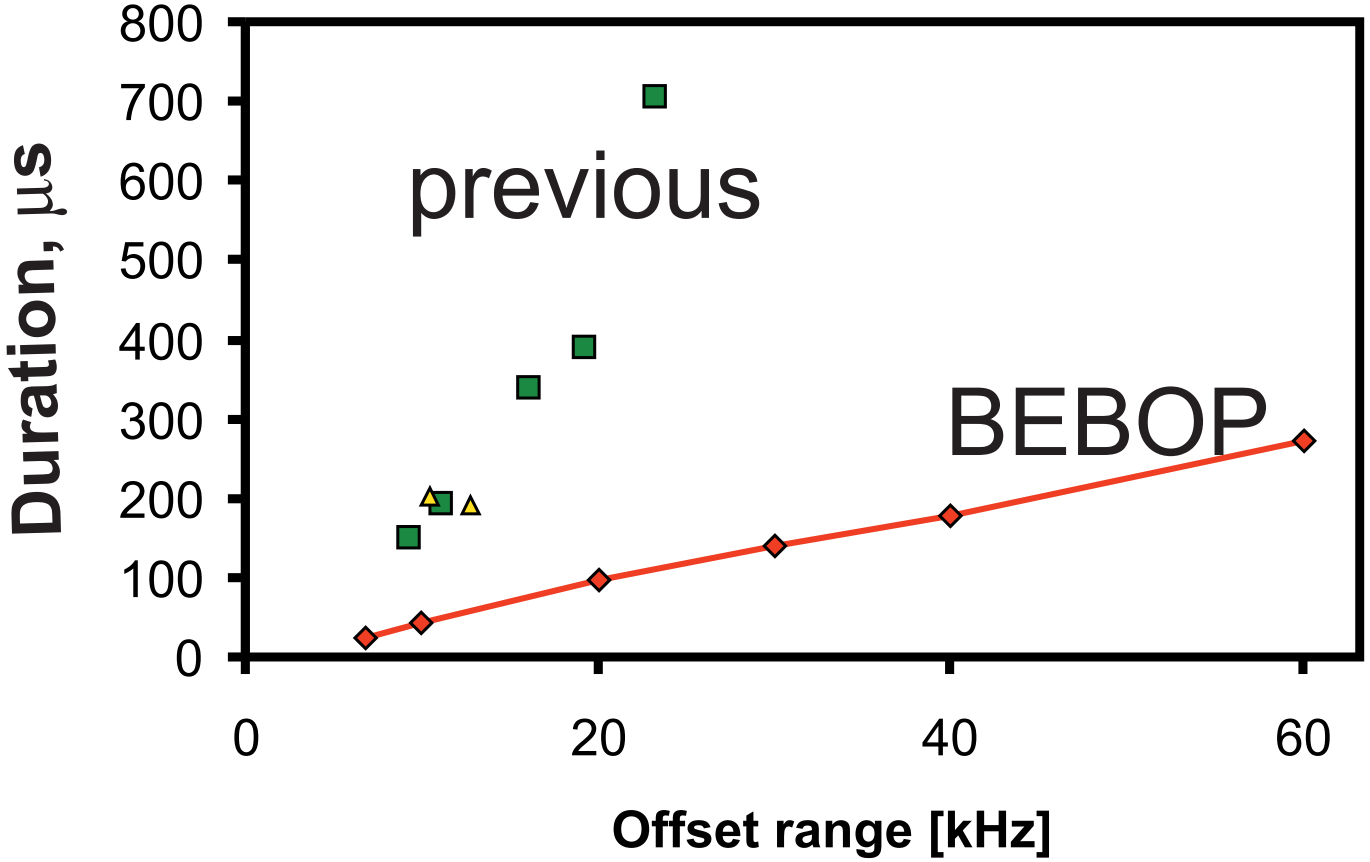




frequency dispersion



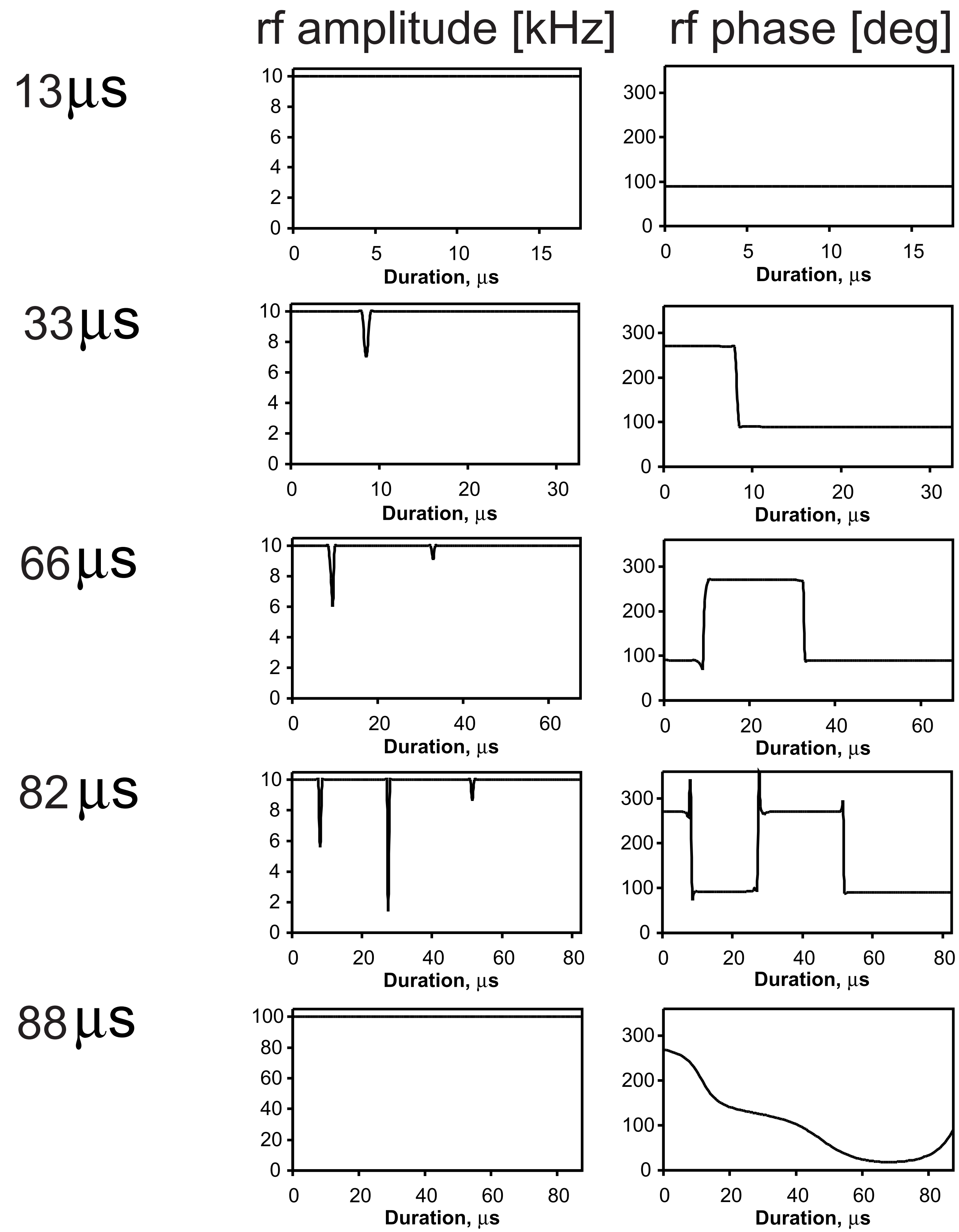




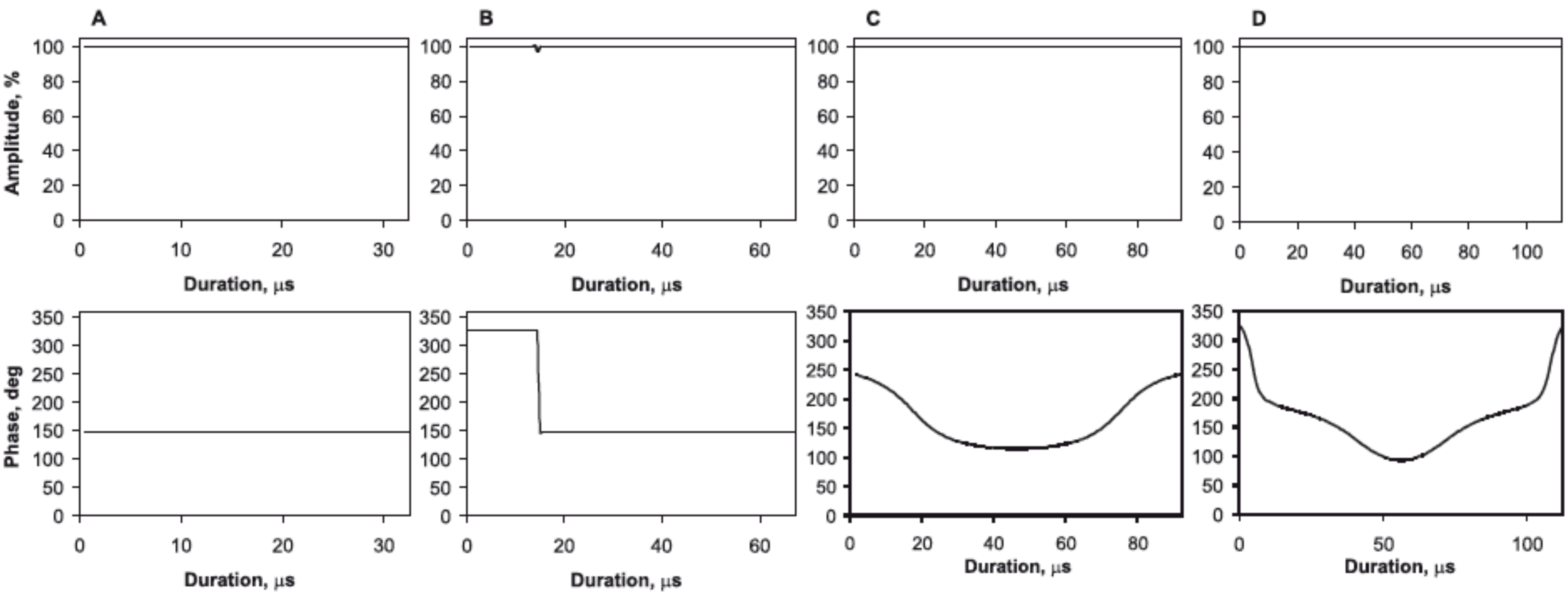
(excitation efficiency: 98%, max. rf amplitude: 10 kHz, no rf inhomogeneity)

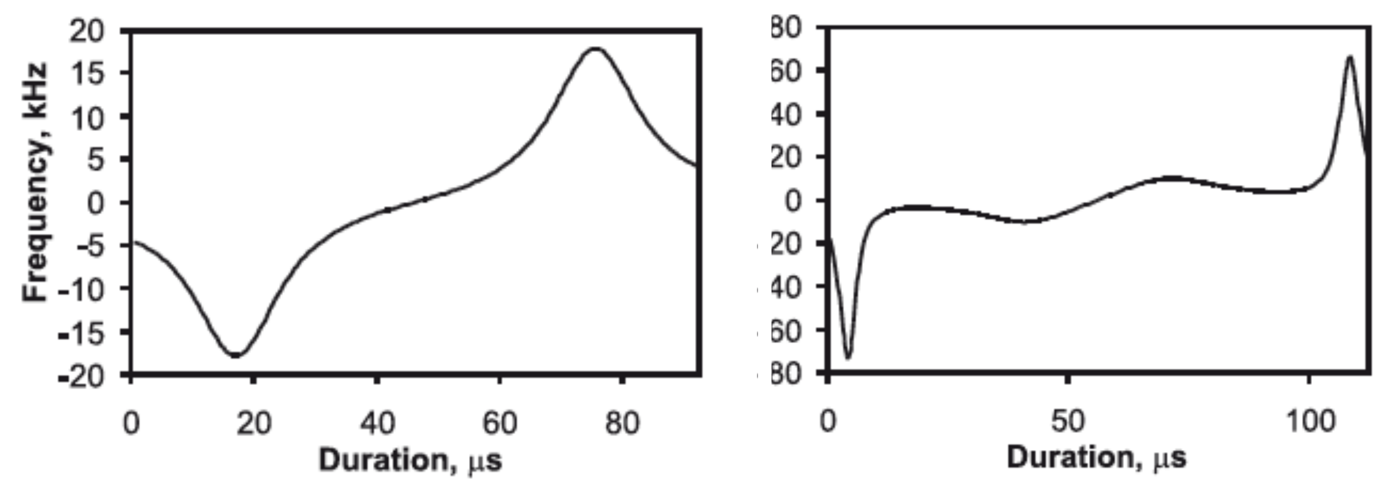
K. Kobzar, T. E. Skinner, N. Khaneja, S. J. Glaser, B. Luy, JMR 170, 236 (2004)

Longer pulse durations allow for more complex phase variations



excitation bandwidth: 20 kHz
no rf inhomogeneity

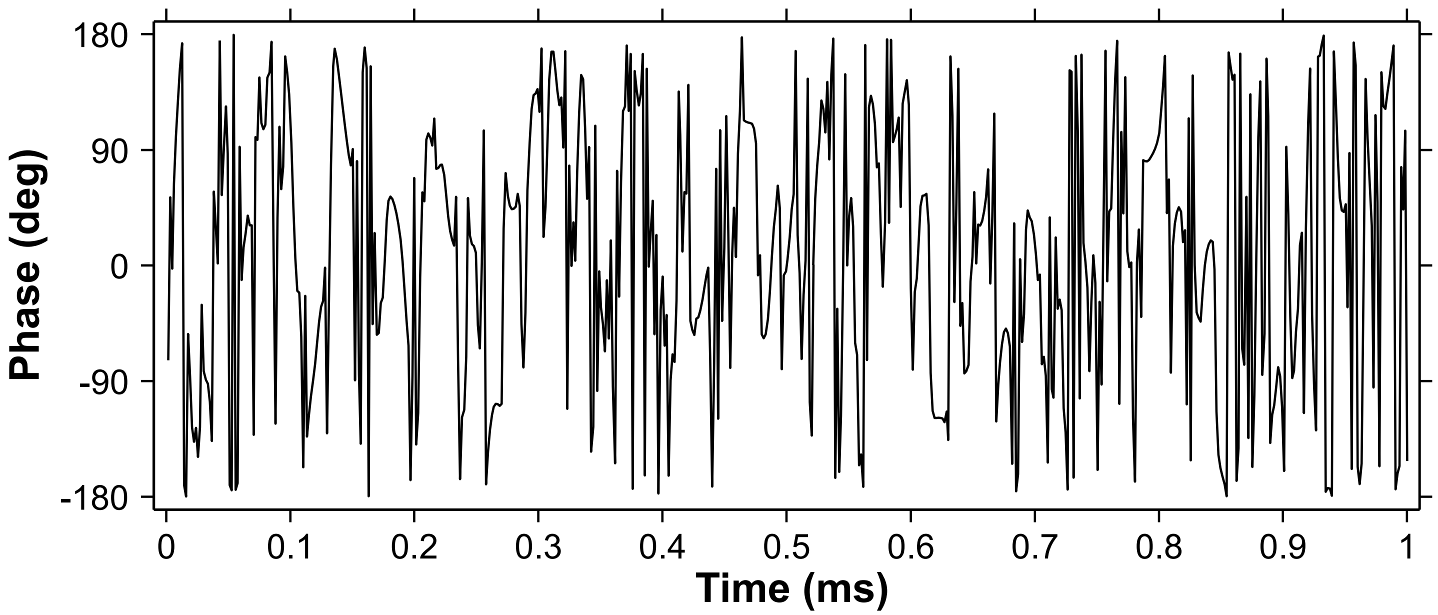




optimal inversion pulses
bandwidth of 20 kHz

Robust broadband excitation pulse

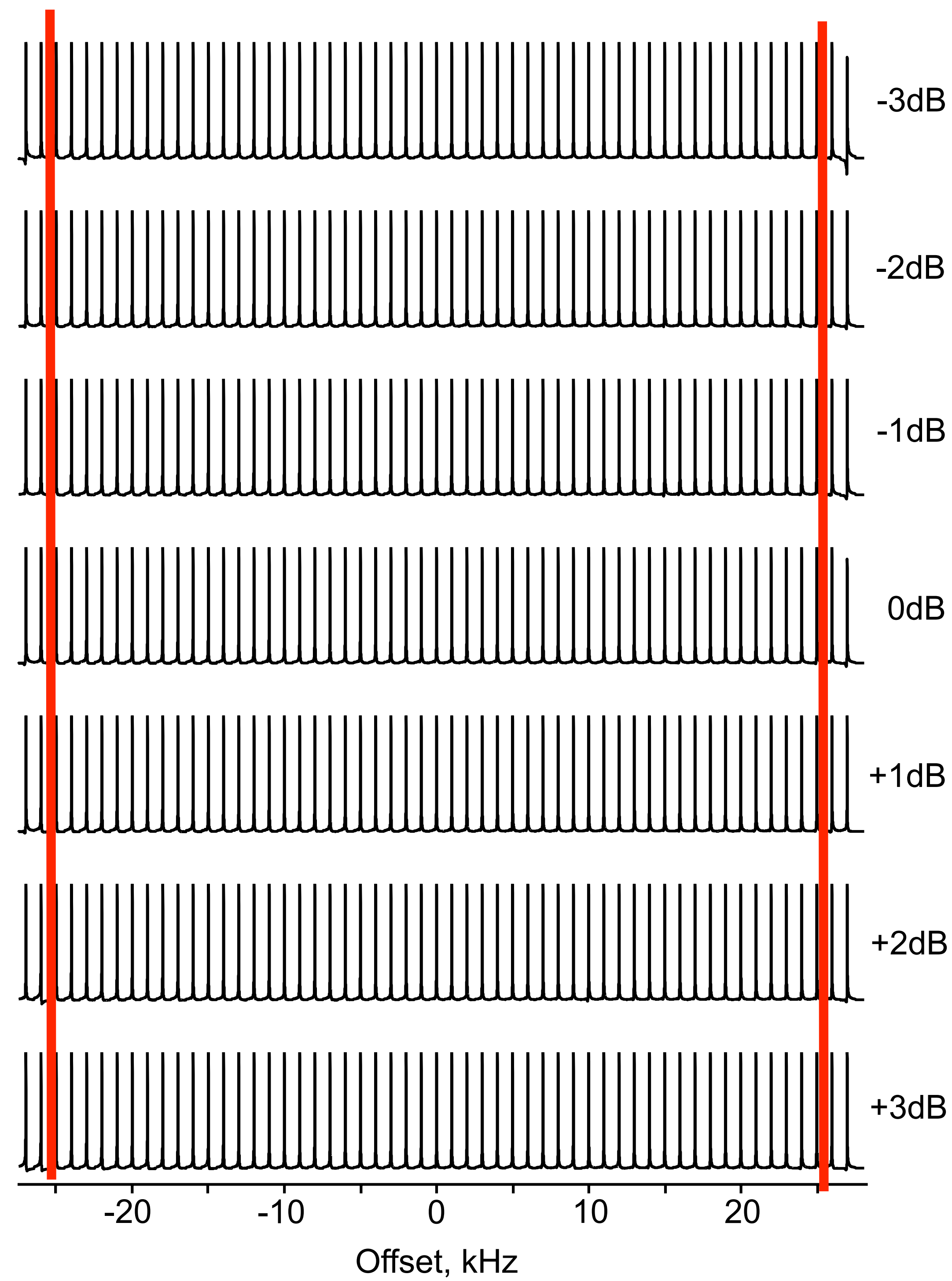
PM-BEBOP

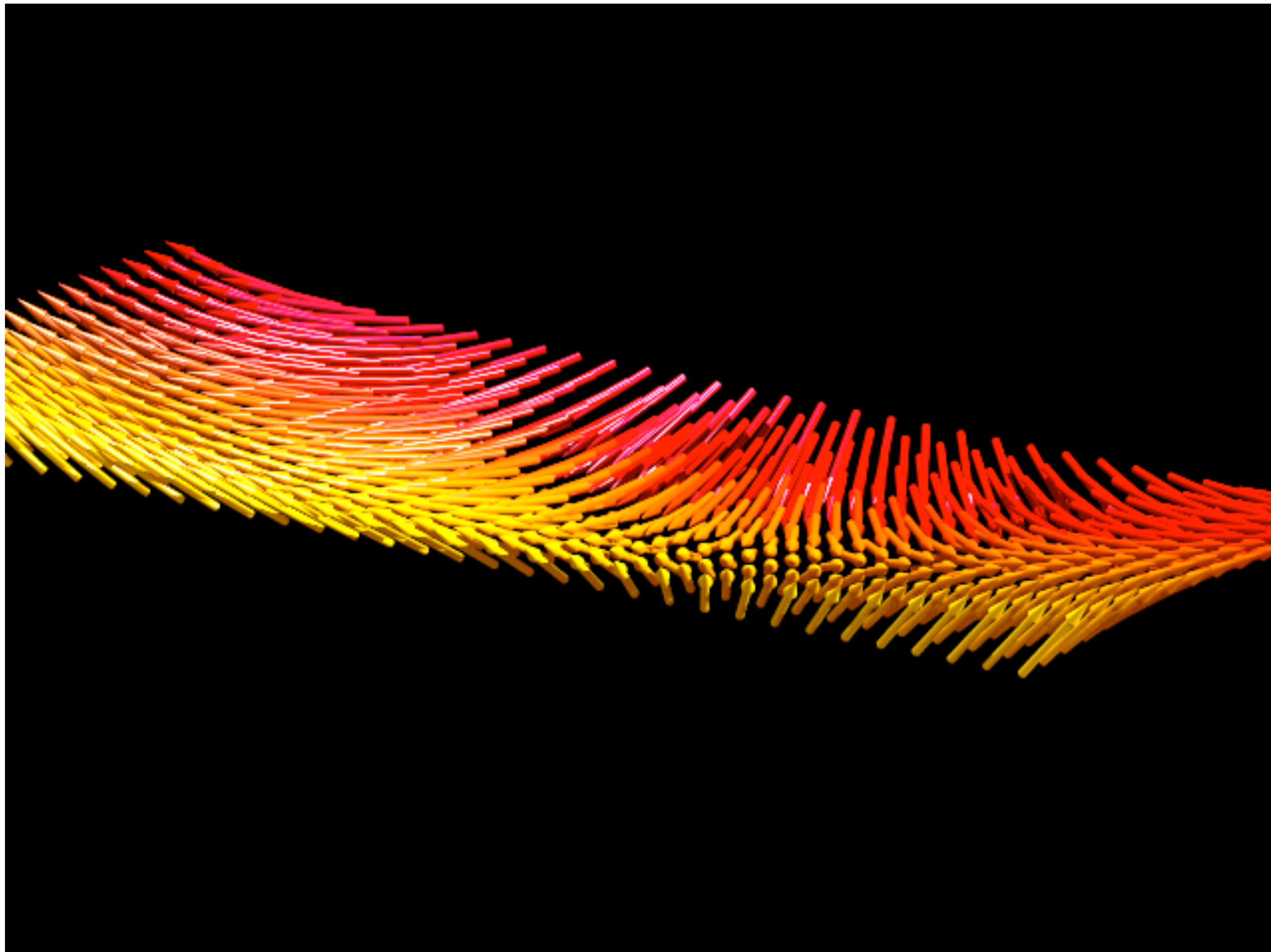


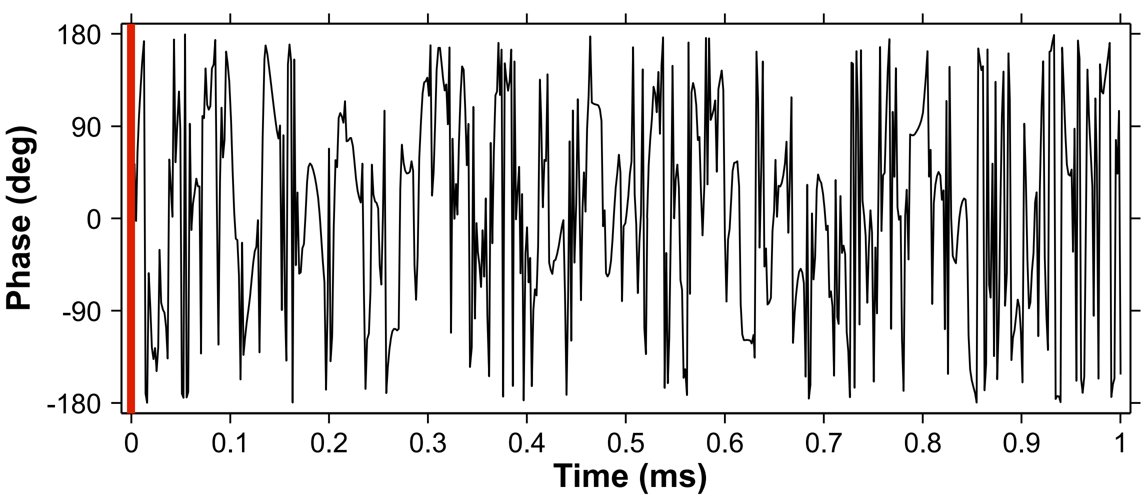
bandwidth: 50 kHz

rf amplitude: 15 kHz

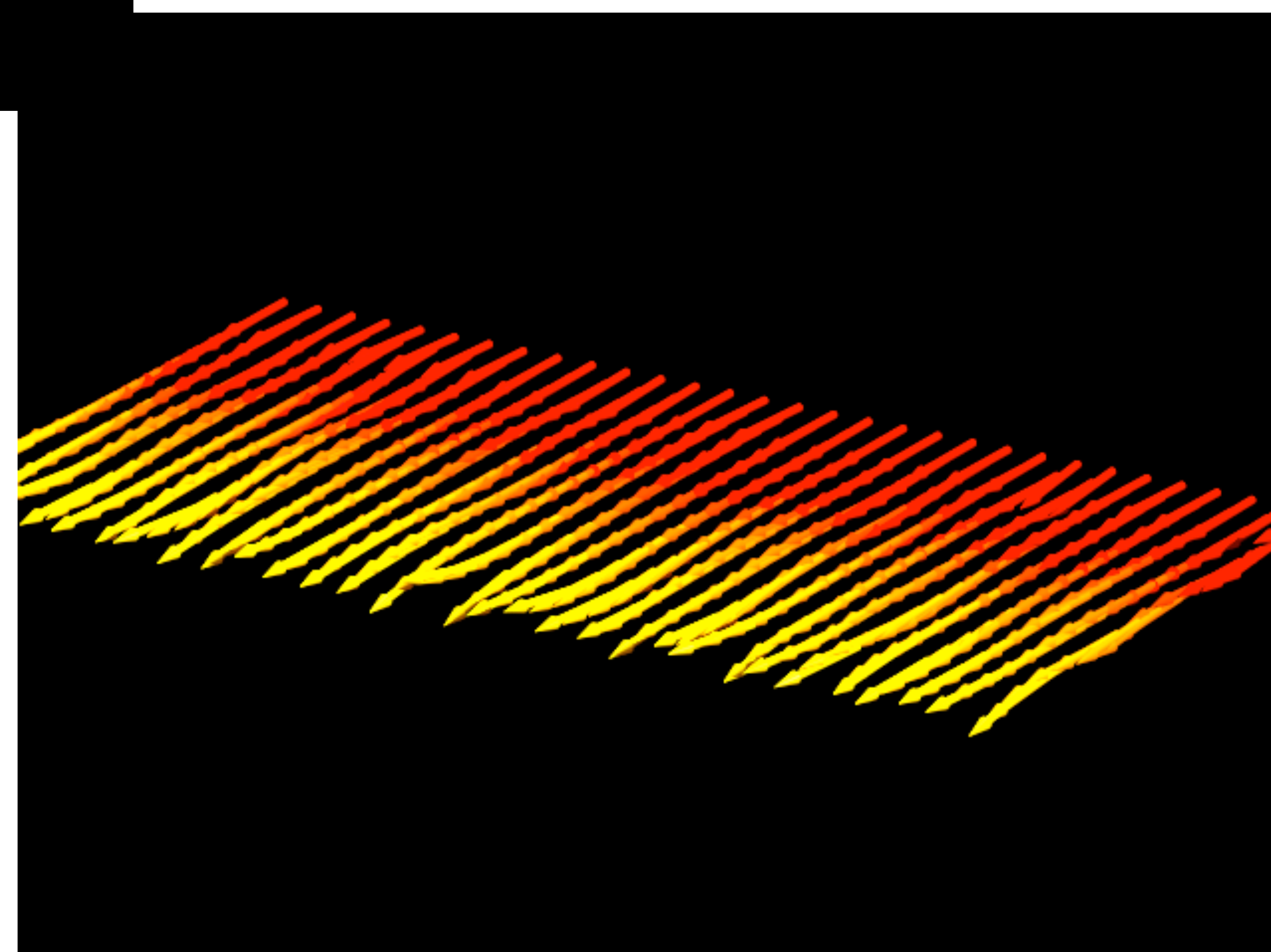
Skinner et al. (2006)







180
90
0
-90
-180
Phase (deg)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time (ms)



Harvard
N. Khaneja
H. Yuan

Wright State Univ.
T. Skinner
N. Gershenzon

TU München
M. Sattler
A. Haase
M. Schwaiger

Dijon
D. Sugny
B. Bonnard

University of Aarhus
N. C. Nielsen

Karlsruhe
B. Luy

UC Santa Barbara
S. Han
M. Sherwin

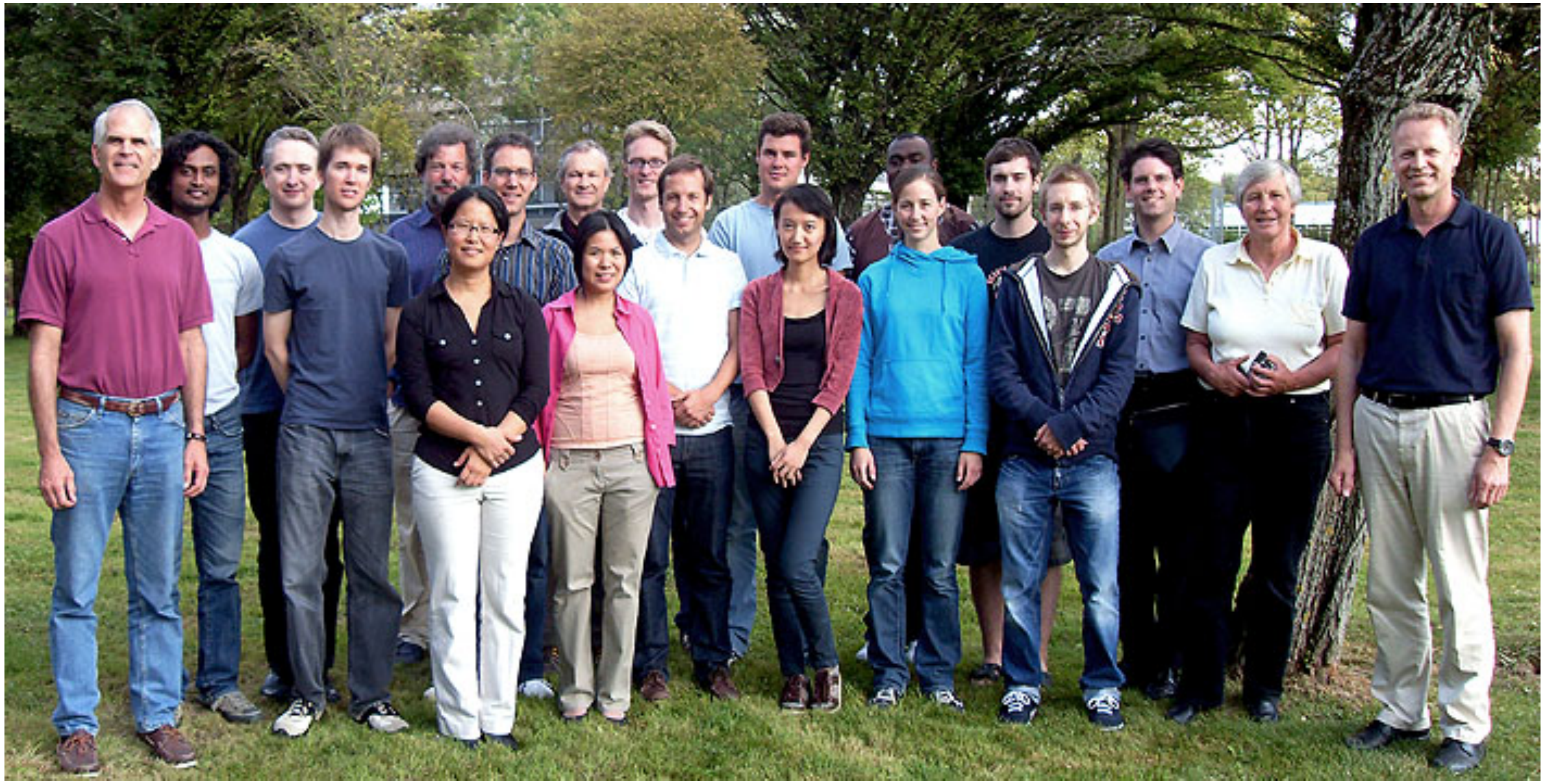
Univ. Frankfurt
T. Prisner
P. Spindler

Stuttgart/Ulm
J. Wrachtrup
F. Jelezko

Aix-Marseille Univ.
S. Caldarelli
M. Reddy

General Electrics
R. Schulte
F. Wiesinger

Bruker
W. Bermel
R. Kümmerle



Technische Universität München (TUM)

Y. Zhang, M. Braun, M. Nimbalkar, F. Schilling, A. Djintchui,
M. Janich, A. Khegai, A. Garon, T. Nguyen, R. Marx, R. Zeier

T. Schulte-Herbrüggen, C. O‘Meara, T. Nguyen, S. Düwel

Funding: DFG, SFB-631, EU (Q-ESSENCE), ENB(QCCC), DAAD, FCI

selected references

"Optimal Control Solutions to the Magnetic Resonance Selective Excitation Problem", *Conolly et al., IEEE Trans. Med. Imag. MI-5, 106 (1986)*

"Optimal Control of Coupled Spin Dynamics: Design of NMR Pulse Sequences by Gradient Ascent Algorithms" *Khaneja et al., J.Magn, Reson. 172, 296 (2005)*

"Exploring the Limits of Broadband Excitation and Inversion Pulses" *Kobzar et al., J. Magn. Reson. 170, 236 (2004)*

"Pattern Pulses: Design of Arbitrary Excitation Profiles as a Function of Pulse Amplitude and Offset", *Kobzar et al., J. Magn. Reson. 173, 229 (2005)*

"Exploring the Limits of Excitation and Inversion Pulses II. RF-Power Optimized Pulses", *Kobzar et al., J. Magn. Reson. 194, 58 (2008)*

"Optimal Control Design of Excitation Pulses that Accomodate Relaxation" *Gershenzon et al., J. Magn. Reson.188, 330 (2007)*

”Cooperative Pulses” *Braun, Glaser, J. Magn. Reson. 207, 114 (2010)*

"Robust Slice-Selective Broadband Refocusing Pulses" *Janich et al., J. Magn. Reson. 213, 126 (2011)*